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Stochastic dynamic stability analysis of composite curved panels subjected to non-uniform partial edge loading

S. Dey [a], T. Mukhopadhyay [b,*], S.K. Sahu [c], S. Adhikari [b]

[a] Mechanical Engineering Department, National Institute of Technology Silchar, India
[b] College of Engineering, Swansea University, United Kingdom
[c] Department of Civil Engineering, National Institute of Technology Rourkela, India

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ABSTRACT

The stochastic dynamic stability analysis of laminated composite curved panels under non-uniform partial edge loading is studied using finite element analysis. The system input parameters are randomized to ascertain the stochastic first buckling load and zone of resonance. Considering the effects of transverse shear deformation and rotary inertia, first order shear deformation theory is used to model the composite doubly curved shells. The stochasticity is introduced in Love's and Donnell's theory considering dynamic and shear deformable theory according to the Sander's first approximation by tracers for doubly curved laminated shells. The moving least square method is employed as a surrogate of the actual finite element model to reduce the computational cost. The results are compared with those available in the literature. Statistical results are presented to show the effects of radius of curvatures, material properties, fibre parameters, and non-uniform load parameters on the stability boundaries.

1. Introduction

The use of composite materials have gained immense popularity over the past few decades for the design of structures in aerospace, automotive, civil and other engineering applications. It has improved the performance and reliability of structural system due to its mechanical advantages of specific modulus and specific strength over monolithic materials, improved fatigue, impact resistance, and design flexibility. Such structures subjected to in-plane periodic forces may lead to parametric resonance because of certain random combinations in the values of uncertain parameters. The instability may occur below the stochastic critical load of the structure under compressive loads over wide ranges of resonance frequencies. Specially the aerospace structures such as skin panels in wings, fuselage, submarine hulls and civil application has practical importance of stability analysis of doubly curved panels/open shells subjected to uncertain non-uniform loading condition. Traditionally, structural analysis is formulated with deterministic behaviour of material properties, loads and other system parameters. However, the real-life structures employed in aerospace, naval, civil, and mechanical applications are always subjected to intrusive uncertainties. The inherent sources of such uncertainties in real structural problems can be due to randomness in material properties, loading conditions, geometric properties and other random input parameters. As an inevitable consequence of the uncertainties in these system parameters, the response of structural system will always exhibit some degree of uncertainty. The traditional deterministic analysis based on an exact reliable model would not help in properly accounting the variation in the response and therefore, the analysis based on deterministic material properties may vary significantly from the real behaviour. The incorporation of randomness of input parameters enables the prediction of the performance variation in the presence of uncertainties and more importantly their sensitivity for targeted testing and quality control. In order to provide useful and accurate information about the safe and reliable design of structures, it is essential to incorporate these uncertainties into account for modelling, design and analysis procedure. The steady development of powerful computational technologies in recent years has led to high-resolution numerical models of real-life engineering structural systems. It is also required to quantify uncertainties and robustness associated with a computational model. Hence, the quantification of uncertainties plays a key role in establishing the credibility of a numerical model. Therefore, the development of an

* Corresponding author.
E-mail address: tanmoy.mukhopadhyay@eng.ox.ac.uk (T. Mukhopadhyay).
URL: http://www.tmukhopadhyay.com

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efficient mathematical model possessing the capability to quantify the uncertainties present in the structures is extremely essential in order to accurately assess the laminated composite structures.

Structural elements under in-plane periodic forces may undergo unstable transverse vibrations, leading to parametric resonance, due to certain combinations of the values of in-plane load parameters and natural frequency of transverse vibration. Several means of combating parametric resonance such as damping and vibration isolation may be inadequate and sometimes dangerous with reverse results (Evan-Iwanowski, 1965). A number of catastrophic incidents can be traced to parametric instability and is often studied in the spectrum of determination of natural frequency and critical load of structures. The stochasticity in the measurement of natural frequencies, critical load and ultimately the excitation frequencies during parametric resonance are of great technical importance in studying the instability behaviour of dynamic systems. Many authors addressed the parametric instability characteristics of laminated composite flat panel subjected to uniform loads (Iwatsubo et al., 1973; Moorthy and Reddy, 1990; Chen and Yang, 1990; Patel et al., 2009; Kochmann and Drugan, 2009; Singha and Daripa, 2009; Kim et al., 2013). In contrast, Bolotin (1964) and Yao (1965) studied the parametric resonance subjected to periodic loads. Stochastic principal parametric resonance of composite laminated beam is numerically investigated by Lan et al. (2014). The influences of transverse shear (Andrzej et al., 2011) and rotary inertia (Ratko et al., 2012) on dynamic instability are studied for cross-ply laminated plates. The parametric dynamic stability analysis is numerically investigated for composite beam (Meng-Kao and Yao, 2004), plates (Dey and Singha, 2006) or shells (Bert and Birman, 1988) and stiffened panel (Sepe et al., 2016). Further studies are also carried out for modelling mesoscopic volume fraction stochastic fluctuations in fiber reinforced composites (Guilleminot et al., 2008) and for parametric instability of graphite-epoxy composite beams under excitation (Yeh and Kuo, 2004). Free vibration and dynamic stability analysis of rotating thin-walled composite beams (Saraviaa et al., 2011) and nonlinear thermal stability of eccentrically stiffened functionally graded truncated conical shells are recently reported (Duc and Cong, 2015). In contrast, many numerical investigations are carried out using response surface methods such as moving least square (MLS) method and other methods for structural analysis (Choi et al., 2004; Wu et al., 2005; Park and Grandhi, 2014; Shu et al., 2007; Kang et al., 2010). Some researchers studied specifically on the moving least squares (MLS) approximation for the regression analysis (Lancaster and Salkauskas, 1981; Breitkpf et al., 2005) instead of the conventional least squares (LS) approximation in conjunction to traditional response surface method (RSM) techniques (Mukhopadhyay et al., 2015; Dey et al., 2015a). Several studies are carried out on uncertainty quantification for dynamic response of structures including different surrogate based analyses of composite beams, plates and shells (Sarrouy et al., 2013; Dey et al., 2015b,c,d, 2016a,b,c,d,e,f, 2018; Mukhopadhyay et al., 2016; Naskar et al., 2017). Few articles have reported the critical comparative assessment of different surrogate models for their performance in dynamic analyses of composite laminates (Dey et al., 2017; Mukhopadhyay et al., 2017).

To the best of authors' knowledge, no literature is reported on uncertainty quantification of parametric instability of doubly curved composite shells. The application of stochastic non-uniform loading on the structural component can significantly alter the global dynamic quantities of interests such as resonance frequency, buckling loads and dynamic stability region (DSR). Thus it is imperative to consider the effect of stochasticity for robust analysis, design and control of the system. The application of moving least square method in this realm as a computationally efficient surrogate of expensive finite element method has not been investigated yet. Even though the perturbation method is an efficient way of stochastic analysis for relatively simpler structures (Kaminski, 2013; Gadade et al., 2016), this intrusive method can be mathematically quite cumbersome for complex problems like stochastic dynamic stability analysis of composite laminates. The main drawback of this method is that it can obtain only the statistical moments (not the entire probability distribution) of the stochastic output quantity of interest. If the nature of the output distribution is known to be Gaussian, the probability distribution can be obtained using the first two moments. However, the nature of distribution of the output parameter may not be known a priori in most engineering problems. Monte Carlo simulation, on the other hand, can obtain the entire probabilistic description of the stochastic output parameter. The main lacuna of traditional Monte Carlo simulation is its computational intensiveness. A surrogate based Monte Carlo simulation approach, as followed in this paper, allows us to quantify the probabilistic descriptions in a computationally efficient manner. In the present study, a moving least square based approach is employed in conjunction with finite element formulation to figure out the random eigenvalue problem and quantify the probabilistic characteristics of the responses related to dynamic stability of composite laminates. The numerical results are shown for first random buckling load and stochastic fundamental resonance frequencies with individual and combined variation of the stochastic input parameters.

2. Importance of stochastic dynamic stability analysis in composite laminates

Engineering structures are often subjected to periodic loads. For examples, aerospace structures are subjected to wind load, rotating machine systems are usually exerted a periodic unbalanced inertia force, bridges are frequently subjected to the cyclic loads from the running vehicles, marine structures are always suffered the periodic wave forces etc. Structural components subjected to in-plane periodic forces undergo an unstable dynamic response known as dynamic instability or parametric instability or parametric resonance. Parametric resonance, may occur for certain combinations of natural frequency of transverse vibration, the frequency of the in-plane forcing functions and the magnitude of the in-plane load. A number of flight accidents can be traced due to parametric instability of structures. In comparison to the principal resonance, the parametric instability can take place not only at a single excitation frequency but even for small excitation amplitudes and combination of frequencies. The difference between good and bad vibration regimes of a structure under in-plane periodic loads can be found from dynamic instability region (DIR) spectra. The computation of these spectra is usually studied in term of natural frequencies and static buckling loads. The parametric instability has a catastrophic effect on structures near critical regions of parametric instability. Hence, the parametric resonance characteristics of structures are of great technical importance for understanding the dynamic characteristics under periodic loads.

As discussed in the preceding paragraph, structures are subjected to dynamic loads more often than static loads. Dynamic load means the load varies with time. Periodic loading is one type of dynamic loading. This type of load occurs in repeated periods or cycles like sine and cosine functions. Structures subjected to in-plane periodic loads can be expressed in the form as suggested by Bolotin (1964): $P(t) = P_s + P_t \cos\Omega t$, where P_s is the static portion of $P(t)$, P_t is the amplitude of the dynamic portion of $P(t)$ and Ω is the frequency of excitation. It can be noted here that the quantities P_s, P_t, Ω possess random values in practical systems. This, in turn, makes the time varying periodic load $P(t)$ random in nature. The present paper aims to account such stochastic character of the time varying load along with other sources of stochasticity for a comprehensive probabilistic analysis of the system. Laminated composites being a complex structural form and susceptible to different forms of uncertainty, the compound effects of stochastic time varying loading and structural and material uncertainties associated with composites can be crucial in the intended performance for various engineering applications.

3. Governing equations

In the present study, a layered graphite-epoxy composite laminated simply supported shallow doubly curved shell is considered with thickness *t*, intensity of loading *C*, principal radii of curvature R_x, R_y along x- and y-direction, respectively and the radius of curvature R_{xy} in x-y plane, as furnished in Fig. 1. Using Hamilton's principle (Meirovitch, 1992) for free vibration of composite shell structure subjected to in-plane loads, the equation of equilibrium can be expressed as

$$[M(\tilde{\omega})]\,[\ddot{q}] + \left([K_e(\tilde{\omega})] - F(\tilde{\omega})\left[K_g(\tilde{\omega})\right]\right)\{q\} = 0 \tag{1}$$

where $M(\tilde{\omega})$, $K_e(\tilde{\omega})$ and $K_g(\tilde{\omega})$ are mass, elastic stiffness and geometric stiffness matrices, respectively. Here $\tilde{\omega}$ is used to denote the element of probability space. Therefore, any quantity expressed as a function of $\tilde{\omega}$ is a random quantity (can be a scalar, vector or a matrix). The in-plane load $[F(\tilde{\omega})(t)]$ is periodic and can be expressed in the stochastic form (Patel et al., 2009)

$$F(\tilde{\omega})(t) = F_s(\tilde{\omega}) + F_t(\tilde{\omega})\; Cos\Omega t \tag{2}$$

where $F_s(\tilde{\omega})$ and $F_t(\tilde{\omega})$ are the random static portion and the amplitude of the dynamic portion of stochastic in-plane load, respectively. The static buckling load of elastic shell $F_{cr}(\tilde{\omega})$ is the measure of the magnitude of $F_s(\tilde{\omega})$ and $F_t(\tilde{\omega})$

$$F_s(\tilde{\omega}) = \alpha(\tilde{\omega})\; F_{cr}(\tilde{\omega}) \quad F_t(\tilde{\omega}) = \beta(\tilde{\omega})F_{cr}(\tilde{\omega}) \tag{3}$$

where $\alpha(\tilde{\omega})$ and $\beta(\tilde{\omega})$ are known as static and dynamic load factors, respectively. The equation of motion can be expressed by employing equation (2) as

$$\begin{aligned}&[M(\tilde{\omega})][\ddot{q}] + \left([K_e(\tilde{\omega})] - \alpha(\tilde{\omega})F_{cr}(\tilde{\omega})\left[K_g(\tilde{\omega})\right]\right.\\ &\quad \left. - \beta(\tilde{\omega})F_{cr}(\tilde{\omega})\left[K_g(\tilde{\omega})\right]\cos\Omega t\right)\{q\}\\ &\quad = 0\end{aligned} \tag{4}$$

It can be noted that the matrices involved in equation (4) are stochastic in nature. Depending on the degree of stochasticity, each element of the matrices is random in nature. The solution of equation (4) would obtain different results for each of the realizations of a Monte Carlo simulation depending on the respective set of input parameters. Thus probabilistic distributions can be obtained based on the results of different realizations following a non-intrusive method. This stochastic equation (4) indicates second order differential equations with periodic Mathieu-Hill type coefficients. The formation of zone of instability arises from Floquet's theory which establishes the existence of periodic solutions. The periodic solutions of period *T* and 2*T* derive the limiting bounds of the dynamic instability regions (where $T = 2\pi/\Omega$). The significant stochastic importance lies in the limiting bounds of primary instability regions with period 2*T* (Chen and Yang, 1990) wherein the solution can be represented as the trigonometric series form

$$q(t) = \sum_{j=1,3,5}^{\infty}\left[\{a_j\}\sin\left(\frac{j\Omega t}{2}\right) + \{b_j\}\sin\left(\frac{j\Omega t}{2}\right)\right] \tag{5}$$

Considering this in equation (4) and first term of the above series, equation (4) can be expressed by equating the coefficients of *Sin* $(\Omega\, t/2)$ and *Cos* $(\Omega\, t/2)$ as

$$\left[[K_e(\tilde{\omega})] - \alpha(\tilde{\omega})\; F_{cr}(\tilde{\omega})\left[K_g(\tilde{\omega})\right]\pm\beta(\tilde{\omega})F_{cr}(\tilde{\omega})\left[K_g(\tilde{\omega})\right] - \frac{\Omega^2}{4}[M]\right]\{q\} = 0 \tag{6}$$

The above equation (6) represents an eigenvalue problem for known values of $\alpha(\tilde{\omega})$, $\beta(\tilde{\omega})$ and $F_{cr}(\tilde{\omega})$ as for $\Omega_j q_j = 0$ for j = 1,2,3 …. Here the two conditions under a plus and minus sign represents the two limiting bounds of the dynamic instability region. The eigenvalues (Ω_j) provide the boundary frequencies of the instability regions for specific values of α and β and the reference stochastic static buckling load is computed accordingly (Ganapathi et al., 1999) and in contrast, exact solution for doubly curved shells can also be carried out (Chaudhuri and Abuarja, 1988). An eight-noded curved isoparametric element is employed with five degrees of freedom u,v,w,θ_x and θ_y per node. The present study employs the first order shear deformation theory and the shear correction coefficient for the nonlinear distribution of the thickness shear strains through the total thickness. The displacement field along mid-plane is assumed to be straight before and after deformation, but it is not necessary to remain normal after deformation. The displacement components can be expressed as

$$\begin{aligned}\overline{u}(x,y,z) &= u(x,y) + z\theta_x(x,y)\\ \overline{v}(x,y,z) &= v(x,y) + z\theta_y(x,y)\\ \overline{w}(x,y,z) &= w(x,y)\end{aligned} \tag{7}$$

where the rotations of the mid-plane surface are represented by θ_x and θ_y. Here the displacement components in the *x*, *y*, *z* directions at any point and at the mid-plane surface are denoted as $\overline{u}$, $\overline{v}$, $\overline{w}$, and u, v and w, respectively. Thus the integrated relationship for the composite curved shell can be represented as

$$\begin{Bmatrix} N_i(\tilde{\omega})\\ M_i(\tilde{\omega})\\ Q_i(\tilde{\omega})\end{Bmatrix} = \begin{bmatrix} A_{ij}(\tilde{\omega}) & B_{ij}(\tilde{\omega}) & 0\\ B_{ij}(\tilde{\omega}) & D_{ij}(\tilde{\omega}) & 0\\ 0 & 0 & S_{ij}(\tilde{\omega})\end{bmatrix}\begin{Bmatrix}\varepsilon_j\\ k_j\\ \gamma_j\end{Bmatrix} \tag{8}$$

where A_{ij}, B_{ij}, D_{ij} (where $i,j = 1,2,6$) and S_{ij}(where $i,j = 4,5$) are the extension-bending coupling, bending and transverse shear stiffness, respectively. The shear correction factor (=5/6) is incorporated in S_{ij} in the numerical calculation. In the present analysis, shear deformable Sander's kinematic relation (Bathe, 1990) is extended for doubly curved shells. The strain displacement

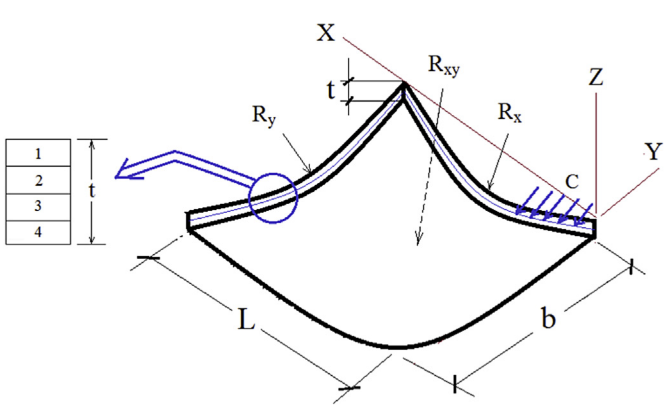

Fig. 1. Laminated composite curved panel.

equations of linear nature can be obtained as

$$
\begin{aligned}
\varepsilon_{xl}(\tilde{\omega}) &= \frac{\partial u}{\partial x} + \frac{w}{R_x(\tilde{\omega})} + z\,\kappa_x \\
\varepsilon_{yl}(\tilde{\omega}) &= \frac{\partial v}{\partial y} + \frac{w}{R_y(\tilde{\omega})} + z\,\kappa_y \\
\gamma_{xyl}(\tilde{\omega}) &= \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + z\,\kappa_{xy} \\
\gamma_{yz}(\tilde{\omega}) &= \frac{\partial w}{\partial y} + \theta_y - C_1 \frac{v}{R_y(\tilde{\omega})} \\
\gamma_{xz}(\tilde{\omega}) &= \frac{\partial w}{\partial x} + \theta_x - C_1 \frac{u}{R_x(\tilde{\omega})}
\end{aligned}
\tag{9}
$$

where

$$
\begin{aligned}
&\kappa_x = \frac{\partial \theta_x}{\partial x} \text{ and } \kappa_y = \frac{\partial \theta_y}{\partial y} \\
&\kappa_{xy}(\tilde{\omega}) = \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} + \frac{1}{2} C_2 \left(\frac{1}{R_x(\tilde{\omega})} - \frac{1}{R_y(\tilde{\omega})} \right) \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right)
\end{aligned}
\tag{10}
$$

Here the formulation can be derived to shear deformable Love's first approximation and Donnell's theories from tracers (C_1 and C_2). Considering nonlinearity in strain, the element geometric stiffness matrix for doubly curved shells can be expressed as

$$
\begin{aligned}
\varepsilon_{xnl}(\tilde{\omega}) &= \frac{1}{2}\left(\frac{\partial u}{\partial x} + \frac{w}{R_x(\tilde{\omega})}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{u}{R_x(\tilde{\omega})}\right)^2 + \frac{1}{2} z^2 \left[\left(\frac{\partial \theta_x}{\partial x}\right)^2 + \left(\frac{\partial \theta_y}{\partial x}\right)^2\right] \\
\varepsilon_{ynl}(\tilde{\omega}) &= \frac{1}{2}\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{2}\left(\frac{\partial v}{\partial y} + \frac{w}{R_y(\tilde{\omega})}\right)^2 + \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{v}{R_y(\tilde{\omega})}\right)^2 + \frac{1}{2} z^2 \left[\left(\frac{\partial \theta_x}{\partial y}\right)^2 + \left(\frac{\partial \theta_y}{\partial y}\right)^2\right] \\
\gamma_{xynl}(\tilde{\omega}) &= \left[\left(\frac{\partial u}{\partial x} + \frac{w}{R_x(\tilde{\omega})}\right)\frac{\partial u}{\partial y} + \left(\frac{\partial v}{\partial y} + \frac{w}{R_y(\tilde{\omega})}\right)\frac{\partial v}{\partial x} + \left(\frac{\partial w}{\partial x} - \frac{u}{R_x(\tilde{\omega})}\right)\left(\frac{\partial w}{\partial y} - \frac{v}{R_y(\tilde{\omega})}\right)\right] + z^2\left[\left(\frac{\partial \theta_x}{\partial x}\right)\left(\frac{\partial \theta_x}{\partial y}\right) + \left(\frac{\partial \theta_y}{\partial x}\right)\left(\frac{\partial \theta_y}{\partial y}\right)\right]
\end{aligned}
\tag{11}
$$

The overall stochastic stiffness and mass matrices i.e., $[K_e(\tilde{\omega})]$, $[K_g(\tilde{\omega})]$ and $[M(\tilde{\omega})]$ are obtained by assembling the corresponding element matrices by using skyline technique. The element mass and stiffness matrices of composite shells are computed wherein the geometric stiffness matrix is obtained as the function of in-plane stress distribution in the element due to applied edge loading. Due to non-uniformity in the stress field, plane stress analysis is carried out by using the finite element formulation. The possible shear locking is avoided by employing the reduced integration technique for the element matrices. The subspace iteration method (Bathe, 1990) is utilized to solve the stochastic eigenvalue problems.

4. Moving least square method

In general, the polynomial regression models give the large errors in conjunction to non-linear responses while give good approximations in small regions wherein the responses are less complex. Such features are found advantageous while implementing the method of moving least squares (MLS). Moreover, the least square method gives a good result to represent the original limit state but it creates a problem if anyone like to fit a highly nonlinear limit function with this technique because this technique uses same factor for approximation throughout the space of interest. To overcome this problem, the moving least square method is introduced. In this method, a weighted interpolation function or limit state function is employed to the response surface and some extra support points are also generated over least square method to represent perfectly the nonlinear limit surface. In stochastic analysis, uncertainties can be expressed as a vector of random variables, $x = [x_1, x_2, x_3, \ldots\ldots x_n]^T$, characterized by a probability density function (PDF) with a particular distribution such as normal or lognormal with limit state function of these random variables. To avoid the curse of dimensionality in dealing with random input variables, response surface methods (RSM) can be utilized to increase the computational efficiency. These methods approximate an implicit limit state function as a response surface function (RSF) in an explicit form, which is evaluated for a set of selected design points throughout a number of deterministic structural analyses. RSM approximates an implicit limit state function as a RSF in explicit form. It selects experimental points by an axial sampling scheme and fits these experimental points using a second order polynomial without cross terms expressed as

$$
y(x) = \beta_o + \sum_{i=1}^{k} \beta_i\, x_i + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij}\, x_i\, x_j + \sum_{i=1}^{k} \beta_{ii}\, x_i^2 \tag{12}
$$

where β_o, β_i, β_{ij} and β_{ii} are the unknown coefficients of the polynomial equation. The least squares approximation commonly used in the conventional RSM allots equal weight to the experimental points in evaluating the unknown coefficients of the RSF. The weights of these experimental points should consider the proximity to the actual limit state function so that MLS enables a higher weight to yield a more accurate output. The approximated RSF can be defined in terms of basis functions $b(x)$ and the coefficient vector $a(x)$ as

$$
\tilde{L}(x) = b(x)^T\, a(x) \tag{13}
$$

The coefficient vector $a(x)$ is expressed as a function of the random variables x to consider the variation of the coefficient vector according to the change of the random variable at each iteration. The local MLS approximation at x is formulated as (Kang et al., 2010)

$$
\tilde{L}(x, x_i) = b(x_i)^T\, a(x) \tag{14}
$$

where x_i denotes experimental points and the basis functions $B(x)$ are commonly chosen as

$$b(x) = \left[1\ x_1 \dots x_n\ x^2 \dots x_n^2\right]^T \tag{15}$$

The vector of unknown coefficients $a(x)$ is determined by minimizing the error between the experimental and approximated values of the limit state function. This error is defined as

$$Err(x) = \sum_{i=1}^{n} w(x - x_i)\left[\tilde{L}(x, x_i) - L(x_i)\right]^2 = (Ba - L)^T W(x)(Ba - L) \tag{16}$$

where $L = [L(x_1), L(x_2), \dots\dots L(x_n)]^T$, $B = [b(x_1), b(x_2), \dots\dots b(x_n)]^T$ and $W(x) = diag.[w_1(x_1 - x), w_2(x_2 - x), \dots\dots w_m(x_n - x)]$. Here $(n+1)$ is the number of sampling points and $(m+1)$ is the number of basis functions. Now for minimization of error with respect to $a(x)$, $\partial(Err)/\partial a = 0$ transforming the coefficient of vector $a(x)$ as

$$a(x) = \left(B^T W(x) B\right)^{-1} B^T W(x) L \tag{17}$$

The approximated response surface function is obtained from equation (14) as

$$\tilde{L}(x) = b(x)^T \left(B^T W(x) B\right)^{-1} B^T W(x) L \tag{18}$$

5. Random input representation

The random input parameters such as ply-orientation angle, radius of curvatures, material properties (both longitudinal and transverse elastic modulus, shear modulus, Poisson ratio, mass density), load, load factors (both static and dynamic) and combined variation of all these parameters are considered for composite doubly curved shells considering layer-wise stochasticity. It is assumed that the uniform random distribution of input parameters exists within a certain band of tolerance with their mean values. The following cases are considered in the present study:

(a) Variation of ply-orientation angle only: $\theta(\tilde{\omega}) = \{\theta_1\ \theta_2\ \theta_3 \dots\dots \theta_i \dots\dots \theta_l\}$
(b) Variation of radius of curvatures only: $R(\tilde{\omega}) = \{R_x(\tilde{\omega}), R_y(\tilde{\omega})\}$
(c) Variation of material properties only: $P(\tilde{\omega}) = \{E_1(\tilde{\omega}), E_2(\tilde{\omega}), G_{12}(\tilde{\omega}), G_{23}(\tilde{\omega}), G_{13}(\tilde{\omega}), \mu(\tilde{\omega}), \rho(\tilde{\omega})\}$
(d) Variation of intensity of load only: $\{F(\tilde{\omega})\}$
(e) Variation of static load factor $\{\alpha(\tilde{\omega})\}$ and dynamic load factor: $\{\beta(\tilde{\omega})\}$
(f) Combined variation of ply orientation angle, radius of curvatures, material properties (namely, elastic moduli, shear moduli, Poisson's ratio and density), applied load and load factors (static and dynamic): $\{\theta, R, P, F, \alpha, \beta\}(\tilde{\omega})$

In the present study, $\pm 5^\circ$ variation for ply orientation angle, $\pm 10\%$ volatility in material properties (as per industry standard), applied load and load factors, respectively are considered from their respective deterministic values unless otherwise specified. Fig. 2 presents a flowchart of the stochastic dynamic stability analysis using MLS method (surrogate based Monte Carlo simulation) as followed in the present study. A brief description of the Monte Carlo simulation method is provided in the following paragraphs.

Uncertainty quantification is part of modern structural analysis problems. Practical structural systems are faced with uncertainty, ambiguity, and variability constantly. Even though one might have unprecedented access to information due to the recent improvement in various technologies, it is impossible to accurately predict future structural behaviour during its service life. Monte Carlo simulation, a computerized mathematical technique, lets us realize all the possible outcomes of a structural system leading to better and robust designs for the intended performances. The technique was first used by scientists working on the atom bomb; it was named after Monte Carlo, the Monaco resort town renowned for its casinos. Since it's introduction in World War II, this technique has been used to model a variety of physical and conceptual systems across different fields such as engineering, finance, project management, energy, manufacturing, research and development, insurance, oil and gas, transportation and environment.

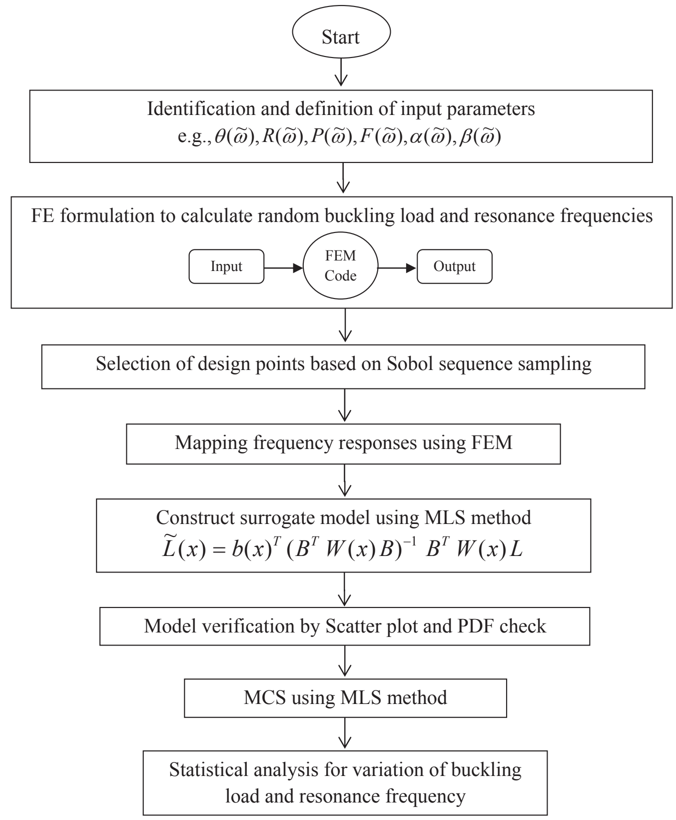

Fig. 2. Flowchart of stochastic dynamic stability analysis using MLS method.

Monte Carlo simulation furnishes a range of prospective outcomes along with their respective probability of occurrence. This technique performs uncertainty quantification by forming probabilistic models of all possible results accounting a range of values from the probability distributions of any factor that has inherent uncertainty. It simulates the outputs over and over, each time using a different set of random values from the probability distribution of stochastic input parameters. Depending upon the nature of stochasticity, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it can provide a converged result depicting the distributions of possible outcome values of the response quantities of interest. Each set of samples is called an iteration or realization, and the resulting outcome from that sample is recorded. In this way, Monte Carlo simulation provides not only a comprehensive view of what could happen, but how likely it is to happen i.e. the probability of occurrence.

The mean or expected value of a function $f(x)$ of a n dimensional random variable vector, whose joint probability density function is given by (x), can be expressed as

$$\mu_f = E[f(x)] = \int_{\Omega} f(x)\phi(x)dx \tag{19}$$

Similarly the variance of the random function $f(x)$ is given by the integral below,

$$\sigma_f^2 = Var[f(x)] = \frac{1}{N-1}\sum_{i-1}^{N}\left(f(x_i) - \mu_f\right)^2 \tag{20}$$

The above multidimensional integrals, as shown in equations (19) and (20) are difficult to evaluate analytically for many types of joint density functions and the integrand function $f(x)$ may not be available in analytical form for the problem under consideration. Thus the only alternative way is to calculate it numerically. The above integral can be evaluated using MCS approach, wherein N sample points are generated using a suitable sampling scheme in the n-dimensional random variable space. The N samples drawn from a dataset must follow the distribution specified by $\varphi(x)$. Having the N samples for x, the function in the integrand $f(x)$ is evaluated at each of the N-sampling points x_i of the sample set $\chi = \{x_1, \ldots\ldots, x_N\}$. Thus, the integral for the expected value takes the form of averaging operator as shown below

$$\mu_f = E[f(x)] = \frac{1}{N}\sum_{i=1}^{N} f(x_i) \tag{21}$$

Similarly, using sampled values of MCS, equation (20) leads to

$$\sigma_f^2 = Var[f(x)] = \frac{1}{N-1}\sum_{i-1}^{N}\left(f(x_i) - \mu_f\right)^2 \tag{22}$$

Thus the statistical moments can be obtained using a brute force Monte Carlo simulation based approach, which is often computationally very intensive due the evaluation of function $f(x_i)$ corresponding to the N-sampling points x_i, where $N \sim 10^3$. The noteworthy fact in this context is the adoption of surrogate based Monte Carlo simulation approach in the present study that reduces the computational burden of traditional (i.e. brute force) Monte Carlo simulation to a significant extent.

6. Results and discussion

The present study considers a simply supported four layered graphite-epoxy angle-ply (45°/-45°/45°/-45°) and cross-ply (0°/90°/0°/90°) composite doubly curved shallow shells. In finite element formulation, an eight noded isoparametric quadratic element is considered. For graphite-epoxy composite shells, the deterministic values of geometric properties are considered as $L = 1$ m, $b = 0.5$ m, $t = 0.005$ m, $C = 0.5$, $R_x = R_y = 10$, (for spherical shell), $\alpha = 0.5$, $\beta = 0.5$ and the material properties are assumed as $E_1 = 141$ GPa, $E_2 = 9.23$ GPa, $G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa, $\rho = 1580$ kg/m^3, $\nu = 0.3$. Table 1 presents the non-dimensional buckling loads for the simply supported singly-curved cylindrical composite (0°/90°) panel for different b/R_y ratios (Baharlou and Leissa, 1987). Table 2 presents the convergence study of non-dimensional fundamental natural frequencies of three layered graphite-epoxy untwisted composite plates (Qatu and Leissa, 1991). A close agreement with benchmarking results are obtained in conjunction to (4 × 4), (8 × 8) and (10 × 10) mesh size. Table 3 presents the non-dimensional natural frequencies for simply-supported symmetric cross-ply composite plates and spherical shells (Reddy, 1984; Chandrashekhara, 1989). It can be noted here that, analysis of small constituent components is worthwhile and insightful to understand the structural behaviour of larger structures. For example, fuselage of aircraft consists of a cylindrical shell stiffened by circumferential frames and longitudinal stringers. Tests on full scale structure showed that adjacent panels across a frame vibrate independently of one another, with the frames acting as rigid boundaries (Clarson and Ford, 1962). Hence, in compliance of the same, the present study considers a simple example problem of a small component of laminated composite curved shells as a representative case to map the zone of dynamic instability due to stochastic variations on input parameters wherein the moving least square (MLS) model is employed to reduce the computational time and cost compared to Monte Carlo Simulation (MCS). However, in future, an extended work of the present study can be carried out to deal with the role of components in the overall stability of the whole large complex structural system.

The moving lease square based approach is validated with original Monte Carlo simulation considering random variations of input parameters within upper and lower bounds (tolerance zone). Fig. 3 presents the scatter plot which establishes the accuracy of

Table 1
Non-dimensional buckling loads for the simply supported singly-curved cylindrical composite (0°/90°) panel with $a = 0.25$ m, $b = 0.25$ m, $t = 0.0025$ m, $a/R_x = 0$, $E_1 = 2.07 \times 10^{11}$ N/m^2, $E_2 = 5.2 \times 10^9$ N/m^2, $G_{12} = 2.7 \times 10^9$ N/m^2, $\nu_{12} = 0.25$.

Structure	$b/R_y = 0.1$	$b/R_y = 0.2$	$b/R_y = 0.3$
Present method	17.612	32.5027	57.117
Baharlou and Leissa (1987)	17.49	32.17	56.62

Table 2
Convergence study for non-dimensional frequencies [$\omega = \omega_n L^2 \sqrt{(\rho/E_1 t^2)}$] without in-plane load of doubly curved (45°/−45°/45°) angle ply composite with $a/b = 1$, $b/t = 100$, $b/R_y = 0.5$, $E_1 = 138$ GPa, $E_2 = 8.96$ GPa, $G_{12} = 7.1$ GPa, $\nu_{12} = 0.3$.

Structure	Present FEM (4 × 4)	Present FEM (8 × 8)	Present FEM (10 × 10)	Qatu and Leissa (1991)
Plate	0.4600	0.4581	0.4577	0.4607
Spherical Shell	1.3507	1.2977	1.2941	1.3063

Table 3
Non-dimensional fundamental frequencies [$\omega = \omega_n a^2 \sqrt{(\rho/E_2 t^2)}$] for the simply supported four layered cross-ply (0°/90°/90°/0°) composite with $E_{11}/E_{22} = 25$, $G_{23} = 0.2E_{22}$, $G_{12} = G_{13} = 0.5E_{22}$, $\nu_{12} = 0.25$.

Analysis	$a/t = 100$		$a/t = 10$	
	Plate	Spherical ($R/b = 1$)	Plate	Spherical ($R/b = 1$)
Present FEM	15.187	126.320	12.228	16.146
Reddy (1984)	15.184	126.330	12.226	16.172
Chandrashekhara (1989)	15.195	126.700	12.233	16.195

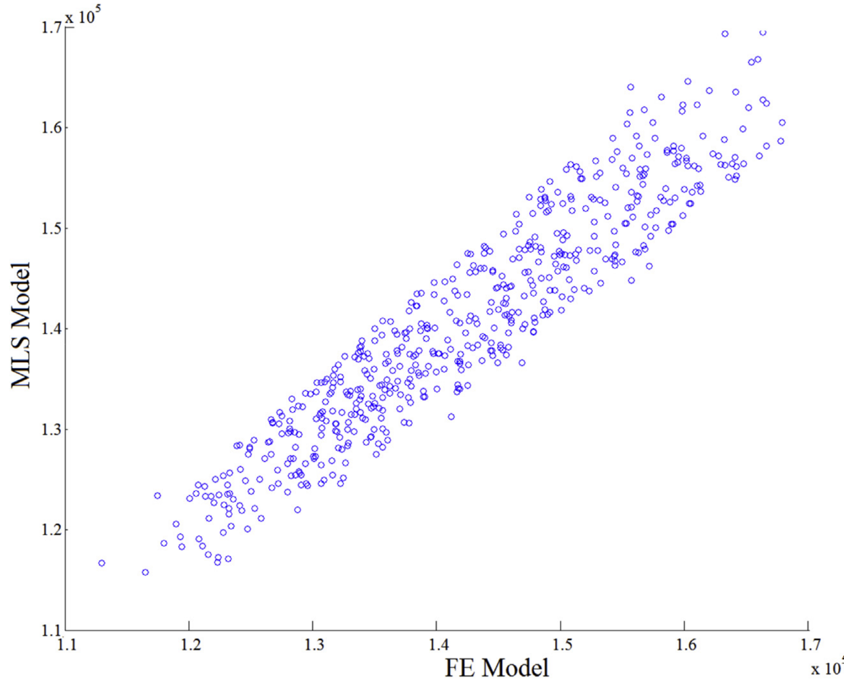

Fig. 3. Scatter plot for stochstic buckling loads corresponding to FE model and MLS model.

present MLS model with respect to original finite element model corresponding to stochastic first buckling load for combined variation of ply-orientation angle, radius of curvatures, material properties (both longitudinal and transverse elstic modulus, shear modulus, poisson ratio, mass density), load, load factors (both static and dynamic). The present MLS surrogate model is used to determine the first stochastic buckling load and resonance frequencies corresponding to given values of input variables, instead of time-consuming deterministic finite element analysis. The probability density function is plotted as the benchmark of bottom line results. The variations of material properties, load intensity and factors are scaled in the range between the lower and the upper limit (tolerance limit) as $\pm 10\%$ with respective mean values while for ply orientation angle as within $\pm 5^{\circ}$ fluctuation (as per standard of composite manufacturing industry) with respective deterministic values. Due to paucity of space, only a few important representative results are furnished.

A sample size of 64 is considered in case of individual variation of stochastic input parameters while due to higher number of input variables for combined random variation, the subsequent sample size of 512 is found to meet the convergence criteria in the present MLS method. The sampling size of 10,000 is considered for direct MCS with 10,000 finite element (FE) iteration. In contrast, comparatively much lesser number of actual FE iteration (equal to

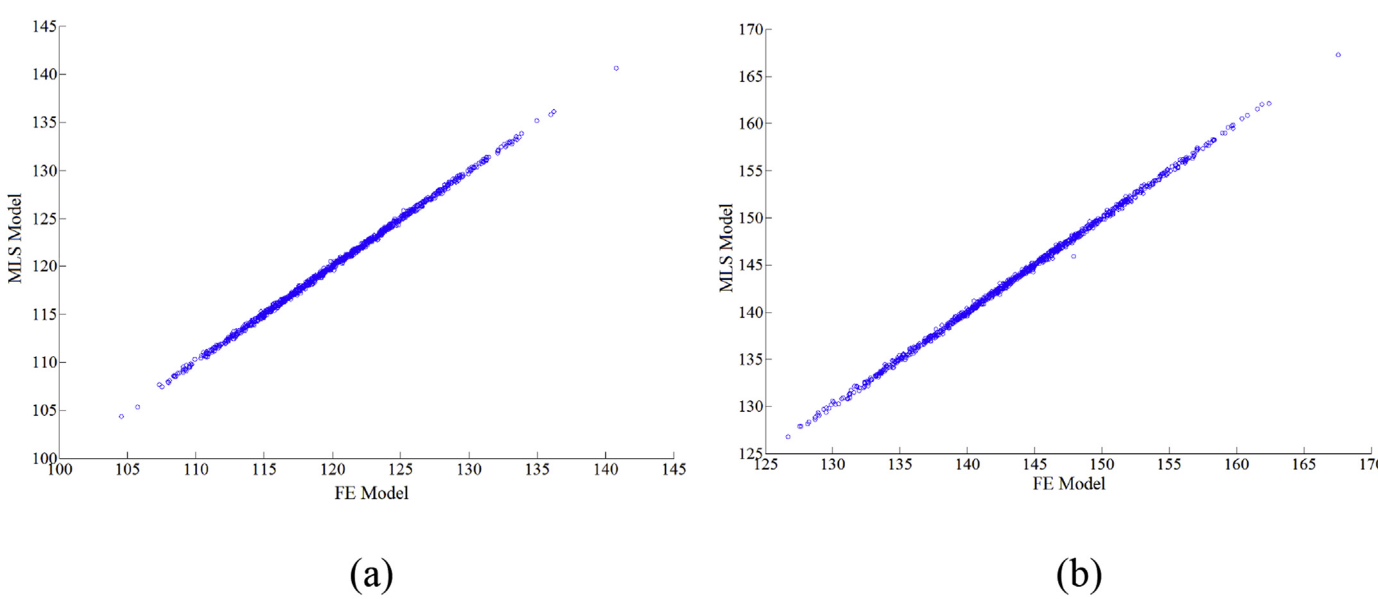

Fig. 4. Scatter plot for (a) lower bound and (b) upper bound of fundamental resonance frequencies corresponding to combined variation.

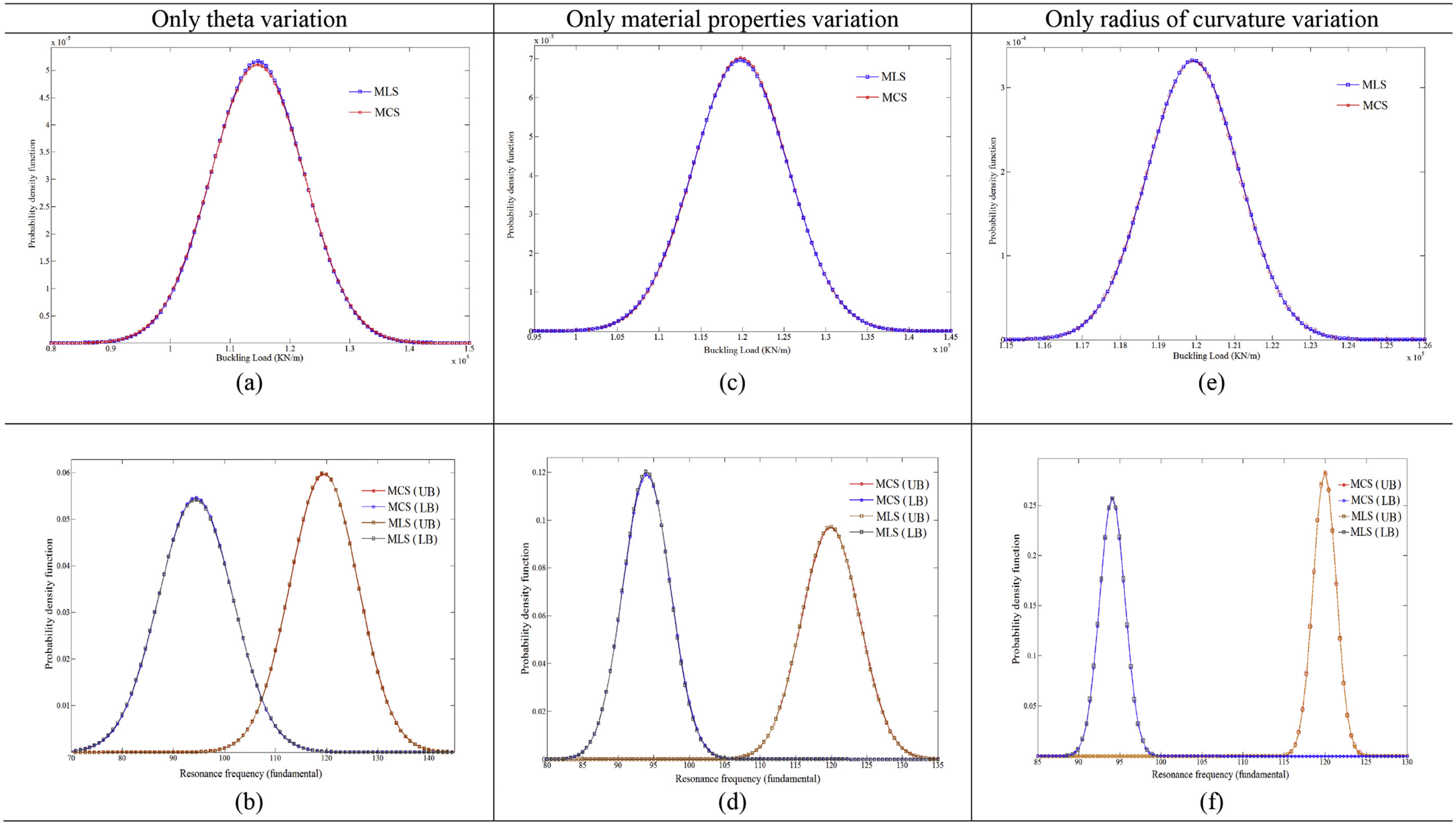

Fig. 5. Probability density function obtained by original Monte Carlo Simulation (MCS) and Moving Least Square (MLS) with respect to buckling load (first) and fundamental resonance frequencies [Upper Bound(UB), Lower Bound(LB)] due to individual variation of ply orientation angle, material properties and radius of curvatures for angle −ply (45°/-45°/45°/-45°) composite spherical shells.

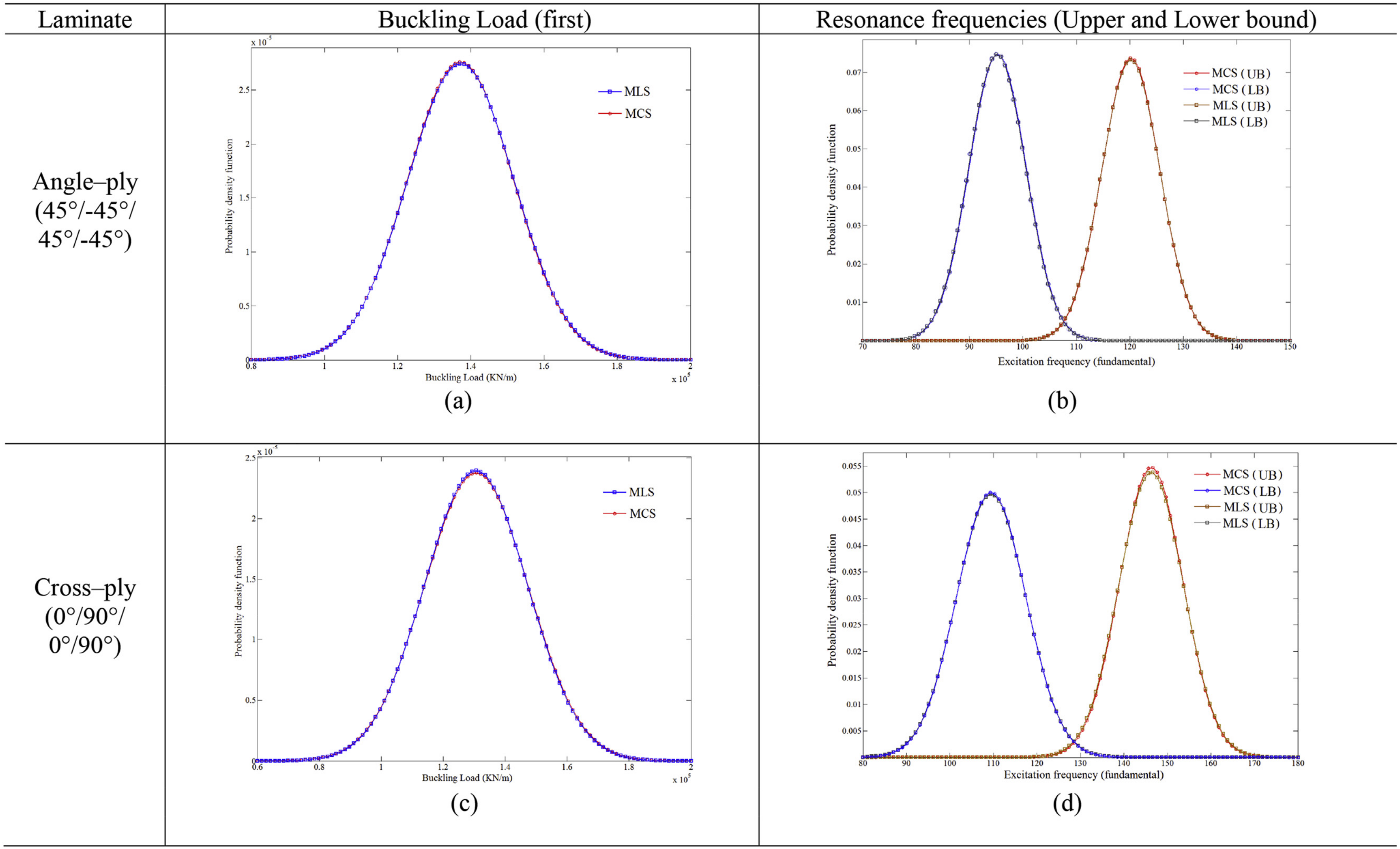

Fig. 6. Probability density function obtained by original Monte Carlo Simulation (MCS) and Moving Least Square (MLS) with respect to (a) buckling load (first) and (b) fundamental resonance frequencies [Upper Bound(UB), Lower Bound(LB)] due to combined variation for simply supported angle–ply and cross-ply composite spherical shells.

Table 4
Confidence interval boundaries for mean and standard deviation of buckling load (KN/m) for samples of direct MCS and MLS model.

Confidence interval (%)		MLS		MCS	
		Mean	SD	Mean	SD
95	Min	1.35240×10^5	1.64351×10^4	1.35869×10^5	1.48764×10^4
	Max	1.36893×10^5	1.68972×10^4	1.36460×10^5	1.52946×10^4
97	Min	1.35205×10^5	1.64110×10^4	1.35837×10^5	1.48545×10^4
	Max	1.36928×10^5	1.69226×10^4	1.36492×10^5	1.53176×10^4
99	Min	1.35137×10^5	1.63645×10^4	1.35776×10^5	1.48124×10^4
	Max	1.36996×10^5	1.69718×10^4	1.36553×10^5	1.53621×10^4

number of design points required to construct the suroagte model) is carried out in case of MLS method. The surrogate model is formed employing MLS method, on which the full sample size of direct MCS is conducted. Hence, the computational time and effort expressed in terms of FE calculation is significantly reduced compared to full scale direct MCS. This provides an efficient and economic way to simulate the uncertainties in buckling load and resonance frequencies (both upper bound and lower bound) for dynamic stability analysis. The scatter plot is also presented for validation of the present MLS model with original FE model with respect to resonance frequencies (fundamental) of lower bound (Fig. 4(a)) and upper bound (Fig. 4(b)) corresponding to combined variation of

Table 5
Stochastic buckling load (first) and resonance frequencies (first and second) with error due to individual and combined variation of simply supported angle-ply (45°/-45°/45°/-45°) composite spherical shells considering L = 1 m, b = 0.5 m, $t = 0.005$ m, c = 0.5, $R_x = R_y = 10$ m, $E_1 = E_2 = 141$ GPa, $G_{12} = G_{13} = 5.95$ GPa, $G_{23} = 2.96$ GPa, $\rho = 1580$ kg/m^3, $\nu = 0.3$

Para-meter	Value	Buckling Load (first)			Resonance frequency (First)						Resonance frequency (Second)					
		MCS	MLS	Err %	Upper bound			Lower bound			Upper bound			Lower bound		
					MCS	MLS	Err %	MCS	MLS	Err %	MCS	MLS	Err %	MCS	MLS	Err %
$\theta(\tilde{\omega})$	Max	128291.9	127596.5	0.54	137.13	136.76	0.27	114.56	115.37	−0.71	154.4566	154.68	−0.14	137.2272	137.7822	−0.40
	Min	88701.2	85767.6	3.31	102.14	101.13	0.99	77.68	76.36	1.70%	126.1999	124.61	1.26	103.3465	101.2563	2.02%
	Mean	114559.3	114586.8	−0.02	119.50	119.45	0.04	94.37	94.35	0.02%	143.71	143.68	0.02	124.66	124.73	−0.06
	SD	7812.6	7718.3	1.21	6.68	6.66	0.30	7.30	7.36	−0.82	4.83	4.86	−0.62	6.53	6.48	0.77%
$R(\tilde{\omega})$	Max	122779.8	122735.2	0.04	123.17	123.12	0.04	97.61	97.59	0.02%	153.2791	153.15	0.08	129.4123	129.3734	0.03%
	Min	117567.8	117573.0	0.00	117.25	117.26	−0.01	91.12	91.12	0.00%	135.4901	135.42	0.05	121.0511	120.9995	0.04%
	Mean	119910.8	119919.8	−0.01	119.94	119.96	−0.02	94.08	94.09	−0.01	144.2207	144.25	−0.02	125.3795	125.3827	0.00
	SD	1202.1	1199.9	0.18	1.41	1.40	0.71	1.55	1.54	0.65%	3.74	3.71	0.80	1.78	1.78	0.00%
$P(\tilde{\omega})$	Max	131650.3	131442.9	0.16	131.05	130.91	0.11	103.32	103.17	0.15%	157.5167	157.29	0.14	137.57	137.23	0.25%
	Min	108278.9	108146.1	0.12	109.54	110.03	−0.45	85.33	85.43	−0.12	131.5795	132.32	−0.56	114.05	114.24	−0.17
	Mean	119787.4	119748.1	0.03	119.92	119.89	0.03	94.01	93.98	0.03%	144.1147	144.07	0.03	125.41	125.36	0.04%
	SD	5678.4	5728.4	−0.88	4.11	4.10	0.24	3.35	3.31	1.19%	5.10	5.08	0.39	4.56	4.51	1.10%
$F(\tilde{\omega})$	Max	157639.4	157447.1	0.12	120.36	120.38	−0.02	96.25	96.28	−0.03	143.9863	144.01	−0.02	126.43	126.42	0.01%
	Min	119790.2	119692.4	0.08	119.81	119.79	0.01	93.94	93.92	0.02%	143.4501	143.43	0.01	125.31	125.34	−0.02
	Mean	137767.5	137644.7	0.09	120.08	120.08	−0.01	95.11	95.11	0.00%	143.7182	143.71	0.01	125.90	125.90	0.00%
	SD	12653.6	12272.1	3.01	0.27	0.27	0.00	0.89	0.87	2.25%	0.26	0.26	0.00	0.31	0.30	3.23%
{θ,R,P,F,α,β}	Max	177886.5	180808.1	−1.64	140.77	139.61	0.82	114.0825	117.94	3.38	169.3019	165.86	2.03	146.10	149.0155	−2.00
	Min	101954	101032.5	0.90	103.79	104.23	−0.42	77.42	78.28	−1.11	124.7183	124.71	0.01	106.07	107.45	−0.30
	Mean	137164.4	137238.2	−0.05	120.20	120.18	0.02	95.27	95.28	−0.01	144.0335	143.99	0.03	125.86	125.87	−0.01
	SD	14468.9	11919.0	17.62	5.40	5.44	−0.74	5.32	5.23	1.69	6.58	6.60	−0.30	6.09	6.02	1.15

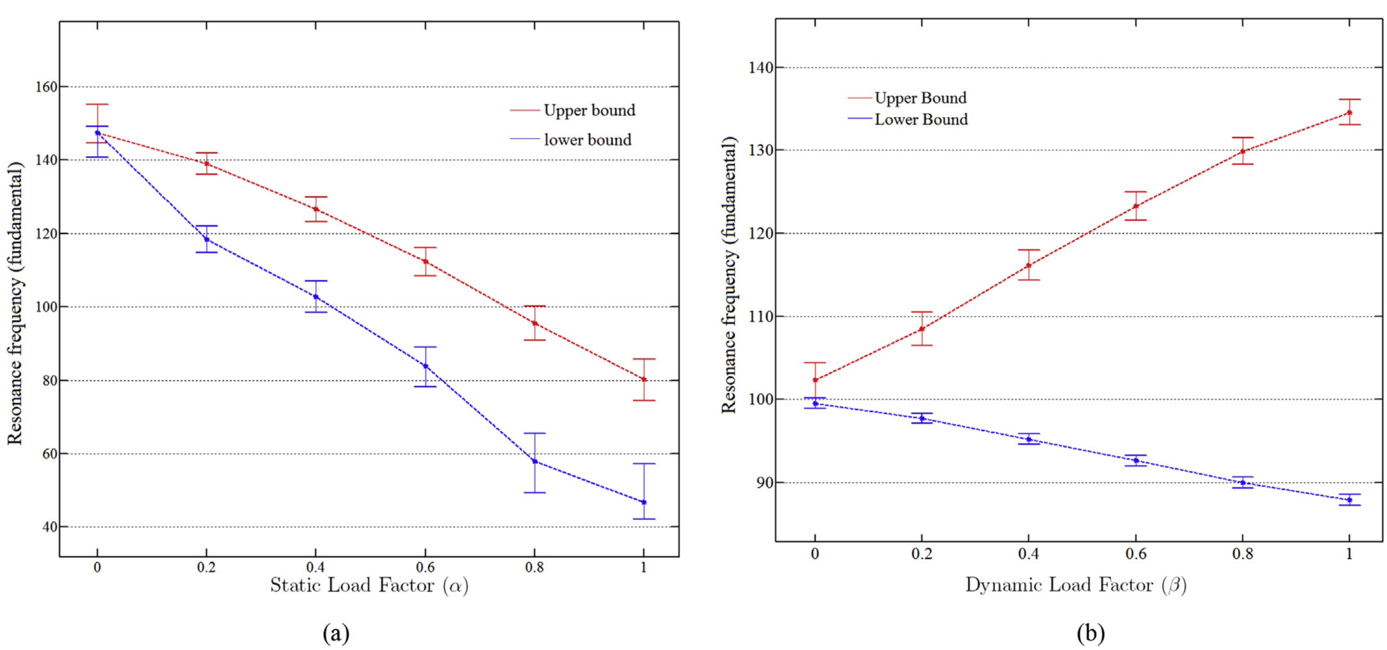

Fig. 7. Effect of static load factor and dynamic load factor on stochastic resonance frequencies (fundamental) due to combined variation of ply-orientation angle, radius of curvatures, material properties, loading for simply supported angle-ply (45°/-45°/45°/-45°) composite spherical shells.

ply-orientation angle, radius of curvatures, material properties, load, load factors (both static and dynamic). The probability density function (PDF) is plotted as the benchmark results due to individual and combined variation as depicted in Fig. 5 and Fig. 6, respectively. The confidence interval boundaries (95%, 97% and 99%) for mean and standard deviation of buckling load are shown in Table 4 for samples of direct MCS and MLS model.

The MLS model is validated extensively for different laminate configurations as well as different forms of stochasticity (individual and combined) so that the computationally efficient surrogate is ensured to obtain accurate results in the uncertainty analysis. The combined variations of stochastic input parameters for both MCS as well as present MLS method are carried out corresponding to both angle-ply (45°/-45°/45°/-45°) and cross-ply (0°/90°/0°/90°) composite spherical shells. Due to random variation of input parameters, the elastic stiffness of the laminated composite plate is found to be varied, which in turn influence the stochastic output irrespective of laminate configuration. Table 5 presents the comparative results of Monte Carlo simulation (MCS) and present MLS method for first buckling load and resonance frequencies (upper bound and lower bound) due to individual and combined variations of ply-orientation angle, radius of curvatures, material properties, intensity of load and load factors of a simply supported angle-ply (45°/-45°/45°/-45°) composite shallow spherical shells. The influence of static load factor and dynamic load factor on stochastic resonance frequencies due to combined variation of ply-orientation angle, radius of curvatures, material properties, loading for angle-ply (45°/-45°/45°/-45°) composite spherical shells are furnished in Fig. 7. It is observed that the width of the instability zone increases with the increase of static and dynamic load factors. Based on the rate of increment of the region of instability, it can be inferred that the dynamic load factor (β) is more sensitive to resonance frequencies than static load factor (α). Further to explore the effect of degree of stochasticity on resonance frequency and the capability of the proposed MLS based approach for higher degree of variations in the stochastic input parameters, three different degree of stochasticities are considered: 5%, 10% and 15% variations in the stochastic input parameters with respect to their respective deterministic values. Fig. 8 presents the validation in resonance frequencies (fundamental) using MLS model corresponding to different degree of stochasticities (5%, 10% and 15%) for combined variation of input parameters considering simply supported angle-ply spherical shells. The figure clearly depicts the increase in sparsity of resonance frequency (fundamental) due to increase in percentage of variations of random input parameters. The figure also affirms that the proposed MLS based uncertainty quantification algorithm for composites produces quite satisfactory results for different degree of stochasticities in input parameters with respect to direct Monte Carlo simulations.

Depending on the geometry of doubly curved shells, a comparative study is carried out for cylindrical, hyperbolic paraboloid and spherical shells as furnished in Fig. 9 for both stochastic buckling load and random resonance frequencies (fundamental) due to combined variation of for cross-ply (0°/90°/0°/90°) composite shells. The zone of resonance frequencies (fundamental) maps the different instability regions for different shell geometries. It is observed that the resonance frequency (fundamental)

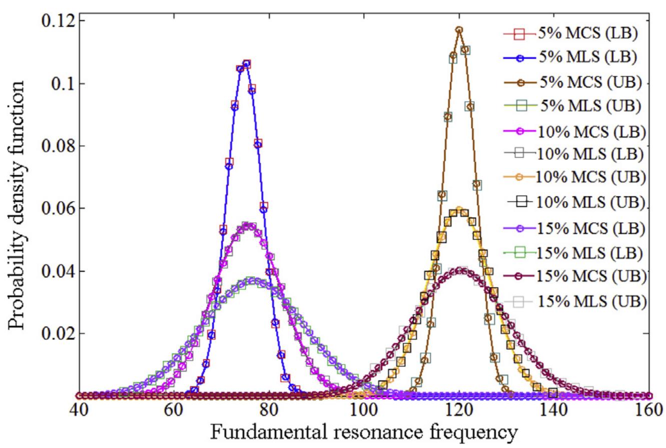

Fig. 8. Effect of percentage variation (5%, 10% and 15%) for combined variations of input parameters on resonance frequencies (fundamental) for simply supported angleply (45°/-45°/45°/-45°) composite spherical shells.

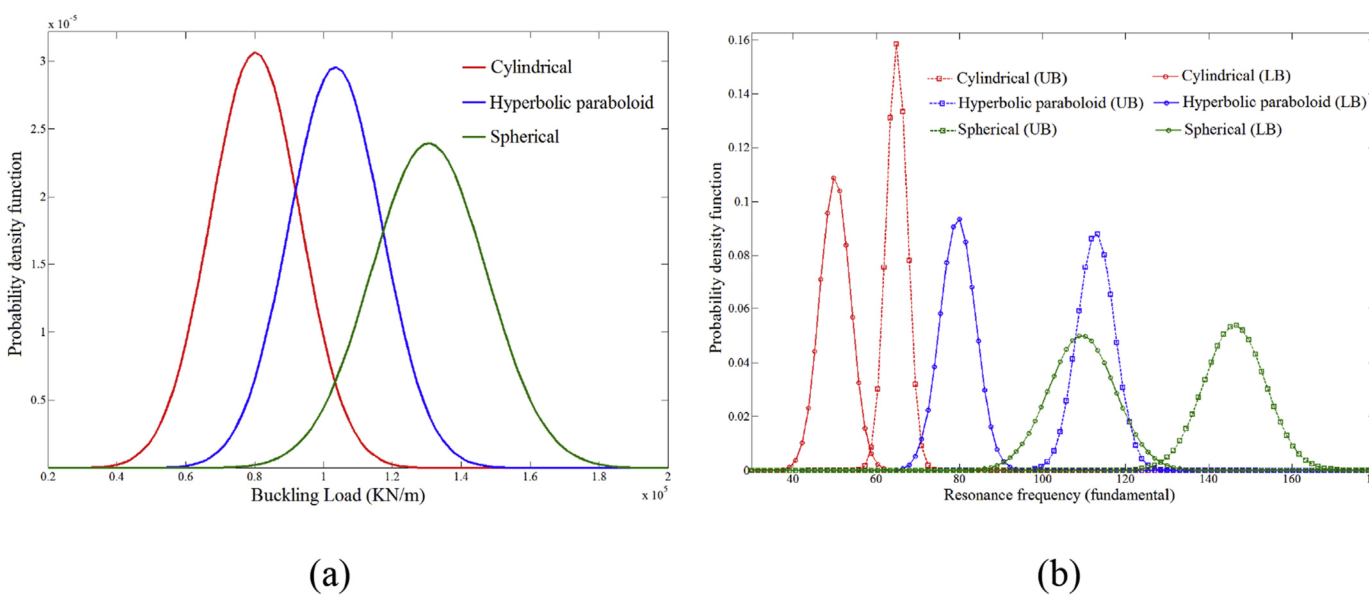

Fig. 9. Effect of shell geometry (Cylindrical, Hyperbolic paraboloid and Spherical) on stochastic (a) buckling load (first) and (b) resonance frequencies (fundamental) due to combined variation of for simply supported cross-ply (0°/90°/0°/90°) composite curved shells.

decreases with reduction of curvatures from spherical shell to hyperbolic paraboloid shells while single cylindrical shell shows the least stiffness compared to the other two. In order to address the influence of degree of shallowness (R_x/a = R_y/b = 5, 10, 20) of the doubly curved shells, a spherical shell is considered to portray the instability regions as furnished in Fig. 10. It is identified that there is an increase of instability resonance frequencies with the decrease in radius of curvature along x and y directions (i.e., R_x and R_y values). The significant effects of degree of orthotropy on stochastic buckling load and resonance frequency (fundamental) due to combined variation of ply-orientation angle, radius of curvatures, material properties, loading for cross-ply composite spherical shells are furnished in Fig. 11. As the static parameter is increased, the dynamic instability zone tend to shift towards lower frequencies and become stipper. The effect of degree of orthotropy is studied for E_1/E_2 ratio = 15, 30, 45, by randomizing the other parameters. The study shows an increase of random resonance frequencies due to increase in degree or orthotropy. The boundary conditions of the composite shells are observed to have a significant influence on the dynamic instability regions. The influence of different boundaries (CCCC, SCSC, SSSS where C − clamped, S-Simply supported) is investigated for stochastic buckling load and first resonance frequencies (lower and upper bounds) due to combined variation of ply-orientation angle, radius of curvatures, material properties, loading for cross-ply composite spherical shells by probability density function as furnished in Fig. 12. This study shows that the stochastic resonance frequencies are minimum for simply supported and maximum for clamped edges due to the restraint at the edges while SCSC boundary condition is found to be intermediate for both stochastic buckling load as well as zone of resonance

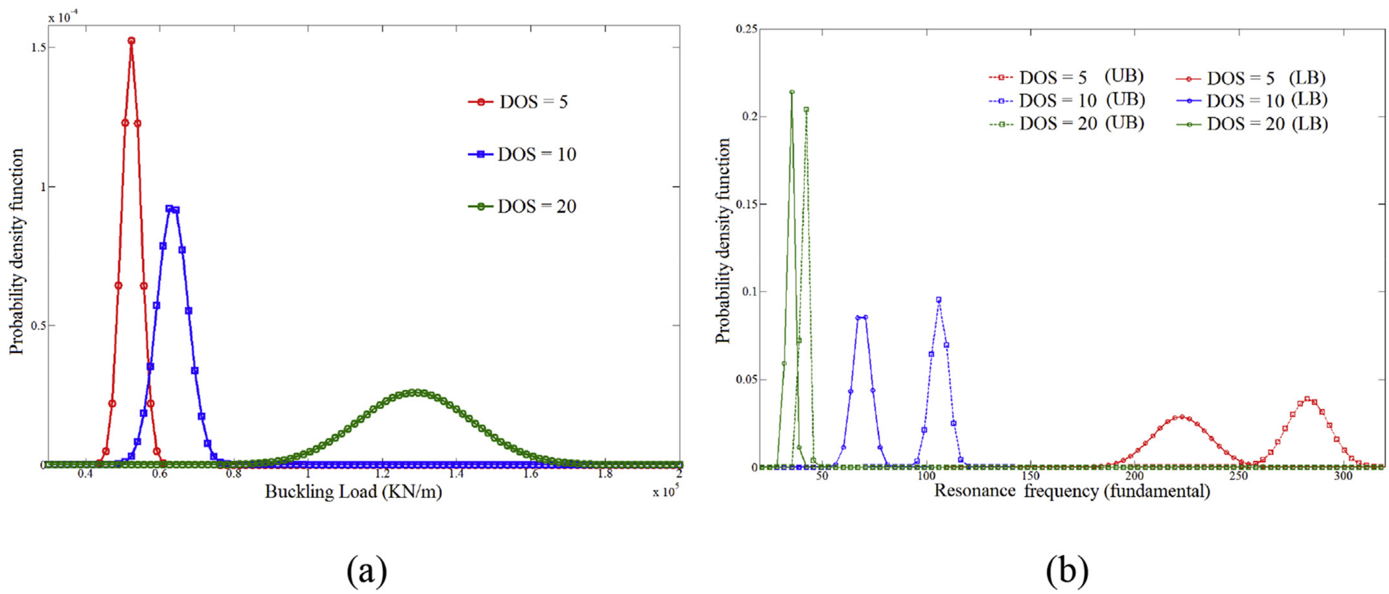

Fig. 10. Effect of degree of shallowness (DOS) (R_x/a = R_y/b) on stochastic (a) buckling load (first) and (b) resonance frequencies (fundamental) due to combined variation of for simply supported cross-ply (0°/90°/0°/90°) composite spherical shells.

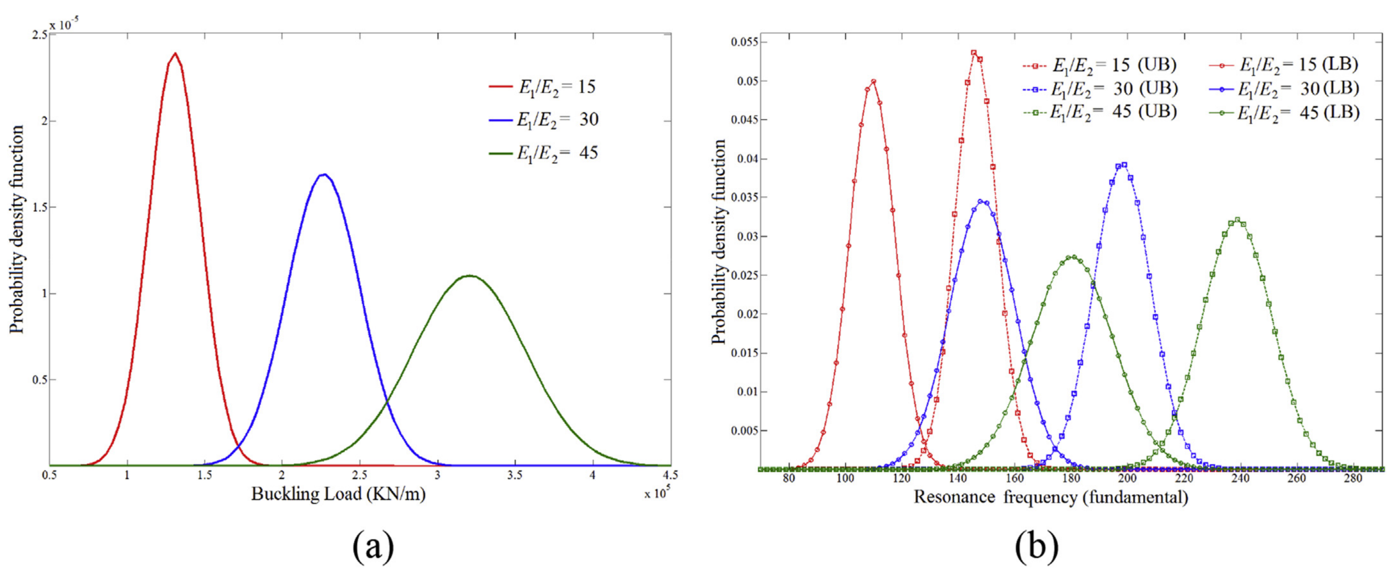

Fig. 11. Effect of degree of orthotropy on stochastic (a) buckling load (first) and (b) resonance frequencies (fundamental) due to combined variation for simply supported cross-ply (0°/90°/0°/90°) composite spherical shells.

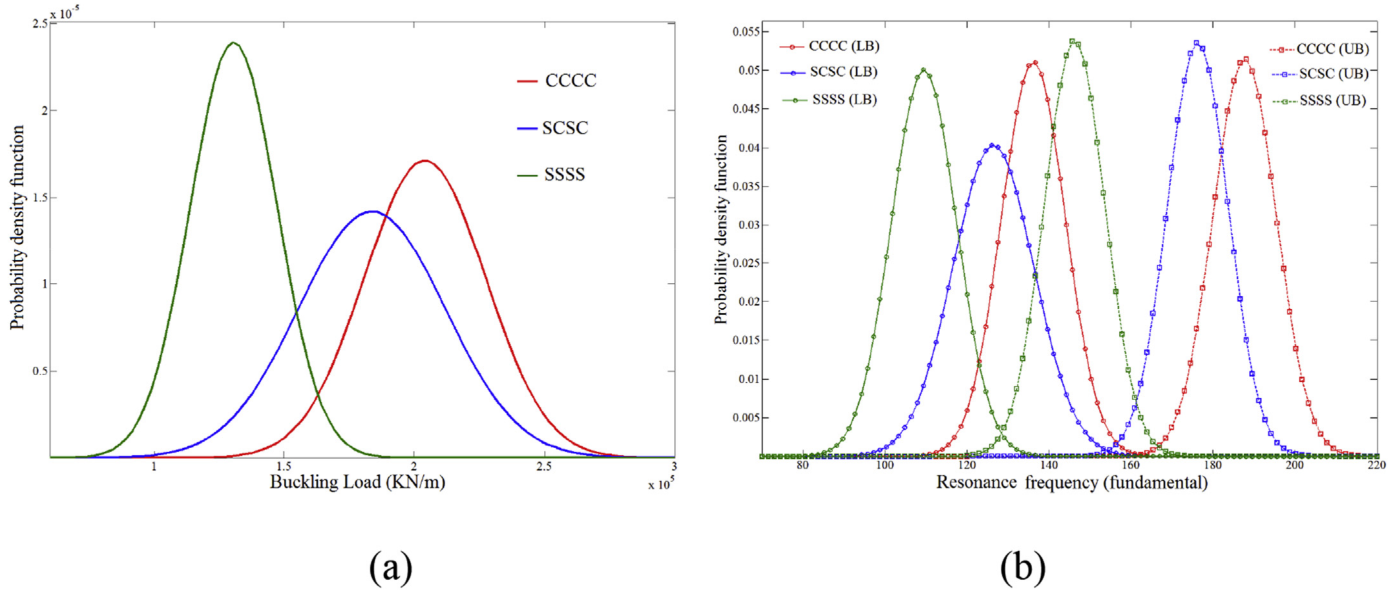

Fig. 12. Effect of boundary end condition (CCCC, SCSC, SSSS) on stochastic (a) buckling load and (b) resonance frequency (fundamental) due to combined variation for cross-ply (0°/90°/0°/90°) composite spherical shells.

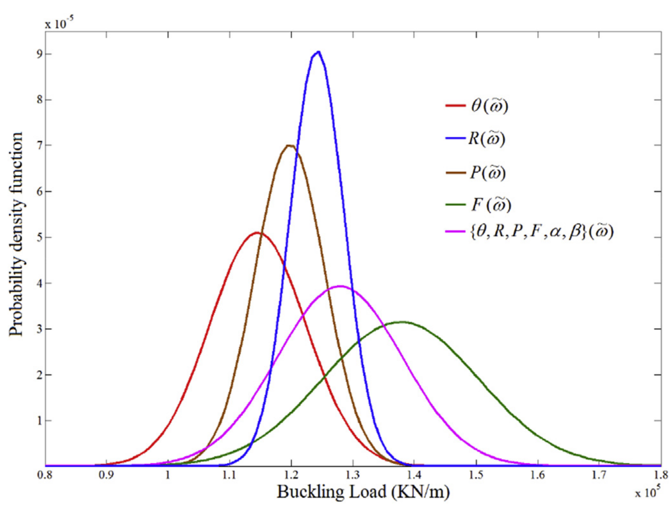

Fig. 13. Stochastic buckling load for combined variation for simply supported cross-ply spherical shells.

frequencies (see Fig. 12).

The effect of individual variations and combined variation of different random parameters for angle-ply composite spherical shells on stochastic first buckling load are furnished in Fig. 13 wherein the maximum sparsity of buckling load is observed for only variation of load-intensity among all the individual parameters. Fig. 14 represents the influence of aspect ratio (AR = a/b) on stochastic buckling load and resonance frequency (fundamental) due to combined variation of ply-orientation angle, radius of curvatures, material properties, loading for cross-ply composite spherical shells. Because of the shear deformation, it is found that the width of instability region narrows down. It is also found that as the aspect ratio (a/b) increases, the resonance frequencies also increase and the width of instability zone becomes wider. In the present study, the relative coefficient of variance (RCV) (normalized mean to standard deviation ratio) due to individual and combined variations is quantified for angle-ply laminate as furnished in Fig. 15. On the basis of individual variation of input parameters, ply orientation angle is found to be comparatively most sensitive, while loading parameter (for resonance frequencies) and radius of curvature (for buckling load) are found to have lesser sensitivity.

7. Conclusions

This study illustrates an efficient stochastic dynamic stability analysis of laminated composite curved panels considering non-uniform partial edge loading. The ranges of variation in first stochastic buckling load and fundamental resonance frequencies are analyzed considering both individual and combined stochasticity of input parameters. Novelty of the present study includes an efficient stochastic dynamic stability analysis with random non-uniform loading. Moving least square method is employed in conjunction with stochastic finite element analysis following a non-intrusive approach to achieve the computational efficiency. After utilizing the surrogate modelling approach, the number of finite element simulations is found to be significantly reduced compared to original Monte Carlo simulation without compromising the accuracy of results. The computational time is reduced to (1/157) times (for individual variation) and (1/20) times (for combined variation) of Monte Carlo simulation. The stochastic instability regions are found to shift to lower frequencies with increase in static load factor showing wider random instability regions indicating destabilization effect on the dynamic stability characteristics of composite spherical shells. It is observed that the zone of stochstic instability has significant influence due to variation in degree of orthotropy, aspect ratio and boundary condition. The width of stochstic instability region increases with the increase of degree of orthotropy and aspect ratio. The ply orientation angle is found to be most sensitive, while the least sensitive parameters are observed as loading parameter (for resonance frequencies) and radius of curvatures (for buckling load) compared to other parameters considered in this analysis.

Laminated composites being a complex structural form and susceptible to different forms of uncertainty, the compound effects of stochastic time varying loading and structural and material uncertainties associated with composites are crucial for the intended

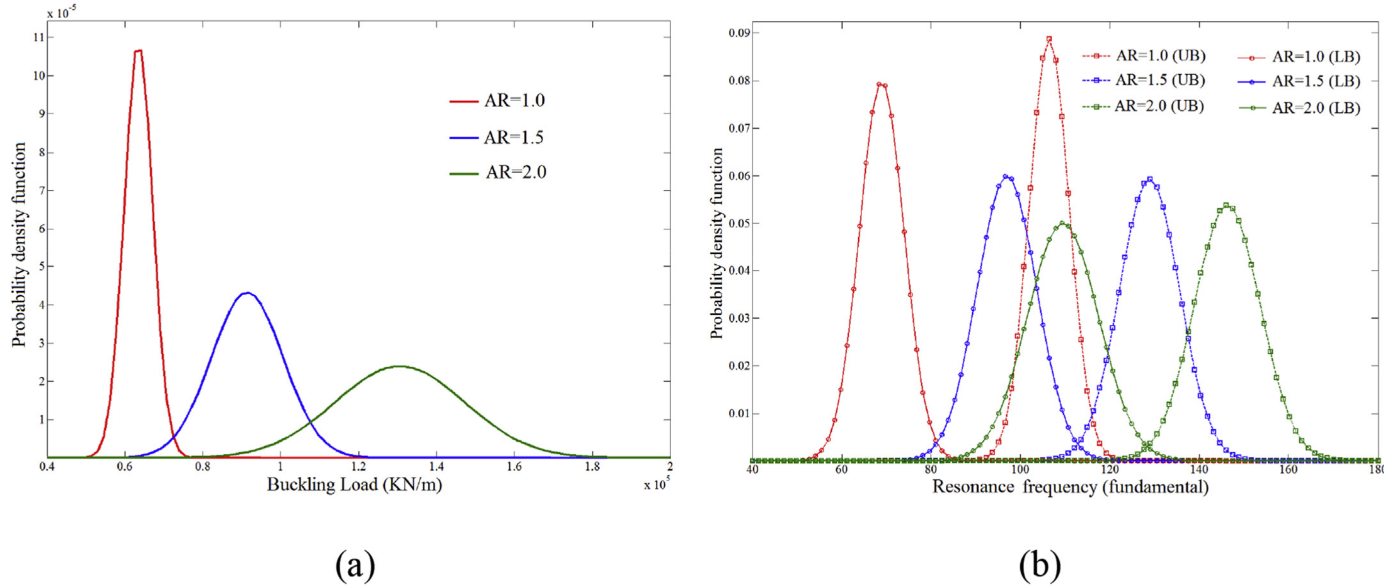

Fig. 14. Effect of aspect ratio (AR) on stochastic (a) buckling load and (b) resonance frequencies (fundamental) for combined variation for simply supported cross-ply spherical shells.

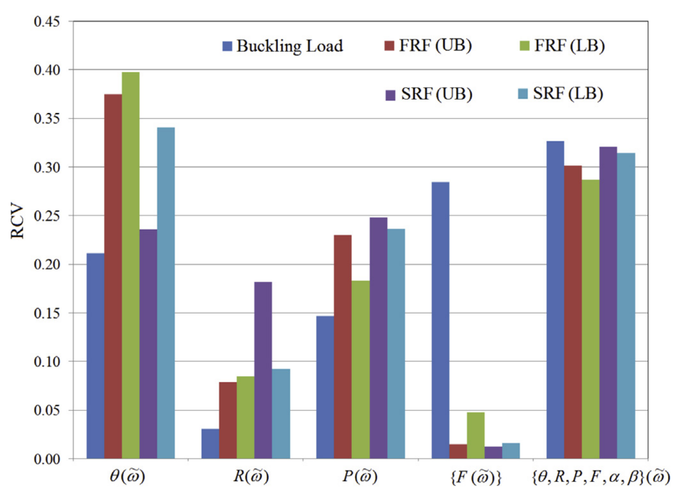

Fig. 15. Relative coefficient of variance (RCV) of buckling load and first resonance frequencies (FRF) and second resonance frequencies (SRF) due to individual variation of ply orientation angle, radius of curvatures, material properties, loading and combined variation for simply supported angle-ply (45°/-45°/45°/-45°) composite spherical shells.

performance in various applications. It is found that stochastic variations of input parameters has significant impact on dynamic instability of composite shell structures and thus such sensitive parameters are to be considered in design for operational safety and serviceability point of view. The numerical results obtained in this study provide a comprehensive idea for design and control of laminated composite curved pamels. The efficient moving least square based approach of uncertainty quantification can be extended further to other computationally intensive analyses of composite structures.

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