From Anisotropy to Efficiency: A Lamination Parameter Approach to Composite Lattices

Sondipon Adhikari
Professor of Engineering Mechanics
James Watt School of Engineering
The University of Glasgow

Coauthors: Jagan Selvaraj & Shuvajit Mukherjee

AIAA SciTech Forum, 12 – 16 January 2026
Orlando, FL, USA

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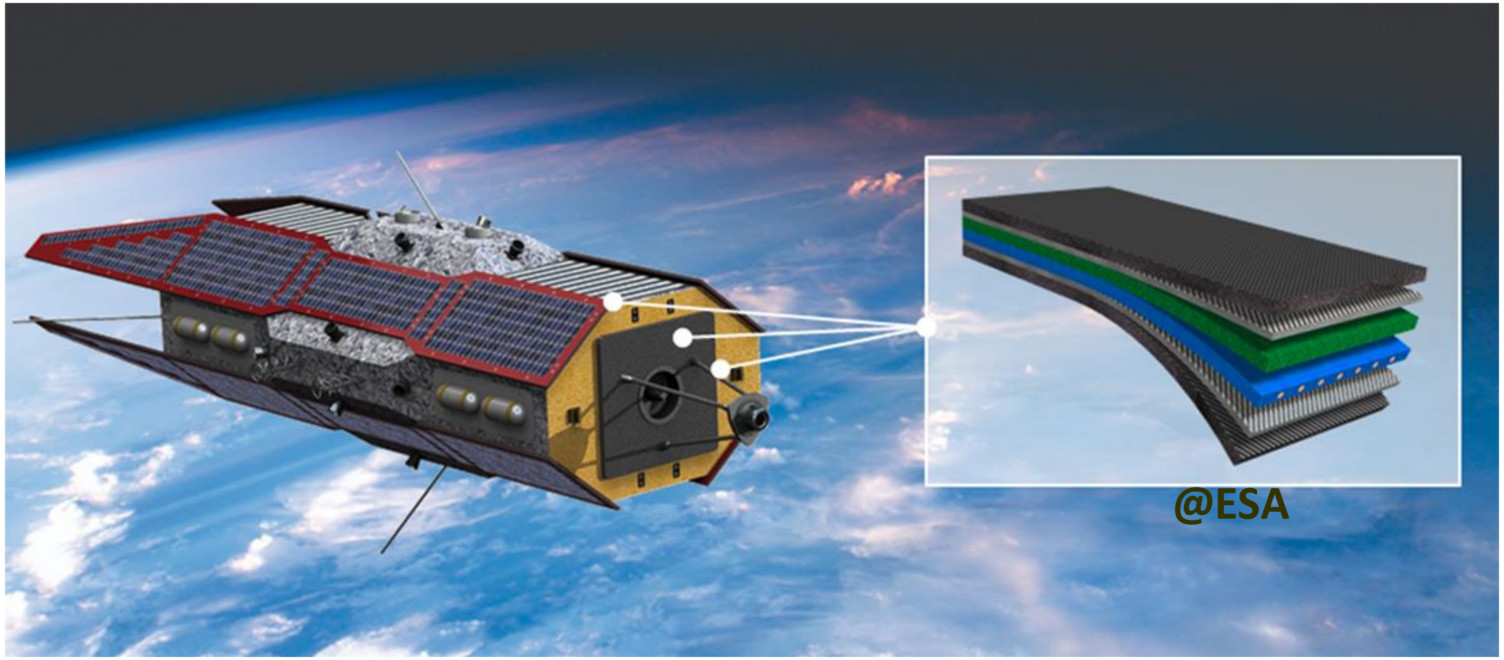
Introduction

- The design of lightweight and high-performance structures remains a cornerstone in aerospace engineering, where reductions in structural mass directly translate to improved energy efficiency and payload capacity.



- Lattice structures, characterized by periodic arrangements of interconnected struts or unit cells, offer a compelling solution due to their ability to deliver high stiffness-to-weight ratios.
- These architectures, composed of members primarily loaded in tension or compression, enable efficient load transfer while minimizing material usage. This quality is highly advantageous in aerospace structures ranging from satellite frames to morphing skins.
- The hierarchical nature of lattice architectures allows for tailoring of mechanical properties across multiple length scales, a feature increasingly exploited in multifunctional aerospace applications.

Introduction

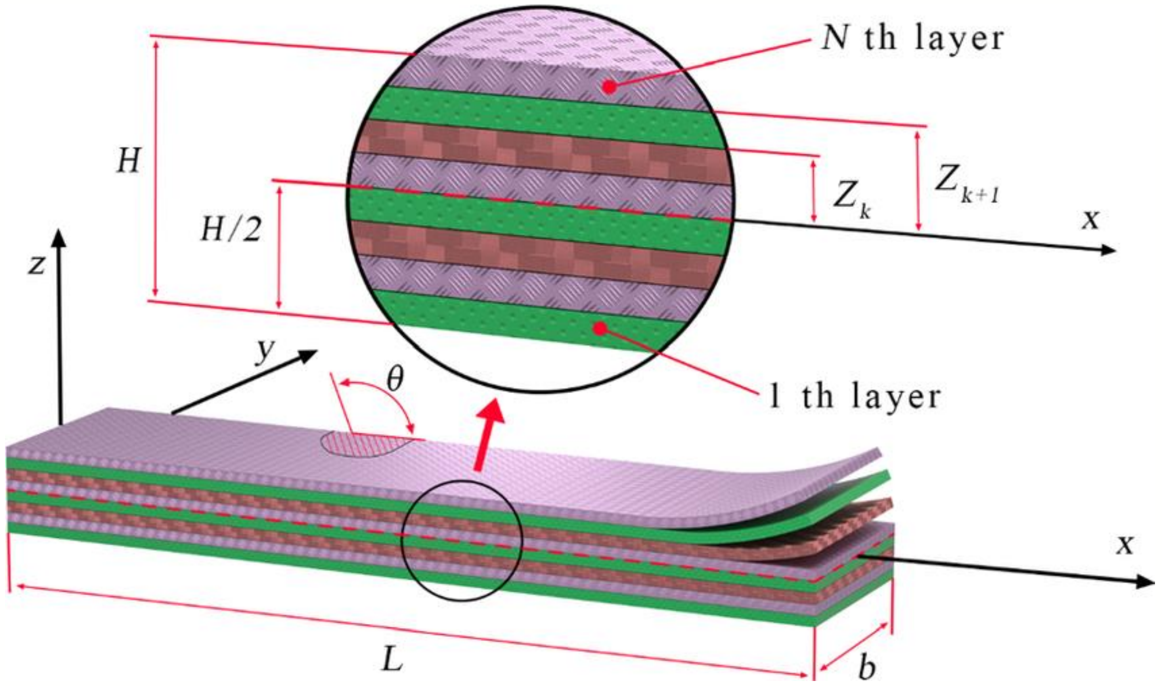
- Recent advances in additive manufacturing and composite material science have opened new avenues for embedding anisotropy into lattice structures
- Composite lattices, unlike their metallic counterparts, offer directional tailoring through ply orientation, enabling alignment of stiffness with principal load paths.
- This synergy between geometry and material anisotropy holds great promise for multifunctional aerospace components, including impact-resistant panels and structurally adaptive elements.
- Topology optimization techniques have been successfully applied to tailor lattice configurations for specific loading conditions, while computational homogenization methods have enabled the prediction of effective properties for periodic architectures
- Recent investigations into graded lattices and functionally graded cellular materials have demonstrated significant improvements in energy absorption and load-bearing capacity.
- The majority of existing research has focused on isotropic lattices, particularly metallic honeycombs, where closed-form expressions for effective moduli are well established.

Background and motivation

- The intrinsic complexity of anisotropic material behavior, coupled with the need for homogenizing laminate layups, has rendered most existing approaches heavily reliant on high-fidelity numerical methods such as Repetitive Volume Element (RVE) simulations.
- Although, computational models provide valuable insights, they are often restricted to narrow regions of the design space and are computationally prohibitive for large-scale optimization.
- The lack of closed-form analytical expressions linking composite layup parameters to lattice-level effective properties represents a significant barrier to rapid design iteration and optimization. This gap is particularly acute in aerospace applications where weight-constrained multiobjective optimization demands exploration of vast design spaces that cannot be efficiently sampled using finite element-based homogenization alone.
- Recent work on irregular lattices has highlighted the importance of analytical frameworks for understanding lattice mechanics, yet the extension to composite material systems with remains unexplored.

Background and motivation

- This paper introduces a analytical approach that leverages lamination parameters to calculate the effective elastic properties of two-dimensional composite lattices.

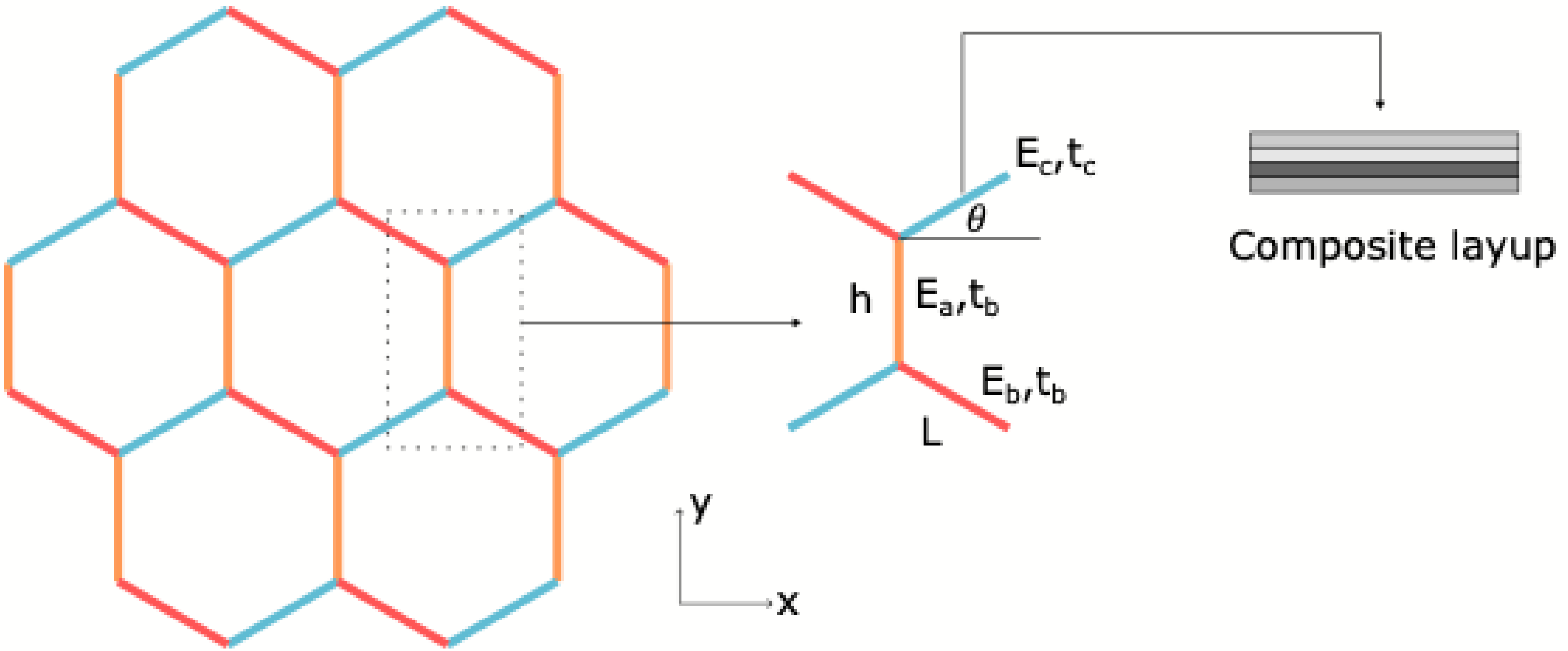


- By integrating classical laminate plate theory (CLPT) with a reduced-order representation of the stacking sequence, the proposed method enables closed-form estimation of axial and transverse stiffness components.
- Lamination parameters provide a compact and continuous parameterization of the design space by decoupling material properties from ply orientations. This approach eliminates the combinatorial complexity associated with discrete ply angles.

Background and motivation

- In the present context, lamination parameters are employed to express the elastic modulus of individual lattice members, which are then incorporated into unit cell homogenization equations to obtain lattice-level effective properties.
- The lattice is parameterized using ply count, ply orientation, lattice angle, and cell topology, providing a compact and continuous design space that eliminates the dependence on computational homogenization.
- *Material disparity ratios* are introduced to quantify stiffness contrast among heterogeneous lattice members, enabling systematic exploration of the anisotropic design space.
- Analytical expressions are derived to prescribe target anisotropy ratios and determine the required material contrast between lattice members.

Unit cell analysis of a heterogeneous lattice



- We consider the hexagonal lattice where the unit cell has three different members with unique composite properties. The equivalent elastic properties of this composite lattice can be calculated using plane stress assumptions.
- The equations for the equivalent properties are derived by assuming the deformation behaviour of the lattice member and follows Castigliano's theorem to calculate the stiffness of the members in a particular direction (*x* and *y*).

Equivalent elastic properties

- The five key equivalent elastic properties of the heterogeneous lattice are expressed in terms of the geometric and material properties of the lattice
- The thickness of all the elements is assumed to be *t*.
- The non-dimensional parameters α and β are defined as

  $$\alpha = \frac{t}{L} \text{ and } \beta = \frac{h}{L}$$

- Here *h* is the height of the vertical link and *L* is the length of the horizontal link.

$$E_x^{eq} = \frac{2\alpha^3 \cos\theta}{(\beta+\sin\theta)\left[\sin^2\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\cos^2\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)\right]}$$

$$E_y^{eq} = \frac{2\alpha^3(\beta+\sin\theta)}{\cos^3\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\sin^2\theta\cos\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\cos\theta\frac{4\beta}{E_c}}$$

$$\nu_{xy}^{eq} = \frac{\cos^2\theta\left[\left(\frac{1}{E_a}+\frac{1}{E_b}\right)-\alpha^2\left(\frac{1}{E_a}+\frac{1}{E_b}\right)\right]}{(\beta+\sin\theta)\sin\theta\left[\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\cot^2\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)\right]}$$

$$\nu_{yx}^{eq} = \frac{\sin\theta(\beta+\sin\theta)\left[\left(\frac{1}{E_a}+\frac{1}{E_b}\right)-\alpha^2\left(\frac{1}{E_a}+\frac{1}{E_b}\right)\right]}{\cos^2\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\sin^2\theta\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\frac{4\beta}{E_c}}$$

$$G_{xy}^{eq} = \frac{4\alpha^3(\beta+\sin\theta)}{\cos\theta\left[\beta^2\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\alpha^2\left(\cos\theta+(\beta+\sin\theta)\tan\theta\right)^2\left(\frac{1}{E_a}+\frac{1}{E_b}\right)+\frac{4\beta^3}{E_c}\right]}$$

Equivalent elastic properties

- We introduce two further non-dimensional parameters quantifying the disparities between the elastic modulus of the three composite elements in the lattice:

$$\eta_1 = \frac{E_b}{E_a} \qquad \eta_2 = \frac{E_c}{E_a}$$

- The material disparity ratios quantify the stiffness contrast between the primary material $(E_a)$and the secondary materials (E_b) and (E_c),respectively.

$$E_x^{eq} = E_a \cdot \frac{\eta_1}{1+\eta_1} \cdot \frac{2\alpha^3 \cos\theta}{(\beta + \sin\theta)} \cdot \frac{1}{\sin^2\theta + \alpha^2 \cos^2\theta}$$

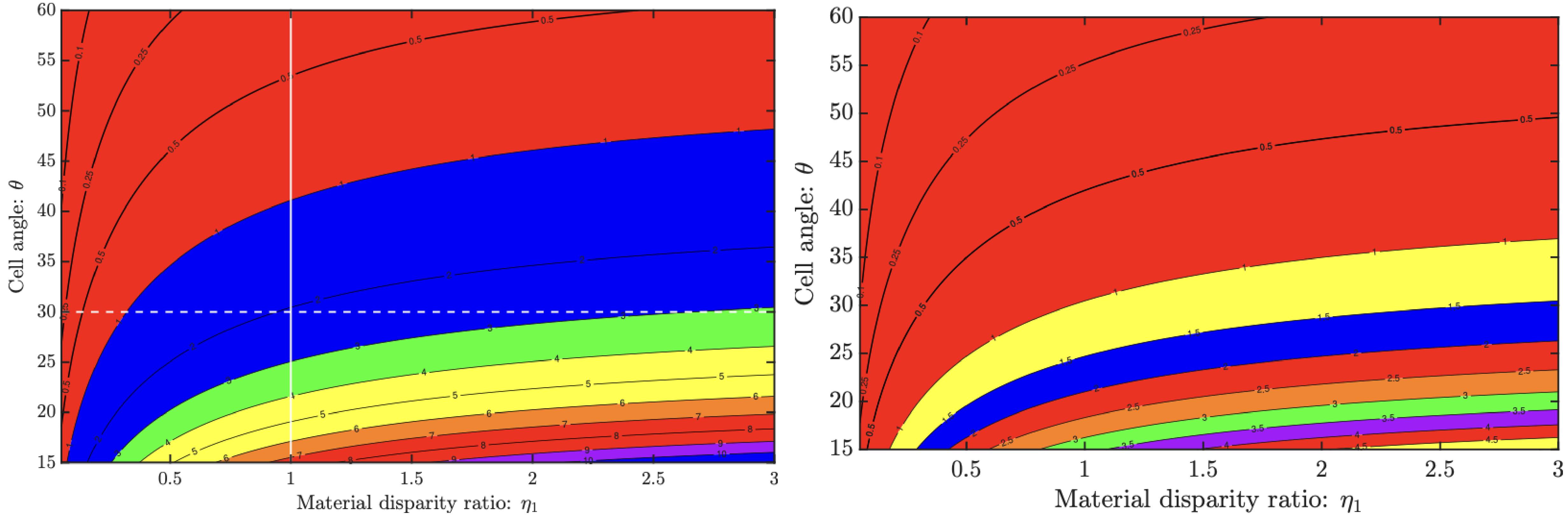
$$E_y^{eq} = E_a \frac{\eta_1}{1+\eta_1} \cdot \frac{2\alpha^3 (\beta + \sin\theta)}{\cos\theta} \cdot \frac{1}{\cos^2\theta + \alpha^2 \sin^2 + \alpha^2 \frac{4\beta}{\eta_2} \cdot \frac{\eta_1}{1+\eta_1}}$$

$$\nu_{xy}^{eq} = \frac{\cos^2\theta (1-\alpha^2)}{(\beta + \sin\theta)\sin\theta(1 + \alpha^2 \cot^2\theta)}$$

$$\nu_{yx}^{eq} = \frac{\sin\theta(\beta + \sin\theta)(1-\alpha^2)}{\left(\cos^2\theta + \alpha^2 \sin^2\theta\right) + \alpha^2 \frac{4\beta}{\eta_2} \cdot \frac{\eta_1}{1+\eta_1}}$$

$$G_{xy}^{eq} = E_a \cdot \frac{\eta_1}{1+\eta_1} \cdot \frac{4\alpha^3(\beta + \sin\theta)}{\cos\theta} \cdot \frac{1}{\beta^2 + \alpha^2(\cos\theta + (\beta + \sin\theta)\tan\theta)^2 + \frac{4\beta^3}{\eta_2} \cdot \frac{\eta_1}{1+\eta_1}}$$

Equivalent elastic properties

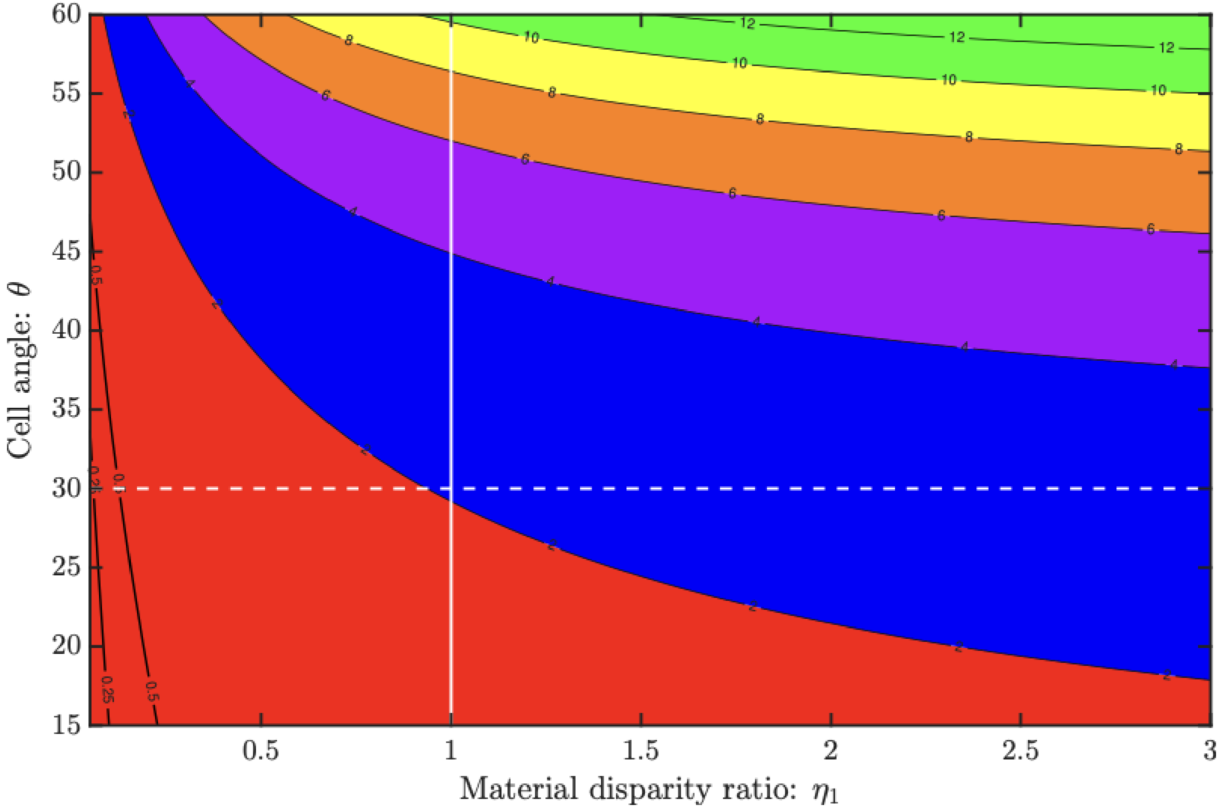


(a) $E_x^{eq}/(E_a\alpha^3)$ ($\beta = 1$, $\eta_2 = 1$)

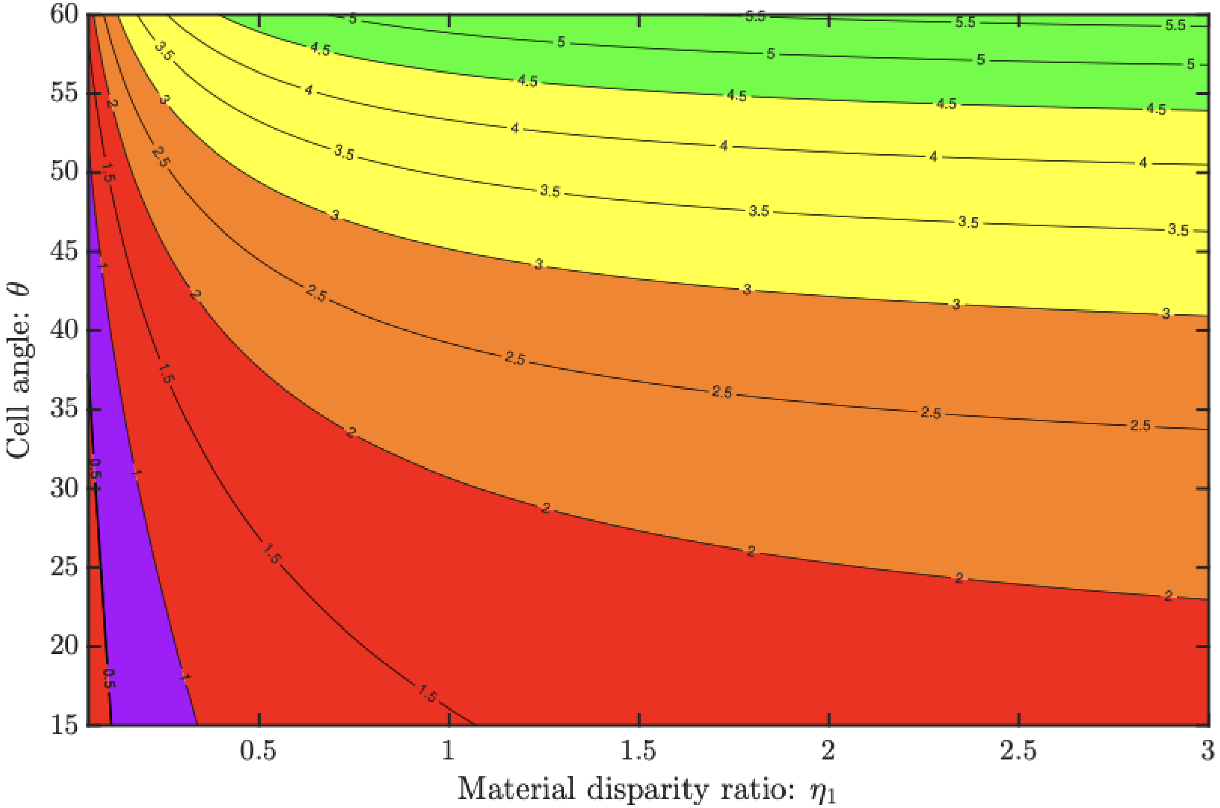
(b) $E_x^{eq}/(E_a\alpha^3)$ ($\beta = 2.5$, $\eta_2 = 0.2$)

Fig. 2 Contours of normalised E_x^{eq} plotted as functions of the material disparity ratio (η_1) and the cell angle (θ) for $\alpha = 0.2$ and different values of the height ratio (β) and material disparity ratio η_2. The case corresponding to the classical single-material regular homogeneous lattice ($\beta = \eta_1 = \eta_2 = 1, \theta = 30°$) is marked with lines in (a).

Equivalent elastic properties



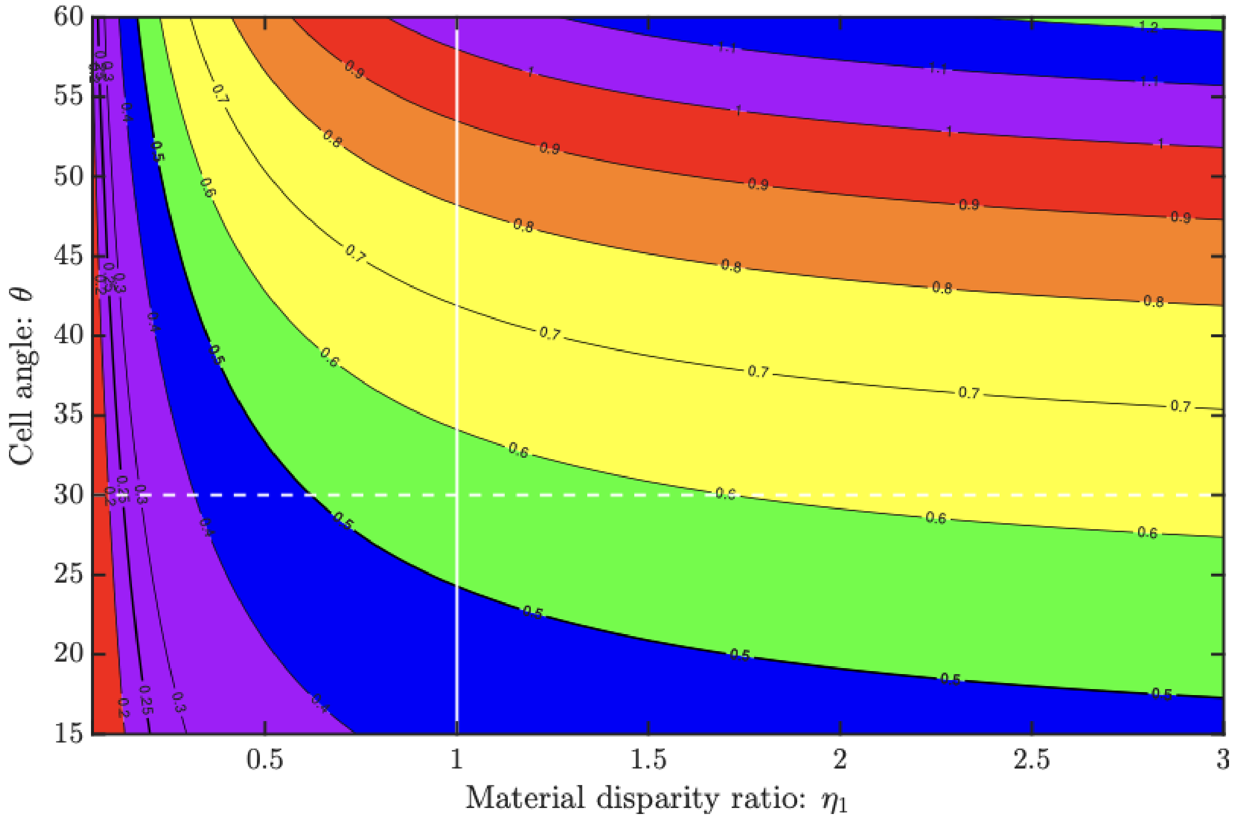
(a) $E_y^{eq}/(E_a\alpha^3)$ ($\beta = 1$, $\eta_2 = 1$)



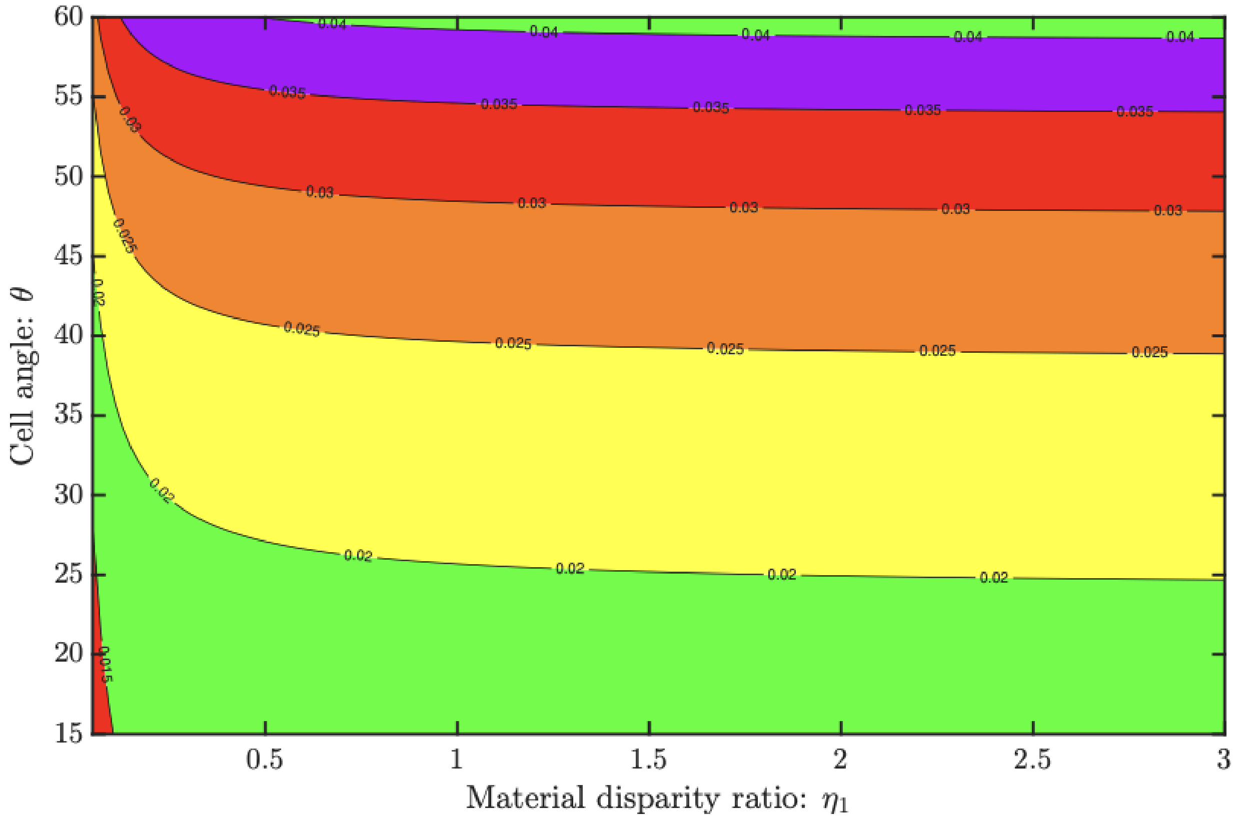
(b) $E_y^{eq}/(E_a\alpha^3)$ ($\beta = 2.5$, $\eta_2 = 0.2$)

Fig. 3 Contours of normalised E_y^{eq} plotted as functions of the material disparity ratio (η_1) and the cell angle (θ) for $\alpha = 0.2$ and different values of the height ratio (β) and material disparity ratio η_2. The case corresponding to the classical single-material regular homogeneous lattice ($\beta = \eta_1 = \eta_2 = 1, \theta = 30°$) is marked with lines in (a).

Equivalent elastic properties



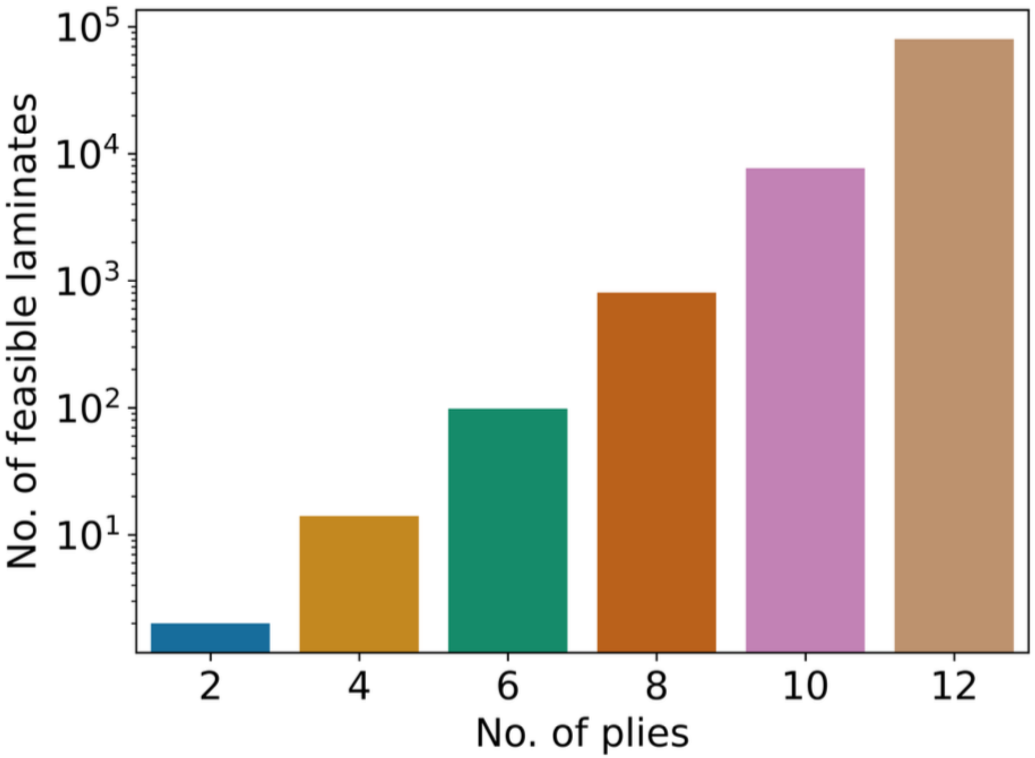
(a) $G_{xy}^{eq}/(E_a\alpha^3)$ ($\beta = 1$, $\eta_2 = 1$)



(b) $G_{xy}^{eq}/(E_a\alpha^3)$ ($\beta = 2.5$, $\eta_2 = 0.2$)

Fig. 4 Contours of normalised G_{xy}^{eq} plotted as functions of the material disparity ratio (η_1) and the cell angle (θ) for $\alpha = 0.2$ and different values of the height ratio (β) and material disparity ratio η_2. The case corresponding to the classical single-material regular homogeneous lattice ($\beta = \eta_1 = \eta_2 = 1, \theta = 30°$) is marked with lines in (a).

Lamination parameters for composites



- In composite material systems, the mechanical properties along specific directions can vary significantly due to the anisotropic nature of individual plies and their stacking sequence.
- Lamination parameters act as a solution to reduce the problem complexity, especially when the design space is large.

Fig. 5 Combinatorial explosion of symmetric and balanced laminate design space with increasing ply count. The number of feasible stacking sequences (vertical axis, logarithmic scale) grows exponentially from 2 configurations at 2 plies to over 10^5 at 12 plies, with approximately order-of-magnitude increase every 2 additional plies. Symmetry eliminates bending-extension coupling while balance minimizes thermal distortion. This exponential growth motivates the lamination parameter approach, which compresses the discrete, intractable space of stacking sequences into a continuous four-dimensional domain $(\xi_1, \xi_2, \xi_3, \xi_4)$ amenable to efficient optimization and inverse design.

Lamination parameters for composites

- One of the key global properties of interest in lattice structures is the longitudinal stiffness. This stiffness can be represented in a simplified form as:

$$E_x = f(\mathbf{U})g(\xi)$$

- Here $\mathbf{U}$ denotes a material invariant that encapsulates the intrinsic stiffness of the lamina, and $g(\xi)$ is a function of the lamination parameters which encode the orientation distribution of plies throughout the laminate thickness.

$$Q_{11} = \frac{E_{11}}{1 - \nu_{12}\nu_{21}} \qquad Q_{22} = \frac{E_{22}}{1 - \nu_{12}\nu_{21}}$$

$$Q_{12} = \frac{\nu_{12}E_{22}}{1 - \nu_{12}\nu_{21}} = \frac{\nu_{21}E_{11}}{1 - \nu_{12}\nu_{21}} \qquad Q_{66} = G_{12}$$

- Here E_1 is the longitudinal modulus, E_2 is the transverse modulus, G_{12} is the shear modulus, and ν_{12} is Poisson's ratio.

Lamination parameters for composites

- In multi-ply symmetric and balanced laminates, the in-plane extensional behaviour is then fully characterized by the $\mathbf{A}$ matrix, with components:

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q_{ij}}\right)_k, (h_k - h_{k-1})$$

- Here h_k and h_{k-1} are the distances from the laminate mid-plane to the top and bottom of the *k*-th ply, and Q_{ij} is the transformed reduced stiffness matrix of the *k*-th ply.
- Computing the $\mathbf{A}$ matrix directly from ply-by-ply orientations and stiffness transformations is computationally expensive.
- This can be rewritten in terms of lamination parameters (ξ_m) and material invariants (**U**) as:

$$\begin{bmatrix} A_{11} \\ A_{12} \\ A_{22} \\ A_{66} \\ A_{16} \\ A_{26} \end{bmatrix} = t \begin{bmatrix} 1 & \xi_1 & \xi_2 & 0 & 0 \\ 0 & 0 & -\xi_2 & 1 & 0 \\ 1 & -\xi_1 & \xi_2 & 0 & 0 \\ 0 & 0 & -\xi_2 & 0 & 1 \\ 0 & \frac{\xi_3}{2} & \xi_4 & 0 & 0 \\ 0 & \frac{\xi_3}{2} & -\xi_4 & 0 & 0 \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ U_5 \end{bmatrix}$$

$$U_1 = \frac{1}{8}(3Q_{11} + 3Q_{22} + 2Q_{12} + 4Q_{66})$$

$$U_2 = \frac{1}{2}(Q_{11} - Q_{22})$$

$$U_3 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} - 4Q_{66})$$

$$U_4 = \frac{1}{8}(Q_{11} + Q_{22} + 6Q_{12} - 4Q_{66})$$

$$U_5 = \frac{1}{8}(Q_{11} + Q_{22} - 2Q_{12} + 4Q_{66})$$

Lamination parameters for composites

- The lamination parameters ξ_m are defined as:

$$\xi_1 = \frac{1}{t}\int_{-t/2}^{t/2} \cos 2\Theta(z)\,dz = \frac{1}{t}\sum_{k=1}^{n} \cos(2\Theta_k)\ (h_{k-1} - h_k),$$

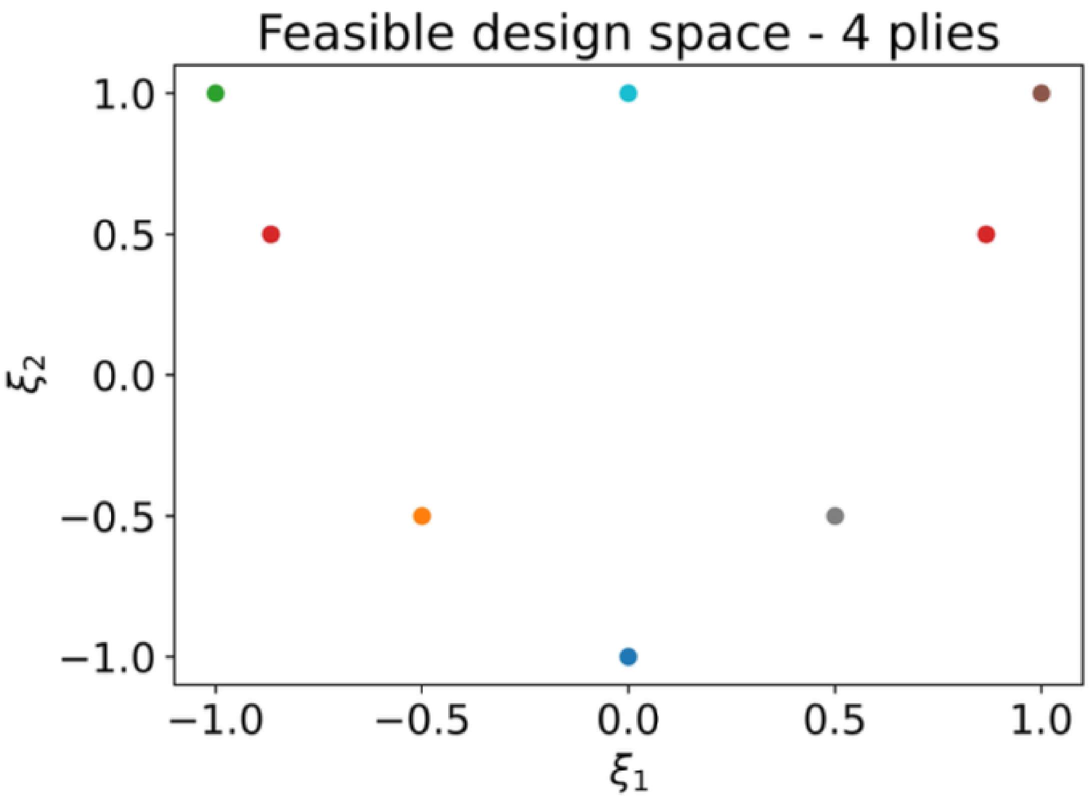
$$\xi_2 = \frac{1}{t}\int_{-t/2}^{t/2} \cos 4\Theta(z)\,dz = \frac{1}{t}\sum_{k=1}^{n} \cos(4\Theta_k)\ (h_{k-1} - h_k),$$

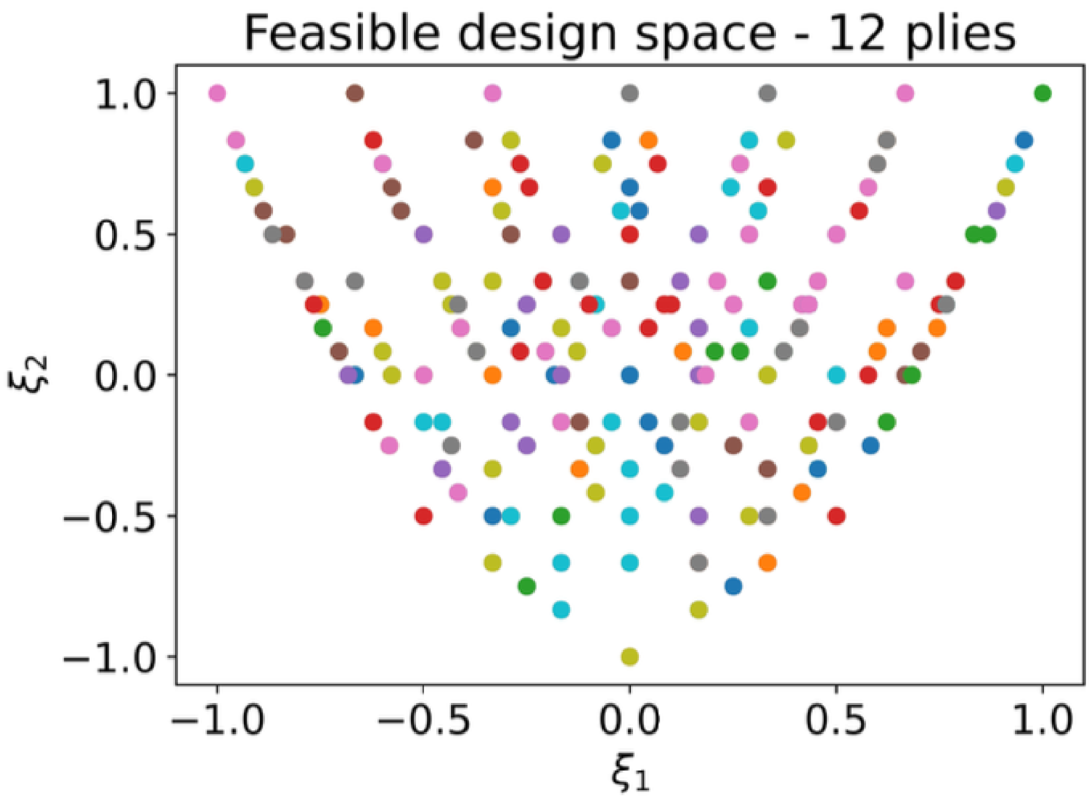
$$\xi_3 = \frac{1}{t}\int_{-t/2}^{t/2} \sin 2\Theta(z)\,dz = \frac{1}{t}\sum_{k=1}^{n} \sin(2\Theta_k)\ (h_{k-1} - h_k),$$

$$\xi_4 = \frac{1}{t}\int_{-t/2}^{t/2} \sin 4\Theta(z)\,dz = \frac{1}{t}\sum_{k=1}^{n} \sin(4\Theta_k)\ (h_{k-1} - h_k),$$

- Here, Θ is the fibre orientation at a distance z from the mid-plane. The integral above is a general definition and is converted to a finite sum, since the laminated composite thickness distribution is piecewise continuous.

Lamination parameters for composites





- Feasible design space of symmetric and balanced laminates in lamination parameter coordinates (ξ_1, ξ_2) for (a) 4-ply and (b) 12-ply configurations. Each point represents a distinct manufacturable stacking sequence satisfying symmetry and balance constraints. The characteristic parabolic boundary, defined by $|\xi_2| \leq 1 - \xi_1^2$, bounds all physically realizable laminates. The design space remains discrete rather than continuous, with increased ply count providing denser coverage and expanded tailoring capability. The sparse distribution at 4 plies (7 feasible points) contrasts with substantially richer design space at 12 plies (thousands of points), enabling finer stiffness control. This discretization motivates the encoder-decoder approach: the neural network learns continuous mappings in this bounded domain, while post-processing retrieves the nearest manufacturable discrete sequence via kd-tree search.

Lamination parameters for composites

- The equivalent elastic modulus can be computed from the **A** matrix by computing its inverse:

$$\mathbf{A} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}$$

$$A_{11} = t(U_1 + \xi_1 U_2 + \xi_2 U_3)$$
$$A_{12} = t(-\xi_2 U_3 + U_4)$$
$$A_{22} = t(U_1 - \xi_1 U_2 + \xi_2 U_3)$$
$$A_{66} = t(-\xi_2 U_3 + U_5)$$

- The equivalent elastic modulus of a lattice member *m = {a,b,c}* is calculated from the **A** matrix as:

$$E_m = \frac{1}{t}\left(\frac{1}{A_{11}^{-1}}\right)$$
$$E_m = \frac{1}{t}\left(\frac{A_{66}.(A_{11}A_{22} - A_{12}A_{21})}{A_{22}A_{66}}\right)$$
$$E_m = \frac{1}{t}\left(\frac{(A_{11}A_{22} - A_{12}A_{21})}{A_{22}}\right)$$
$$E_m = \frac{1}{t}\left(\frac{A_{11}A_{22} - A_{12}^2}{A_{22}}\right)$$

Summary: Composite lattices

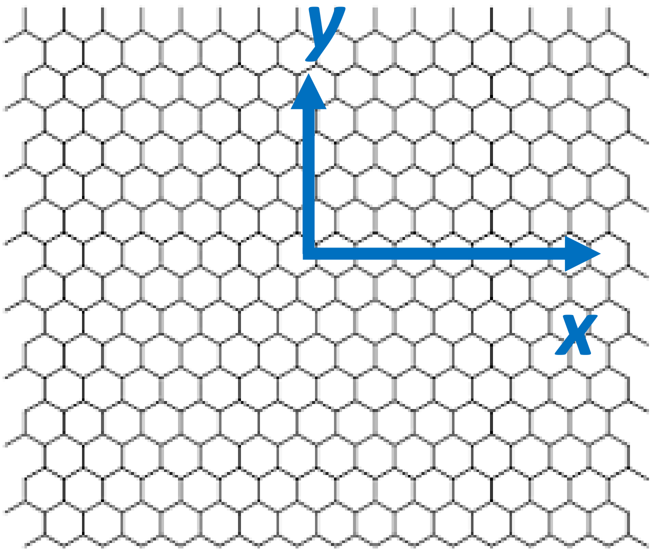
$$\eta_1 = \frac{E_b}{E_a}, \quad \eta_2 = \frac{E_c}{E_a},$$

$$\alpha = \frac{t}{L} \text{ and } \beta = \frac{h}{L}$$

$$E_m = \frac{1}{t}\left(\frac{t^2(U_1+\xi_1U_2+\xi_2U_3)(U_1-\xi_1U_2+\xi_2U_3)-t^2(-\xi_2U_3+U_4)^2}{t(U_1-\xi_1U_2+\xi_2U_3)}\right)$$

$m = \{a,b,c\}$

$$E_m = \frac{U_1^2+2\xi_2U_1U_3-(\xi_1U_2)^2+2\xi_2U_3U_4-U_4^2}{U_1-\xi_1U_2+\xi_2U_3}$$



$$E_x^{eq} = E_a \cdot \frac{\eta_1}{1+\eta_1} \cdot \frac{2\alpha^3\cos\theta}{(\beta+\sin\theta)} \cdot \frac{1}{\sin^2\theta+\alpha^2\cos^2\theta}$$

$$E_y^{eq} = E_a \frac{\eta_1}{1+\eta_1} \cdot \frac{2\alpha^3(\beta+\sin\theta)}{\cos\theta} \cdot \frac{1}{\cos^2\theta+\alpha^2\sin^2+\alpha^2\frac{4\beta}{\eta_2}\cdot\frac{\eta_1}{1+\eta_1}}$$

$$\nu_{xy}^{eq} = \frac{\cos^2\theta(1-\alpha^2)}{(\beta+\sin\theta)\sin\theta(1+\alpha^2\cot^2\theta)}$$

$$\nu_{yx}^{eq} = \frac{\sin\theta(\beta+\sin\theta)(1-\alpha^2)}{\left(\cos^2\theta+\alpha^2\sin^2\theta\right)+\alpha^2\frac{4\beta}{\eta_2}\cdot\frac{\eta_1}{1+\eta_1}}$$

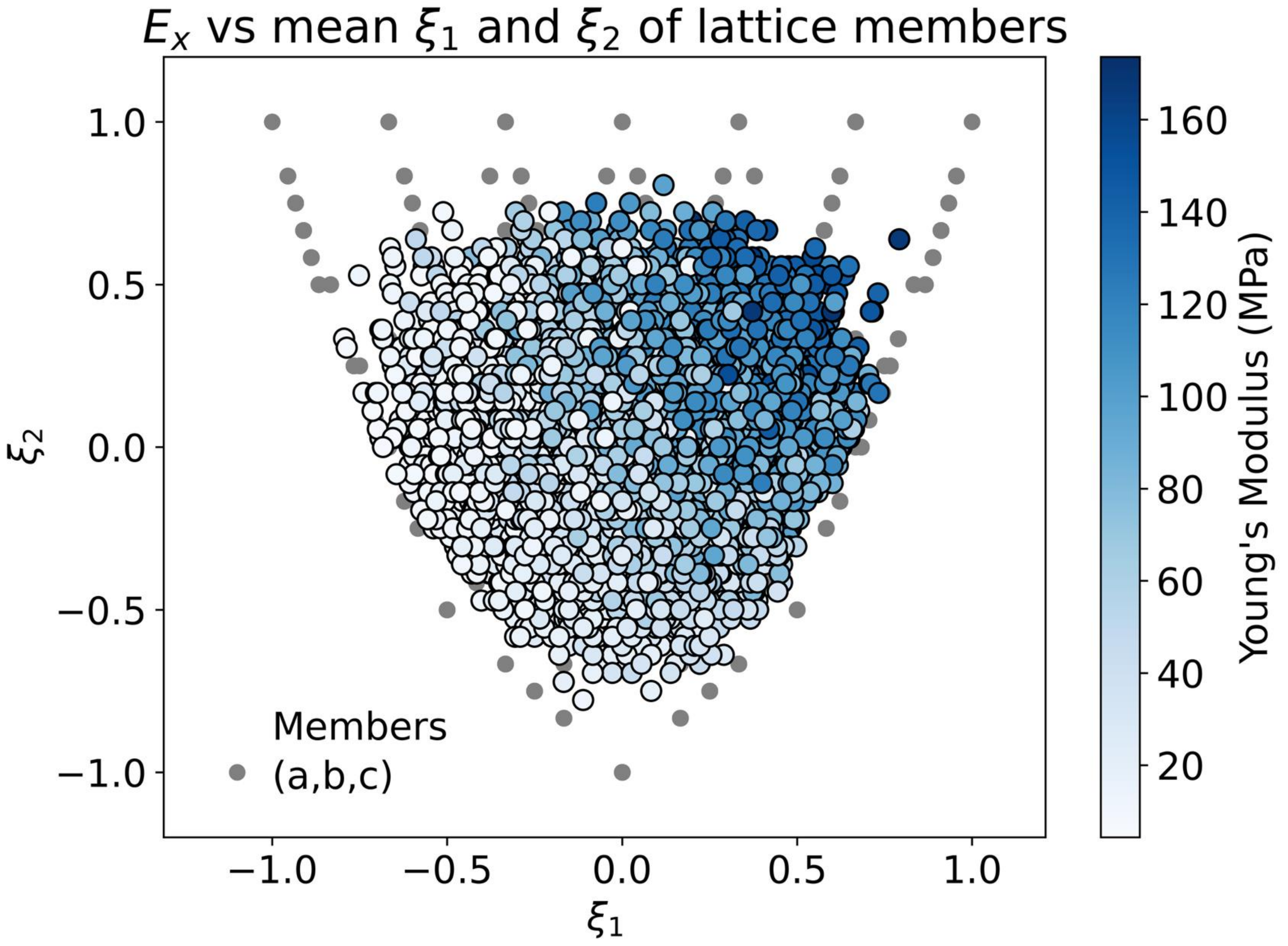
$$G_{xy}^{eq} = E_a \cdot \frac{\eta_1}{1+\eta_1} \cdot \frac{4\alpha^3(\beta+\sin\theta)}{\cos\theta} \cdot \frac{1}{\beta^2+\alpha^2(\cos\theta+(\beta+\sin\theta)\tan\theta)^2+\frac{4\beta^3}{\eta_2}\cdot\frac{\eta_1}{1+\eta_1}}$$

Composite lattice elastic properties

- To demonstrate the applicability of the proposed homogenization approach, a composite lattice structure comprising five plies is considered.
- The constituent ply properties are based on a unidirectional Glass/Epoxy composite with the following orthotropic parameters: longitudinal modulus $E_{11} = 38600$ MPa, transverse modulus $E_{22} = 8270$MPa, major Poisson's ratio $\nu_{12} = 0.26$ and in-plane shear modulus $G_{12} = 4140$MPa.
- The geometry of the lattice is defined by a unit cell with height $h = 50$ mm, length $L = 50$ mm and an effective core thickness derived to achieve a relative density of $\rho^*/\rho_s = 0.0722$.

Composite lattice elastic properties

- Grey markers show individual member parameters (ξ_1^m, ξ_2^m);colored markers show lattice-averaged parameters with color intensity indicating modulus magnitude (20-160 MPa).
- Lamination parameter variation enables continuous stiffness tailoring from 20 to 160 MPa at fixed geometry and relative density.



Variation of Elastic modulus E_x(MPa) of the composite lattice as a function of lamination parameters

Composite lattice elastic properties

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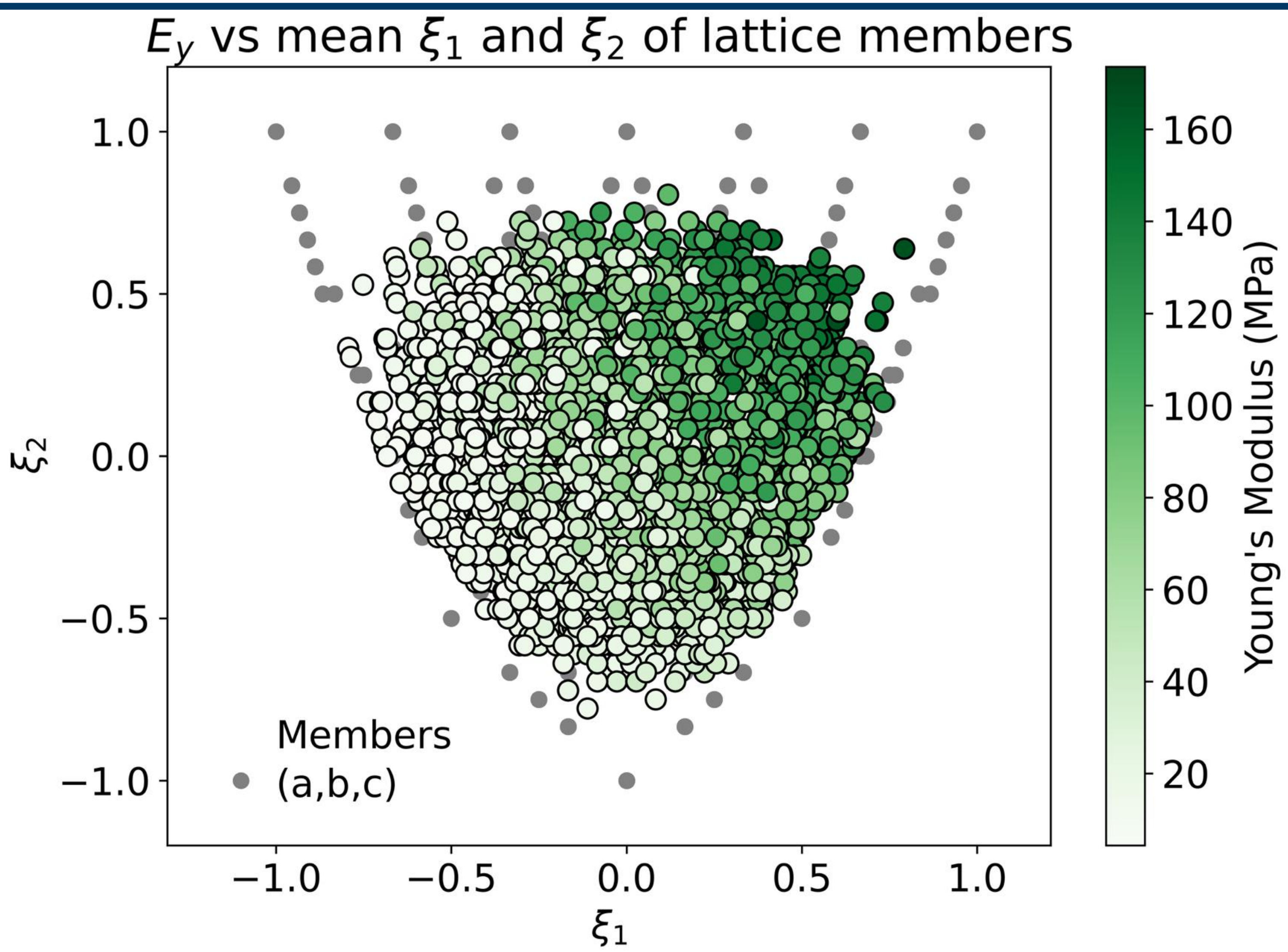


Variation of Elastic modulus E_y(MPa) of the composite lattice as a function of lamination parameters

Anisotropy ratio

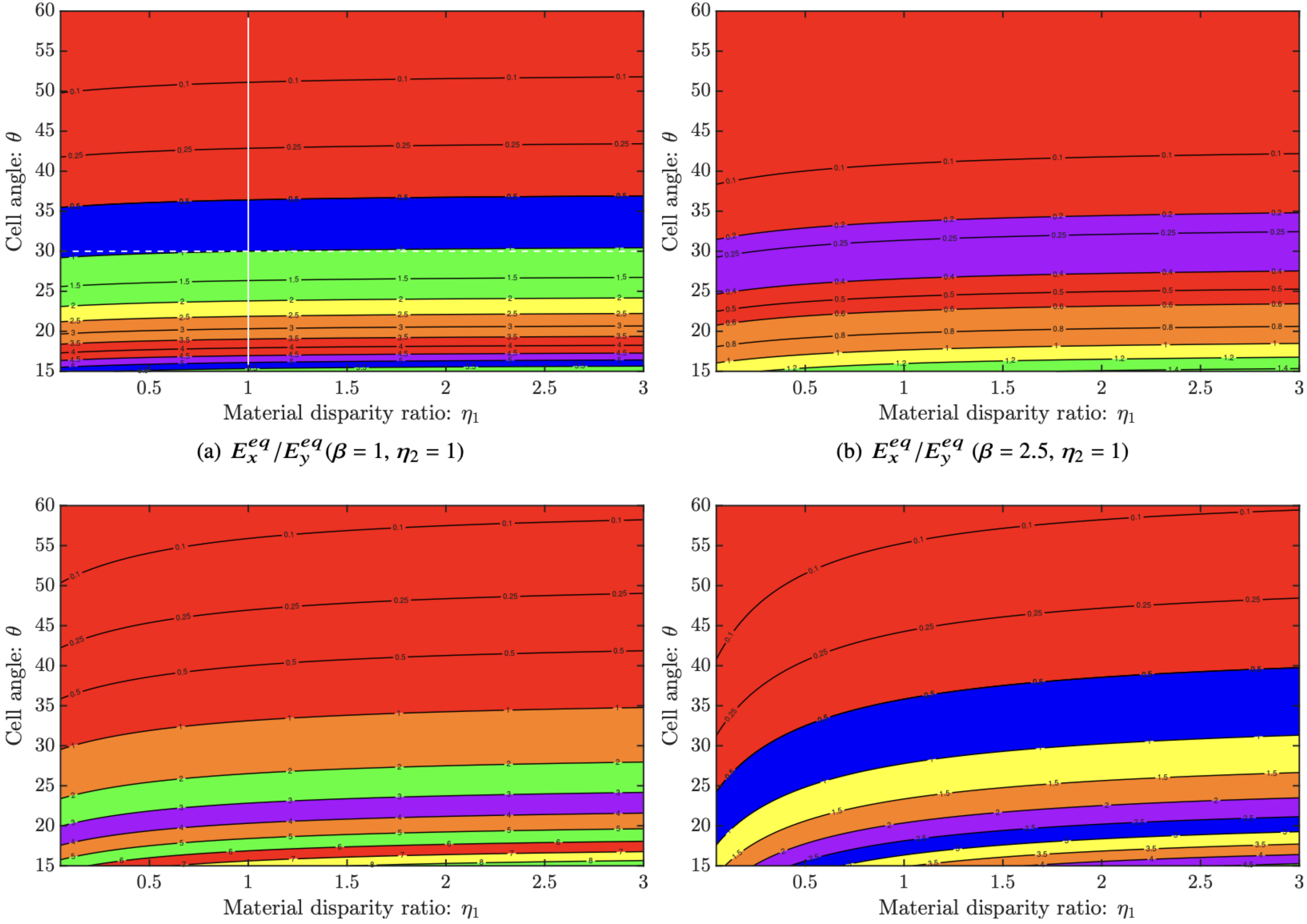
- The anisotropy ratio is defined as the ratio of the equivalent Young's modulus in the longitudinal direction to that in the transverse direction, denoted by E_x^{eq}/E_y^{eq}.
- The ratio, denoted by *m* can be expressed as

$$m = \frac{E_x^{eq}}{E_y^{eq}} = \frac{\cos^2\theta(\alpha^2(\sin^2\theta) + \cos^2\theta) + 4\alpha^2\beta\cos^2\theta\,\dfrac{\eta_1}{\eta_2(1+\eta_1)}}{(\beta + \sin\theta)^2(\alpha^2\cos^2\theta + \sin^2\theta)}$$

- This form offers a clear interpretation of the directional stiffness contrast in terms of geometry $((\alpha), (\beta), (\theta))$and material disparity ratios $((\eta_1), (\eta_2))$.
- The expression highlights how tuning these parameters controls the degree of anisotropy in a composite architected lattice.

Anisotropy ratio

- Contours of anisotropy ratio E_x^{eq}/E_y^{eq} plotted as functions of the material disparity ratio (η_1) and the cell angle (θ) for $\alpha = 0.2$ and different values of the height ratio (\beta) and material disparity ratio η_2. The case corresponding to the classical single-material regular homogeneous lattice $\beta = \eta_1 = \eta_2 = 1, \theta = 30°)$ is marked with lines in (a).



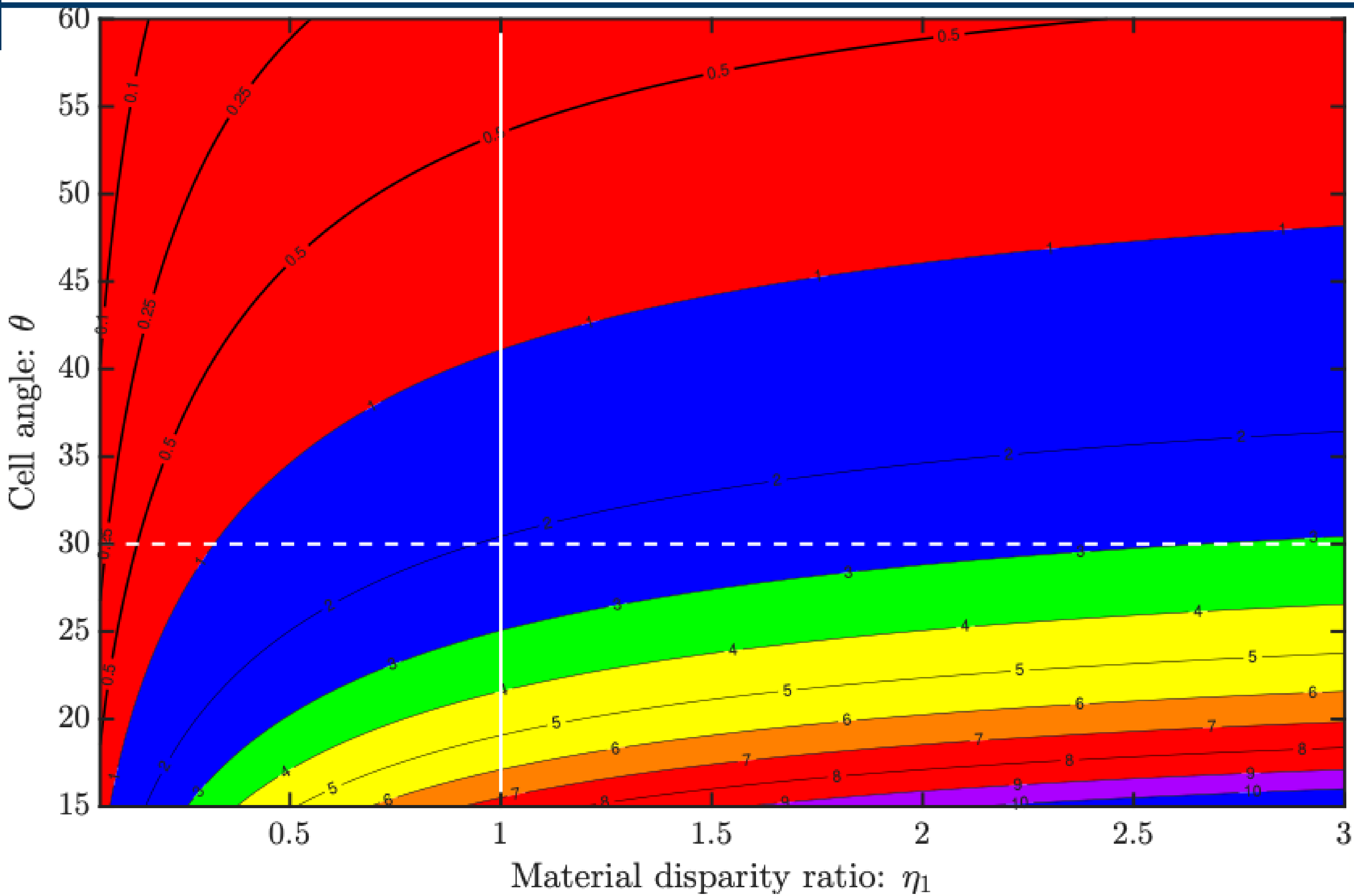
(a) $E_x^{eq}/E_y^{eq}(\beta = 1, \eta_2 = 1)$

(b) E_x^{eq}/E_y^{eq} $(\beta = 2.5, \eta_2 = 1)$

(c) E_x^{eq}/E_y^{eq} $(\beta = 1, \eta_2 = 0.2)$

(d) E_x^{eq}/E_y^{eq} $(\beta = 2.5, \eta_2 = 0.1)$

Anisotropy ratio



Tailoring based on the anisotropy ratio

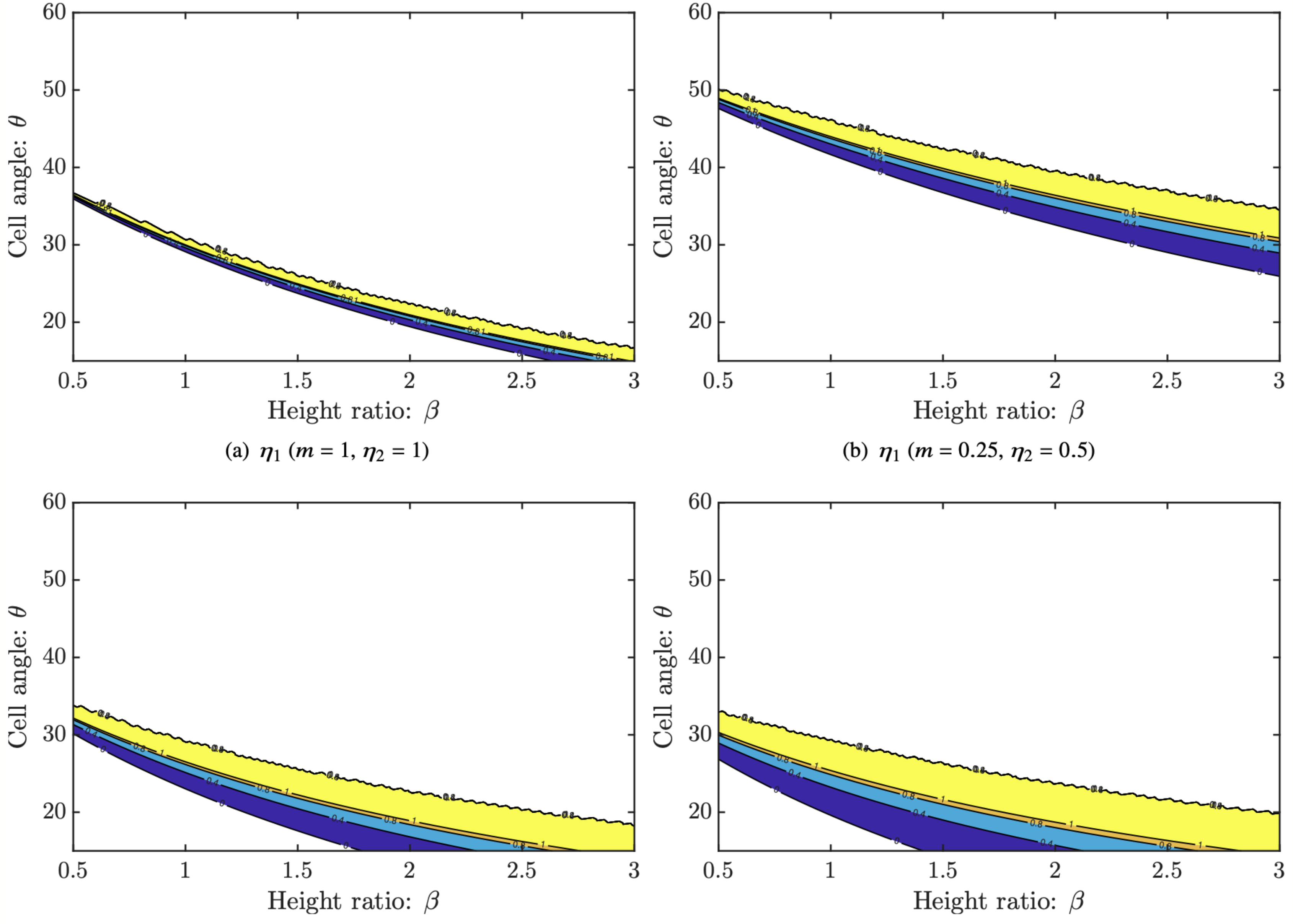
- The equation can be rearranged to explicitly obtain the material disparity ratios necessary to achieve a given anisotropy ratio as

$$\frac{\eta_2(1+\eta_1)}{\eta_1} = \frac{4\cos^2\theta\,\alpha^2\beta}{-(\cos^4\theta) + \left((m-1)\sin^2\theta + 2\beta m\sin\theta + m,\beta^2\right)\alpha^2\cos^2\theta + \sin^2\theta\, m(\beta+\sin\theta)^2}$$

- This expressions provides a direct analytical relationship between the material disparity ratios $\eta_1$and η_2,the geometric parameters $(\alpha,,\beta,,\theta)$,and the target anisotropy ratio $m = E_x^{eq}/E_y^{eq}$.
- This expression is significant because it enables designers to prescribe a desired anisotropy level and explicitly determine the required contrast in constituent stiffness among the lattice members, without resorting to iterative numerical optimisation.
- In other words, it transforms the traditionally forward problem of evaluating anisotropy into an inverse design tool that links structural geometry and material composition in closed form.
- This analytical tractability allows the anisotropy to be tuned independently of lattice topology, facilitating systematic exploration of design trade-offs between stiffness, weight, and manufacturability. As a result, this serves as the foundational relation for anisotropy-controlled design of composite lattices.

Parametric landscape for designed anisotropy

- Contours of the material disparity ratio η_1 plotted as functions of the material disparity ratio $(\eta_2)$and the cell angle (θ) for $\alpha = 0.2$ and different values of the designed anisotropy ratio (m). The zone outside the marked area implies that no feasible solution is possible



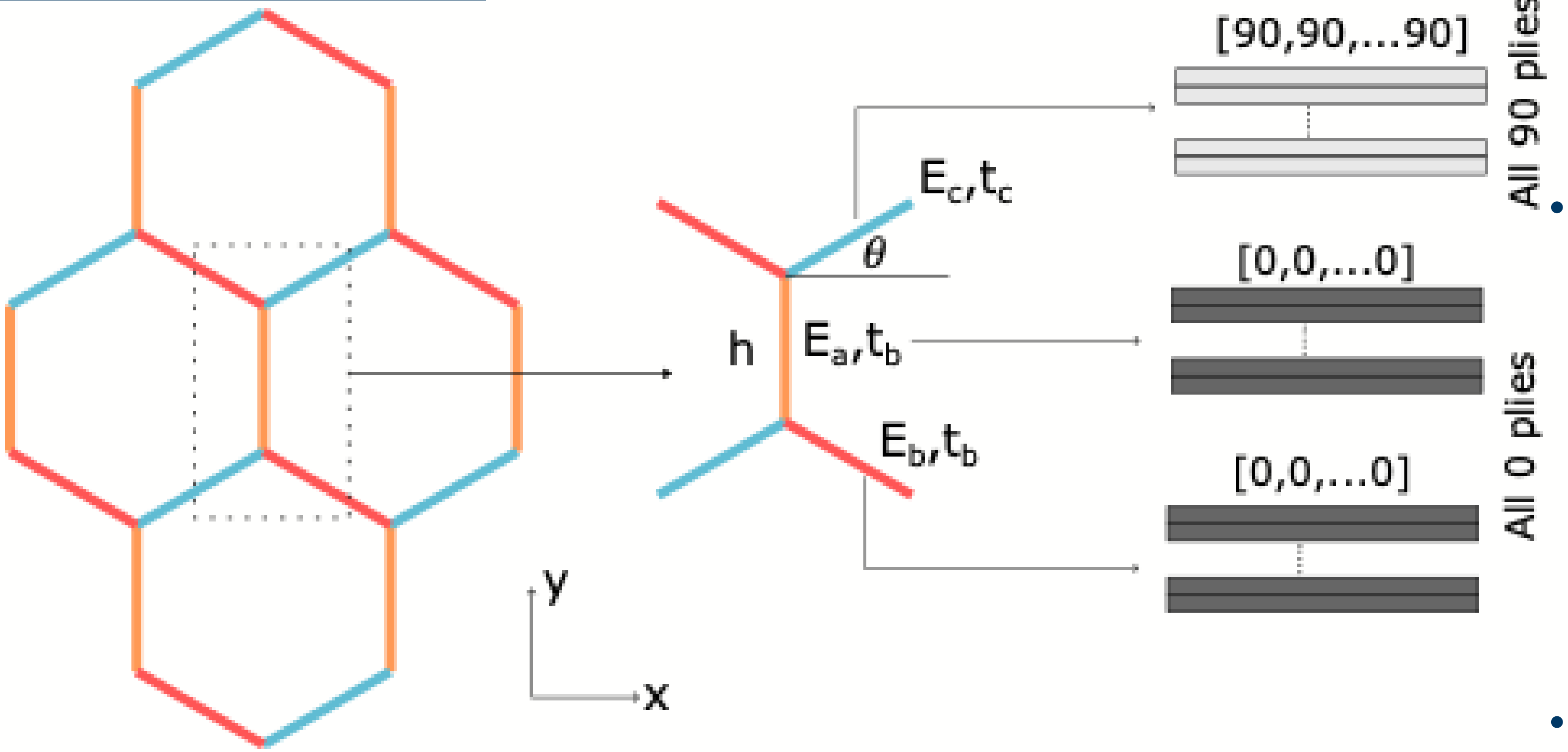
(a) η_1 ($m = 1$, $\eta_2 = 1$)

(b) η_1 ($m = 0.25$, $\eta_2 = 0.5$)

(c) η_1 ($m = 2$, $\eta_2 = 0.2$)

(d) η_1 ($m = 3$, $\eta_2 = 0.1$)

Tailoring based on the anisotropy ratio



Material System	Maximum anisotropy
XN80 + Epoxy	1.49
M55 + Epoxy	1.34
Boron + Epoxy	1.10
E-glass + Epoxy	1.03

- Material-dependent upper bounds on anisotropy for hexagonal lattices $\theta = 30°$ with extreme unidirectional stacking sequences.
- Members are assigned either all-0° or all-90° plies to establish theoretical maximum directional stiffness contrast achievable through layup design alone. High-modulus carbon systems (XN80, M55) reach maximum anisotropy of 1.49 and 1.34 respectively, while Boron-epoxy achieves 1.10 and E-glass-epoxy approaches isotropy at 1.03.
- These bounds define fundamental limits on lattice anisotropy tailoring for each material system, providing essential design constraints for inverse optimization and material selection criteria for applications requiring specific directional property ratios.

Conclusions: Key Contributions

- **Lamination parameter framework integrated with lattice homogenization** eliminates the exponential growth of design space, enabling continuous analytical optimisation without finite element simulations
- **Closed-form expressions derived** for effective elastic properties (*Ex, Ey, Gxy, vxy*) as functions of lamination parameters, material invariants, and geometric ratios
- **Material disparity ratios (η_1, η_2) were introduced** as compact parameters quantifying stiffness contrast among heterogeneous lattice members, enabling systematic heterogeneous lattice design
- **Inverse design capability established** through analytical expressions that prescribe required material disparity ratios to achieve target anisotropy ratios, transforming trial-and-error into direct calculation
- **Fundamental non-uniqueness revealed** in the inverse mapping from target properties to lamination parameters, providing designers flexibility to satisfy secondary constraints such as manufacturability and buckling resistance
- **Feasibility boundaries mapped** across the parametric landscape, identifying regions where physical solutions exist and quantifying geometric and material limitations on achievable anisotropy

Conclusions: Impact and Future Directions

- **Material selection criteria established**: High-modulus carbon ($E_1/E_2 = 49.25$) enables anisotropy ratios up to 1.49, while E-glass ($E_1/E_2 = 4.66$) limits anisotropy to 1.03 regardless of configuration
- **Maximum anisotropy depends on absolute stiffness levels**: Extreme directional contrast requires high member modulus disparity, coupling anisotropy targets with overall lattice stiffness
- **Computational acceleration achieved**: Replaces expensive repetitive volume element simulations with instant analytical queries, enabling real-time design space exploration and optimization
- **Aerospace applications enabled**: Morphing wing skins with spanwise stiffness and chordwise compliance, impact-resistant panels with directional energy absorption, and satellite structures with mass-efficient anisotropy
- **Optimization integration pathway**: Framework ready for coupling with multiobjective algorithms to simultaneously optimize stiffness, weight, manufacturability, and multiphysics performance metrics

Sondipon Adhikari (Infrastructure & Environment)
James Watt School of Engineering
Rankine Building
The University of Glasgow
Phone: + 44 (0) 141 330 3317
Email: Sondipon.Adhikari@glasgow.ac.uk
http://www.gla.ac.uk/schools/engineering/staff/sondiponadhikari

Google Scholar page:
http://scholar.google.co.uk/citations?user=tKM35S0AAAAJ

@ ProfAdhikari