Composite Structures 96 (2013) 57–63

Contents lists available at SciVerse ScienceDirect

Composite Structures

journal homepage: www.elsevier.com/locate/compstruct

In-plane magnetic field affected transverse vibration of embedded single-layer graphene sheets using equivalent nonlocal elasticity approach

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ARTICLE INFO

Article history:
Available online 17 September 2012

Keywords:
Graphene
Nonlocal elasticity
Magnetic field
Vibration
Lorentz force

ABSTRACT

The behaviour of graphene in a magnetic field has attracted considerable attention among the scientific community. In this paper, the effect of an in-plane magnetic field on the transverse vibration of a magnetically sensitive single-layer graphene sheet (SLGS) is examined using equivalent continuum nonlocal elastic plate theory. The SLGS is considered to be embedded in an elastic medium. Governing equations for nonlocal transverse vibration of the SLGS under an in-plane magnetic field are derived considering the Lorentz magnetic force obtained from Maxwell's relation. Numerical results from the model show that the in-plane magnetic field exerted on the SLGS increases the natural frequencies of the SLGS. This is in line with some reported results in the literature where macroscopic plates under an in-plane magnetic field are considered. Further, the nonlocal effects decrease the natural frequencies of the SLGS. The effects of the in-plane magnetic field on higher natural frequencies and different aspect ratios of SLGS are also presented.

1. Introduction

The study of the mechanical, magnetic and other physical properties of nanotubes and nanoplates (graphene) under a magnetic field is important for future technological applications such as in NEMS, MEMS, nanosensors, spintronics and nanocomposites etc. This has created increasing attention among the scientific researchers. The literature contains a large number of studies on the effect of magnetic fields on single-walled and multi-walled carbon nanotubes [1–10]. Nanotubes embedded in an elastic matrix under a magnetic field have also recently been reported in [11,12]. The effect of magnetic fields on the properties of graphene sheets has also been examined in [13–18]. The emergence of magnetism in graphene and nanostructures has been found [19] and Wang et al. [20] experimentally illustrated the ferromagnetism of graphene-based materials at room temperature. The observed room-temperature ferromagnetism is considered to have come from defects in the graphene. Research into magnetism behaviour in graphene is in its infancy stage and requires further attention.

Similar to nanotubes, graphene sheets are recently used in composites as filler materials. In some cases graphene based composites are compared with nanotube based composites to explore the better nanocomposites option between the two. These graphene nanomaterials possess enhanced mechanical, thermal and electrical properties. Graphene sheets embedded in polymer matrix are important in developing and fabricating future composites such as graphene polymer composites. However homogenous and desired dispersion and embedment of graphene sheets in polymer matrix is an open challenge in recent research. To address the proper bonding between the graphene sheets and the polymer, functionalised graphenes are also being used. Similar to various studies of graphenes, the study of embedded graphene sheet in magnetic field is worth studying.

Experimental [21–24] and atomistic simulations [25,26] have shown a significant 'size-effect' in the mechanical and physical properties when the dimensions of structures become 'small'. Here, by 'small' we mean in the order of nanoscale. Size-effects are related to atoms and molecules that constitute the materials. Molecular dynamics (MD) simulations [27–30] are computationally expensive and classical continuum models are questionable in the analysis of 'smaller' structures. Thus size-dependent continuum theories are gaining importance [31–34]. These theories bring in the size-effects or scale-effects within the formulation by amending the traditional classical continuum mechanics. One widely used size-dependant theory is the nonlocal elasticity theory pioneered by Eringen [35]. Nonlocal elasticity accounts for the small-scale effects arising at the nanoscale level. In nonlocal elasticity theory, the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain [35]. This is unlike classical elasticity theory. Nonlocal theory considers long-range inter-atomic interaction and yields

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http://dx.doi.org/10.1016/j.compstruct.2012.09.005

results dependent on the size of a body. The application of nonlocal elasticity in the mechanical analysis of nanostructures is highly reported in the literature [33,36–49]. The reliability of the nonlocal approaches has also been validated by molecular dynamics simulations [50].

In the present paper we study the effects of an external in-plane magnetic field on the transverse vibration of nanoplates such as graphene. Nonlocal elasticity is used to address the small scale-effects. The graphene sheets are considered magnetically sensitive and assumed to be embedded in an elastic medium such as polymer matrix. Using nonlocal plate theory, governing equations for nonlocal bending-vibration are derived, considering the Lorentz magnetic forces applied on graphene sheets induced by an in-plane magnetic field through Maxwell equations. The nonlocal natural frequencies of graphene under an in-plane magnetic field are obtained analytically by solving the governing equation. Combined effects of the magnetic field and elastic matrix, together with nonlocal effects on the vibration response of magnetically sensitive graphene are illustrated and discussed.

2. Maxwell's relations

Denoting **J** as current density, **h** as distributing vector of the magnetic field, and **e** as strength vectors of the electric field, the Maxwell relation according to [51] is given as

$$\mathbf{J} = \nabla \times \mathbf{h} \tag{1}$$

$$\nabla \times \mathbf{e} = -\eta \frac{\partial \mathbf{h}}{\partial t} \tag{2}$$

$$\nabla \cdot \mathbf{h} = 0 \tag{3}$$

$$\mathbf{e} = -\eta\left(\frac{\partial \mathbf{U}}{\partial t} \times \mathbf{H}\right) \tag{4}$$

$$\mathbf{h} = \nabla \times (\mathbf{U} \times \mathbf{H}) \tag{5}$$

where η is the magnetic field permeability. ∇ is the Hamilton operator and is expressed as $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$. ($\mathbf{i}$,$\mathbf{j}$,$\mathbf{k}$) are the unit vectors. For the present analysis, we consider the in-plane uniaxial magnetic field as a vector $\mathbf{H} = (H_x, 0, 0)$ acting on the SLGS. Let the displacement vector be $\mathbf{U} = (u, v, w)$, then

$$\mathbf{h} = \nabla \times (\mathbf{U} \times \mathbf{H}) = -H_x\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\mathbf{i} + H_x\frac{\partial v}{\partial x}\mathbf{j} + H_x\frac{\partial w}{\partial x}\mathbf{k} \tag{6}$$

$$\mathbf{J} = \nabla \times \mathbf{h} = H_x\left(-\frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y}\right)\mathbf{i} - H_x\left(\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right)\mathbf{j} + H_x\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right)\mathbf{k} \tag{7}$$

The Lorentz force induced by the in-plane uniaxial magnetic field is given as

$$\mathbf{f} = f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k} = \eta(\mathbf{J} \times \mathbf{H}) = \eta\left[0\mathbf{i} + H_x^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right)\mathbf{j} + H_x^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right)\mathbf{k}\right] \tag{8}$$

Therefore the Lorentz forces along the x, y and z directions are

$$f_x = 0 \tag{9}$$

$$f_y = \eta H_x^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right) \tag{10}$$

$$f_z = \eta H_x^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right) \tag{11}$$

For the present transverse vibration analysis, we assume that the only deflection is $w = w(x,y,t)$, and that the Lorentz force acting on the magnetically sensitive graphene sheet in the z direction is

$$f_z = \eta H_x^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \tag{12}$$

For a one-dimensional vibrating CNT we have the Lorentz force in the z direction as $f_z = \eta H_x^2 \frac{\partial^2 w}{\partial x^2}$ [11]. It should be noted that in the present study the effective Lorentz force is a function of magnetic permeability and H_x.

3. A brief review of nonlocal elasticity

A brief review of nonlocal elasticity applied to nano structural analysis is presented here. In nonlocal elasticity theory [35], the basic equations for an isotropic linear homogenous nonlocal elastic body neglecting the body force are given as

$$\begin{aligned}
&\sigma_{ij,i} + \rho(f_j - \ddot{u}_j) = 0\\
&\sigma_{ij}(\mathbf{x}) = \int_V \phi(|\mathbf{x} - \mathbf{x}'|, \alpha)\sigma^c_{ij}(\mathbf{x}')dV(\mathbf{x}')\\
&\sigma^c_{ij}(\mathbf{x}') = C_{ijkl}\varepsilon_{kl}(\mathbf{x}')\\
&\varepsilon_{kl}(\mathbf{x}') = \frac{1}{2}\left(\frac{\partial u_i(\mathbf{x}')}{\partial x'_j} + \frac{\partial u_j(\mathbf{x}')}{\partial x'_i}\right)
\end{aligned} \tag{13}$$

The terms σ_{ij}, σ^c_{ij}, ε_{kl}, C_{ijkl} are the nonlocal stress, classical stress, classical strain and fourth order elasticity tensors respectively. The volume integral is over the region **V** occupied by the body. The kernel function $\phi(|\mathbf{x} - \mathbf{x}'|, \alpha)$ is known as the nonlocal modulus. The nonlocal modulus acts as an attenuation function, incorporating into the constitutive equations the nonlocal effects at the reference point $\mathbf{x}$ produced by local strain at the source $\mathbf{x}'$. The term $|\mathbf{x} - \mathbf{x}'|$ represents the distance in the Euclidean form and α is a material constant that depends on the internal (e.g. lattice parameter, granular size, distance between the C–C bonds) and external characteristic lengths (e.g. crack length, wave length). One example of a nonlocal modulus as given by Eringen [35] is

$$\phi(|\mathbf{x}|, \alpha) = (2\pi\ell^2\alpha^2)^{-1}K_0\left(\sqrt{\mathbf{x} \cdot \mathbf{x}}/\ell\alpha\right) \tag{14}$$

The material constant α is defined as $\alpha = e_0 a/\ell$. Here e_0 is a constant for calibrating the model with experimental results and other validated models. The parameter e_0 is estimated such that the relations of the nonlocal elasticity model can provide a satisfactory approximation to the atomic dispersion curves of the plane waves obtained from atomistic lattice dynamics. According to Eringen [35], the value of e_0 is reported as 0.39. Details on the various values of nonlocal parameter e_0 as reported by various researchers is discussed in [52]. The terms a and ℓ are the internal (e.g. lattice parameter, granular size, distance between C–C bonds) and external characteristics lengths (e.g. crack length, wave length) of the nanostructure, respectively.

As Eq. (13) is difficult to solve, using Eq. (14) a differential form is popularly used as

$$\left(1 - \alpha^2\ell^2\nabla_L^2\right)\sigma_{ij}(\mathbf{x}) = \sigma^c_{ij}(\mathbf{x}) = C_{ijkl}\varepsilon_{kl}(\mathbf{x}) \tag{15}$$

Here ∇_L is the Laplacian. For two-dimensional nanostructures such as graphene Eq. (15) is simplified as

$$\left[1 - (e_0 a)^2\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\right]\sigma = \sigma^c_{ij} \tag{16}$$

4. Governing equation based on nonlocal elasticity

Using the nonlocal elastic relation (Eq. (16)), the governing equation for a vibrating flat graphene sheet as an elastic plate in an elastic medium (Fig. 1), ignoring surface effects, is derived as [53]

$$\begin{aligned} &D\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)+\rho h\frac{\partial^2 w(x,y,t)}{\partial t^2}\\ &+kw(x,y,t)-q(x,y,t)+(e_0a)^2\left(\frac{\partial^2 q(x,y,t)}{\partial x^2}+\frac{\partial^2 q(x,y,t)}{\partial y^2}\right)\\ &-k(e_0a)^2\left(\frac{\partial^2 w(x,y,t)}{\partial x^2}+\frac{\partial^2 w(x,y,t)}{\partial y^2}\right)\\ &-(e_0a)^2\rho h\left(\frac{\partial^4 w(x,y,t)}{\partial x^2\partial t^2}+\frac{\partial^4 w(x,y,t)}{\partial y^2\partial t^2}\right)=0 \end{aligned} \tag{17}$$

where $w(x,y,t)$ is the deflection of the graphene sheet and k is the stiffness of the embedded elastic medium. Here D is the bending rigidity of the graphene sheet and is expressed as $D=Eh^3/12(1-\vartheta^2)$. Terms ρ, ϑ and h are the density, Poisson's ratio and the thickness of graphene sheet, respectively.

The term f_z is due to the in-plane Lorentz magnetic field, from which the $q(x,y,t)$, the effective transverse Lorentz magnetic force is obtained as

$$q(x,y,t)=\int_{-h/2}^{h/2} f_z dz = hf_z=\eta h H_x^2\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right) \tag{18}$$

Thus the governing equation of vibrating graphene under magnetic field can be written as

$$\begin{aligned} &D\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)+\rho h\frac{\partial^2 w(x,y,t)}{\partial t^2}\\ &+kw(x,y,t)-\eta h H_x^2\left(\frac{\partial^2 w(x,y,t)}{\partial x^2}+\frac{\partial^2 w(x,y,t)}{\partial y^2}\right)\\ &-k(e_0a)^2\left(\frac{\partial^2 w(x,y,t)}{\partial x^2}+\frac{\partial^2 w(x,y,t)}{\partial y^2}\right)\\ &-(e_0a)^2\rho h\left(\frac{\partial^4 w(x,y,t)}{\partial x^2\partial t^2}+\frac{\partial^4 w(x,y,t)}{\partial y^2\partial t^2}\right)\\ &+(e_0a)^2\eta h H_x^2\left(\frac{\partial^4 w(x,y,t)}{\partial x^4}+2\frac{\partial^4 w(x,y,t)}{\partial x^2\partial y^2}+\frac{\partial^4 w(x,y,t)}{\partial y^4}\right)=0 \end{aligned} \tag{19}$$

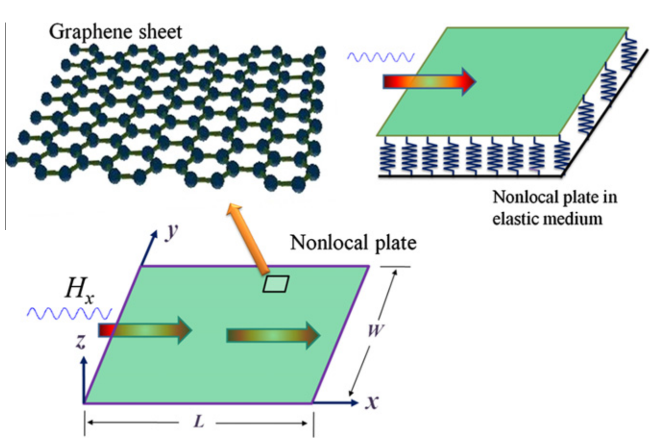

Fig. 1. Schematic diagram of graphene sheet subjected to in-plane magnetic field modelled as nonlocal plate.

When the nonlocal and the magnetic field effects are ignored in Eq. (19), the equation reverts back to the equation of conventional classical plate theory. The consideration of in-plane forces or other magnetic body force behaviour due to any inherent magnetic properties in simple graphene, charged graphene, ferromagnetic graphene [20] or graphene-derived systems [19] is a topic of open study. It should be noted that Eq. (19) is an approximate theoretical relation to represent the realistic graphene sheets in a strong magnetic field where some conditions are assumed.

5. Nonlocal vibration response of graphene under in-plane magnetic field

Considering that the graphene sheet has nonlocal simply supported boundary conditions, Eq. (19) can be solved by the Navier method assuming the solution to be in the form

$$w=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}W_{mn}\sin(a_m x)\sin(b_n y)e^{i\omega_{mn}t} \tag{20}$$

where

$$a_m=\frac{m\pi}{L},\quad b_n=\frac{n\pi}{W} \tag{21}$$

Terms L and W are the length and width of the graphene sheet, respectively and ω_{mn} is the natural frequency.

Substituting Eq. (20) into Eq. (19) and simplifying yields

$$\begin{aligned} &-D\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{W}\right)^2\right]^2 W_{mn}-k\left(1+(e_0a)^2\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{W}\right)^2\right]\right)W_{mn}\\ &-\eta h H_x^2\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{W}\right)^2\right]\left(1+(e_0a)^2\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{W}\right)^2\right]\right)\\ &=-\rho h\left(1+(e_0a)^2\left[\left(\frac{m\pi}{L}\right)^2+\left(\frac{n\pi}{W}\right)^2\right]\right)\omega_{mn}^2 W_{mn} \end{aligned} \tag{22}$$

We define the following parameters for the sake of convenience and generality:

$$\Omega_{mn}=\omega_{mn}L^2\sqrt{\frac{\rho h}{D}},\quad R=\frac{L}{W},\quad K=\frac{kL^4}{D},\quad \mu=\frac{e_0a}{L},\quad MP=\frac{\eta h H_x^2 L^2}{D} \tag{23}$$

Consider

$$\xi_{mn}=[(m\pi)^2+R^2(n\pi)^2],\quad m,n=1,2,\ldots \tag{24}$$

Using the parameters of Eqs. (23) and (24), the frequency of graphene in an in-plane magnetic field is analytically obtained as a function of stiffness of elastic medium, magnetic parameter and nonlocal parameter:

$$\Omega_{mn}=\sqrt{\frac{\xi_{mn}^2+K(1+\mu^2\xi_{mn})+MP\xi_{mn}(1+\mu^2\xi_{mn})}{1+\mu^2\xi_{mn}}} \tag{25}$$

For long strip of nanoribbons we have

$$\xi_{mn}=(r\pi)^2,\quad r=1,2,\ldots \tag{26}$$

We define the nonlocal frequency ratio as the ratio of the natural frequency of graphene with nonlocal effects to the natural frequency of graphene without nonlocal effects. The Nonlocal Frequency Ratio is expressed as

$$\text{Nonlocal frequency ratio}=\sqrt{\frac{1}{1+\mu^2\xi_{mn}}\left(1+\frac{K\mu^2\xi_{mn}+\mu^2 MP\xi_{mn}^2}{\xi_{mn}^2+K+MP\xi_{mn}}\right)} \tag{27}$$

Similarly we define the magnetic frequency ratio as the ratio of the natural frequency of graphene with a magnetic field to the natural frequency of graphene without a magnetic field. The magnetic frequency ratio is expressed as

$$\text{Magnetic Frequency Ratio} = \sqrt{1 + \frac{MP\xi_{mn}(1+\mu^2\xi_{mn})}{\xi_{mn}^2 + K(1+\mu^2\xi_{mn})}} \qquad (28)$$

Graphs are plotted and the insights of the present nonlocal model are discussed in the next section.

6. Results and discussion

The effect of increasing strength of magnetic field on the vibration characteristics of a SLGS embedded in an elastic medium is examined via nonlocal elastic model. A strong magnetic field is mathematically assumed in the analysis. The effective SLGS properties considered are: Young's modulus, E = 1.06 TPa, Poisson ratio ϑ = 0.25, density ρ = 2300 kg/m^3 and the thickness h = 0.34 nm. For generality, the vibration solutions of the SLGS are presented in terms of the parameters defined in Eq. (23). The present results are provided in terms of frequency parameter, nonlocal parameter and magnetic parameter. This is for capturing the effect for broad range of values and understanding the characteristic properties of the system. Further it will help in assessing the relative importance of the terms. As the exact value of the nonlocal parameter for SLGS is still an open topic of study, we consider the nonlocal parameter (μ) range as 0–1. The stiffness of the elastic medium is assumed as K = 1.

Fig. 2 shows the variation of frequency parameter with the change of both magnetic parameter and nonlocal parameter. The magnetic parameter depicts the relative strength of the magnetic field and is considered in the range of MP = 0–50. Some dimensional values of H_x related to MP are reported in [11]. A square SLGS is considered (R = 1). It is seen from the figure that the frequency parameter decreases as the nonlocal parameter increases. This decrease can be attributed to the distributed transverse force due to: (1) the curvature change in the nanoplates, and (2) the surface stress due to the nonlocal atom–atom interaction as described in Wang et al. [54]. It can also be seen from the figure that the frequency parameter increases as the strength of in-plane magnetic field increases. This is due to the coupling effect of the vibrating SLGS and the magnetic field.

A long graphene nanoribbon having aspect ratio $R = 10^{-5}$ is also considered. Fig. 3 shows the variation of frequency parameter with the change of both magnetic parameter and nonlocal parameter for a nanoribbon. A similar variation of the frequency parameter with magnetic parameter and nonlocal parameter is observed as for the square SLGS. However for the same ranges of magnetic parameter and nonlocal parameter, the frequency parameter is numerically less. Thus in magnetic field, graphene can be made high frequency resonators than nanoribbon.

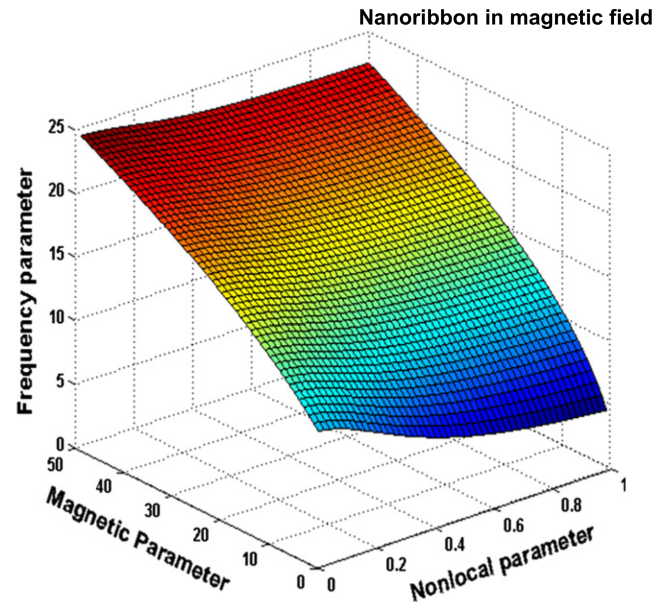

Fig. 3. Effect of magnetic parameter and nonlocal parameter on fundamental frequency parameter for long nanoribbon.

Higher modes of natural frequencies are also considered for a square SLGS. Figs. 4 and 5 depict the variation of the frequency parameter with a change of both magnetic parameter and nonlocal parameter for m = 2, n = 2 and m = 3, n = 3, respectively. It is seen that, for higher frequency modes, the frequency parameter is strongly affected by the nonlocal parameter, but is less affected by changes in the strength of the magnetic field.

The effect of the aspect ratio of rectangular SLGS on its vibration response is next investigated. Fig. 6 shows the variation of frequency parameter with the change of magnetic parameter and aspect ratio (R) of SLGS. The range considered is from a very low

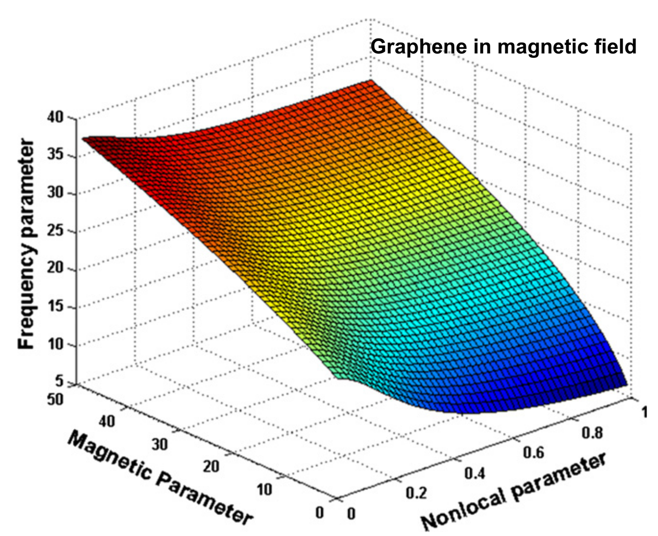

Fig. 2. Effect of magnetic parameter and nonlocal parameter on fundamental frequency parameter for single-walled square graphene sheets.

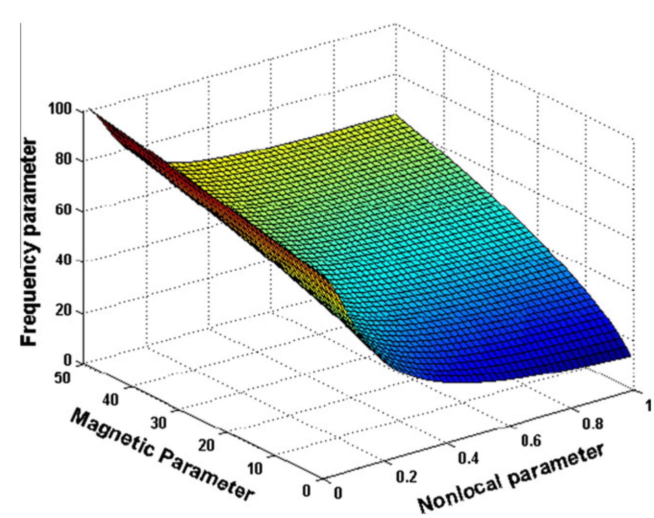

Fig. 4. Effect of magnetic parameter and nonlocal parameter on second frequency parameter for single-walled square graphene sheets.

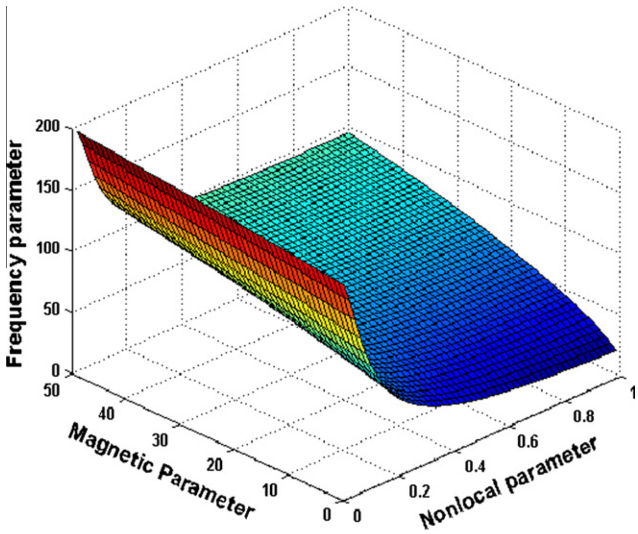

Fig. 5. Effect of magnetic parameter and nonlocal parameter on third frequency parameter for single-walled square graphene sheets.

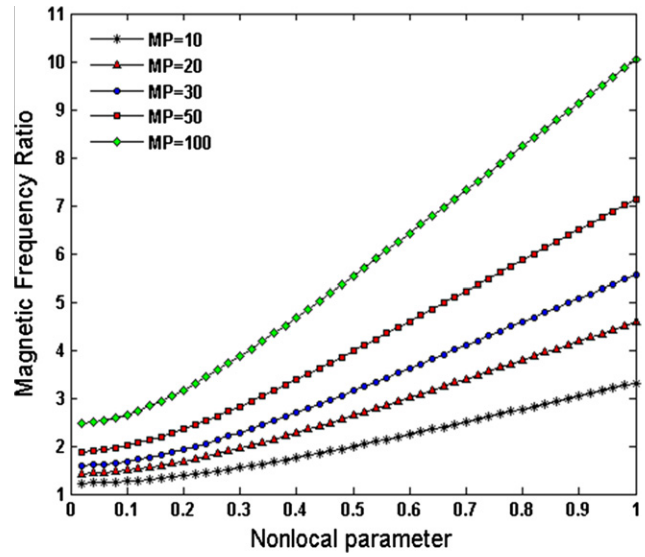

Fig. 7. Change of magnetic frequency ratio with nonlocal parameter for different strength of magnetic field in single-walled graphene sheets.

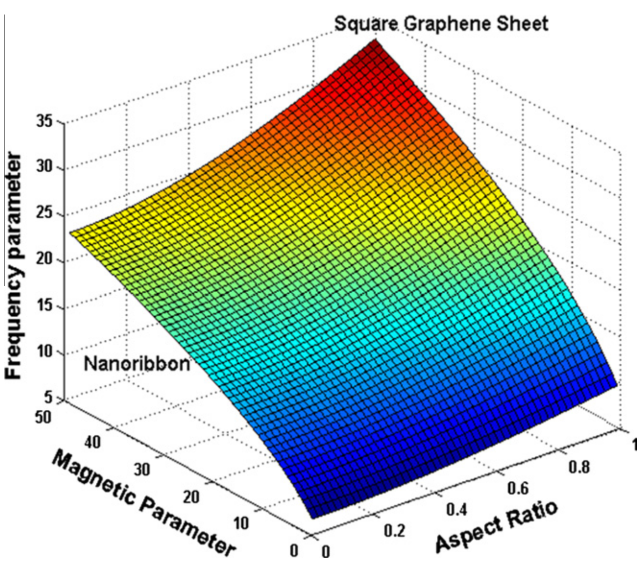

Fig. 6. Effect of magnetic parameter and aspect ratio on fundamental frequency parameter for graphene sheets.

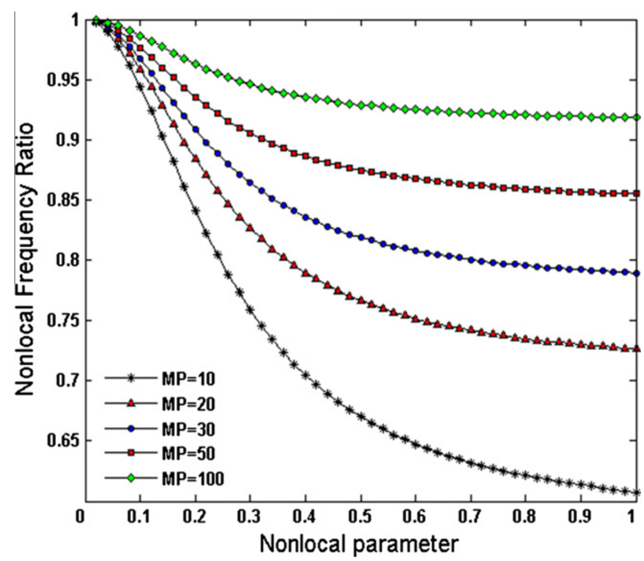

Fig. 8. Change of nonlocal frequency ratio with nonlocal parameter for different strength of magnetic field in single-walled graphene sheets.

aspect ratio $R = 10^{-4}$(nanoribbon) to $R = 1.0$ (square graphene). It is observed from the figure that the frequency parameter increases for both the increase of magnetic parameter and aspect ratio (R). The effect of aspect ratio on frequency parameter is more pronounced at high strength of the magnetic field.

Figs. 7 and 8 depict the change of magnetic frequency ratio (MFR) and nonlocal frequency ratio (NFR) with nonlocal parameter, respectively. Various magnetic field strengths (MP) are considered. It is noticed that the MFR increases almost linearly as the nonlocal parameter increases (Fig. 7), while NFR decreases nonlinearly as the nonlocal parameter increases (Fig. 8). The degree of nonlinearity decreases with increasing magnetic field strength.

Experimental or molecular dynamics results for the vibrating system in the present specific study are so far unavailable in the literature. This represents future scope of work. In the present study the in-plane magnetic field increases the frequency. Similarly for a macroscopic beam-plate under the effect of an in-plane magnetic field, Yang et al. [55], Lee and Lin [56], Zhou and Miya [57] have reported that the natural frequency increases as the strength of the in-plane magnetic field increases. Takagi et al. [58] have also reported experimentally an increase in natural frequency as the strength of the in-plane magnetic field increases. This is due to the coupling effect of vibrating plate and the magnetic field.

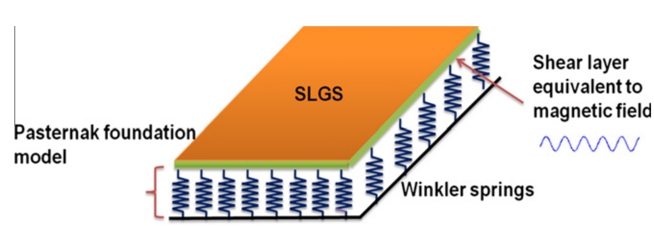

Fig. 9. Embedded SLGS under in-plane magnetic field is shown analogous to nonlocal plate coupled by equivalent Pasternak foundation model.

From the present study it is found that an in-plane magnetic field increases the frequencies of the SLGS, which can be explained as follows. The magnetic field can be considered as a shear layer parameter in a Pasternak foundation model. The coupling elastic medium is thus analogous to a Pasternak foundation, where the Winkler modulus is the stiffness of the springs and the magnetic parameter is the Pasternak shear modulus (see Fig. 9). The Pasternak model can be seen as a membrane having a surface tension laid on a system of elastic springs which increases the frequencies.

7. Conclusion

The effect of an in-plane magnetic field on the transverse vibration of magnetically sensitive embedded single-layer graphene sheet (SLGS) is examined using equivalent continuum nonlocal elastic plate theory. Results reveal that nonlocal effects decrease the frequency of SLGS. However nonlocal effects are dampened by the in-plane magnetic field exerted on SLGS. The in-plane magnetic field increases the natural frequencies of the SLGS. This is in-line with some reported results in literature where a macroscopic plate under an in-plane magnetic field is considered.

Acknowledgement

TM acknowledges the support from the Irish Research Council for Science, Engineering & Technology (IRCSET) for the present work.

References

[1] Bellucci S, Gonzalez J, Guinea F, Onorato P, Perfetto E. Magnetic field effects in carbon nanotubes. J Phys – Condens Matter 2007:19.
[2] Chang L, Lue J. Magnetic properties of multi-walled carbon nanotubes. J Nanosci Nanotechnol 2009;9:1956.
[3] Chen J, Liu H, Ahmad N, Li Y, Chen Z, Zhou W, et al. Effect of external magnetic field on magnetic properties of Co–Pt nanotubes and nanowires. J Appl Phys 2011:109.
[4] Jang B, Sakka Y. Influence of shape and size on the alignment of multi-wall carbon nanotubes under magnetic fields. Mater Lett 2009;63:2545.
[5] Kibalchenko M, Payne M, Yates J. Magnetic response of single-walled carbon nanotubes induced by an external magnetic field. Acs Nano 2011;5:537.
[6] Lee C, Hsue Y, Chen R, Li T, Lin M. Electronic structures of finite double-walled carbon nanotubes in a magnetic field. J Phys – Condens Matter 2008:20.
[7] Roche S, Saito R. Effects of magnetic field and disorder on the electronic properties of carbon nanotubes. Phys Rev B 1999;59:5242.
[8] Sebastiani D, Kudin K. Electronic response properties of carbon nanotubes in magnetic fields. Acs Nano 2008;2:661.
[9] Zhang Z, Guo W, Guo Y. The effects of axial magnetic field on electronic properties of carbon nanotubes. Acta Phys Sin 2006;55:6526.
[10] Li S, Xie H, Wang X. Dynamic characteristics of multi-walled carbon nanotubes under a transverse magnetic field. Bull Mater Sci 2011;34:45.
[11] Wang H, Dong K, Men F, Yan Y, Wang X. Influences of longitudinal magnetic field on wave propagation in carbon nanotubes embedded in elastic matrix. Appl Math Modell 2010;34:878.
[12] Wang X, Shen J, Liu Y, Shen G, Lu G. Rigorous van der Waals effect on vibration characteristics of multi-walled carbon nanotubes under a transverse magnetic field. Appl Math Modell 2012;36:648.
[13] Peng F. Magnetic field induced thermal effect of phonons in graphene. Phys Status Solidi B – Basic Solid State Phys 2011;248:1388.
[14] Faugeras C, Kossacki P, Basko D, Amado M, Sprinkle M, Berger C, et al. Effect of a magnetic field on the two-phonon Raman scattering in graphene. Phys Rev B 2010:81.
[15] Fu Z, Wang Z, Li S, Zhang P. Magnetic quantum oscillations in a monolayer graphene under a perpendicular magnetic field. Chinese Phys B 2011:20.
[16] Kumar S, Jalil M, Tan S, Liang G. The effect of magnetic field and disorders on the electronic transport in graphene nanoribbons. J Phys – Condens Matter 2010:22.
[17] Lopez-Urias F, Rodriguez-Manzo J, Munoz-Sandoval E, Terrones M, Terrones H. Magnetic response in finite carbon graphene sheets and nanotubes. Opt Mater 2006;29:110.
[18] Shizuya K. Electromagnetic response and effective gauge theory of graphene in a magnetic field. Phys Rev B 2007:75.
[19] Yazyev O. Emergence of magnetism in graphene materials and nanostructures. Reports Prog Phys 2010:73.
[20] Wang Y, Huang Y, Song Y, Zhang X, Ma Y, Liang J, et al. Room-temperature ferromagnetism of graphene. Nano Lett 2009;9:220.
[21] Bauer S, Pittrof A, Tsuchiya H, Schmuki P. Size-effects in TiO(2) nanotubes: diameter dependent anatase/rutile stabilization. Electrochem Commun 2011;13:538.
[22] Kiang C, Endo M, Ajayan P, Dresselhaus G, Dresselhaus M. Size effects in carbon nanotubes. Phys Rev Lett 1998;81:1869.
[23] Xiao S, Hou W. Studies of size effects on carbon nanotubes' mechanical properties by using different potential functions. Fulleren Nanotub Carbon Nanostruct 2006;14:9.
[24] Zienert A, Schuster J, Streiter R, Gessner T. Transport in carbon nanotubes: contact models and size effects. Phys Status Solidi B – Basic Solid State Phys 2010;247:3002.
[25] Tang C, Meng L, Sun L, Zhang K, Zhong J. Molecular dynamics study of ripples in graphene nanoribbons on 6H–SiC(0001): temperature and size effects. J Appl Phys 2008:104.
[26] Tang C, Guo W, Chen C. Molecular dynamics simulation of tensile elongation of carbon nanotubes: temperature and size effects. Phys Rev B 2009:79.
[27] Bi K, Chen Y, Yang J, Wang Y, Chen M. Molecular dynamics simulation of thermal conductivity of single-wall carbon nanotubes. Phys Lett A 2006;350:150.
[28] Brodka A, Koloczek J, Burian A. Application of molecular dynamics simulations for structural studies of carbon nanotubes. J Nanosci Nanotechnol 2007;7:1505.
[29] Han T, He P, Wang J, Wu A. Molecular dynamics simulation of a single graphene sheet under tension. New Carbon Mater 2010;25:261.
[30] Li R, Hu Y, Wang H, Zhang Y. Molecular dynamics simulation of motion of single-walled carbon nanotubes on graphite substrate. Acta Phys Sin 2006;55:5455.
[31] Akgoz B, Civalek O. Strain gradient elasticity and modified couple stress models for buckling analysis of axially loaded micro-scaled beams. Int J Eng Sci 2011;49:1268.
[32] Beni Y, Koochi A, Abadyan M. Theoretical study of the effect of Casimir force, elastic boundary conditions and size dependency on the pull-in instability of beam-type NEMS. Phys E – Low-Dimen Syst Nanostruct 2011;43:979.
[33] Simsek M. Dynamic analysis of an embedded microbeam carrying a moving microparticle based on the modified couple stress theory. Int J Eng Sci 2010;48:1721.
[34] Kong S, Zhou S, Nie Z, Wang K. Static and dynamic analysis of micro beams based on strain gradient elasticity theory. Int J Eng Sci 2009;47:487.
[35] Eringen A. On differential-equations of nonlocal elasticity and solutions of screw dislocation and surface-waves. J Appl Phys 1983;54:4703.
[36] Ansari R, Rouhi H, Sahmani S. Calibration of the analytical nonlocal shell model for vibrations of double-walled carbon nanotubes with arbitrary boundary conditions using molecular dynamics. Int J Mech Sci 2011;53:786.
[37] Aydogdu M. Axial vibration of the nanorods with the nonlocal continuum rod model. Phys E – Low-Dimen Syst Nanostruct 2009;41:861.
[38] Aydogdu M, Filiz S. Modeling carbon nanotube-based mass sensors using axial vibration and nonlocal elasticity. Phys E – Low-Dimen Syst Nanostruct 2011;43:1229.
[39] Filiz S, Aydogdu M. Axial vibration of carbon nanotube heterojunctions using nonlocal elasticity. Comput Mater Sci 2010;49:619.
[40] Heireche H, Tounsi A, Benzair A, Maachou M, Bedia E. Sound wave propagation in single-walled carbon nanotubes using nonlocal elasticity. Phys E – Low-Dimen Syst Nanostruct 2008;40:2791.
[41] Hsu J, Lee H, Chang W. Longitudinal vibration of cracked nanobeams using nonlocal elasticity theory. Curr Appl Phys 2011;11:1384.
[42] Wang Q. Wave propagation in carbon nanotubes via nonlocal continuum mechanics. J Appl Phys 2005:98.
[43] Wang Q, Varadan V. Vibration of carbon nanotubes studied using nonlocal continuum mechanics. Smart Mater Struct 2006;15:659.
[44] Wang Q, Liew K. Application of nonlocal continuum mechanics to static analysis of micro- and nano-structures. Phys Lett A 2007;363:236.
[45] Wang Q, Wang C. The constitutive relation and small scale parameter of nonlocal continuum mechanics for modelling carbon nanotubes. Nanotechnology 2007:18.
[46] Aghababaei R, Reddy J. Nonlocal third-order shear deformation plate theory with application to bending and vibration of plates. J Sound Vib 2009;326:277.
[47] Ansari R, Sahmani S, Arash B. Nonlocal plate model for free vibrations of single-layered graphene sheets. Phys Lett A 2010;375:53.
[48] Ansari R, Ramezannezhad H. Nonlocal Timoshenko beam model for the large-amplitude vibrations of embedded multiwalled carbon nanotubes including thermal effects. Phys E – Low-Dimen Syst Nanostruct 2011;43:1171.
[49] Ansari R, Arash B, Rouhi H. Vibration characteristics of embedded multi-layered graphene sheets with different boundary conditions via nonlocal elasticity. Compos Struct 2011;93:2419.
[50] Murmu T, Adhikari S. Nonlocal vibration of carbon nanotubes with attached buckyballs at tip. Mech Res Commun 2011;38:62.
[51] Kraus J. Electromagnetics. USA: McGrawHill Inc.; 1984.
[52] Narendar S, Gopalakrishnan S. Spectral finite element formulation for nanorods via nonlocal continuum mechanics. J Appl Mech – Trans ASME 2011:78.
[53] Pradhan S, Phadikar J. Small scale effect on vibration of embedded multilayered graphene sheets based on nonlocal continuum models. Phys Lett A 2009;373:1062.
[54] Wang C, Murmu T, Adhikari S. Mechanisms of nonlocal effect on the vibration of nanoplates. Appl Phys Lett 2011:98.

[55] Yang W, Pan H, Zheng D, Cai Q. Vibration and dynamic instability of ferromagnetic thin plates in magnetic fields. Mech Res Commun 1999;26:239.
[56] Lee J, Lin C. The magnetic viscous damping effect on the natural frequency of a beam plate subject to an in-plane magnetic field. J Appl Mech – Trans ASME 2010:77.
[57] Zhou Y, Miya K. A theoretical prediction of natural frequency of a ferromagnetic beam plate with low susceptibility in an in-plane magnetic field. J Appl Mech – Trans ASME 1998;65:121.
[58] Takagi T, Tani J, Matsubara Y, Mogi I. Dynamic behavior of fusion structural components under strong magnetic-fields. Fusion Eng Des 1995;27:481.