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Nonlocal vibration of bonded double-nanoplate-systems

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ABSTRACT

Nonlocal vibration of a double-nanoplate-system is considered. The two nanoplates are assumed to be bonded by an enclosing elastic medium. Situation of this type would arise in multiple graphene sheets dispersed in nanocomposites. Expressions for free bending-vibration of double-nanoplate-system are established utilising nonlocal elasticity. An analytical method is introduced for determining the natural frequencies of nonlocal double-nanoplate-system (NDNPS). Explicit closed-form expressions for natural frequencies are derived for the case when all four ends are simply-supported. Two single-layered graphene sheets coupled within bonding polymer matrix are considered. The study highlights that the small-scale effects considerably influence the transverse vibration of NDNPS. The small-scale effects in NDNPS are higher with the increasing values of nonlocal parameter for the case of synchronous modes of vibration than in the asynchronous modes. The increase of the stiffness of the coupling springs in NDNPS reduces the small-scale effects during the asynchronous modes of vibration. Further, the effect of aspect ratio and higher modes on the natural frequencies of NDNPS is studied in this manuscript. Present work may provide an analytical scale-based nonlocal approach which could serve as the starting point for further investigation of more complex *n*-nanoplates systems arising in future generation graphene based nanocomposites.

1. Introduction

For realistic designing of nanodevices and nanocomposites [1–3] one must incorporate the small-scale effects and the atomic forces in the analysis of nano-components (carbon nanotubes, graphene) to achieve solutions with acceptable accuracy. At the small-scale, the sizes of nanostructures often become prominent. Both experimental [4–7] and atomistic simulation [8] results have shown a significant 'size-effect' in the mechanical properties when the dimensions of these structures become small. Ignoring the small-scale effects and the atomic forces in sensitive nano-designing fields may cause completely incorrect solutions and hence erroneous designs. Atomistic methods such as molecular mechanics simulation [9] are able to capture the small-scale effects and atomic forces. However these approaches are computationally prohibitive for mechanical analyses of nanostructures with large numbers of atoms. Thus analyses have been generally carried out by using the classical mechanics [10]. Though classical mechanics delivers reasonable solutions, it is independent of small-scale effects and thus may not be always reliable for analysis of nanostructures.

Recently one such promising theory which contains information about the forces between atoms, and the internal length scale is the nonlocal elasticity theory [11]. Nonlocal elasticity theory has been applied in various structural studies of nanostructures. Nonlocal elasticity accounts for the small-scale effects arising at the nanoscale level. Recent literature shows that the theory of nonlocal elasticity is being increasingly used for reliable and fast analysis of nanostructures [12–50].

A large amount of investigations have been conducted on one dimensional nanostructure such as CNTs using nonlocal elasticity [13,15,18–20,28,32,33,36–38,41,43,44,46–48]. However, compared to one-dimensional nonlocal nanostructures (nanobeams and nanorods), very limited number of studies have been reported on the nonlocal nanoplates (graphene sheets), even though nanoplates possesses many superior properties. Nanoplates such as graphene would be one of the prominent new materials for the next generation nano-electronic devices. Reports related to its use as strain sensor, mass and pressure sensors, atomic dust detectors, enhancer of surface image resolution are observed. Studies on graphene sheets include vibration studies on single-walled and multi-walled graphene sheets.

It is reported that the study of transversal vibration of an elastically connected double-plate system is important for both theoretical and practical reasons [51]. Many important structures can be modelled as composite structures. Similar to macro plates, an important technological extension of the concept of the single-nanoplate-system would be that of the complex-nanoplate-systems. Complex-nanoplate-systems may find applications in nanooptomechanical systems (NOMS). Vibration of double-nanobeam systems in NOMS is reported [52–55]. Vibration analysis of double-nanoplate systems with small aspect ratio is very relevant

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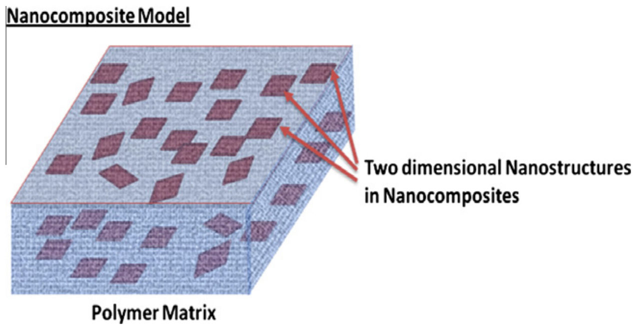

Fig. 1. Nanoplates dispersed in polymer matrix in nanocomposites.

to NOMS. Elastically connected double-nanoplate system can also be used for the acoustic and vibration isolation.

Further complex-nanoplate-systems can be important in nanosensors applications and in nanocomposites. Vibration characteristics of multiple nanoplates (graphene sheets) dispersed in nanocomposites can be important from the structural performance of nanocomposites [56–58]. Vibration of double-nanoplate-system coupled by elastic medium is worth understanding in this respect. The different stiffness of elastic medium will impart different vibration characteristic of the system in nanocomposites. A schematic model of nanocomposites is shown in Fig. 1. Though complex-nanoplate-systems are important in nanodevices and nanocomposites, no works appear related to the study of its vibration characteristics.

Therefore, based on the above discussion there is a strong encouragement to gain an understanding of the vibration of complex-nanoplate-system and the mathematical modelling of such phenomena. In the present paper attempt is made to study the vibration of double-nanoplates-systems. The two nanoplates are elastically bonded by enclosing elastic medium. It should be noted that similar model is developed for a different problem of double-layered [14] and multi-layered nanoplates [30] with constant van der Waals forces between nanoplates. Here we will study the effect of bonding elastic medium. The elastic medium is modelled by vertical springs. Expressions for free bending-vibration of double-nanoplate-system are established within the framework of Eringen's nonlocal elasticity by updating the Kirchhoff's plate theory. An analytical method is introduced for determining the natural frequencies of nonlocal double-nanoplate-system (NDNPS). Explicit closed-form expressions for natural frequencies are derived for the case when all four ends are simply-supported. Further, this paper presents a unique yet simple method of obtaining the exact solution for the free vibration of double-nanoplate system. The simplification in the computation is achieved based on the change of variables to decouple the set of two fourth-order partial differential equations. Two single-layered graphene sheets enclosed by polymer matrix are considered for the study. Isotropic assumption of graphene sheet is considered in the study. The study highlights that the small-scale or nonlocal effects considerably influence the transverse vibration of NDNPS. Further, the effect of aspect ratio and higher modes on the natural frequencies of NDNPS is also investigated. In summary, this work may provide an analytical scale-based nonlocal approach which could serve as the starting point for further investigation of more complex n-nanoplates systems for nanocomposites.

2. Review of nonlocal plate theory

In the nonlocal elasticity theory, the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain. Nonlocal theory considers long-range inter-atomic interaction and yields results dependent on the size of a body. According to the nonlocal elasticity, the basic equations for an isotropic linear homogenous nonlocal elastic body neglecting the body force are given as [11]

$$\begin{aligned} &\sigma_{ij,j} = 0, \\ &\sigma_{ij}(\mathbf{x}) = \int_v \phi(|\mathbf{x}-\mathbf{x}'|,\alpha) t_{ij} d\mathbf{V}(\mathbf{x}'), \forall \mathbf{x} \in \mathbf{V} \\ &t_{ij} = H_{ijkl}\varepsilon_{kl}, \\ &\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) \end{aligned} \tag{1}$$

The terms σ_{ij}, t_{ij}, ε_{kl}, H_{ijkl} are the nonlocal stress, classical stress, classical strain and fourth order elasticity tensors respectively. The volume integral is over the region **V** occupied by the body. The above equation (Eq. (1)) couples the stress due to nonlocal elasticity and the stress due to classical elasticity. The kernel function $\phi(|\mathbf{x}-\mathbf{x}'|,\alpha)$ is the nonlocal modulus. The nonlocal modulus acts as an attenuation function incorporating into constitutive equations the nonlocal effects at the reference point **x** produced by local strain at the source $\mathbf{x}'$. The term $|\mathbf{x}-\mathbf{x}'|$ represents the distance in the Euclidean form and α is a material constant that depends on the internal (e.g. lattice parameter, granular size, distance between the C—C bonds) and external characteristics lengths (e.g. crack length, wave length). Material constant α is defined as $\alpha = e_0 a/\ell$ Here e_0 is a constant for calibrating the model with experimental results and other validated models. The parameter e_0 is estimated such that the relations of the nonlocal elasticity model could provide satisfactory approximation to the atomic dispersion curves of the plane waves with those obtained from the atomistic lattice dynamics. The terms a and ℓ are the internal (e.g. lattice parameter, granular size, distance between C—C bonds) and external characteristics lengths (e.g. crack length, wave length) of the nanostructure.

Eq. (1) is in partial-integral form and generally difficult to solve analytically. Thus a differential form of nonlocal elasticity equation is often used. According to Eringen, the expression of nonlocal modulus can be given as [11]

$$\phi(|\mathbf{x}|,\alpha) = (2\pi\ell^2\alpha^2)^{-1} K_0(\sqrt{\mathbf{x}.\mathbf{x}}/\ell\alpha) \tag{2}$$

where K_0 is the modified Bessel function.

The equation of motion in terms of nonlocal elasticity can be expressed as

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i \tag{3}$$

where f_i, ρ and u_i are the components of the body forces, mass density, and the displacement vector, respectively. The terms i,j take up the symbols x,y and z.

Assuming the kernel function ϕ as the Green's function, Eringen [11] proposed a differential form of the nonlocal constitutive relation as

$$\sigma_{ij,j} + \mathcal{L}(f_i - \rho\ddot{u}_i) = 0 \tag{4}$$

where

$$\mathcal{L} = [1 - (e_0 a)^2 \nabla^2] \tag{5}$$

and ∇^2 is the Laplace operator and is represented by $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Using Eq. (5) the nonlocal constitutive stress–strain relation can be simplified as [11,31]

$$(1 - \alpha^2\ell^2\nabla^2)\sigma_{ij} = t_{ij} \tag{6}$$

Employing the nonlocal constitutive stress–strain relation (Eq. (6)), the equation of motion of a nonlocal nanoplate can be derived as

$$D\nabla^4 w(x,y,t) - q + \rho h \ddot{w}(x,y,t) + (e_0 a)^2 \nabla^2 q - \rho h (e_0 a)^2 (\nabla)^2 \ddot{w}(x,y,t) = 0 \tag{7}$$

where

$$\nabla^4 = (\nabla^2)^2 = \frac{\partial^4}{\partial x^4} + 2\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} \tag{8}$$

and w denotes the deflection of the nonlocal plate. The terms D and ρ are the bending rigidity and density of the nonlocal plate, respectively. Term q is the distributed transverse load on the nonlocal plate.

3. Equations of motion for nonlocal double-nanoplate-system (NDNPS)

Consider the nonlocal double nanoplate system (NDNPS) as shown in Fig. 2a. The two nanoplates of the NDNPS are referred to as nanoplate-1 and nanoplate-2. The two nanoplates are coupled by an elastic medium (polymer matrix). Here we will develop equations based on the change of variables. For mathematical modelling it is assumed that vertically distributed identical Winkler springs couples the two nanoplates (Fig. 2b). In generality the springs may be used to substitute the elastic medium, forces due to nanooptomechanical effect [52–55], or van der Waals forces between the two nanoplates. The springs are assumed to have stiffness k. Different values of k for different polymer matrix can be used for the study. The nanoplates are considered to be of length L and width W. Generally, the two nanoplates are different where the length, width, mass per unit length and bending rigidity of the ith plate are L_i, W_i, m_i and D_i ($i = 1, 2$) respectively. These parameters are assumed to be constant along each nanoplate.

The bending displacements over the two nanoplates are denoted by $w_1(x,y,t)$ and $w_2(x,y,t)$, respectively (Fig. 2).

Using nonlocal plate equations, i.e. Eq. (7), the individual governing equations for NDNPS can be written as

$$\text{(nanoplate-1)}\ D_1 \nabla^4 w_1(x,y,t) + \rho_1 h_1 \ddot{w}(x,y,t) + k[w_1(x,y,t) - w_2(x,y,t)] - k(e_0 a)^2 \nabla^2 [w_1(x,y,t) - w_2(x,y,t)] - (e_0 a)^2 \rho_1 h_1 \nabla^2 \ddot{w}_1(x,y,t) = f(x,y,t) - (e_0 a)^2 \nabla^2 f(x,y,t) \tag{9}$$

where the bending rigidity of nanoplate-1 can be expressed as

$$D_1 = E_1 h_1^3 / 12(1 - \nu_1^2). \tag{10}$$

The terms E_1, h_1 and ν_1 are the Young's modulus, thickness and Poisson's ratio of the nanoplate. Term $f(x, y, t)$ is the forcing function. Subscript 1 denotes the properties of the nanoplate-1.

$$\text{(nanoplate-2)}\ D_2 \nabla^4 w_2(x,y,t) + \rho_2 h_2 \ddot{w}_2(x,y,t) - k[w_1(x,y,t) - w_2(x,y,t)] + k(e_0 a)^2 \nabla^2 [w_1(x,y,t) - w_2(x,y,t)] - (e_0 a)^2 \rho_2 h_2 \nabla^2 \ddot{w}_2(x,y,t) = 0$$

where the bending rigidity of nanoplate-2 is expressed as

$$D_2 = E_2 h_2^3 / 12(1 - \nu_2^2) \tag{12}$$

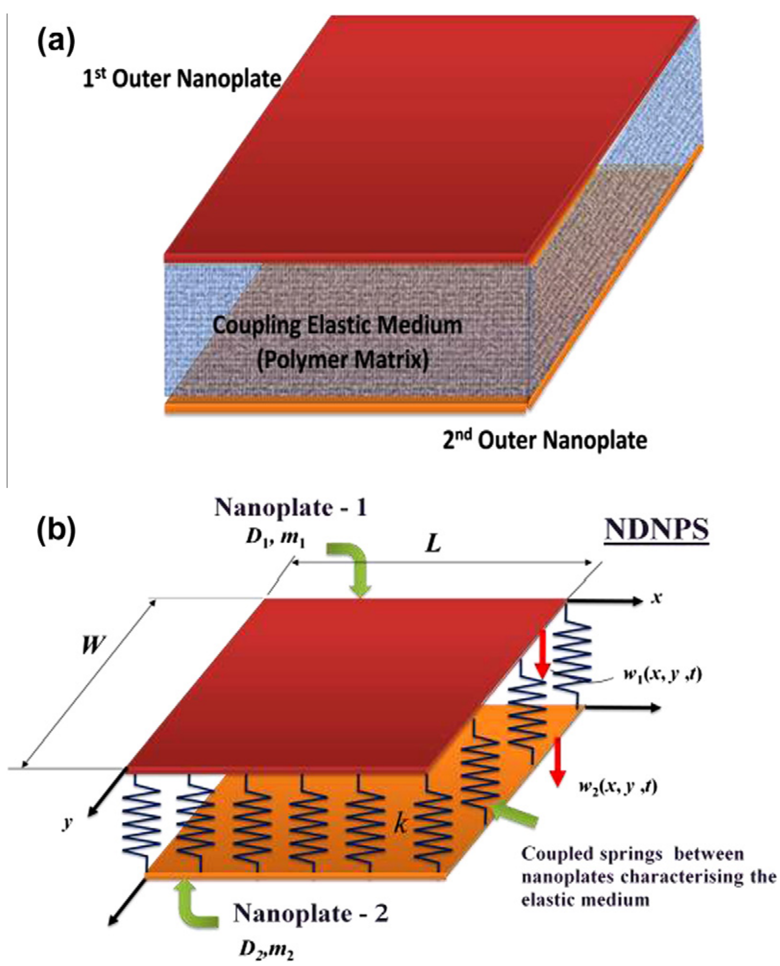

Fig. 2. (a) Double-nanoplate system coupled by an elastic medium. (b) Mathematical idealisation of nanoplate-system characterised by the coupling springs.

For the present analysis we assume that

$$\begin{aligned} &D_1 = D_2 = D \equiv \text{constant} \\ &\rho_1 h_1 = \rho_2 h_2 = \rho h \equiv \text{constant} \\ &f(x,y,t) = 0 \quad (\text{free vibration}) \end{aligned} \tag{13}$$

Substituting the assumptions (Eq. (13)) in Eqs. (9) and (11) we get

$$\begin{aligned} &(\text{nanoplate-1})\ D\nabla^4 w_1(x,y,t) + \rho h \ddot{w}(x,y,t) + k[w_1(x,y,t) \\ &\quad - w_2(x,y,t)] - k(e_0 a)^2 \nabla^2 [w_1(x,y,t) - w_2(x,y,t)] \\ &\quad - (e_0 a)^2 \rho h \nabla^2 \ddot{w}_2(x,y,t) = 0 \end{aligned} \tag{14}$$

$$\begin{aligned} &(\text{nanoplate-2})\ D\nabla^4 w_2(x,y,t) + \rho h \ddot{w}_2(x,y,t) - k[w_1(x,y,t) \\ &\quad - w_2(x,y,t)] + k(e_0 a)^2 \nabla^2 [w_1(x,y,t) - w_2(x,y,t)] \\ &\quad - (e_0 a)^2 \rho h \nabla^2 \ddot{w}(x,y,t) = 0 \end{aligned} \tag{15}$$

Next, for the NDNPS we employ a change of variables by considering *w(x, y, t)* as the relative displacement of the nanoplate-1 with respect to the nanoplate-2:

$$w(x,y,t) = w_1(x,y,t) - w_2(x,y,t), \tag{16}$$

such that for nanoplate-1, the displacement is expressed as

$$w_1(x,y,t) = w(x,y,t) + w_2(x,y,t) \tag{17}$$

Subtracting Eq. (14) and (15) leads to

$$\begin{aligned} &D\nabla^4 [w_1(x,y,t) - w_2(x,y,t)] + 2k[w_1(x,y,t) - w_2(x,y,t)] \\ &\quad + \rho h[\ddot{w}_1(x,y,t) - \ddot{w}_2(x,y,t)] - 2k(e_0 a)^2 \nabla^2 [w_1(x,y,t) - w_2(x,y,t)] \\ &\quad - \rho h (e_0 a)^2 \nabla^2 [\ddot{w}_1(x,y,t) - \ddot{w}_2(x,y,t)] = 0 \end{aligned} \tag{18}$$

The use of Eq. (16) in Eqs. (18) and (15) results

$$\begin{aligned} &D\nabla^4 w(x,y,t) + 2kw(x,y,t) + \rho h \ddot{w}(x,y,t) - 2k(e_0 a)^2 \nabla^2 w(x,y,t) \\ &\quad - \rho h (e_0 a)^2 \nabla^2 \ddot{w}(x,y,t) = 0 \end{aligned} \tag{19}$$

and

$$\begin{aligned} &D\nabla^4 w_2(x,y,t) + \rho h \ddot{w}_2(x,y,t) - (e_0 a)^2 \rho h \nabla^2 \ddot{w}(x,y,t) \\ &\quad = k(e_0 a)^2 w(x,y,t) - k(e_0 a)^2 \nabla^2 w(x,y,t) \end{aligned} \tag{20}$$

For the present analysis of coupled NDNPS, we observe the simplicity in using Eqs. (19) and (20). It should be noted that when the nonlocal effects are ignored ($e_0 a = 0$) and a single nanoplate is considered, the above equations revert to the equations of classical scale-free Kirchhoff's plate theory.

4. Boundary conditions in nonlocal double-nanoplate-system (NDNPS)

Now we present the explicit mathematical expressions of the boundary conditions of the double-nanoplate-system. Different boundary condition can be studied. Here we assume that all the edges in the nanoplate system are simply supported. At each ends of the nanoplates in NDNPS, the displacement and the nonlocal moments are considered to be zero. They can be mathematically expressed as

(nanoplate-1) Displacement condition

$$w_1(0,y,t) = 0;\ w_1(L,y,t) = 0;\ w_1(x,0,t) = 0;\ w_1(x,W,t) = 0; \tag{21}$$

Nonlocal moment condition

$$M_1(0,y,t) = 0;\ M_1(L,y,t) = 0;\ M_1(x,0,t) = 0;\ M_1(x,W,t) = 0; \tag{22}$$

(nanoplate-2) Displacement condition

$$w_2(0,y,t) = 0;\ w_2(L,y,t) = 0;\ w_2(x,0,t) = 0;\ w_2(x,W,t) = 0; \tag{23}$$

Nonlocal moment condition

$$M_2(0,y,t) = 0;\ M_2(L,y,t) = 0;\ M_2(x,0,t) = 0;\ M_2(x,W,t) = 0; \tag{24}$$

Now we use Eq. (16) in the boundary conditions (21)–(24), and obtain

$$w(0,y,t) = w_1(0,y,t) - w_2(0,y,t) = 0 \tag{25a}$$

$$w(L,y,t) = w_1(L,y,t) - w_2(L,y,t) = 0 \tag{25b}$$

$$w(x,0,t) = w_1(x,0,t) - w_2(x,0,t) = 0 \tag{25c}$$

$$w(x,W,t) = w_1(x,W,t) - w_2(x,W,t) = 0 \tag{25d}$$

$$M(0,y,t) = M_1(0,y,t) - M_2(0,y,t) = 0 \tag{26a}$$

$$M(L,y,t) = M_1(L,y,t) - M_2(L,y,t) = 0 \tag{26b}$$

$$M(x,0,t) = M_1(x,0,t) - M_2(x,0,t) = 0 \tag{26c}$$

$$M(x,W,t) = M_1(x,W,t) - M_2(x,W,t) = 0 \tag{26d}$$

Similarly one can obtain the nonlocal boundary conditions for all edges clamped (CCCC) of NDNPS

$$w(0,y,t) = w_1(0,y,t) - w_2(0,y,t) = 0 \tag{27a}$$

$$w(L,y,t) = w_1(L,y,t) - w_2(L,y,t) = 0 \tag{27b}$$

$$w(x,0,t) = w_1(x,0,t) - w_2(x,0,t) = 0 \tag{27c}$$

$$w(x,W,t) = w_1(x,W,t) - w_2(x,W,t) = 0 \tag{27d}$$

$$w'(0,y,t) = w_1'(0,y,t) - w_2'(0,y,t) = 0 \tag{28a}$$

$$w'(L,y,t) = w_1'(L,y,t) - w_2'(L,y,t) = 0 \tag{28b}$$

$$w'(x,0,t) = w_1'(x,0,t) - w_2'(x,0,t) = 0 \tag{28c}$$

$$w'(x,W,t) = w_1'(x,W,t) - w_2'(x,W,t) = 0 \tag{28d}$$

Eqs. (19) and (20) along with boundary conditions (Eqs. (25) and (26)) will yield the frequencies of NDNPS.

5. Exact solutions of the frequency equations

5.1. Both nanoplates of NDNPS are vibrating out-of-phase; ($w_1 - w_2 \neq 0$)

Consider the case of the NDNPS when both the nanoplates are vibrating with in-phase sequence (synchronous) and out-of-phase (asynchronous) sequence. The configuration of the NDNPS with the out-of-phase sequence of vibration ($w_1 - w_2 \neq 0$) is shown in Fig. 3. In this Section we solve the frequency for the out-of-phase (asynchronous) vibration. Here we consider the case when all the ends have simply supported boundary conditions. For the present case we deal with Eq. (19).

This equation system (Eq. (19) with nonlocal boundary conditions (Eqs. (25) and (26)) can be solved by the Navier method assuming the solutions in the form [51]

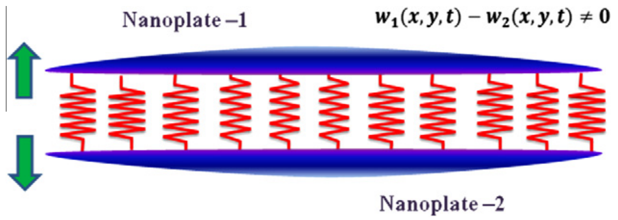

Fig. 3. Out-of-phase type or synchronous vibration of the double-nanoplate-system.

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn}\sin(a_m x)\sin(b_n y)e^{i\omega_{mn}^t} \quad (29)$$

where

$$a_m = m\pi/L, \; b_n = n\pi/W \quad (30)$$

and m and n are half wave numbers.

Substituting Eq. (29) into Eq. (19) yields

$$-D[(m\pi/L)^2 + (n\pi/W)^2]^2 W_{mn} - 2k(1 + (e_0a)^2[(m\pi/L)^2 + (n\pi/W)^2])W_{mn} = -\rho h(1 + (e_0a)^2[(m\pi/L)^2 + (n\pi/W)^2])\omega_{mn}^2 W_{mn} \quad (31)$$

We define the following parameters for the sake of convenience and generality

$$\Omega_{mn} = \omega_{mn}L^2\sqrt{\frac{\rho h}{D}}; \; R = L/W; \; K = \frac{kL^4}{D}, \; \mu = \frac{e_0a}{L} \quad (32)$$

Using the parameters in Eq. (32) and substituting in Eq. (31) we obtain the natural frequencies of the NDNPS for asynchronous vibration as

$$\Omega_{mn} = \sqrt{\frac{[(m\pi)^2 + R^2(n\pi)^2]^2 + 2K + 2K\mu^2[(m\pi)^2 + R^2(n\pi)^2]}{1 + \mu^2[(m\pi)^2 + R^2(n\pi)^2]}}, m, n = 1, 2 \ldots \quad (33)$$

5.2. Both nanoplates of NDNPS are vibrating in-phase ($w_1 - w_2 = 0$)

Next, the in-phase sequence (synchronous) of vibration is considered (Fig. 4). For the present NDNPS, the relative displacements between the two nanoplates are absent($w_1 - w_2 = 0$). Here we solve Eq. (20) for the vibration of NDNPS. The vibration of nanoplate-2 here would represent the vibration of the coupled vibrating system. We apply the same procedure for solving Eq. (20). Using Eq. (29) we can obtain the natural frequencies. The natural frequencies for the NDNPS in this case can be expressed as

$$\Omega_{mn} = \sqrt{\frac{[(m\pi)^2 + R^2(n\pi)^2]^2}{1 + \mu^2[(m\pi)^2 + R^2(n\pi)^2]}} \; m, n = 1, 2, 3 \ldots \quad (34)$$

For this case the vibration, we see that NDNPS is independent of the stiffness of the connecting springs and therefore the NDNPS can be effectively treated as a single nanoplate.

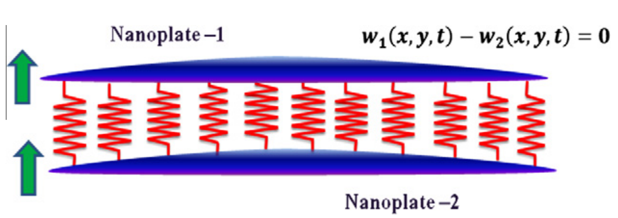

Fig. 4. In-phase type or synchronous vibration of the double-nanoplate-system.

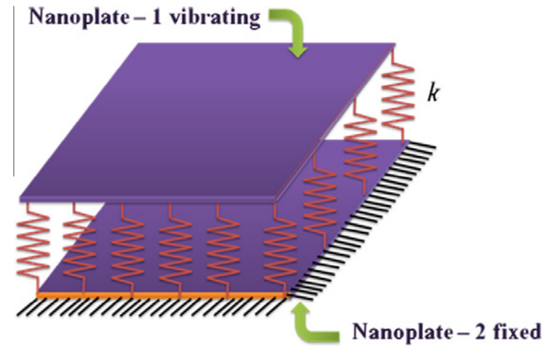

Fig. 5. Vibration of NDNPS with one nanoplate stationary.

5.3. One nanoplate of NDNPS is stationary ($w_2(x, y, t) = 0$)

Consider a special case of NDNPS when one of the two nanoplates (viz. nanoplate-2) is stationary ($w_2 = 0$) The schematic diagram is shown in Fig. 5.For this case, using the equations from nonlocal elasticity (Eqs. (1)–(6)), the governing equation for the NDNPS (Eq. (19)) with Eq. (17) reduces to

$$D\nabla^4 w(x,y,t) + kw(x,y,t) + \rho h\ddot{w}(x,y,t) - k(e_0a)^2\nabla^2 w(x,y,t) - \rho h(e_0a)^2\nabla^2\ddot{w}(x,y,t) = 0 \quad (35)$$

The boundary conditions are expressed as

$$w(0,y,t) = 0; \; w(L,y,t) = 0; \; w(x,0,t) = 0; \; w(x,W,t) = 0; \quad (36)$$

and the nonlocal moment conditions are given by

$$M(0,y,t) = 0; \; M(L,y,t) = 0; \; M(x,0,t) = 0; \; M(x,W,t) = 0; \quad (37)$$

In this case, the NDNPS behaves as if the nanoplate is embedded in an elastic medium. The elastic medium can be modelled as Winkler elastic foundation. The stiffness of the elastic medium is denoted by k. By following the same procedure as solution of Eq. (19), the nonlocal frequency of NDNPS can be explicitly expressed as

$$\Omega_{mn} = \sqrt{\frac{[(m\pi)^2 + R^2(n\pi)^2]^2 + K + K\mu^2[(m\pi)^2 + R^2(n\pi)^2]}{1 + \mu^2[(m\pi)^2 + R^2(n\pi)^2]}} \quad (38)$$

where K is the stiffness parameter of the coupling springs and μ is nondimensional nonlocal parameter as defined in Eq. (32). In fact when one of the nanoplate (viz. nanoplate-2) in NDNPS is fixed ($w_2 = 0$), the NDNPS behaves as a single nanoplate on an elastic medium.

6. Results and discussions

6.1. Bonded double-graphene-sheet-system

The nonlocal plate theory for NDNPS illustrated here is a generalised theory and can be applied for the bending-vibration analysis of coupled graphene sheets, gold nanoplates, etc. The applicability of nonlocal elasticity theory in the analysis of nanostructures (nanotubes and graphene sheet) can be observed in various earlier works [12–50].

As an illustration, the properties of the nanoplates are considered that of a single-layered graphene sheets. The two single-layered graphene sheets (GS) are coupled by embedded polymer as shown in Fig. 6. The Young's modulus of the GS is considered as $E = 1.06$ TPa, the Poisson ratio $\nu = 0.25$, and the mass density as $\rho = 2250$ kg/m^3.The thickness of the GS is taken as $h = 0.34$ nm. Similar use of material and geometrical values can be seen in [59].

The frequency results of the NDNPS are presented in terms of the frequency parameters (Eq. (32)). The nonlocal parameter and the stiffness of the springs are computed as given in Eq. (32). Different values of spring parameters, K, are considered. Spring stiffness represents the stiffness of the enclosing elastic medium. Both high and large stiffness of springs are assumed. Values of K range from 10 to 100. Both the graphene sheets (GS-1 and GS-2) are assumed to have the same geometrical and material properties.

The nonlocal parameters are taken as $e_0 = 0.39$ [11] and $a = 0.142$ nm (distance between carbon-carbon atoms). For carbon nanotubes and graphenes, the range of $e_0a = 0$–2.0 nm has been widely used. In the present study we take the scale coefficient μ or nonlocal parameter in the similar range as $\mu = 0$–1.

6.2. Effect of small-scale on vibrating NDNPS

To see the influence of small-scale on the natural frequency of the coupled-graphene sheet-systems, curves have been plotted for frequency parameter and scale coefficient (nonlocal parameter, μ). The GSs are coupled by a polymer matrix of stiffness $K = 100$. To signify the small-scale effect we introduce a parameter frequency reduction percent (FRP). Frequency reduction percent (FRP) is defined as

$$\mathrm{FRP} = 100 \times \left(\frac{\Omega_{\mathrm{Local}} - \Omega_{\mathrm{Nonocal}}}{\Omega_{\mathrm{Local}}} \right) \tag{39}$$

Fig. 7 shows the variation of the frequency parameter with the scale coefficient for different cases of NDNPS ($m = 1$, $n = 1$). Here NDNPS denotes the coupled-graphene sheet-systems. The results for the frequency parameter Ω are in the dimensionless form as in Eq. (32) .The stiffness parameter of the springs is assumed to be constant ($K = 100$). Unless stated the frequency parameter would denote the parameter associated with the first natural frequencies (in-phase and out-of-phase type vibration). From Fig. 7 it can be observed that as the scale coefficient μ increases the FRP increases. This implies that for increasing scale coefficient the value of frequency parameter decreases. The reduction in frequency parameter is due to the assimilation of small-scale effects in the NDNPS in the material properties of the graphene sheets. The small-scale effect reduces the stiffness of the material and hence the comparative lower frequencies. Therefore by the nonlocal elastic model the size-effects are reflected in the NDNPS.

Three different cases of NDNPS are considered. Case 1, Case2 and Case 3 depicts the conditions (i) when both the GS vibrates in the out-of-phase sequence ($w_1 - w_2 \neq 0$) (ii) when one of the GS in NDNPS is stationary ($w_2 = 0$) (iii) when both the GS vibrates with in-phase sequence($w_1 - w_2 = 0$), respectively. Comparing the three cases of coupled-graphene-sheet-system, we observe that the FRP for case 3 (in-phase vibration) is larger than the FRP for case 1 (out of phase vibration) and case 2 (one-GS fixed). In other words, the scale coefficient significantly reduces the in-phase natural frequencies (thus higher FRP) compared to other cases considered. The relative higher FRP in in-phase vibration is due to the absence of coupling effect of the spring and the two nanoplates (GSs) and the whole NDNPS can be treated as a vibrating single GS. In general, it is worth noticing that the small-scale effects in NDNPS are higher with increasing nonlocal parameter in the in-phase vibration than in the out-of-phase vibration. This is because the stiffness of the springs in the out-of-phase (asynchronous) vibration reduces the nonlocal effects. In addition, it can be seen that the values of the FRP for case-2 (one-GS fixed) is larger than the values of the FRP for case-1 (out-of-phase vibration). For Case-2 the coupled-graphene-sheet-system becomes similar to the vibration characteristic of the single GS with the effect of elastic medium.

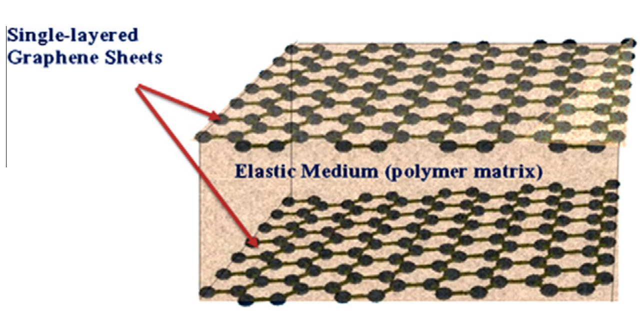

Fig. 6. Coupled graphene sheet system in polymer matrix environment.

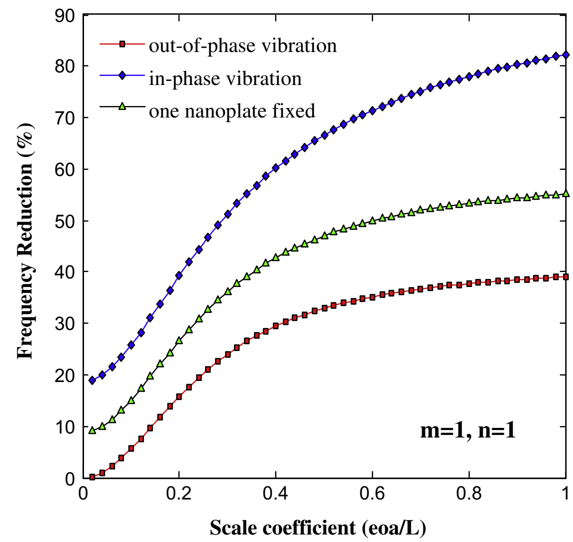

Fig. 7. Change of frequency reduction percent with scale coefficient for out-of-phase, in-phase and one nanoplate fixed in NDNPS.

6.3. Effect of stiffness of coupling springs in NDNPS

To illustrate the influence of stiffness of the springs on the natural frequencies of the coupled-GS-systems, curves have been plotted for the FRP against the scale coefficient. Spring stiffness represents the stiffness of the enclosing elastic medium. Different values of stiffness parameter of the coupling springs are considered. Fig. 8a–f depicts the stiffness of the springs on the FRP of coupled systems. The stiffness parameter of the coupling springs are taken as $K = 0$, 10, 20, 50, 80, 100. As the stiffness parameter of the coupling springs increases the FRP decreases. Aspect ratio is taken as unity. Considering all values of the stiffness parameter; and comparing the three cases of coupled-GS system, it is noticed that the FRP for case 3 (in-phase vibration) is larger than the FRP for case 1 (out of phase vibration) and case 3 (one-GS fixed). These different changes of FRP with the increasing scale coefficient for the three different cases are more amplified as the stiffness parameter of the spring's increases. For case 1(out-of-phase vibration) and case 2 (one-GS-fixed), the FRP reduces with increasing values of stiffness parameter. This observation implies that case 1 (out-of-phase vibration) and case 2 (one GS fixed) are less affected by scale-effects. Comparing case 1 and case 2 it can be seen the FRP is lesser for out of phase vibration than for vibration in case 2. Thus the out-of-phase vibration is less affected by the small-scale or nonlocal effects. This out-of-phase vibration can be attributed to the fact that the coupling springs in the vibrating system dampens the nonlocal effects.

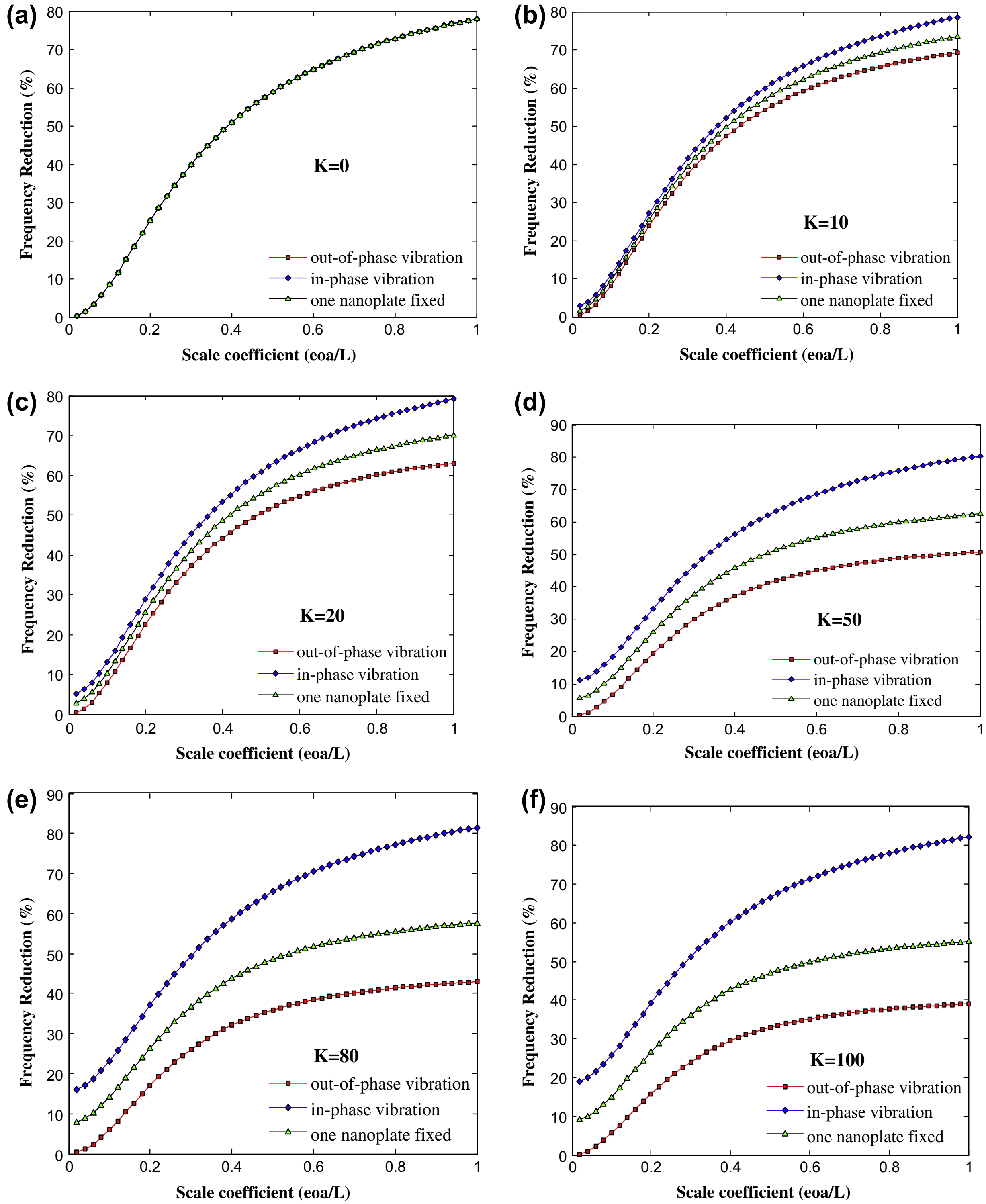

Fig. 8. Change of frequency reduction percent with scale coefficient for different coupling springs in NDNPS; (a) $K = 0$, (b) $K = 10$, (c) $K = 20$, (d) $K = 50$, (e) $K = 80$, (f) $K = 100$.

In-phase vibration of coupled-system is unchangeable with increasing stiffness of springs. This is accounted due to the in-phase vibration mode of behaviour. For in-phase type of vibration the coupled system behaves as if a single SWGS without the effect of internal elastic medium. In other words the whole coupled system can be treated as a single nano element and the coupling internal structure is effect less. In summary, it should be noted that the in-phase vibration of coupled-system are more affected by small-scale effects compared to out-of-phase vibration.

6.4. Effect of aspect ratio on NDNPS

Next we illustrate the influence of aspect ratios (L/W) of the nanoplates (GS) on the natural frequencies of the coupled-GS-systems. The GSs are assumed to be coupled by a polymer matrix of stiffness $K = 100$. Curves have been plotted for the FRP against the scale coefficient for different aspect ratios. Different values of aspect ratios of the NDNPS are considered. Fig. 9a–f depicts the effect of aspect ratio on the FRP of coupled systems. The aspect ratios are taken as $L/W = 0.1$, 0.5, 1, 2, 5, 10.

From Fig. 9 we see that with the increase of aspect ratios (L/W) of NDNPS, the FRP for all case of vibration increases. It is noticed that the difference between the in-phase type vibration, out-of-phase type vibration and vibration with one GS fixed become less for increasing aspect ratios (L/W) of NDNPS. Thus it can be concluded that although the small scale effects are more in higher aspect ratios (L/W) of NDNPS, the effect of stiffness of coupling springs are reduced in higher aspect ratios of NDNPS. And thus less difference in curves between the in-phase type vibration, out-of-phase type vibration and vibration with one GS fixed become less for higher aspect ratios of NDNPS. The FRP between the different

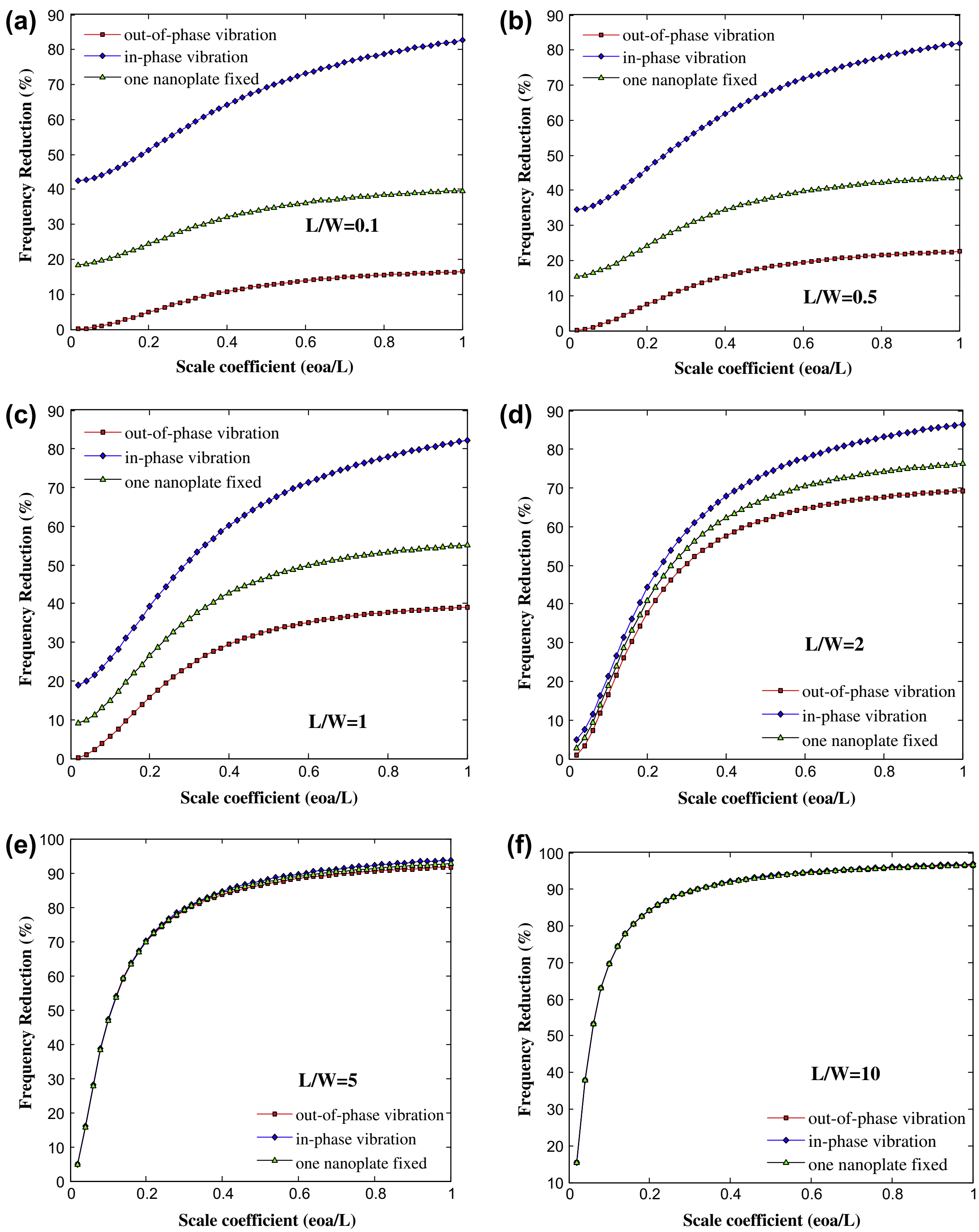

Fig. 9. Change of frequency reduction percent with scale coefficient for different aspect ratios of NDNPS; (a) $L/W = 0.1$, $L/W = 0.5$, $L/W = 1$, $L/W = 2$, $L/W = 5$, $L/W = 10$.

cases of vibrations (in-phase, out-of-phase and with one GS stationary) are reduced in the following order:

$$(L/W = 0.1) > (L/W = 0.5) > (L/W = 1) > (L/W = 2) > (L/W = 5) > (L/W = 10).$$

6.5. Effect of small-scale on NDNPS on higher natural frequencies

To see the influence of small-scale effects on the higher natural frequencies of coupled system, curves have been plotted for FRP against scale coefficient with higher FRPs. Similarly to Section 6.2, three cases of vibration characteristics are considered here; case 1: out-of-phase vibration; case 2: vibration with one GS fixed; case 3: in-phase vibration. The plots are shown in Fig. 10a–f. The stiffness parameter of the coupling springs between SWCNT is assumed to be $K = 100$. The higher natural frequencies are plotted for ($m = 1$, $n = 2$), ($m = 2$, $n = 2$), ($m = 2$, $n = 3$), ($m = 3$, $n = 3$), ($m = 3$, $n = 4$) and ($n = 4$, $m = 4$). Here it should be noted that we are considering in-phase and out-of-phase vibration as submodes.

From Fig. 10 we see that with the increase of higher natural frequencies, the FRP for all case of vibration increases. This implies

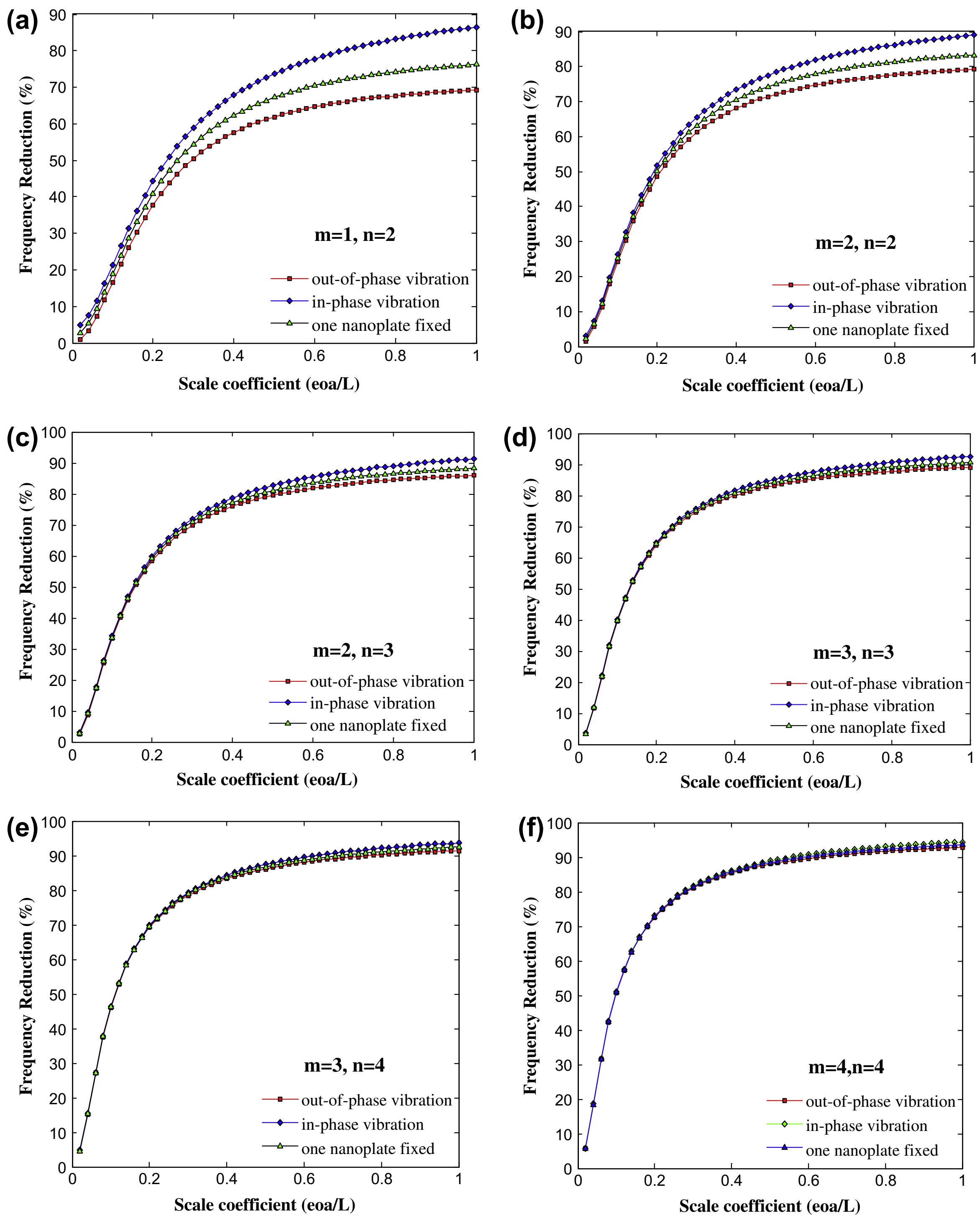

Fig. 10. Change of frequency reduction percent with scale coefficient for higher frequencies of NDNPS (a) $m = 1$, $n = 2$; (b) $m = 2$, $n = 2$; (c) $m = 2$, $n = 3$; (d) $m = 3$, $n = 3$; (e) $m = 3$, $n = 4$; (f) $m = 4$, $n = 4$.

that the higher natural frequencies of the coupled system are significantly reduced due to the nonlocal effects. These results are in-line with earlier results on nonlocal elasticity [60]. Further, it can be also noticed that the difference between the in-phase type vibration, out-of-phase type vibration and vibration with one GS fixed. Thus it can be concluded that although the small scale effects are more in higher modes of frequencies, the effect of stiffness of coupling springs are reduced in higher frequencies. And thus less difference in curves between the in-phase type vibration, out-of-phase type vibration and vibration with one GS fixed. The effect of stiffness of coupling springs in NDNPS is reduced in the following order of natural frequencies:

$$(m=1,n=2) > (m=2,n=2) > (m=2,n=3) > (m=3,n=3) > (m=3,n=4) > (m=4,n=4).$$

6.6. Nonlocal double-nanobeam-system vs. nonlocal double-nanoplate-system

Next we present the vibration behaviour of nonlocal double-nanoplate-systems (NDNPS) with respect to double-nanobeam system (NDNBS). Studies of double-nanobeam system by nonlocal elasticity can be seen in [61]. Similar to previous analysis three cases of vibration are considered, out-of-phase vibration, in-phase vibration and one nano-entity fixed. Lower and higher stiffness of coupling springs are assumed here, i.e. $K = 10$ and $K = 100$. Fig. 11 shows the change of FRP against scale coefficient for NDNBS and NDNPS with $K = 10$. From the figure it is observed that the FRP for double-nanoplate-systems are larger than the FRPs for double-nanobeam-systems. This is true for majority of scale coefficient or nonlocal parameter considered. However it should be noted that if different cases of vibration (in-phase, out-of-phase and one nano-entity fixed) are considered, then double-beam-systems have prominent behaviour compared to double-nanoplate-systems. The effect of spring stiffness is more prominent in double-nanobeam-systems which reduce the small scale effect. However for higher stiffness parameter, $K = 100$, FRP in double-nanoplate-systems would result in prominent difference in vibration with in-phase, out-of-phase and one nano-entity fixed. This is illustrated in Fig. 12.

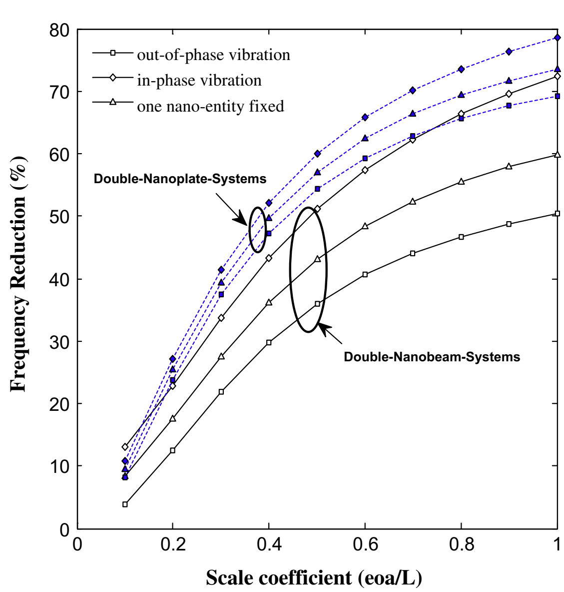

Fig. 11. Change of frequency reduction percent with scale coefficient for NDNBS and NDNPS ($K = 10$).

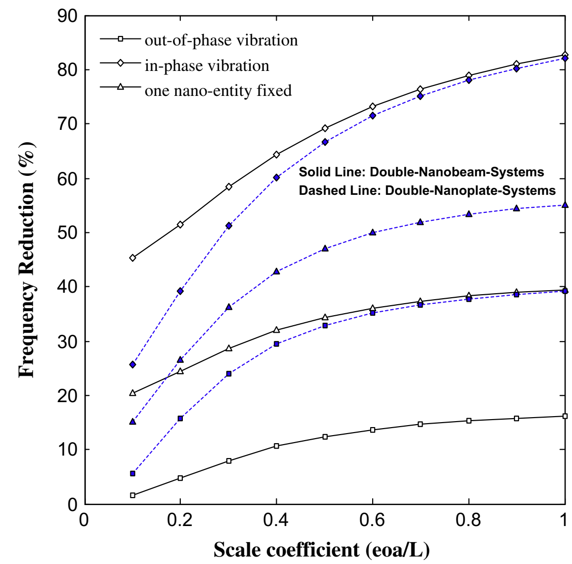

Fig. 12. Change of frequency reduction percent with scale coefficient for NDNBS and NDNPS ($K = 100$).

In the end we say, further to nanocomposites application, this double GS system can be used as nanoresonators. For different set of boundary conditions this behaviour may be different. As small-scale effects reduce the natural frequencies of the vibrating nanosystem, nanoresonators can be designed so as to vibrate in the out-of-phase modes. Thus the nanoresonators would have higher resonant frequencies. This implies the significance of the use of small-scale effects in NDNPS. If some damping properties are present within the NDNPS, then the damping behaviour could be effective in the out-of-phase mode vibration (similar to stiffening NDNPS). Thus one of the nanoplates would act as a vibration absorber. However, the damping behaviour would be ineffective in the in-phase mode vibration.

7. Conclusions

In this paper, the expressions for free bending-vibration of bonded double-nanoplate-system are established utilising nonlocal elasticity. A simple analytical method is introduced for determining the natural frequencies of bonded nonlocal double-nanoplate-system (NDNPS). Explicit closed-form expressions for natural frequencies are derived for the case when all four ends are simply-supported. Two single-layered graphene sheets coupled by polymer matrix are considered for the study. The double-nanoplate-system executes two kinds of vibrations: the synchronous vibrations with lower frequencies and the asynchronous vibrations with higher frequencies. The study highlights that the small-scale effects considerably influence the transverse vibration of NDNPS. The small-scale effects in NDNPS are higher with increasing values of nonlocal parameter for the case of synchronous (in-phase) modes of vibration than in the asynchronous (out-of-phase) modes of vibration. The increase of the stiffness of the coupling springs in NDNPS reduces the small-scale effects during the asynchronous modes of vibration. The synchronous natural frequencies are not dependent on the stiffness parameter of the elastic medium. In this case, the double-nanoplate system oscillates as a single plate with the same natural frequencies. For rectangular NDNPS with increasing aspect ratios, FRP in the synchronous and asynchronous becomes similar. Finally we say that the analytical scale-based nonlocal approach applied here can serve as the starting point for further investigation of more complex n-nanoplates systems arising in future generation graphene based nanocomposites.

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