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Frictional Energy Harvesting Vibration Absorbers

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ABSTRACT

Structural vibration control is essential for enhancing the safety and longevity of engineering systems. Traditional Tuned Mass Dampers (TMDs) effectively mitigate vibrations but are limited by their reliance on large auxiliary masses. To address this challenge, we propose the Frictional Energy Harvesting Vibration Absorber (FEHVA), a novel hybrid absorber that integrates frictional damping with energy harvesting. The governing equations of FEHVA are derived using the statistical linearisation method, and optimal design parameters are determined via H_2 optimisation. Dynamic response evaluations reveal that FEHVA achieves up to 14.20% improved vibration reduction over conventional TMDs under harmonic and random excitations. Furthermore, comparisons with the Inerter Energy Harvesting Vibration Absorber (IEHVA) show that FEHVA provides 85.97% superior vibration attenuation, demonstrating its efficiency in broadband vibration mitigation. The hybrid combination of frictional damping and energy harvesting significantly enhances structural resilience while generating electrical power. These results establish FEHVA as a robust, sustainable alternative for vibration control in civil engineering applications. By offering enhanced damping and energy harvesting simultaneously, FEHVA presents a step forward in the development of multifunctional vibration absorbers, advancing future engineering solutions for infrastructure subjected to dynamic loads.

1. Introduction

Structural vibration control is a crucial aspect of modern engineering, playing a fundamental role in ensuring the safety, durability, and optimal performance of various structural and mechanical systems subjected to dynamic loads [1]. From civil infrastructure to aerospace and automotive applications, excessive vibrations can lead to structural fatigue, reduced operational efficiency, and even catastrophic failure [2]. As a result, numerous vibration mitigation strategies have been developed to enhance structural resilience and operational reliability [3]. Among these, passive, active, and hybrid vibration control methods have received significant attention, with Tuned Mass Dampers (TMDs) being one of the most widely adopted passive solutions [4]. TMDs operate by introducing a secondary mass tuned to a specific frequency range, effectively absorbing vibrational energy from the primary structure [5]. While these systems have proven to be effective, their reliance on a substantial auxiliary mass often imposes limitations, particularly in large-scale applications or under high-amplitude vibrations [6]. The need for enhanced vibration attenuation without excessive weight has led researchers to explore innovative solutions that combine vibration control with additional functionalities, such as energy harvesting [7]. The concept of energy harvesting vibration absorbers has emerged as a promising approach to improving system efficiency while also converting mechanical energy into usable electrical power [8].

Recent advancements in hybrid absorbers have focused on integrating energy harvesting mechanisms, such as piezoelectric and electromagnetic transducers, into vibration control systems [9]. These absorbers leverage the principles of electromechanical coupling, wherein vibrational energy is converted into electrical power while simultaneously mitigating structural oscillations [10]. Piezoelectric materials, in particular, have gained prominence due to their high power density, scalability, and direct electromechanical conversion capabilities [11]. Similarly, electromagnetic energy harvesters offer advantages in low-frequency applications, making them suitable for civil and mechanical systems [12]. Recent studies have investigated the integration of frictional damping with energy harvesting. Huang et al. (2023) proposed a hybrid absorber combining friction [13] and electromagnetic damping for improved vibration mitigation [14]. Xiang et al. (2023) explored piezoelectric energy harvesting driven by friction-induced vibrations [15], while Chen et al. (2024) employed a PZT-based absorber to suppress stick–slip behaviour [16]. However, despite these developments, the interaction between frictional damping and energy harvesting remains largely unexplored in existing studies [17]. Frictional damping elements have long been recognised as effective energy

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dissipation mechanisms, capable of significantly enhancing vibration attenuation in passive dynamic systems [18]. The ability of frictional damping to provide robust and tunable energy dissipation makes it an attractive candidate for hybrid vibration absorbers [19]. However, excessive frictional dissipation can counteract the energy harvesting process by reducing the relative motion required for optimal electromechanical conversion [20]. Thus, achieving a balance between friction-induced damping and energy harvesting efficiency presents a significant challenge in the design of advanced vibration absorbers. While previous research has extensively explored energy harvesting through piezoelectric and electromagnetic mechanisms, the integration of frictional damping with energy harvesting remains underexplored. Most existing absorbers either focus on vibration reduction or energy conversion, but not both in a harmonised framework. This limited integration leaves a significant research gap in the design of multifunctional devices capable of robust vibration mitigation while simultaneously harvesting energy from structural motion. In addition, conventional Friction Tuned Mass Dampers (FTMDs) require large auxiliary masses and are often ineffective under broadband and low-frequency excitations. Therefore, an analytical optimisation scheme such as H_2 optimisation scheme needs to be employed to provide specific optimal design parameters. These parameters allow specific mass for the damper. In addition, friction absorbers contain nonlinear damping in the governing equations of motion which needs to be linearised to apply H_2 optimisation. The statistical linearisation method [21] is provided linearised governing equations of motion with zero error. Statistical methods have been widely applied to analyse nonlinear dynamic behaviour in similar systems. Such approaches, as demonstrated by recent studies, provide valuable insights into probabilistic responses and system stability under random excitations [22]. Similarly, Inerter-based Energy Harvesting Vibration Absorbers (IEHVAs) improve performance through added inertance but suffer from mechanical complexity and limited energy conversion efficiency.

In contrast, the proposed Frictional Energy Harvesting Vibration Absorber (FEHVA) offers a compact, efficient solution by introducing a frictional interface that enhances damping without excessive mass and allows meaningful energy harvesting. The primary objective of this paper is to develop a hybrid vibration absorber, FEHVA, that leverages frictional damping and electromechanical energy conversion in a single device. By optimally tuning the frictional characteristics and coupling parameters, the FEHVA aims to maximise vibration attenuation and electrical energy output under both harmonic and stochastic excitations. The theoretical formulation, based on statistical linearisation and H_2 optimisation, enables analytical characterisation of system performance, offering a novel framework for designing multifunctional vibration control devices. In addition, the analytical expressions for the optimal tuning parameters are derived and validated under both harmonic and stochastic excitations, providing a theoretical framework that extends beyond the predominantly empirical or simulation-based approaches in prior studies. To evaluate its effectiveness, the vibration reduction and energy harvesting capacities of FEHVA are compared with those of conventional TMDs and IEHVAs. This comparison framework enables a comprehensive assessment of FEHVA's potential as an effective solution for advanced vibration control and integrated energy harvesting.

2. Structural diagram and equations of motion

Frictional Energy Harvesting Vibration Absorbers (FEHVA) are introduced in this paper to overcome the limitations of conventional vibration absorbers (VA). This novel absorber is installed at the top of a single degree of freedom (SDOF) system to control its vibration subjected to base excitation. The structural diagram of the FEHVA-coupled SDOF system is shown in Fig. 1. m_s, $c_s = 2m_s\zeta_s\omega_s$, and $k_s = m_s\omega_s^2$ define the mass, damping, and stiffness of the SDOF system. The mass and stiffness of FEHVA define as m_d and $k_d = m_d\omega_d^2$. θ defines the coupling between the electrical and mechanical parts of the harvester. C_p and R_v define the electrical capacitance and resistance of the harvester. Newton's second law is employed to derive the governing equations of motion of the FEHVA-coupled SDOF system and expressed as

$$\begin{aligned} &m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s - k_d x_d - f_d = -m_s\ddot{x}_g \\ &m_d\ddot{x}_s + m_d\ddot{x}_d + k_d x_d + f_d - \theta v_d = -m_d\ddot{x}_g \\ &\theta\dot{x}_d + C_p\dot{v}_d + \frac{v_d}{R_v} = 0 \end{aligned} \tag{1}$$

f_d defines the nonlinear frictional force of FEHVA and is expressed as

$$f_d = \mu m_d g \,\mathrm{sgn}\left(\dot{x}_d\right) \tag{2}$$

$x_s = u_s - x_g$ and $x_d = u_d - u_s$ define the relative displacements of SDOF system and FEHVA. $v_d = w_d - u_s$ defines the voltage across the load resistor. x_g defines the base displacement. $(\bullet)$ defines the derivative of the variables with respect to time. The statistical linearisation method is employed to linearise each nonlinear element of Eq. (2) and is expressed as follows:

$$c_{eq} = E\left\{\frac{\partial\left(\mu m_d g\,\mathrm{sgn}\left(\dot{x}_d\right)\right)}{\partial\dot{x}_d}\right\} = \sqrt{\frac{2}{\pi}}\frac{\mu m_d g}{\sigma_{\dot{x}_d}} \tag{3}$$

μ defines the frictional coefficient. g defines the gravitational acceleration. The root mean square (RMS) velocity of the damper, denoted as $\sigma_{\dot{x}_d}$, represents the statistical measure of the damper's velocity fluctuations. When employing the statistical linearisation method to approximate the nonlinear damping term, an error may arise during the transition from the original nonlinear damping factor to its linearised counterpart. This error can be quantified as:

$$\epsilon_d = \mu m_d g\,\mathrm{sgn}\left(\dot{x}_d\right) - \underbrace{\sqrt{\frac{2}{\pi}}\frac{\mu m_d g}{\sigma_{\dot{x}_d}}}_{c_{eq}}\dot{x}_d \tag{4}$$

The equivalent damping of the damper is defined by c_{eq}.

$$\frac{\partial\epsilon_d^2}{\partial c_{eq}} = E\left\{\left(\mu m_d g\,\mathrm{sgn}\left(\dot{x}_d\right) - c_{eq}\dot{x}_d\right)^2\right\} = 0 \tag{5}$$

Therefore, no errors have been identified throughout the statistical linearisation procedure. The equivalent damping of the absorber is further conceptualised as:-

$$c_{eq} = \sqrt{\frac{2}{\pi}}\frac{\mu m_d g}{\sigma_{\dot{x}_d}} \equiv 2m_d\zeta_d\omega_d \tag{6}$$

The value of ζ_d has been derived as:-

$$\zeta_d = \frac{1}{\sqrt{2\pi}}\left(\frac{\mu g}{\omega_d\sigma_{\dot{x}_d}}\right) = \frac{1}{\sqrt{2\pi}}\left(\frac{\mu g}{\eta_d\left(\omega_s\sigma_{\dot{x}_d}\right)}\right) \tag{7}$$

According to the above-mentioned derivations, it has been considered that $f_d = c_{eq}$ and the initial value of is considered zero to derive the value of $\sigma_{\dot{x}_d}$. The closed-form expression for $\sigma_{\dot{x}_d}$ has been derived using H_2 optimisation method. To apply this optimisation scheme, the transfer function of the controlled SDOF needs to be derived. Hence, the transfer matrix of Eq. (1) has been derived using Laplace transformation and expressed as

$$\begin{bmatrix} -\eta^2 + 2\,\mathrm{i}\zeta_s\eta + 1 & -\mu_d\eta_d^2 & 0 \\ -\eta^2 & -\eta^2 + \eta_d^2 & -\frac{\theta\,\eta_d^2}{k_d} \\ 0 & \frac{\mathrm{i}\eta\alpha\theta}{C_p} & \mathrm{i}\eta\alpha + \eta_d \end{bmatrix}\begin{Bmatrix} X_s \\ X_d \\ V_d \end{Bmatrix} = \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}\left(\frac{X_g}{\omega_s^2}\right) \tag{8}$$

The transfer function of the SDOF system has been derived as:-

$$H_s(\eta) = \left(\frac{X_s}{X_g}\right)\omega_s^2 = \frac{-\eta_d^3\mu_d + \eta^2\eta_d - \eta_d^3 + \mathrm{i}\left(-\kappa^2\eta_d^2\eta\alpha - \alpha\eta\,\eta_d^2\mu_d + \alpha\,\eta^3 - \alpha\eta\,\eta_d^2\right)}{\Delta}$$

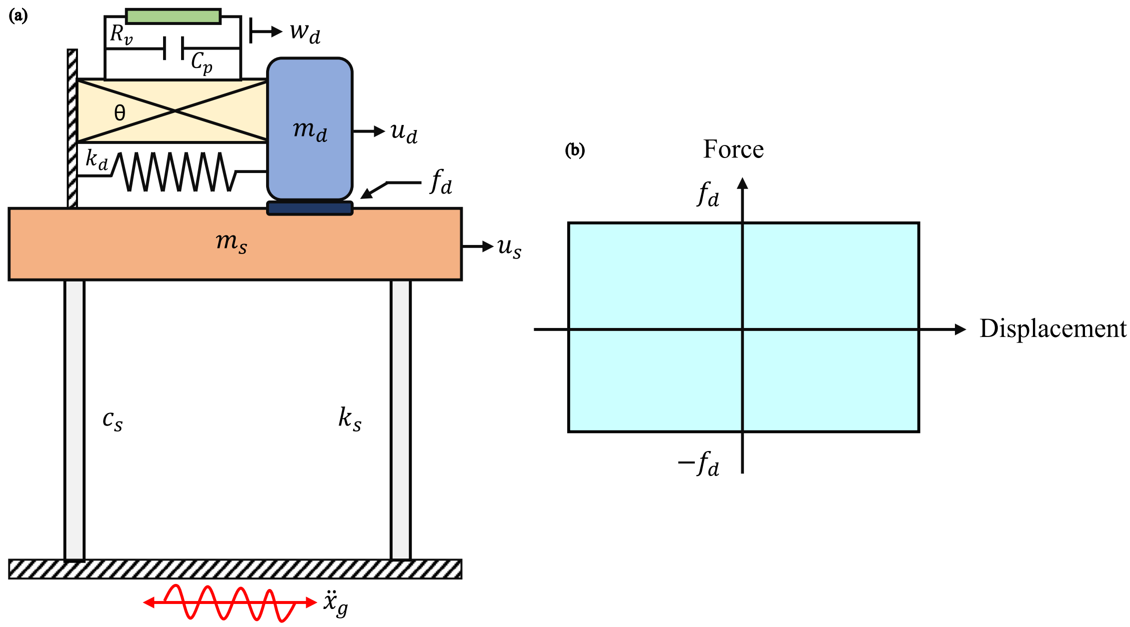

Fig. 1. (a) An SDOF system is controlled by the FEHVA subjected to base excitation. (b) Modelling of frictional force in the damper.

(9)

The transfer function of the FEHVA has been derived as:-

$$H_d(\eta) = \left(\frac{X_d}{X_g}\right)\omega_s^2 = \frac{2\eta^2\alpha\zeta_s - \eta_d + \mathrm{i}\left(-2\eta_d\zeta_s\eta - \alpha\eta\right)}{\Delta} \tag{10}$$

The transfer function of the voltage has been derived as:-

$$P_d(\eta) = \left(\frac{V_d}{X_g}\right)\omega_s^2 = \frac{\theta\eta\left(-2\zeta_s\eta + \mathrm{i}\right)\alpha}{C_p\Delta} \tag{11}$$

The denominator of Eqs. (9)–(11) has been derived as:-

$$\Delta = \begin{array}{c} 2\alpha\,\eta^4\zeta_s - 2\alpha\,\eta^2\eta_d^2\zeta_s - \eta^2\eta_d^3\mu_d - 2\alpha\,\eta^2\eta_d^2\kappa^2\zeta_s \\ +\eta^4\eta_d - \eta^2\eta_d^3 - \eta^2\eta_d + \eta_d^3 \\ +\mathrm{i}\left(\begin{array}{c} -\alpha\,\eta^3\eta_d^2\kappa^2 - \alpha\,\eta^3\eta_d^2\mu_d + \alpha\,\eta^5 - \alpha\,\eta^3\eta_d^2 + \kappa^2\eta_d^2\eta\alpha \\ -2\eta^3\eta_d\zeta_s + 2\eta\,\eta_d^3\zeta_s - \alpha\,\eta^3 + \alpha\eta\,\eta_d^2 \end{array}\right) \end{array} \tag{12}$$

Eqs. (9) and (12) are further utilised to derive optimal design parameters for FEHVA. To achieve the optimal design parameters analytically in terms of closed-form expressions, the H_2 optimisation method is employed. The random white noise excitation is applied at the base of the SDOF system to apply this method. The random excitation is modelled as a zero-mean stationary Gaussian white-noise process with a constant power spectral density S_0. This consideration enables analytical evaluation of the system's stochastic response through the H_2 optimisation framework, following the methodology of Roberts and Spanos [21].

3. Optimal design parameter

The damping ratio of the main structure is considered zero, i.e., $\zeta_s = 0$, in the H_2 optimisation scheme to simplify mathematical derivations and enable closed-form analytical expressions for the damper's optimal tuning frequency and damping ratio. This assumption eliminates interference from structural damping, isolating the damper's contribution to vibration control and ensuring precise and effective optimisation parameters. The standard deviation of the velocity of the FEHVA is derived using the H_2 optimisation framework, which analytically evaluates the system response under a zero-mean stationary Gaussian white-noise excitation with constant spectral density S_0. In this approach, the response variance is expressed in terms of the system's transfer function, enabling the determination of closed-form expressions for the optimal parameters. The standard deviation [23] of the velocity of the FEHVA has been derived as

$$\sigma_{\dot{x}_d}^2 = \frac{S_0\pi\left(\alpha^2 + \eta_d^2\right)}{\alpha\,\eta_d^5\kappa^2\mu_d} \tag{13}$$

The standard deviation of the displacement of the SDOF system has been derived as:-

$$\sigma_{x_s}^2 = \frac{S_0\pi\left(\begin{array}{c}\left(\mu_d+1\right)^3\eta_d^6 + \left(\left(\kappa^2+\mu_d+1\right)^2\alpha^2 - 2\mu_d - 2\right)\eta_d^4 \\ + \left(1 + \left(-2\kappa^2 - \mu_d - 2\right)\alpha^2\right)\eta_d^2 + \alpha^2\end{array}\right)}{\alpha\,\eta_d^5\kappa^2\mu_d} \tag{14}$$

The closed-form expressions for the optimal values of α, κ, and η_d have been derived using the expressions presented below.

$$\frac{\partial\sigma_{x_s}^2}{\partial\eta_d} = 0 \tag{15}$$

Eq. (14) is substituted in Eq. (15) and the closed-form expressions for optimal frequency ratio of the FEHVA are derived as:-

$$\begin{aligned} &A_1\eta_d^6 + A_2\eta_d^4 + A_3\eta_d^2 + A_4 = 0 \\ &\eta_d^2 = \frac{G_1}{6A_1} - \frac{2\left(3A_3A_1 - A_2^2\right)}{3A_1G_1} - \frac{A_2}{3A_1} \end{aligned} \tag{16}$$

$$G_1 = \left(\begin{array}{c} -108A_4A_1^2 + 36A_3A_2A_1 - 8A_2^3 \\ +12\sqrt{3}\sqrt{27A_1^2A_4^2 - 18A_1A_2A_3A_4 + 4A_1A_3^3 + 4A_2^3A_4 - A_2^2A_3^2}\,A_1 \end{array}\right)^{\frac{1}{3}} \tag{17}$$

The closed-form expressions of A_1, A_2, A_3, and A_4 are derived as:-

$$\begin{aligned} A_1 &= -\mu_d^3 - 3\mu_d^2 - 3\mu_d - 1 \\ A_2 &= \alpha^2\kappa^4 + 2\alpha^2\kappa^2\mu_d + 2\alpha^2\kappa^2 + \alpha^2\mu_d^2 + 2\alpha^2\mu_d + \alpha^2 - 2\mu_d - 2 \\ A_3 &= -6\alpha^2\kappa^2 - 3\alpha^2\mu_d - 6\alpha^2 + 3 \\ A_4 &= 5\alpha^2 \end{aligned} \tag{18}$$

Eq. (16) is further substituted in Eqs. (13) and (7) to obtain the optimal damping ratio of FEHVA. Fig. 2(a) illustrates the variations in optimal frequency ratios as a function of the absorber mass ratio. The frequency ratio exhibits a decreasing trend with an increase in the absorber mass ratio, indicating enhanced and robust vibration reduction capabilities

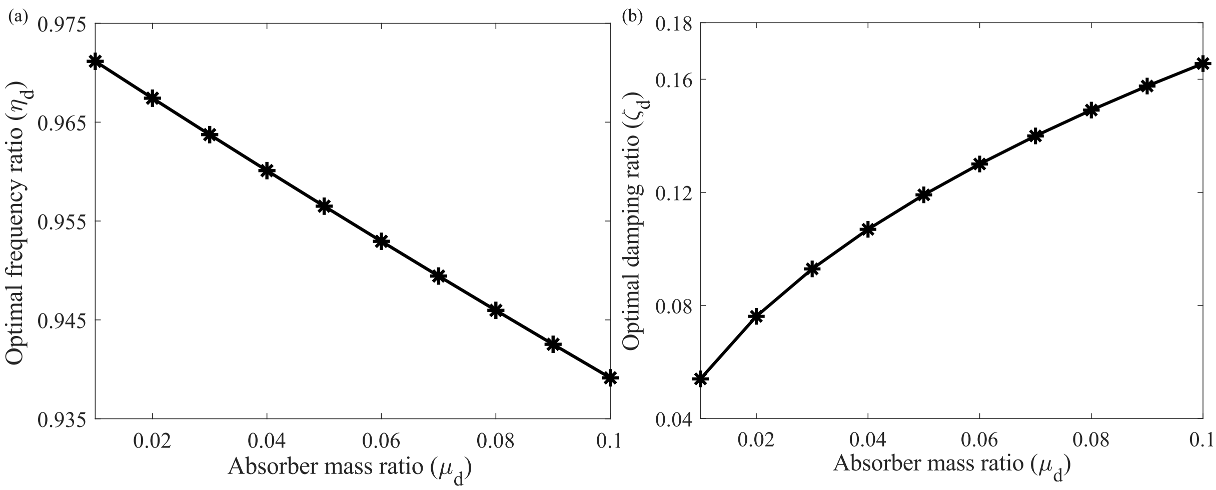

Fig. 2. The variations of optimal (a) frequency and (b) damping ratios of the FEHVA versus absorber mass ratio.

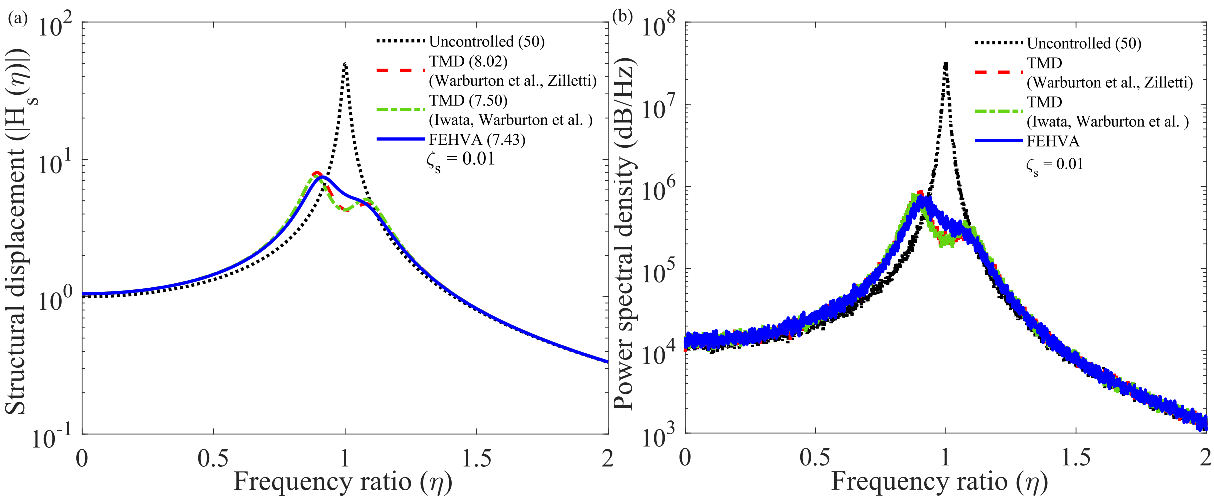

Fig. 3. The variations of optimal structural displacements of structures controlled by TMD and FEHVA subjected to (a) harmonic and (b) random white-noise excitations.

of the absorbers. Fig. 2(b) depicts the variations in optimal damping ratios as a function of the absorber mass ratio. The damping ratio shows an increasing trend with the absorber mass ratio, although the rate of increase is relatively moderate. This characteristic makes the design feasible for practical applications. Incorporating the frictional element enhances energy dissipation, thereby improving the damping characteristics. The coefficient of friction is set to 0.07 (i.e., μ = 0.07), which is relatively low. This ensures that the harvested energy remains unaffected. Furthermore, robust vibration reduction performance is achieved even at this lower friction level.

4. Dynamic response evaluation

The vibration reduction capacity of the FEHVA is derived by conducting frequency domain analysis when the controlled SDOF systems are subjected to harmonic and random excitations. The frequency domain results are further validated through time history analysis. Near-field earthquake records are applied at the base of the controlled SDOF systems. Newmark-beta method is employed to perform this numerical analysis.

4.1. Frequency domain results

This section evaluates the dynamic responses of structures equipped with the proposed FEHVA under various excitation conditions, emphasising its effectiveness in vibration reduction and energy dissipation. Both harmonic and random white noise excitations are considered in the analysis. Table 1 lists the optimal design parameters for the study. The damping ratio of the main structure is set to 0.01 (ζ_s = 0.01). Fig. 3(a) shows the variations in optimal structural displacements of structures controlled by TMD and FEHVA under harmonic excitation. The maximum displacement of the uncontrolled structure is found to be 50, while the maximum displacements of the structures controlled by TMD and FEHVA are 8.02, 7.50, and 7.43, respectively. These results highlight the superior performance of FEHVA, with vibration reduction capacities 7.35% and 0.92% better than TMD. This improvement signifies that FEHVA not only mitigates structural vibrations effectively but also offers a more efficient solution for maintaining structural integrity under harmonic loads. Subsequently, random excitation is applied for further analysis, as shown in Fig. 3(b). The random excitation was modelled as a zero-mean and standard deviation = 2 stationary Gaussian white-noise process with a constant power spectral density, and the output power spectral density (PSD) of the structural response was

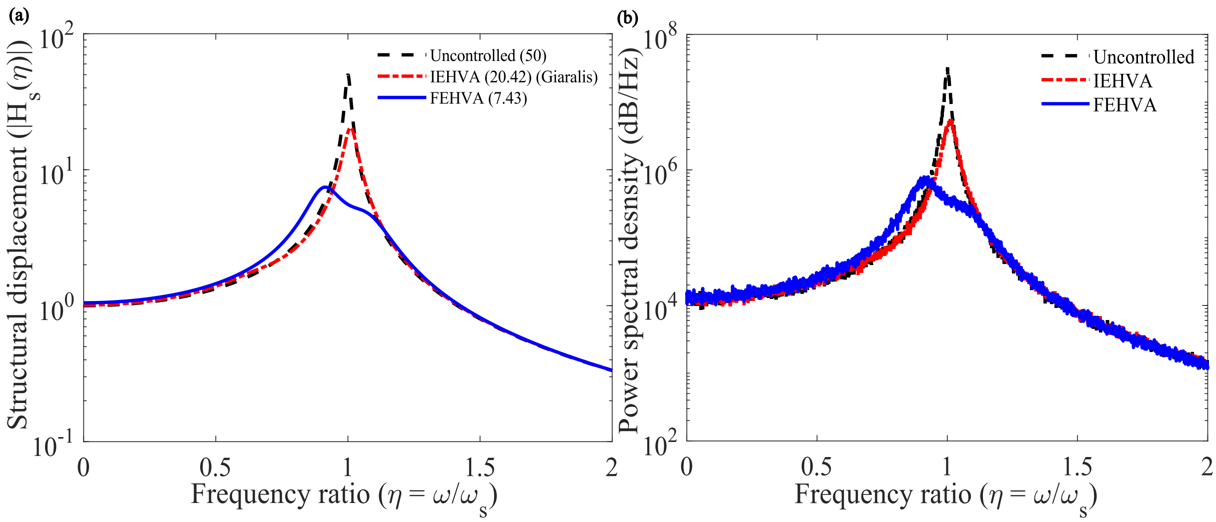

Fig. 4. The variations of optimal structural displacements of structures controlled by IEHVA and FEHVA subjected to (a) harmonic and (b) random white-noise excitations.

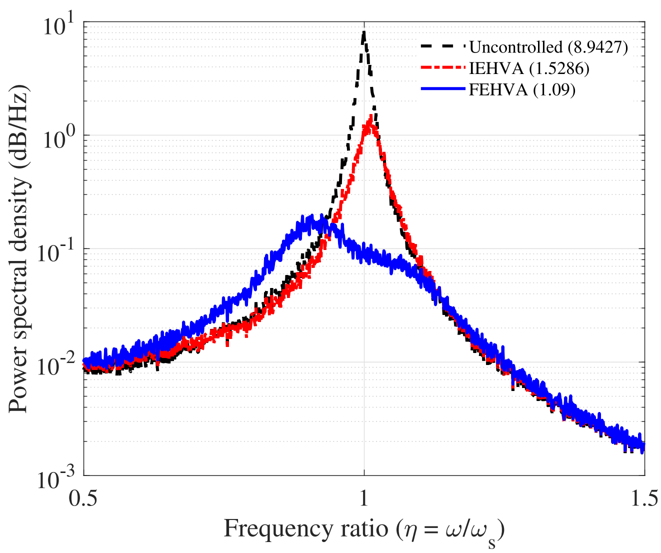

Fig. 5. Power spectral density of the structural response under pink noise excitation for the uncontrolled system, IEHVA, and FEHVA. The FEHVA demonstrates significantly lower spectral amplitude across the frequency range, particularly around resonance ($\eta \approx 1$), highlighting its superior broadband vibration mitigation capability compared to IEHVA.

computed using the system's transfer function obtained through the H_2 optimisation framework. The simulation was carried out in MATLAB and the averaged PSDs were plotted to demonstrate the vibration attenuation characteristics under random excitation. The maximum displacement of the uncontrolled structure is calculated as 3.27×10^7 dB/Hz, while the maximum displacements of the structures controlled by TMD and FEHVA are 8.95×10^5 dB/Hz, 8.27×10^5 dB/Hz, and 7.68×10^5 dB/Hz, respectively. The FEHVA achieves vibration reduction capacities that are 14.20% and 7.16% superior to TMD. These findings underscore the robustness of FEHVA in addressing real-world random excitations, where unpredictable and broadband forces are common. Its ability to dissipate energy efficiently and enhance vibration control can lead to prolonged service life and reduced maintenance costs for structures, making it a practical and economically viable solution for infrastructure subjected to dynamic loads. The hybrid configuration of the piezoelectric and frictional elements embedded within the core material of the FEHVA enables the TMD to harvest energy and improve vibration attenuation without the need for excessive static mass. The inclusion of the piezoelectric element allows for the conversion of mechanical energy into electrical energy, contributing to energy harvesting and enhancing the overall efficiency of the system. Meanwhile, the frictional element aids in dissipating additional energy, thereby improving the damping characteristics. This dual-function approach ensures robust performance under both harmonic and random excitations, providing a sustainable solution for high-resilience structural designs. Fig. 4 illustrates the variations in optimal structural displacements of structures controlled by IEHVA (Inerter Energy Harvesting Vibration Absorber) [24] and FEHVA (Frictional Energy Harvesting Vibration Absorber) under harmonic and random white-noise excitations. In the harmonic excitation case (Fig. 4a), the uncontrolled structure exhibits the highest peak displacement at resonance, indicating significant vibration amplification. The IEHVA reduces the peak displacement but still allows some amplification, while the FEHVA demonstrates superior performance by significantly minimising structural displacement. The numerical values in parentheses indicate the maximum response levels, further highlighting the effectiveness of FEHVA. The maximum displacement of the uncontrolled structure is found to be 50, while the maximum displacements of the structures controlled by IEHVA and FEHVA are 20.42 and 7.43, respectively. These results highlight the superior performance of FEHVA, with vibration reduction capacities 63.58% better than IEHVA. Under random white-noise excitation (Fig. 4b), a similar trend is observed, with the uncontrolled structure showing the highest spectral peak at resonance. The IEHVA provides moderate attenuation, whereas the FEHVA achieves the most substantial reduction in spectral density, demonstrating its superior efficiency in mitigating broadband vibrations. The results emphasise that FEHVA outperforms IEHVA, making it the most effective choice for structural vibration control and energy harvesting applications. The maximum displacement of the uncontrolled structure is calculated as 3.27×10^7 dB/Hz, while the maximum displacements of the structures controlled by TMD and FEHVA are 5.6393×10^6 dB/Hz and 7.9117×10^5 dB/Hz, respectively. The FEHVA achieves vibration reduction capacities that are 85.97% superior to IEHVA. To enhance the realism of the stochastic simulations and better represent broadband environmental excitations, pink noise was employed as the random input for frequency-domain response analysis. Unlike white noise, which assumes a flat power spectral density (PSD) across all frequencies, pink noise exhibits a $1/f$ spectral decay, allocating more energy to lower frequencies. f represents frequency in Hz. This characteristic aligns more closely

Table 1
H_2 optimised design parameters.

System	Introduced by	H_2 optimisation	
		η_d	ζ_d
FEHVA	Eqs. (16) and (7)	0.9565021213	0.11912147
TMD	Warburton et al. [25], Zilletti [26]	$\frac{1}{\sqrt{1+\tilde{\mu}_d}}$	$\frac{\sqrt{\tilde{\mu}_d}}{2}$
TMD	Iwata [27], Warburton et al. [25]	$\frac{1}{1+\tilde{\mu}_d}\sqrt{\frac{2+\tilde{\mu}_d}{2}}$	$\sqrt{\frac{\tilde{\mu}_d(4+3\tilde{\mu}_d)}{8(1+\tilde{\mu}_d)(2+\tilde{\mu}_d)}}$

Conventional tuned mass dampers (TMD): absorber mass ratio ($\tilde{\mu}_d$) = 0.05.

with natural excitation sources which typically dominate at low to mid frequencies in civil structures. Incorporating pink noise thus provides a more physically meaningful evaluation of absorber performance under stochastic conditions. All three systems, uncontrolled SDOF, IEHVA, and FEHVA, were subjected to identical realisations of pink noise, enabling a fair comparison of vibration attenuation and spectral energy reduction across the frequency range. Fig. 5 presents the power spectral density (PSD) of the structural response when subjected to pink noise excitation, comparing the uncontrolled system, IEHVA, and the proposed FEHVA. The uncontrolled system exhibits the highest spectral amplitude near resonance ($\eta \approx 1$), indicating significant energy concentration in that frequency band. Introduction of IEHVA leads to noticeable reduction in the spectral amplitude, yet a considerable peak remains. In contrast, the FEHVA demonstrates superior broadband attenuation, with significantly lower PSD values across the entire frequency range and especially around resonance. The uncontrolled system exhibits a peak spectral amplitude of approximately 8.9427, indicating significant vibration energy accumulation near the resonance frequency. With the introduction of IEHVA, the peak response drops to 1.5286, demonstrating a clear improvement in vibration suppression. The FEHVA further reduces this peak to 1.09, achieving an additional 28.69% reduction compared to the IEHVA case. This substantial reduction in vibration energy highlights the effectiveness of frictional damping combined with energy harvesting in attenuating low-frequency excitations typical of pink noise. The hybrid configuration not only suppresses resonance peaks but also improves overall spectral energy distribution, reinforcing the suitability of FEHVA for real-world applications involving broadband environmental excitations.

The harvested power of the optimal inerter-based energy harvesting dynamic vibration absorber, along with the novel absorbers, is determined using the following equation.

$$P_h = \frac{|P_d|^2}{R_v} \tag{19}$$

Fig. 6 presents a comparative analysis of the harvested power $P_h(\eta)$ as a function of the frequency ratio $\eta = \frac{\omega}{\omega_s}$ for two different energy harvesting dynamic vibration absorbers: the Inerter Energy Harvesting Vibration Absorber (IEHVA) and the Frictional energy harvesting vibration absorbers (FEHVA). The plot uses a logarithmic scale for power to emphasise variations across different frequencies.

- The FEHVA (solid blue line) demonstrates significantly higher harvested power across the entire frequency range compared to the IEHVA.
- At low-frequency ratios ($\eta < 0.5$), the FEHVA rapidly increases in harvested power, whereas the IEHVA remains nearly constant with a relatively low power level.
- Around the resonance region ($\eta \approx 1$), the FEHVA reaches its peak harvested power, indicating superior energy conversion efficiency at this critical frequency.
- The IEHVA (dashed red line) exhibits a relatively lower harvested power throughout the frequency range, with only a minor peak near $\eta \approx 1$. However, its power output remains considerably lower than that of the FEHVA.

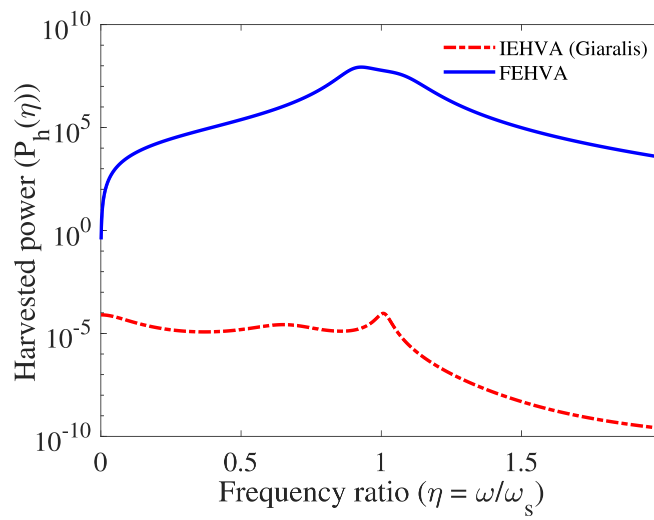

Fig. 6. The harvested power of the optimum IEHVA and FEHVA.

- At higher frequencies ($\eta > 1.5$), the harvested power of both absorbers starts to decline. However, FEHVA still maintains a significantly higher performance than IEHVA.

Overall, this comparison highlights the superior energy harvesting capability of FEHVA over the conventional IEHVA, making it a more effective solution for vibration energy harvesting applications.

5. Conclusion

In this study, we developed the Frictional Energy Harvesting Vibration Absorber (FEHVA), a novel hybrid vibration control system that integrates frictional damping with energy harvesting. Using the statistical linearisation method, we derived the governing equations and optimised the design through H_2 optimisation. Comparative dynamic response evaluations under harmonic and random excitations demonstrated that FEHVA significantly outperforms conventional Tuned Mass Dampers (TMDs) and Inerter Energy Harvesting Vibration Absorbers (IEHVA), achieving superior vibration mitigation and efficient energy conversion. The key novelties are illustrated below.

- FEHVA achieved up to 14.20% greater vibration reduction compared to TMDs and 85.97% superior performance over IEHVA.
- Successfully integrated frictional damping with energy harvesting, enabling simultaneous vibration attenuation and power generation.
- Developed a closed-form optimisation framework for determining optimal tuning parameters.
- Outperformed conventional absorbers under broadband excitations, proving its efficiency for both harmonic and random disturbances.
- Enhanced structural resilience and energy sustainability, making it ideal for civil engineering applications.

FEHVA provides a lightweight, multifunctional alternative to traditional dampers, reducing dependency on heavy auxiliary masses while contributing to energy harvesting efforts. Its superior performance under stochastic excitations suggests a strong potential for sustainable infrastructure, renewable energy, and smart structural applications. A potential future scope of this research is the experimental realisation of the proposed FEHVA system.

CRediT authorship contribution statement

Sudip Chowdhury: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Sondipon Adhikari:** Writing – review & editing, Visualization, Supervision, Software, Resources, Project administration, Methodology, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

All data, models, and code generated or used during the study appear in the submitted article.

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