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Axial instability of double-nanobeam-systems

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ABSTRACT

This Letter considers the axial instability of double-nanobeam-systems. Eringen's nonlocal elasticity is utilized for modelling the double-nanobeam-systems. The nonlocal theory accounts for the small-scale effects arising at the nanoscale. The small-scale effects substantially influence the instability (or buckling) of double-nanobeam-systems. Results reveal that the small-scale effects are higher with increasing values of nonlocal parameter for the case of in-phase (synchronous) buckling modes than the out-of-phase (asynchronous) buckling modes. The increase of the stiffness of the coupling elastic medium in double-nanobeam-system reduces the small-scale effects during the out-of-phase (asynchronous) buckling modes. Analysis of the scale effects in higher buckling loads of double-nanobeam-system with synchronous and asynchronous modes is also discussed in this Letter. The theoretical development presented herein may serve as a reference for nonlocal theories as applied to the instability analysis of complex-nanobeam-system such as complex carbon nanotube system.

1. Introduction

Structural beams fabricated from nanomaterials [1] and of nanometer dimension are referred to as nanobeams (viz. nanowires, nanotubes, etc.). It is being extensively utilized as nanostructure components for nanoelectromechanical (NEMS) and microelectromechanical systems (MEMS) [2]. Being important from the experimental and theoretical standpoint, the stability and dynamic problems involving nanobeams and nanowires [3] have drawn great deal of attention in the engineering and physics community. An important technological extension of the concept of the single nanobeam [4] is that of the complex-nanobeam-systems. One such nanobeam system is the double-nanobeam-system. Like macroscopic double-beam systems [5,6], the double-nanobeam-system is a complex system consisting of two one-dimensional nanobeams joined by a coupling medium (Van der Waals forces, elastic medium, etc.). The coupling medium is represented by distributed vertical springs. Also it is reported that double-nanobeam-systems are important in nano-optomechanical systems (NOMS) [7–10].

Experimental [11–13] and atomistic simulations [14] have evidenced a significant 'size-effect' in the mechanical and physical properties when the dimensions of the nanostructures become 'small'. Size-effects are related to atoms and molecules that constitute the materials. The application of classical continuum models [15] thus may be questionable in the analysis of 'smaller' nanostructures. Classical continuum theories are scale-free theory. Therefore, recently there have been research efforts to bring in the scale-effects [11–13] within the formulation by amending the traditional classical continuum mechanics. When the length-scale reduces to the nanoscale, the physics of materials becomes prominent and cannot be ignored. One upcoming scale-dependant theory is the nonlocal elasticity theory pioneered by Eringen [16]. According to Eringen [16], using nonlocal elasticity excellent approximation can be provided for large class of physical phenomenon with characteristic lengths ranging from atomic scale to macroscopic scale. Nonlocal elasticity accounts for the small-scale effects arising at the nanoscale level. Recent literature [17–33] shows that the theory of nonlocal elasticity is being extensively used for reliable and computationally efficient analysis of nanostructures viz. nanobeams, nanoplates, nanorings, carbon nanotubes, graphenes, nanoswitches and microtubules. The majority of existing works on nonlocal elasticity are pertaining to the analysis of single nanobeams (nanotubes) [18–20,23,24,29]. Though the mechanical studies of nanobeams may include buckling and vibration of multiple-walled nanotubes, the study of discrete double-nanobeams has not been reported in literature. Recently, Murmu and Adhikari [34] presented a longitudinal vibration analysis of double-nanorods systems using nonlocal elasticity. Further, the authors [35] have also discussed on the nonlocal transverse vibration of double-nanobeam-systems.

Based on the above discussion, in this Letter an investigation is carried out to illustrate the small-scale effects in the axial instability of nonlocal double-nanobeam-system (NDNBS) subjected to longitudinal compression. Understanding axial instability or buckling of NDNBS is important from structural integrity of the nano-

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system. Further, this Letter provides a unique yet simple method of obtaining the exact solution for the buckling of double-nanobeam system. Equations for buckling load of a double-nanobeam-system (NDNBS) are formulated within the framework of Eringen's nonlocal elasticity [16]. The two nanobeams are assumed to be attached by distributed vertical transverse springs. These springs may represent the stiffness of an enclosed elastic medium, forces due to nano-optomechanical effects [7–10] or Van der Waals forces. An exact analytical method is proposed for solving the compressive load of NDNBS at which buckling occurs. Simply-supported boundary conditions are employed in this study. The results are obtained for various buckling modes of the NDNBS. The buckling phenomenon includes in-phase (synchronous) and out-of-phase (asynchronous) modes of buckling. The effects of (i) nonlocal parameter or scale coefficient, (ii) stiffness of the springs and (iii) the higher modes, on the buckling behaviour of the NDNBS are discussed. This work is aimed at providing an analytical scale-based nonlocal approach which could serve as the starting point for further investigation of more scale-dependent complex n-nanobeam systems.

2. Review of nonlocal elasticity

For the sake of completeness, we provide a brief review of the theory of nonlocal elasticity [16]. According to nonlocal elasticity, the basic equations for an isotropic linear homogenous nonlocal elastic body neglecting the body force are given as

$$\begin{aligned} &\sigma_{ij,j}=0,\\ &\sigma_{ij}(\mathbf{x})=\int_{\mathbf{V}}\phi\big(\left|\mathbf{x}-\mathbf{x}'\right|,\alpha\big)t_{ij}\,d\mathbf{V}\big(\mathbf{x}'\big),\quad \forall\mathbf{x}\in\mathbf{V},\\ &t_{ij}=H_{ijkl}\varepsilon_{kl},\\ &\varepsilon_{ij}=\frac{1}{2}(u_{i,j}+u_{j,i}). \end{aligned} \tag{1}$$

The terms σ_{ij}, t_{ij}, ε_{ij}, H_{ijkl} are the nonlocal stress, classical stress, classical strain and fourth order elasticity tensors respectively. The volume integral is over the region $\mathbf{V}$ occupied by the body. The above equation (Eq. (1)) couples the stress due to nonlocal elasticity and the stress due to classical elasticity. The kernel function $\phi(|\mathbf{x}-\mathbf{x}'|,\alpha)$ is the nonlocal modulus. The nonlocal modulus acts as an attenuation function incorporating into constitutive equations the nonlocal effects at the reference point $\mathbf{x}$ produced by local strain at the source $\mathbf{x}'$. The term $|\mathbf{x}-\mathbf{x}'|$ represents the distance in the Euclidean form and α is a material constant that depends on the internal (e.g. lattice parameter, granular size, distance between the C–C bonds) and external characteristics lengths (e.g. crack length, wave length). Material constant α is defined as

$$\alpha=e_0a/\ell. \tag{2}$$

Here e_0 is a constant for calibrating the model with experimental results and other validated models. The parameter e_0 is estimated such that the relations of the nonlocal elasticity model could provide satisfactory approximation to the atomic dispersion curves of the plane waves with those obtained from the atomistic lattice dynamics. The terms a and ℓ are the internal (e.g. lattice parameter, granular size, distance between C–C bonds) and external characteristics lengths (e.g. crack length, wave length) of the nanostructure.

Eq. (1) is in partial-integral form and generally difficult to solve analytically. Thus a differential form of nonlocal elasticity equation is often used. According to Eringen [16], the expression of nonlocal modulus can be given as

$$\phi\big(|\mathbf{x}|,\alpha\big)=\big(2\pi\ell^2\alpha^2\big)^{-1}K_0(\sqrt{\mathbf{x}\cdot\mathbf{x}}/\ell\alpha) \tag{3}$$

where K_0 is the modified Bessel function.

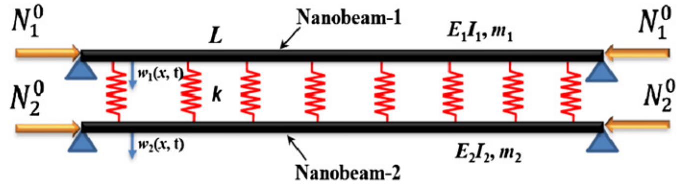

Fig. 1. Schematic diagram of elastically connected double-nanobeam-system subjected to compressive axial load.

The equation of motion in terms of nonlocal elasticity can be expressed as

$$\sigma_{ij,j}+f_i=\rho\ddot{u}_i \tag{4}$$

where f_i, ρ and u_i are the components of the body forces, mass density, and the displacement vector, respectively. The terms i, j takes up the symbols x, y and z.

Assuming the kernel function ϕ as the Green's function, Eringen [16] proposed a differential form of the nonlocal constitutive relation as

$$\sigma_{ij,j}+\mathcal{L}(f_i-\rho\ddot{u}_i)=0 \tag{5}$$

where

$$\mathcal{L}=\big[1-(e_0a)^2\nabla^2\big] \tag{6}$$

and ∇^2 is the Laplacian.

Using Eq. (5) the nonlocal constitutive stress-strain relation at small scale can be simplified as

$$\big(1-\alpha^2\ell^2\nabla^2\big)\sigma_{ij}=t_{ij}. \tag{7}$$

In the next section we present the formulations for double-nanobeam-systems based on nonlocal elasticity.

3. Buckling equations of nonlocal double-nanobeam-systems

Consider a nonlocal double-nanobeam-system (NDNBS) under compression as shown in Fig. 1. The two nanobeams are denoted as nanobeam-1 and nanobeam-2. The nanobeams are assumed to be slender and satisfy Euler-Bernoulli beam theory. Vertically distributed springs attaches the two nanobeams. The stiffness of the springs is equivalent to the Winkler constant in a Winkler foundation model [36]. The springs can be used to represent elastic medium, forces due to nano-optomechanical effects [7–10] or Van der Waals forces between the two nanobeams. These mentioned forces arise when the dimension of system approaches nanoscale. The springs are assumed to have stiffness, k. The two nanobeams are different where the length, mass per unit length and bending rigidity of the ith beam are L_i, m_i and E_iI_i $(i=1,2)$ respectively. These parameters are assumed to be constant along each nanobeam. The bending displacements over the two nanobeams are denoted by $w_1(x,t)$ and $w_2(x,t)$, respectively (Fig. 1).

Let the nanobeams be subjected to axial forces N_1^0 and N_2^0 as

$$N_1^0=A_1\sigma_x^0;\qquad N_2^0=A_2\sigma_x^0 \tag{8}$$

where σ_x^0 is the axial stress and; A_1 and A_2 are the cross sectional areas of the nanobeams.

Employing the theory of nonlocal elasticity (Section 2), the buckling equations of the two compressed nanobeams can be expressed as

nanobeam-1

$$\begin{aligned} &E_1I_1w_1''''(x,t)+k\big[w_1(x,t)-w_2(x,t)\big]\\ &\quad-(e_0a)^2k\big[w_1''(x,t)-w_2''(x,t)\big]+N_1^0w_1''(x,t)\\ &\quad-(e_0a)^2N_1^0w_1''''(x,t)=0; \end{aligned} \tag{9}$$

nanobeam-2

$$E_2I_2w_2''''(x,t) - k\big[w_1(x,t) - w_2(x,t)\big] + (e_0a)^2k\big[w_1''(x,t) - w_2''(x,t)\big] + N_2^0 w_2''(x,t) - (e_0a)^2 N_2^0 w_2''''(x,t) = 0. \tag{10}$$

Here prime (′) denote partial derivatives with respect position coordinate x. For the complete derivation of the equation of a single nonlocal Euler–Bernoulli beam, one can see Ref. [22].

For simplifying the analysis for sake of clarity, we assume that both the nanobeams and the compressive axial forces are identical, i.e.

$$E_1I_1 = (E_2I_2) = EI \equiv \text{constant}, \tag{11}$$

$$N_1^0 = N_2^0 = \hat{N} \equiv \text{constant} \quad \text{(uniformly prestressed)}. \tag{12}$$

Considering Eqs. (9)–(10) and assumptions from Eqs. (11)–(12), we obtain the individual buckling equations for the nanobeam:

nanobeam-1

$$EIw_1''''(x,t) + k\big[w_1(x,t) - w_2(x,t)\big] - (e_0a)^2k\big[w_1''(x,t) - w_2''(x,t)\big] + \hat{N}w_1''(x,t) - (e_0a)^2\hat{N}w_1''''(x,t) = 0; \tag{13}$$

nanobeam-2

$$EIw_2''''(x,t) - k\big[w_1(x,t) - w_2(x,t)\big] + (e_0a)^2k\big[w_1''(x,t) - w_2''(x,t)\big] + \hat{N}w_2''(x,t) - (e_0a)^2\hat{N}w_2''''(x,t) = 0. \tag{14}$$

For the NDNBS we propose a change of variables

$$w(x,t) = w_1(x,t) - w_2(x,t) \tag{15}$$

such that

$$w_1(x,t) = w(x,t) + w_2(x,t). \tag{16}$$

Here $w(x,t)$ is the relative displacement of the nanobeam-1 with respect to the nanobeam-2.

Subtracting Eq. (13) from Eq. (14) gives

$$EI\big[w_1''''(x,t) - w_2''''(x,t)\big] + 2k\big[w_1(x,t) - w_2(x,t)\big] - 2(e_0a)^2k\big[w_1''(x,t) - w_2''(x,t)\big] + \hat{N}\big(w_1''(x,t) - w_2''(x,t)\big) - (e_0a)^2\hat{N}\big(w_1''''(x,t) - w_2''''(x,t)\big) = 0. \tag{17}$$

By introducing Eq. (15) using Eq. (17) we obtain two equations

$$EIw''''(x,t) + 2kw(x,t) - 2(e_0a)^2kw''(x,t) + \hat{N}w''(x,t) - (e_0a)^2\hat{N}w''''(x,t) = 0, \tag{18}$$

$$EIw_2''''(x,t) + \hat{N}w_2''(x,t) - (e_0a)^2\hat{N}w_2''''(x,t) = kw(x,t) - (e_0a)^2kw''(x,t). \tag{19}$$

We see that when the nonlocal effects are ignored ($e_0a = 0$) and a single nanobeam is considered, the above equations (Eqs. (18) and (19)) revert to the equations of classical Euler–Bernoulli beam theory. For the present analysis of coupled NDNBS, we see the simplicity in using Eq. (18). Here we will be dealing with Eq. (18) for coupled NDNBS.

4. Nonlocal boundary conditions of NDNBS

Now we present the mathematical expressions of the boundary conditions in NDNBS. The boundary conditions within the frame of nonlocal elasticity for the simply supported case are described here. At each end of the nanobeams in NDNBS the displacement and the nonlocal moments are considered to be zero [22]. They can be mathematically expressed as

(*nanobeam*-1): at $x = 0$

$$w_1(0,t) = 0, \tag{20}$$

$$M_1(0,t) = -EIw_1''(0,t) + (e_0a)^2k\big[w_1(0,t) - w_2(0,t)\big] + (e_0a)^2\hat{N}w_1''(0,t) = 0; \tag{21}$$

(*nanobeam*-1): at $x = L$

$$w_1(L,t) = 0, \tag{22}$$

$$M_1(L,t) = -EIw_1''(L,t) + (e_0a)^2k\big[w_1(L,t) - w_2(L,t)\big] + (e_0a)^2\hat{N}w_1''(L,t) = 0; \tag{23}$$

(*nanobeam*-2): at $x = 0$

$$w_2(0,t) = 0, \tag{24}$$

$$M_2(0,t) = -EIw_2''(0,t) - (e_0a)^2k\big[w_1(0,t) - w_2(0,t)\big] + (e_0a)^2\hat{N}w_2''(0,t) = 0; \tag{25}$$

(*nanobeam*-2): at $x = L$

$$w_2(L,t) = 0, \tag{26}$$

$$M_2(L,t) = -EIw_2''(L,t) - (e_0a)^2k\big[w_1(L,t) - w_2(L,t)\big] + (e_0a)^2\hat{N}w_2''(L,t) = 0. \tag{27}$$

Now we utilise Eq. (15); and the above boundary conditions simplify to

NDNBS: at $x = 0$

$$w(0,t) = w_1(0,t) - w_2(0,t) = 0, \tag{28}$$

$$\begin{aligned} M_1(0,t) - M_2(0,t) &= -EIw_1''(0,t) + (e_0a)^2\big[k\big[w_1(0,t) - w_2(0,t)\big]\big] \\ &\quad + EIw_2''(0,t) - (e_0a)^2\big[-k\big[w_1(0,t) - w_2(0,t)\big]\big] \\ &\quad + (e_0a)^2\hat{N}\big[w_1''(0,t) - w_2(0,t)''\big] = 0; \end{aligned} \tag{29}$$

NDNBS: at $x = L$

$$w(L,t) = w_1(L,t) - w_2(L,t) = 0, \tag{30}$$

$$\begin{aligned} M_1(L,t) - M_2(L,t) &= -EIw_1''(L,t) + (e_0a)^2\big[k\big[w_1(L,t) - w_2(L,t)\big]\big] \\ &\quad + EIw_2''(L,t) - (e_0a)^2\big[-k\big[w_1(L,t) - w_2(L,t)\big]\big] \\ &\quad + (e_0a)^2\hat{N}\big[w_1''(L,t) - w_2''(L,t)\big] = 0. \end{aligned} \tag{31}$$

By the use of Eq. (15) and Eqs. (20)–(31), the boundary conditions effectively reduce to

$$w(0) = 0 \quad \text{and} \quad w''(0) = 0, \qquad w(L) = 0 \quad \text{and} \quad w''(L) = 0. \tag{32}$$

Here it can be seen that the boundary conditions due to local elasticity and nonlocal elasticity are equivalent.

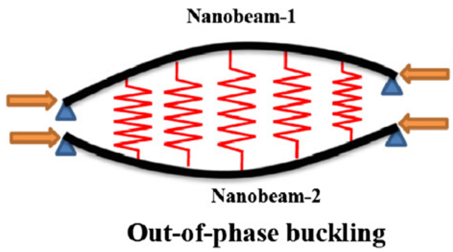

Fig. 2. Out-of-phase buckling of double-nanobeam-system.

5. Different buckling states of double-nanobeam-system

We consider different cases of nonlocal buckling in the NDBNS, i.e. when the nanobeams buckle with out-of-phase (asynchronous); in-phase (synchronous) sequence; and when one of the nanobeams is considered to be fixed.

5.1. Out-of-phase buckling state: $(w_1 - w_2 \neq 0)$

The configuration of NDNBS with the out-of-phase sequence of buckling $(w_1 - w_2 \neq 0)$ is shown in Fig. 2 (first out-of-phase buckling phenomenon). In out-of-phase sequence of buckling the nanobeams is buckled in opposite directions. In this section we evaluate the critical buckling load for the out-of-phase type buckling. Using the boundary condition (Eq. (32)), let us assume that the buckling mode is given as

$$w = W \sin\left(\frac{m\pi}{L}\right)x. \tag{33}$$

Substituting Eq. (33) into Eq. (18) yields

$$EI\left(\frac{m\pi}{L}\right)^4 w + 2kw + 2(e_0a)^2k\left(\frac{m\pi}{L}\right)^2 w - \hat{N}\left(\frac{m\pi}{L}\right)^2 w - (e_0a)^2\hat{N}\left(\frac{m\pi}{L}\right)^4 w = 0. \tag{34}$$

We introduce the following parameters for sake of simplicity and generality

$$K = \frac{kL^4}{EI}, \qquad \mu = \frac{e_0a}{L}, \qquad F = \frac{\hat{N}L^2}{EI}. \tag{35}$$

Using Eqs. (34) and (35) we get the expression of buckling load F in out-of-phase sequence as

$$F = \frac{(m\pi)^4 + 2K + 2K(\mu)^2(m\pi)^2}{(m\pi)^2[1 + (\mu)^2(m\pi)^2]}, \quad m = 1, 2, 3, \ldots. \tag{36}$$

5.2. In-phase buckling state: $(w_1 - w_2 = 0)$

Here the in-phase (synchronous) sequence of buckling will be considered. The schematic illustration is shown in Fig. 3 (first in-phase type buckling). For the present NDNBS, the relative displacements between the two nanobeams are absent i.e. $(w_1 - w_2 = 0)$ and the nanobeams are buckled in the same direction (synchronous).

In in-phase buckling state, the NDNBS can be considered to be as one of the nanobeam (viz. nanobeam-2). Here we solve Eq. (19) for the in-phase sequence of buckling. We apply the same procedure as earlier for solving Eq. (19). The buckling load of the NDNBS is evaluated as

$$F = \frac{(m\pi)^2}{[1 + (\mu)^2(m\pi)^2]}, \quad m = 1, 2, 3, \ldots. \tag{37}$$

Here we see for this case, the buckling phenomenon in the NDNBS is independent of the stiffness of the connecting springs and therefore the NDNBS can be effectively treated as a single nanobeam.

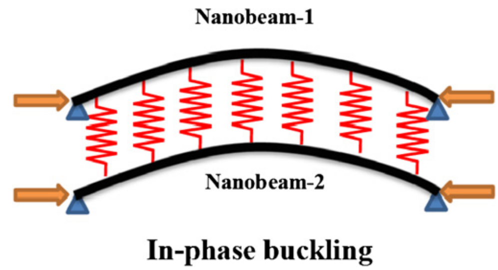

Fig. 3. In-phase buckling of double-nanobeam-system.

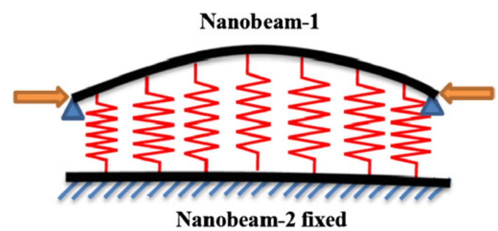

Fig. 4. Buckling of double-nanobeam-system when one nanobeam is fixed.

5.3. One nanobeam is fixed in NDNBS: $(w_2 = 0)$

Consider the case of NDNBS when one of the two nanobeams (viz. nanobeam-2) is stationary $(w_2 = 0)$. The schematic configuration of the NDNBS is shown in Fig. 4.

Using the equations from nonlocal elasticity (Eqs. (1)–(7)), the governing equation for the NDNBS in this case reduces to

$$EIw''''(x,t) + kw - (e_0a)^2kw''(x,t) + \hat{N}w'' - (e_0a)^2\hat{N}w = 0. \tag{38}$$

And the boundary conditions are expressed as

$$w(0) = 0 \quad \text{and} \quad w''(0) = 0, \qquad w(L) = 0 \quad \text{and} \quad w''(L) = 0. \tag{39}$$

Here it is worth noting that in this case, the NDNBS behaves as if nanobeam embedded or supported on an elastic medium or other forces in nanoscale (Fig. 4). The elastic medium can be modelled as Winkler elastic foundation. The stiffness of the elastic medium is denoted by k. By following the same procedure as solution of Eq. (18), the explicit nonlocal buckling load of NDNBS can be easily obtained. The buckling load is evaluated as

$$F = \frac{(r\pi)^4 + K + K(\mu)^2(r\pi)^2}{(r\pi)^2[1 + (\mu)^2(r\pi)^2]}, \quad r = 1, 2, 3, \ldots. \tag{40}$$

In fact when one of the nanobeam (viz. nanobeam-2) in NDNBS is fixed $(w_2 = 0)$, the NDNBS behaves as nanobeam on an elastic medium.

6. Coupled carbon nanotube systems

The nonlocal theory for NDNBS illustrated here is a generalised theory and can be applied for the instability analysis of coupled carbon nanotubes, double ZnO nanobeam systems and in nanocomposites. The applicability of nonlocal elasticity theory in the analysis of single nanostructures (nanotubes and graphene sheet) has been well established in various previous works [17–33]. Here we present the nonlocal behaviour of double-nanostructure-system.

For the present study we assume two carbon nanotubes being elastically attached by an elastic medium (Fig. 5). The elastic medium is substituted by virtual springs. The properties of the

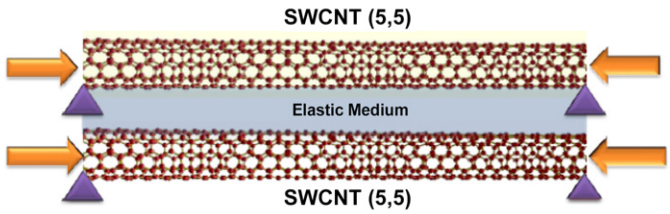

Fig. 5. Schematic diagram of coupled-carbon-nanotubes-system.

nanobeams considered are that of a single-walled carbon nanotube (SWCNT) [37]. An armchair SWCNT with chirality (5, 5) is considered. The radius of each individual SWCNT is assumed as 0.34 nm. Young's modulus, E, is taken as 0.971 TPa [37]. The length is taken as 20 nm. Density ρ is taken as 2300 kg/m^3. The buckling load of the NDNBS is presented in terms of the load parameters (Eq. (35)). The nonlocal parameter and the stiffness of the springs are computed as given in Eq. (35). Spring stiffness represents the stiffness of the enclosing elastic medium. Different values of spring stiffness parameters, K, are considered. This is because the elastic medium can be of low as well as of high stiffness. The values of K range from 1 to 100. Both the nanotubes (nanotube-1 and nanotube-2) are assumed to have the same geometrical and material properties. It should be noted that the coupled carbon nanotubes system is different from the conventional double-walled carbon nanotubes.

The nonlocal parameters are generally taken as $e_0 = 0.39$ [16] and $a = 0.142$ nm (distance between carbon–carbon atoms). For carbon nanotubes and graphene sheets the range of $e_0a =$ 0–2.0 nm has been widely used. For generality, in the present study we take the scale coefficient μ or nonlocal parameter in the range as $\mu = 0$–1 [21].

7. Results and discussions on the scale-dependent buckling phenomenon

To show the influence of small-scale on the critical buckling load of the coupled-carbon-nanotube-systems subjected to compressive forces, curves have been plotted for the buckling load against the scale coefficient (nonlocal parameter, μ). To quantify the small-scale effect we introduce a parameter Buckling Load Reduction Percent (BLRP). Buckling Load Reduction Percent (BLRP) is defined as

$$\text{BLRP} = \frac{F_{\text{Local Theory}} - F_{\text{Nonlocal Theory}}}{F_{\text{Local Theory}}} \times 100. \tag{41}$$

Fig. 6 shows the variation of Buckling Load Reduction Percent (BLRP) against the scale coefficient (μ). From Fig. 6 it can be observed that as the scale coefficient μ increases the BLRP increases. This implies that for increasing scale coefficient the value of buckling load decreases. The reduction in buckling load is due to the incorporation of nonlocal effects in the material properties of the carbon nanotubes. The nonlocal effect reduces the stiffness of the material and hence the comparative lower buckling loads.

Three cases of axial instability are considered here:

Case 1: out-of-phase buckling (asynchronous);
Case 2: buckling with one SWCNT fixed;
Case 3: in-phase (synchronous) buckling.

The stiffness parameter of the coupling springs between SWCNT is assumed to be $K = 30$. Comparing the three cases of coupled-carbon nanobeam system, we observe that the BLRP for case 3 (in-phase buckling) is larger than the BLRP for case 1 (out of phase buckling) and case 2 (one-SWCNT fixed). In other words, the scale coefficient significantly reduces the in-phase buckling load (thus higher BLRP) compared to other cases considered. The relative higher BLRP in case 3 (synchronous buckling) is due to the absence of coupling effect of the spring and the two nanobeams (nanotubes). In addition it can be seen that the values of the BLRP for case 2 (one-SWCNT fixed) is larger than the values of the BLRP for case 1 (out of phase buckling). For case 2 the coupled carbon nanotubes system becomes similar to the buckling phenomenon of the single SWCNT with the effect of elastic medium.

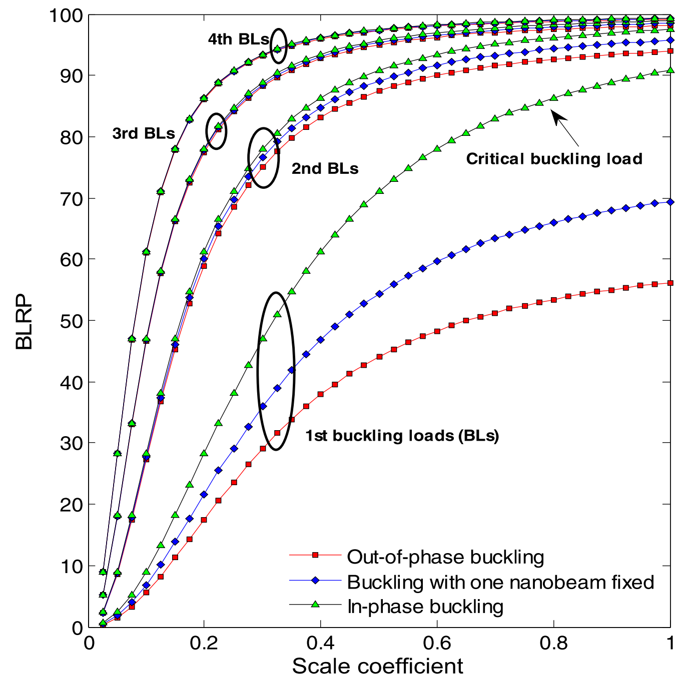

Fig. 6. Effect of scale-coefficient ($\mu = e_0a/L$) on Buckling Load Reduction Percent (BLRP) for critical and higher buckling loads in coupled-SWCNT-systems.

To see the influence of small-scale effects on the higher buckling loads of the coupled system, curves have also been plotted for BLRP against the scale coefficient with higher buckling loads. From Fig. 6 we observe that with the increase of higher buckling loads, the BLRP for all case of buckling phenomenon increases. This implies that the higher buckling loads in the coupled nano-system are significantly reduced due to the nonlocal effects. These results are in-line with earlier instability results on nonlocal elasticity [22]. Further, it can be also noticed that the difference between the in-phase type buckling state, out-of-phase type buckling and buckling with one SWCNT fixed become less for higher modes of buckling loads. Thus it can be concluded that although the small scale effects are more in higher modes of buckling (modes), the effect of stiffness of coupling springs reduces the nonlocal effects. Also, it should be noted that the lowest buckling mode (critical buckling load) is the buckling mode in first in-phase type buckling (Fig. 6). The in-phase and out-of-phase buckling modes are sometimes referred as sub modes.

To illustrate the influence of stiffness of the springs (elastic medium) on the critical buckling load of the coupled-carbon-nanotube-systems, curves have been plotted for the buckling load against the scale coefficient. Spring stiffness represents the stiffness of the enclosing elastic medium. Different values of stiffness parameter of the coupling springs are considered. Fig. 7(a)–(f) depicts the stiffness of the springs on the critical buckling loads of coupled systems. The stiffness parameter of the coupling springs are taken as $K = 2$, 10, 20, 30, 50, 100. As the stiffness parameter of the coupling springs increases the BLRP decreases. Considering all values of the stiffness parameter; and comparing the three cases of coupled-carbon nanobeam system, it is noticed that the BLRP for case 3 (in-phase buckling) is larger than the BLRP

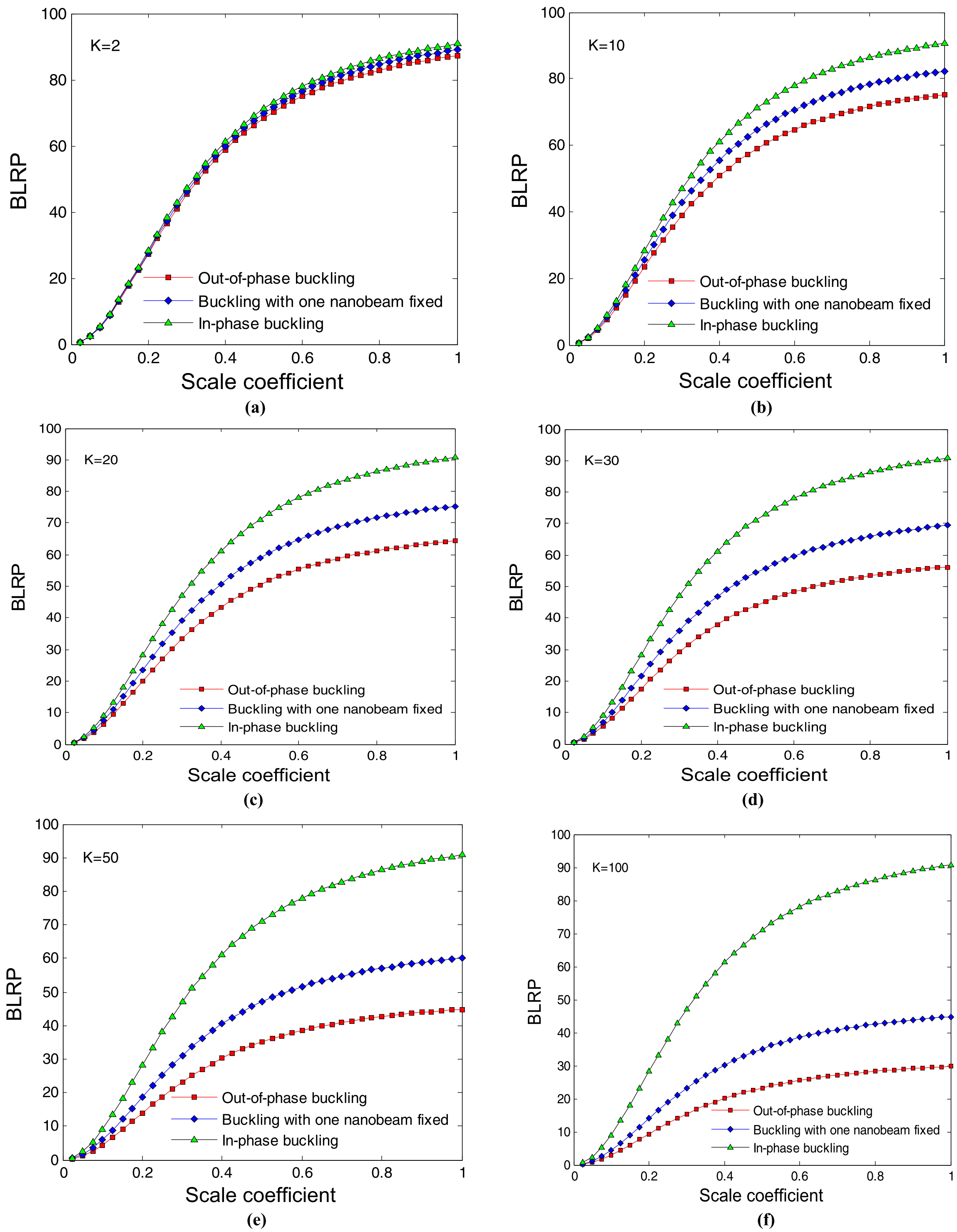

Fig. 7. Effect of scale-coefficient ($\mu = e_0a/L$) on Buckling Load Reduction Percent (BLRP) for higher different values of stiffness (K) of springs in coupled-SWCNT-systems; (a) $K = 2$; (b) $K = 10$; (c) $K = 20$; (d) $K = 30$; (e) $K = 50$; (f) $K = 100$.

for case 1 (out of phase buckling) and case 2 (one-SWCNT fixed). These different changes of BLRP with the increasing scale coefficient for the three different cases are more amplified as the stiffness parameter of the spring's increases. For case 1 (out-of-phase buckling) and case 2 (one SWCNT fixed), the BLRP reduces with increasing values of stiffness parameter. This observation implies that case 1 (out-of-phase buckling) and case 2 (one SWCNT fixed) are less affected by scale-effects. Comparing case 1 and case 2 it can be seen the BLRP is lesser for out of phase buckling than for buckling in case 2. Thus the out-of-phase buckling phenomenon is less affected by the small-scale or nonlocal effects. This out-of-phase buckling phenomenon can be attributed to the fact that the coupling springs in the vibrating system dampens the nonlocal effects. In-phase buckling of coupled-system is unchangeable with increasing stiffness of springs. This is accounted due to the in-phase buckling mode of behaviour. For in-phase type of buckling the coupled system behaves as if a single SWCNT without the effect of internal elastic medium. In other words the whole coupled

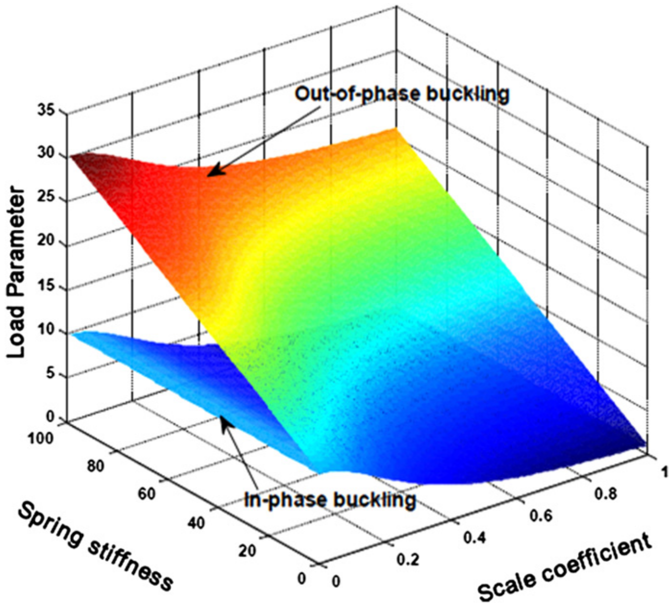

Fig. 8. The variation of the buckling load for the 1st in-phase and out-of-phase buckling of NDNBS as a function of the spring stiffness (K) and the scale-coefficient (μ).

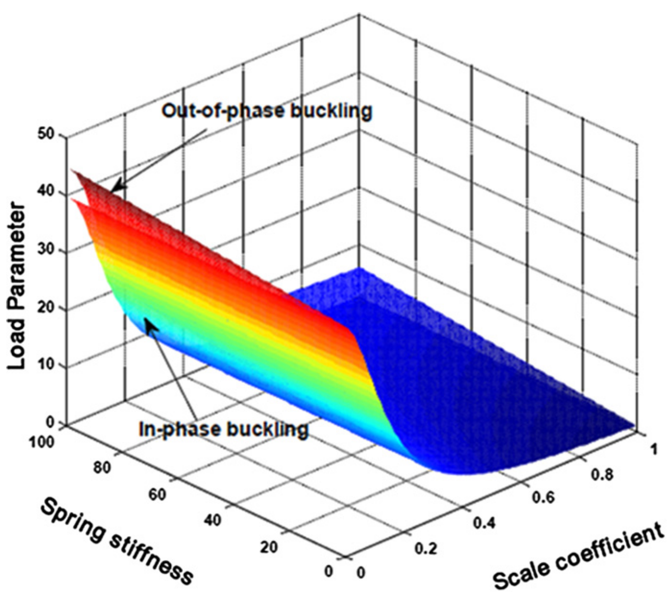

Fig. 9. The variation of the buckling load for the 2nd in-phase and out-of-phase buckling of NDNBS as a function of the spring stiffness (K) and the scale-coefficient (μ).

system can be treated as a single nano element and the coupling internal structure is effect less.

Finally, for the comprehension of the buckling phenomenon of NDNBS with small-scale effects, the buckling load parameter F of the double nanobeam system for different nonlocal parameters (scale coefficient) and spring stiffness parameter (K) are plotted in a three-dimensional graph. The stiffness parameter K of the coupling springs are considered in the range of 0–100. Figs. 8, 9 and 10 show the plots for the first, second and third in-phase and out-of-phase buckling phenomenon respectively. The nonlocal parameter or scale coefficient μ is varied from 0 to 1. From Fig. 8 it is observed that the increase of the scale coefficient has a reducing effect on the nonlocal buckling load of the NDNBS. However this reduction cannot be experienced at higher stiffness parameters values. The stiffness of the springs has a reducing effect on

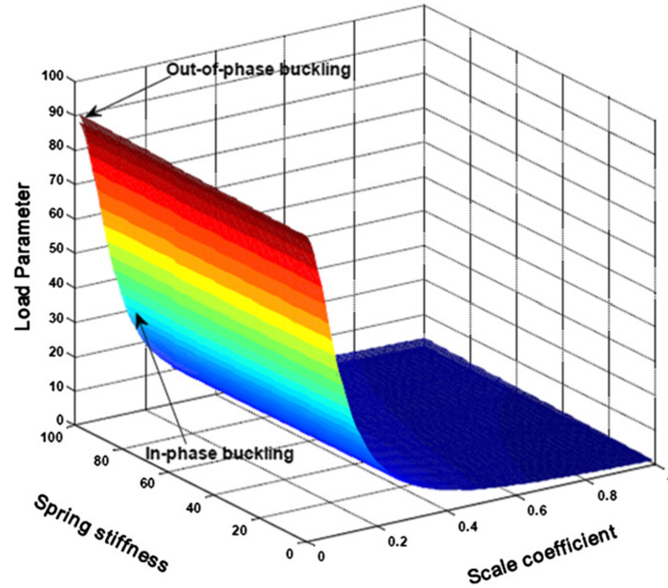

Fig. 10. The variation of the buckling load for the 3rd in-phase and out-of-phase buckling of NDNBS as a function of the spring stiffness (K) and the scale-coefficient (μ).

the small-scale effects of the NDNBS. It should be noted that for in-phase buckling, the load are independent of coupling springs. Fig. 9 shows the plot for the second in-phase and out-of-phase buckling respectively. Compared to the first buckling loads (Fig. 8) the buckling loads for second modes (Fig. 9) are more affected by the scale effects. This is in line with the buckling of 'uncoupled' single nanobeam. In addition it is observed that the stiffness effect is less in both second in-phase and out-of-phase modes of buckling. The stiffness effect is further reduced between the third in-phase and out-of-phase buckling loads (Fig. 10). Thus it can be concluded that although the small scale effects are more in higher modes, the effect of stiffness of coupling springs reduces the nonlocal effects.

8. Conclusions

Axial elastic instability study of nonlocal double-nanobeam-system is presented. Double single-walled carbon nanotube system coupled by elastic medium is considered. Scale-effects in the in-phase (synchronous) and the out-of-phase (asynchronous) buckling phenomenon are examined in details. The study shows that nonlocal or scale effects are important in the instability of double-nanobeam-system. Nonlocal effects reduce the buckling loads in the system. Increasing the stiffness of the springs in coupled nano systems reduces the nonlocal effects. The small-scale or nonlocal effects in coupled nano systems are more prominent with the increasing scale coefficient in the in-phase buckling than in the out-of-phase condition. The buckling loads are independent of the stiffness of the springs in the in-phase type buckling. On the other hand, the buckling load of coupled nano systems in the out-of-phase buckling increase with the increasing stiffness parameter of the coupling springs. The nonlocal theory for nonlocal double-nanobeam-system illustrated here is a generalised theory and can be applied for the instability analysis of short coupled carbon nanotubes, double ZnO nanobeam systems and complex nano-systems.

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