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Reliability analysis of uncertain dynamical systems using correlated function expansion

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ABSTRACT

Reliability analysis of uncertain dynamical systems is considered. The excitations are modeled as non-stationary Gaussian processes, whereas parametric uncertainties due to structural randomness are modeled as non-Gaussian random variables. The structural responses are, therefore, non-Gaussian processes. The limit state is formulated in terms of the extreme value distribution of the response process. Developing these extreme value statistics analytically is not straightforward, which makes failure probability estimations difficult. An alternative procedure is investigated for computing exceedance probabilities. Proposed approach involves generating a full functional operational model, which approximates the original limit surface. Once the approximate form of the original limit state is defined, the failure probability can be obtained by statistical simulation. Thus, the method can be integrated with commercial finite element software, which permits uncertainty analysis of large structures with complexities that include material and geometric nonlinear behavior.

1. Introduction

A vibrating structural/mechanical system with parametric uncertainty is deemed to be safer if its response process lies well below the specified thresholds over a given duration of the load process. Thus, extreme values of responses play a crucial role in the estimation of structural reliability. We refer to a recent review paper [1] and a special issue of a journal [2] for further works on structural dynamics with uncertainty. Stochastic analysis of dynamic systems can be conducted either using the random eigenvalue problem (e.g. [3]) or using the dynamic stiffness approach [4]. In this context, the theory of asymptotic distributions [5,6] can be used to study the extreme values of random processes. As an alternative, extreme value theory of random processes over a given period can be related to the probability distribution of first passage times. For stationary Gaussian processes, above two approaches lead to Gumbel models for the extreme responses, provided the joint probability density function (PDF) of the process and its derivative at a given instant are known. It can also be noted that, determination of the joint PDF for Gaussian random responses is straightforward, while for non-Gaussian process evaluation of joint statistics is extremely difficult task.

Several authors have considered reliability and response moments of linear systems under uncertainty, see for example [7–10]. Few studies on the exceedance probabilities of non-Gaussian random processes are reported in the literature [11–13]. von Mises stress, defined by $V(t)=\sqrt{\sigma(t)\mathbf{C}\sigma(t)}$, $\sigma(t)$ is stress tensor [14], is a commonly studied non-Gaussian process, the exceedance probabilities of which are often required for estimating reliabilities [15] of ductile structure. Analytical expressions for the mean outcrossing rate of von Mises stress are developed [16] for linear structures using outcrossing approximations. Several approaches are also developed [17–19] for computing the root mean square of von Mises stress. However, in real-life problems, where the finite element (FE) method is an essential tool for handling the modeling complexities, it is difficult to apply these methods as the performance function is defined in an implicit form. For this class of problems, surrogate models [14,20] provide a promising alternative solution scheme for estimating the exceedance probability of the response. The method developed in this paper belongs to this class of solution technique.

Recently, the authors developed a new computational framework for stochastic FE analysis and time-invariant reliability analysis [21–23]. In the present study we investigate the scope of the correlated function expansion (CFE) approach, by considering the reliability analysis of nonlinear, randomly parametered dynamical systems, subjected to non-stationary Gaussian excitations. The structure is modeled and analyzed using a commercial FE software (e.g. ANSYS). External software builds the full functional operational model using CFE and is interfaced with the FE

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model. The treatment of the problem includes one or more of the following features: (i) Parametric uncertainties in structural parameters—such as Young's modulus, density and material strength. (ii) Nonlinearity in geometric/structural, material and response. (iii) Non-stationary random excitations. The procedures developed are illustrated through a numerical example and are validated with the help of limited Monte Carlo simulations (MCS) and existing results [14,20].

2. The problem statement

The governing equations of a randomly vibrating structure are well known to be of the form:

$$\mathbf{M}\ddot{\mathbf{y}}(t)+\mathbf{C}\dot{\mathbf{y}}(t)+\mathbf{K}[\mathbf{y}(t),\dot{\mathbf{y}}(t)]=\mathbf{F}(t) \tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are, respectively, the global mass, damping and stiffness matrices. $\ddot{\mathbf{y}}(t)$, $\dot{\mathbf{y}}(t)$ and $\mathbf{y}(t)$ are, respectively, vectors of nodal acceleration, velocity and displacements. $\mathbf{F}(t)$ represents a vector of random load processes characterized by the power spectral density (PSD) matrix, $\mathbf{S_{FF}}(\omega)$. For stationary Gaussian process, $\mathbf{F}(t)$ is expressed as a sum of harmonic functions with random coefficients and is of the form

$$\mathbf{F}(t)=\sum_{p=1}^{P} a_p\cos(\omega_p t)+b_p\sin(\omega_p t) \tag{2}$$

Here, P denotes the number of terms required for discretizing the PSD function, ω_p are the discretized frequencies and σ_p^2 is the variance of the pth segment of the discretized PSD. a_p and b_p are Gaussian random variables with zero mean and standard deviation σ_p. When the random process is modeled as non-stationary, $\mathbf{F}(t)$ is multiplied by a deterministic envelope function [14] of the form

$$e(t)=\alpha[\exp(-\beta t)-\exp(-\chi t)] \tag{3}$$

where the parameters β and χ, determine the shape of the envelope function and α is a normalization factor.

Let $V(t)$ be the response quantity of interest, which, in its most general form, is written as

$$V(t)=h[\ddot{\mathbf{x}}(t),\dot{\mathbf{x}}(t),\mathbf{x}(t)] \tag{4}$$

Thus, even in linear structures under Gaussian excitations, if $h[\cdot]$ is a nonlinear function. Determining the probability distribution of $V(t)$ is generally not easy, even though the probability distributions of $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are known exactly. Examples of such processes are the principal stress components and von Mises stress. For structures which behave nonlinearly, or when $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are randomly parametered, the task is even more difficult as the distributions of $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{y}(t)$ are not available explicitly. For computing the failure probability against yielding, the probability of von Mises stress exceeding the specified threshold, $V_{\lim}$, over $[t_0,\ t_0+T]$, is defined as

$$P_F=P[V(t)\le V_{\lim},\ \forall t\in(t_0,t_0+T)] \tag{5}$$

where $V_{\lim}=2\sigma_y/3$ is the permissible stress and $P[\cdot]$ is the probability measure. Since $V(t)$ is obtained as a nonlinear combination of the stress components, the probability distribution of $V(t)$ is unknown and hence, it is difficult to obtain analytical expressions for P_F. Essentially, $V(t)$ is a function of random variables considers. In the following section, CFE approach is developed for an approximation of $V(t)$, which is then used to estimate the exceedance probability. To reduce the complexity of notations, we denote $V(t)=f(\mathbf{x})$ for $\forall t\in(t_0,t_0+T)$.

3. Generalized correlated function expansion

Correlated function expansion (CFE) of an arbitrary M-dimensional function $f(\mathbf{x})$, $\mathbf{x}\in\mathbb{R}^M$ can be derived by partitioning the identity operator $\mathcal{I}$, called $\mathcal{I}_M$ in the M-dimensional case and also in the 1D case hereafter, with respect to the projectors $\mathcal{P}_1,\mathcal{P}_2,\ldots,\mathcal{P}_M$. This can be expressed as follows [24–32]:

$$\begin{aligned}\mathcal{I}_M&=\prod_{m=1}^{M}(\mathcal{P}_m+(\mathcal{I}_1-\mathcal{P}_m))\\&=\underbrace{\prod_{m=1}^{M}\mathcal{P}_m}_{1\text{term}}+\underbrace{\sum_{m=1}^{M}(\mathcal{I}_1-\mathcal{P}_m)\prod_{s\neq m}\mathcal{P}_s}_{\binom{M}{1}\text{terms}}\\&\quad+\underbrace{\sum_{m=1}^{M}\sum_{s=m+1}^{M}(\mathcal{I}_1-\mathcal{P}_m)(\mathcal{I}_1-\mathcal{P}_s)\prod_{p\neq m,s}\mathcal{P}_p}_{\binom{M}{2}\text{terms}}\\&\quad+\cdots+\underbrace{\sum_{m=1}^{M}\mathcal{P}_m\prod_{s\neq m}(\mathcal{I}_1-\mathcal{P}_s)}_{\binom{M}{M-1}\text{terms}}+\underbrace{\prod_{m=1}^{M}(\mathcal{I}_1-\mathcal{P}_m)}_{1\text{term}}\end{aligned} \tag{6}$$

composed of 2^M mutually orthogonal terms. The orthogonal representation of Eq. (6) is a manifestation of the CFE and can be rewritten as [22,33]

$$\begin{aligned}f(\mathbf{x})&=f_0+\sum_{i=1}^{M}f_i(x_i)+\sum_{1\le i<j\le M}f_{ij}(x_i,x_j)\\&\quad+\cdots+f_{123\ldots M}(x_i,x_2,\ldots,x_M)=\sum_{l=0}^{M}\eta_l(\mathbf{x})\end{aligned} \tag{7}$$

where f_0 is a constant term representing the zeroth-order component function or the mean response of any response function $f(\mathbf{x})$. f_i is the first-order term expressing the effect of variable x_i acting alone upon the output $f(\mathbf{x})$, and this function is generally nonlinear. The function $f_{ij}(x_i,x_j)$ is a second-order term which describes the cooperative effects of the variables x_i and x_j upon the response. The higher order terms gives the cooperative effects of increasing numbers of input variables acting together to influence the output. The last term $f_{123\ldots M}(x_i,x_2,\ldots,x_M)$ contains any residual dependence of all the input variables locked together in a cooperative way to influence the output. Once all of the relevant component functions in Eq. (7) are determined and suitably represented, then the component functions constitute the CFE, thereby replacing the original computationally expensive method of calculating the response by the computationally efficient meta model. Usually the higher order terms in Eq. (7) are negligible [26,34] such that a CFE with only a few low order correlations amongst the input variables is adequate to describe the output behavior. This in turn results in rapid convergence of the CFE expansion [27,28], which has a finite number of terms and is always exact [26,30] in the least-square senses. Other popular expansions (e.g., polynomial chaos) have been suggested [35], but they commonly have an infinite number of terms with some specified functions, such as Hermite polynomials [24,25,30].

To generate the CFE approximation of any function, more precisely the cut-center based CFE, first a reference point $\overline{\mathbf{x}}=(\overline{x}_1,\overline{x}_2,\ldots,\overline{x}_M)$ has to be defined in the variable space. In the convergence limit, where all correlated functions in Eq. (7) are considered, the cut-CFE is invariant to the choice of reference point $\overline{\mathbf{x}}$. However in practice the choice of reference point $\overline{\mathbf{x}}$ is

important for the cut-CFE, especially if only the first few terms, say up to first- and second-order, in Eq. (7) are considered. Sobol [27] showed that the reference point $\overline{\mathbf{x}}$ at the middle of the input domain appears to be the optimal choice. The expansion functions are determined by evaluating the input–output responses of the system relative to the defined reference point along the associated lines, surfaces or sub-volumes in the input variable space. This process reduces to the following relationship for the component functions in Eq. (7):

$$f_0 = \int dx f(\mathbf{x})$$

$$f_i(x_i) = \int dx^i f(\mathbf{x}) - f_0$$

$$f_{ij}(x_i,x_j) = \int dx^{ij} f(\mathbf{x}) - f_i(x_i) - f_j(x_j) - f_0 \tag{8}$$

where $\int dx^i$ means to integrate over all M variables except x_i and $\int dx^{ij}$ means to integrate over all M variables except x_i and x_j, etc. These integrals are generally evaluated using numerical integration techniques. Substituting the component functions defined in Eq. (8) into Eq. (7), the general expression of the CFE can be expressed as

$$\begin{aligned} f(\mathbf{x}) = & \sum_{1 \le i_1 < \cdots < i_\theta \le M} f(x_{i_1},\ldots,x_{i_\theta};\overline{x}^{i_1,\ldots,i_\theta}) - (M-\theta) \sum_{1 \le i_1 < \cdots < i_{\theta-1} \le M} \\ & \times f(x_{i_1},\ldots,x_{i_{\theta-1}};\overline{x}^{i_1,\ldots,i_{\theta-1}}) + \frac{(M-\theta+1)!}{2!(M-\theta-1)!} \sum_{1 \le i_1 < \cdots < i_{\theta-2} \le M} \\ & \times f(x_{i_1},\ldots,x_{i_{\theta-2}};\overline{x}^{i_1,\ldots,i_{\theta-2}}) - \cdots \mp \frac{(M-2)!}{(\theta-1)!(M-\theta-1)!} \sum_{1 \le i \le M} \\ & \times f(x_i;\overline{x}^i) \pm \frac{(M-1)!}{\theta!(M-\theta-1)!} f(\overline{\mathbf{x}}) \end{aligned} \tag{9}$$

where θ is the order of the CFE approximation, $1 \le \theta \le (M-1)$ and the + or − sign of the last term in Eq. (9) corresponds to θ being even or odd, respectively. Considering the weak role of the higher-order correlation effects, the approximation is likely to converge at a lower CFE order, say, $\theta \ll M$. The particular form of Eq. (9) for $\theta = 1$, 2, or 3 corresponds to first-, second- or third-order CFE can be explicitly given as

$$\hat{f}(\mathbf{x}) = \sum_{1 \le i \le M} f(x_i;\overline{x}^i) - (M-1) f(\overline{\mathbf{x}}), \quad \theta = 1 \tag{10}$$

$$\begin{aligned} \hat{f}(\mathbf{x}) = & \sum_{1 \le i < j \le M} f(x_i,x_j;\overline{x}^{i,j}) - (M-2) \sum_{1 \le i \le M} f(x_i;\overline{x}^i) \\ & + \frac{(M-1)!}{2!(M-3)!} f(\overline{\mathbf{x}}), \quad \theta = 2 \end{aligned} \tag{11}$$

$$\begin{aligned} \hat{f}(\mathbf{x}) = & \sum_{1 \le i < j < k \le M} f(x_i,x_j,x_k;\overline{x}^{i,j,k}) - (M-3) \sum_{1 \le i < j \le M} f(x_i,x_j;\overline{x}^{i,j}) \\ & + \frac{(M-2)!}{2!(M-4)!} \sum_{1 \le i \le M} f(x_i;\overline{x}^i) + \frac{(M-1)!}{3!(M-4)!} f(\overline{\mathbf{x}}), \quad \theta = 3 \end{aligned} \tag{12}$$

The term $f(x_i;\overline{x}^i)$ is a function of the single x_i component (i.e., a cut along x_i through the reference point in the function space), while the other variables, $x_j \equiv \overline{x}_j, j \ne i$, are fixed at the reference point. In the same manner, $f(x_i,x_j;\overline{x}^{ij})$ is the observed response for all the variables, $x_k \equiv \overline{x}_k$, $k \ne i,j$, fixed at the cut center except for x_i and x_j. A similar interpretation would apply to higher order CFE terms.

The notion of first-, second-order, etc. used in the CFE does not imply the terminology commonly used either in the Taylor series or in the conventional polynomial based approximation formulae. In CFE-based approximation, these terminologies are used to define the constant term, or for example, terms with one variable or two variables only. It is recognized that the lower order (e.g., first-order or second-order) function expansions in the CFE, do not generally translate to linear or quadratic functions [28]. Each of the lower-order terms in the CFE is sub-dimensional, but they are not necessarily low degree polynomials. The computational savings afforded by the CFE are easily estimated. If the CFE converges at θ order with acceptable accuracy and considering s sample points for each variable, then the total number of numerical analyses needed to determine the CFE is $\sum_{l=0}^{\theta}[M!/l!(M-l)!](s-1)^l$.

4. Implementation of the CFE-based approximation

Based on the discussion in the previous section, the proposed methodology can be easily implemented using the following steps:

1. Select the reference point $\overline{\mathbf{x}}$, generally the mean value of the random variables.
2. Determine f_0, which is a constant term, representing the response at reference point $\overline{\mathbf{x}}$.
3. Generate regularly spaced sample points, as $\overline{x}_i-(s-1)n_i/2$, $\overline{x}_i-(s-3)n_i/2,\ldots,\overline{x}_i,\ldots,\overline{x}_i+(s-3)n_i/2$, $\overline{x}_i+(s-1)n_i/2$, along the variable axis x_i with reference $\overline{x}_i$ and regular distance n_i.
4. Estimate the responses at all these sample points.
5. Construct the CFE approximation using the following steps:
   - Interpolate each of the low dimensional CFE expansion terms $f(x_i;\overline{x}^i)$ and $f(x_{i_1},x_{i_2};\overline{x}^{i_1,i_2})$ as $f(x_i;\overline{x}^i) = \sum_{j=1}^{s} \varphi_j(x_i) f(x_i^j;\overline{x}^i)$ and $f(x_{i_1},x_{i_2};\overline{x}^{i_1,i_2}) = \sum_{j_1=1}^{s}\sum_{j_2<j_1}^{s} \varphi_{j_1j_2}(x_{i_1},x_{i_2}) f(x_{i_1}^{j_1}, x_{i_2}^{j_2};\overline{x}^{i_1,i_2})$, respectively. The response values are calculated in the previous step where $\varphi_j(x_i)$ and $\varphi_{j_1j_2}(x_{i_1},x_{i_2})$ represent interpolation/shape functions. Presently we use the moving least-square interpolation function, details of which can be found in [36].
   - Sum the interpolated values of the CFE expansion terms. This leads to first-order CFE (FO-CFE) and second-order CFE (SO-CFE) approximation of the function $f(\mathbf{x})$ as follows:

$$\hat{f}(\mathbf{x}) = \sum_{1 \le i \le M} \sum_{j=1}^{s} \varphi_j(x_i) f(x_i^j;\overline{x}^i) - (M-1) f(\overline{\mathbf{x}}), \quad \theta = 1 \tag{13}$$

and

$$\begin{aligned} \hat{f}(\mathbf{x}) = & \sum_{1 \le i < j \le M} \sum_{j_1=1}^{s} \sum_{j_2<j_1}^{s} \varphi_{j_1j_2}(x_{i_1},x_{i_2}) f(x_{i_1}^{j_1},x_{i_2}^{j_2};\overline{x}^{i_1,i_2}) \\ & - (M-2) \sum_{1 \le i \le M} \sum_{j=1}^{n} \varphi_j(x_i) f(x_i^j;\overline{x}^i) + \frac{(M-1)!}{2!(M-3)!} f(\overline{\mathbf{x}}), \quad \theta = 2 \end{aligned} \tag{14}$$

6. Perform Monte Carlo simulation on the approximated response function $\hat{f}(\mathbf{x})$.
7. Estimate the failure probability P_F using

$$P_F \cong \mathbb{E}[\mathcal{J}_{\hat{\Omega}_{FS}}(\mathbf{x})] = \lim_{N_S \to \infty} \frac{1}{N_S} \sum_{i=1}^{N_S} \mathcal{J}_{\hat{\Omega}_{FS}}(\mathbf{x}^i) \tag{15}$$

where $\hat{\Omega}_{FS} = \{\mathbf{x} : \hat{f}(\mathbf{x}) < 0\}$

The input–output relationships in the context of the proposed method consist of the statistics of structural responses and the input random variables. This relationship is already nonlinear irrespective of the linearity or nonlinearity of the material properties. Material nonlinearity (assuming the usual smoothness properties) effectively makes the existing nonlinear mapping to a 'higher degree' of nonlinearity. This in turn increases the computational expense as higher degree of CFE need to be used to represent this more nonlinear function. In the numerical example we consider a problem where the output quantity (namely the von Mises stress) is a highly

nonlinear function of the input random variables (namely Young's modulus and the yield strength).

5. Numerical example

A numerical example is presented here to illustrate the performance of the proposed approach and compared with existing results. The failure estimates have been compared through the following three procedures: (1) An estimate of P_F is obtained from the full scale MCS on the exact performance function. This involves the analysis of an ensemble of response time histories obtained by direct integration of Eq. (1), for a set of sample time histories of excitation $F(t)$. (2) An estimate of P_F is obtained by adopting the improved response surface [14] procedure. (3) An estimate of P_F is obtained by Bucher and Bourgund algorithm [20]. When comparing computational efforts in evaluating the response uncertainty, the number of actual FE analysis is chosen as the primary comparison tool in this paper. This is because of the fact that, number of FE analysis indirectly indicates the CPU time usage. For the full-scale MCS, the number of original FE analysis is same as the sampling size. While evaluating the responses through the full-scale MCS, the CPU time is more because it involves significant number of repeated FE analyses. However, in the proposed method, MCS is conducted within the CFE framework. Here, although the same sampling size as in the direct MCS is considered, the number of FE analysis is much less. Hence, the computational effort expressed in terms of FE calculation alone should be carefully interpreted. Since the CFE approximation leads to an explicit representation of the system responses, the MCS can be conducted for large sampling size. The total cost of original FE analysis entails a maximum of $(s-1)M+1$ and $(s-1)^2(M-1)M/2+(s-1)M+1$, respectively, for FO-CFE and SO-CFE approximation. s being the number of sample points of CFE.

A 1.5 m long cantilever beam subjected to non-stationary random excitation is studied. Cross-sectional dimensions of the beam is taken as 0.15 m × 0.03 m. Commercial FE software (ANSYS) is used to model and analyze this structure assuming plane stress condition. The beam is discretized into 10 plane stress elements. The PSD of the support excitation $\mathbf{F}(t)$ is taken as

$$S_{\mathbf{FF}}(\omega) = S_0 \frac{1+4\eta_g^2(\omega/\omega_g)^2}{(1-(\omega/\omega_g)^2)^2+4\eta_g^2(\omega/\omega_g)^2} \tag{16}$$

where $\omega_g = 1500$ rad/s, $\eta_g = 1.2$ and ω is the frequency of excitation. Non-stationary load histories for the support excitation are generated from Eqs. (2) and (3), with $P=5$. The parameters α, β and γ in Eq. (3) are taken to be 7.18, 60 and 41, respectively, to illustrate the performance of the proposed approach with existing methods [14].

Failure is defined as the exceedance of von Mises stress over the material strength, which, for all practical purposes, denotes the limit of linear material behavior in ductile materials. The performance function is defined in terms of the von Mises stress developed at the root of the cantilever beam and is of the form given in Eq. (5). Here, $V(t)$ is the von Mises stress, given by $V(t) = \sqrt{\sigma(t)\mathbf{C}\sigma(t)}$, the time duration $T=0.04$ s, $\sigma(t)$ is the stress tensor and

$$\mathbf{C} = \begin{bmatrix} 1 & -0.5 & -0.5 & 0 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & 0 \\ -0.5 & -0.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} \tag{17}$$

The structure material is assumed to be structural steel and proportional damping is assumed to be 5%. The randomness in

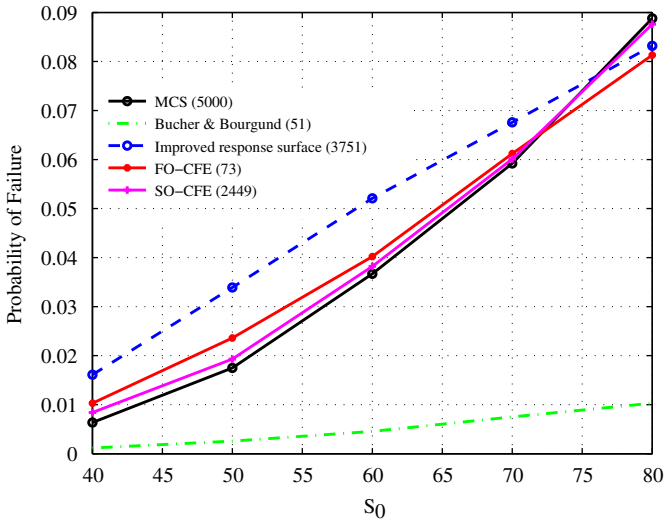

Fig. 1. Estimation of the exceedance probability for randomly parametered structure subjected to non-stationary excitations.

Young's modulus (E) and yield strength of the material (σ_y) are expressed as $E = E_0(1+\varepsilon_1 z_1)$ and $\sigma_y = \sigma_{y0}(1+\varepsilon_2 z_2)$, where, E_0 and σ_{y0}, respectively, denote the mean values of E and σ_y. The stochastic 'strength parameters' ε_1 and ε_2 are small deterministic constants, taken to be equal to 0.05. The random variables z_1 and z_2 are assumed to be lognormal random variables with $\langle z_1 \rangle = 2.0$, $\langle z_2 \rangle = 0.1$, $\langle z_1 z_1 \rangle = 1.0$ and $\langle z_2 z_2 \rangle = 0.05$. In reality it is possible to have statistical correlation between Young's modulus (E) and the yield strength of the material (σ_y). However, to be best of our knowledge numerical data is not available. If the correlation information were available, then it would be possible to incorporate it within the proposed algorithm as statistical simulation is used to implement the method [21]. The only difference would be in the sampling which would take the correlations into account. The limit state/performance function is, thus, involved 12 random variables. Correlated function expansion of the original limit state/performance function is generated using $s=7$ sample points. The constant S_0 in Eq. (16) is varied from 40 to 80 m^2/s^3 and the corresponding failure probability is computed and presented in Fig. 1. A sample size of 5000 has been considered for the direct MCS. Estimation of the failure probabilities using other approaches such as Bucher and Bourgund's algorithm [20] and improved response surface method [14] are also presented in Fig. 1 to show the performance of the proposed methodology. Numbers inside the parentheses represent the number of actual FE analysis required for each method. From these results one can observe that the proposed CFE approach results in acceptable accuracy using only 73 and 2449 FE calls for the first- and second-order expansion, respectively.

6. Conclusions

The computation of structural reliability under stochastic load process is considered. The probability of exceedance of response processes during the specified time evolution is used in this paper. The estimation of these exceedance probabilities needs an explicit knowledge of the mean outcrossing rate of the response process. This, in turn, requires the knowledge of joint statistics of the response and its time derivative. In most of the cases, the response processes are non-Gaussian in nature. Determination of their joint PDF and mean outcrossing rates can be difficult. The main novelty of the paper is the introduction of the correlated function expansion method for this problem. This

approach, originally proposed for multivariate function approximation, has been used for obtaining the exceedance probabilities in an efficient manner.

The proposed approach sidesteps the requirement of the joint statistics of the response processes. As the method requires only conditional responses at selected sample points, this can easily be integrated with a commercial FE software, and has been demonstrated in this paper. Thus, complexities arising out of geometric and nonlinear material behavior, uncertainties in the structural properties and loads can be handled efficiently. The proposed method is applied to a test problem with 12 random variables. Numerical results showed that acceptable accuracy in the failure probability can be obtained with 73 (first-order CFE) and 2449 (second-order CFE) FE evaluations. The computational effort required in the correlated function expansion is economical compared with the full scale MCS (in this case 5000 FE evaluations), when estimates of low failure probabilities are desired and when the evaluation of the performance function requires significant computer time.

The method proposed here is applied to structural/mechanical systems as examples. However, as shown in the paper, the method has the potential for application in other scientific disciplines and multiphysics problems. Further work is, however, needed to realize this potential in addition to the importance measure of the system parameters.

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References

[1] Schueller G, Pradlwarter H. Uncertain linear systems in dynamics: retrospective and recent developments by stochastic approaches. Engineering Structures 2009;31(11):2507–17.
[2] Mace B, Worden K, Manson G. Uncertainty in structural dynamics. Journal of Sound and Vibration 2005;288(3):423–9.
[3] Adhikari S, Friswell MI. Random matrix eigenvalue problems in structural dynamics. International Journal for Numerical Methods in Engineering 2007;69(3):562–91.
[4] Adhikari S, Manohar CS. Transient dynamics of stochastically parametered beams. ASCE Journal of Engineering Mechanics 2000;126(11):1131–40.
[5] Castillo E. Extreme value theory in engineering. Massachusetts: Academic Press; 1988.
[6] Kotz S, Nadarajah S. Extreme value distributions. London: Imperial College Press; 2000.
[7] Hosseini S, Khadem S. Vibration and reliability of a rotating beam with random properties under random excitation. International Journal of Mechanical Sciences 2007;49(12):1377–88.
[8] Onkar A, Upadhyay C, Yadav D. Generalized buckling analysis of laminated plates with random material properties using stochastic finite elements. International Journal of Mechanical Sciences 2006;48(7):780–98.
[9] Khodaparast HH, Mottershead JE, Badcock KJ. Propagation of structural uncertainty to linear aeroelastic stability. Computers & Structures 2010; 88(3–4):223–36.
[10] Adhikari S. Reliability analysis using parabolic failure surface approximation. ASCE Journal of Engineering Mechanics 2004;130(12):1407–27.
[11] Gupta S, Manohar CS. An improved response surface method for the determination of failure probability and importance measures. Structural Safety 2004;26(2):123–39.
[12] Gupta S, Manohar CS. Probability distribution of extremes of von Mises stress in randomly vibrating structures. Journal of Vibration and Acoustics ASME 2005;127(6):547–55.
[13] Dunne LW, Dunne JF. An FRF bounding method for randomly uncertain structures with or without coupling to an acoustic cavity. Journal of Sound and Vibration 2009;322(1–2):98–134.
[14] Gupta S, Manohar CS. Improved response surface method for time-variant reliability analysis of nonlinear random structures under non-stationary excitations. Nonlinear Dynamics 2004;36(2–4):267–80.
[15] Manohar CS, Adhikari S. Statistical analysis of vibration energy flow in randomly parametered trusses. Journal of Sound and Vibration 1998;217(1): 43–74.
[16] Madsen H. Extreme value statistics for nonlinear stress combination. Journal of Engineering Mechanics ASCE 1985;111(9):1121–9.
[17] Segalman D, Fulcher C, Reese G, Field R. An efficient method for calculating r.m.s. von Mises stress in a random vibration environment. Journal of Sound and Vibration 2000;230(2):393–410.
[18] Segalman D, Reese R, Field C, Fulcher C. Estimating the probability distribution of von mises stress for structures undergoing random excitation. Journal of Vibration and Acoustics ASME 2000;112(1):42–8.
[19] Reese G, Field R, Segalman D. A tutorial on design analysis using von Mises stress in random vibration environments. The Shock and Vibration Digest 2000;32(6):466–74.
[20] Bucher CG, Bourgund U. A fast and efficient response surface approach for structural reliability problem. Structural Safety 1990;7(1):57–66.
[21] Chowdhury R, Adhikari S. High dimensional model representation for stochastic finite element analysis. Applied Mathematical Modelling 2010;34(12):3917–32.
[22] Rao BN, Chowdhury R. Enhanced high-dimensional model representation for reliability analysis. International Journal for Numerical Methods in Engineering 2009;77(5):719–50.
[23] Chowdhury R, Rao BN. Hybrid high dimensional model representation for reliability analysis. Computer Methods in Applied Mechanics and Engineering 2009;198(5–8):753–65.
[24] Rabitz H, Alis O. General foundations of high-dimensional model representations. Journal of Mathematical Chemistry 1999;25(2–3):197–233.
[25] Rabitz H, Alis O, Shorter J, Shim K. Efficient input–output model representations. Computer Physics Communications 1999;117(1–2):11–20.
[26] Li G, Rosenthal C, Rabitz H. High dimensional model representations. Journal of Physical Chemistry A 2001;105(33):7765–77.
[27] Sobol I. Theorems and examples on high dimensional model representation. Reliability Engineering & System Safety 2003;79(2):187–93.
[28] Chowdhury R, Rao BN, Prasad AM. High-dimensional model representation for structural reliability analysis. Communications in Numerical Methods in Engineering 2009;25(4):301–37.
[29] Chowdhury R, Rao BN. Assessment of high dimensional model representation techniques for reliability analysis. Probabilistic Engineering Mechanics 2009; 24(1):100–15.
[30] Li G, Wang S, Rabitz H, Wang S, Jaffe P. Global uncertainty assessments by high dimensional model representations (HDMR). Chemical Engineering Science 2002;57(21):4445–60.
[31] Shorter J, Ip P, Rabitz H. An efficient chemical kinetics solver using high dimensional model representation. Journal of Physical Chemistry A 1999;103(36):7192–8.
[32] Rao BN, Chowdhury R. Factorized high dimensional model representation for structural reliability analysis. Engineering Computations 2008;25(7–8): 708–38.
[33] Yaman Y, Demiralp M. A new rational approximation technique based on transformational high dimensional model representation. Numerical Algorithms 2009;52(3):385–407.
[34] Rabitz H, Shim K. Multicomponent semiconductor material discovery guided by a generalized correlated function expansion. Journal of Chemical Physics 1999;111(23):10640–51.
[35] Ghanem R, Spanos P. Stochastic finite elements: a spectral approach. New York, USA: Springer-Verlag; 1991.
[36] Chowdhury R, Rao BN, Prasad AM. High dimensional model representation for piece-wise continuous function approximation. Communications in Numerical Methods in Engineering 2008;24(12, Sp. Iss. SI):1587–609.