# Scale-dependent vibration analysis of prestressed carbon nanotubes undergoing rotation

T. Murmu[a] and S. Adhikari
*School of Engineering, Swansea University, Singleton Park, Swansea, SA2 8PP Wales, United Kingdom*

(Received 14 September 2010; accepted 20 October 2010; published online 20 December 2010)

Understanding the dynamic behavior of rotating nanostructures is important for practical development of nanomachines. At the nanoscale, the nonlocal effects often become prominent. In this study, we investigate the nonlocal effects in bending-vibration of an initially prestressed single-walled carbon nanotube via nonlocal elasticity. The carbon nanotube is assumed to be attached to a molecular hub and is undergoing rotation. Nonlocal Euler–Bernoulli beam theory is employed to formulate the governing equations. Differential quadrature method is being utilized and the nonlocal bending frequencies of the rotating system are determined. The effects of the initial preload on vibration characteristics of rotating carbon nanotube are examined. Further, influence of (a) nonlocal effects (b) angular velocities, (c) hub radii and (d) higher mode frequencies are studied. It is explicitly shown that the bending vibration of a rotating carbon nanotube is significantly influenced by the existence of a preload, angular velocity and the nonlocal parameter. © *2010 American Institute of Physics*. [doi:10.1063/1.3520404]

## I. INTRODUCTION

Because of future promising exploration of nanotechnology, focus is being put in the miniaturization of mechanical and electromechanical devices. Attention is sought toward the development of nanodevices and nanomachines.[1] Nanomachines are systems in the nanometer realm with moving parts.[2] Nanostructures undergoing rotation include nanoturbines, nanoscale molecular bearings, shaft and gear, and multiple gear systems. These nanostructure machines are expected to receive considerable attention in the near future. Researchers have thus reported the feasibility of nanoscale rotating structures. Examples include study of molecular gears,[1] fullerene gears,[3] and carbon nanotubes gears.[4,5] Srivastava[6] has reported the rotational dynamics of carbon nanotubes and carbon nanotubes gears under a single applied laser field. A typical carbon nanotubes gear is shown in Fig. 1.

Lohrasebi and Tabar[7] carried out computational modeling of rotating nanomotor. Dynamics of the rotary nanomotor was simulated using stochastic molecular dynamics (MD) method. Zhang *et al.*[8] carried out atomistic simulations of double-walled carbon nanotubes as rotational bearings. Recently Fennimore *et al.*[9] reported the feasibility of rotating nanostructures. They showed the construction and successful operation of a nanoscale electromechanical actuator. The rotating nanostructural system consists of a rotatable metal plate, with a multiwalled carbon nanotube serving as the key motion-enabling element.

For efficient design of these rotating nanomachines, proper understanding of its mechanical behavior such as bending, vibration, and buckling is required. The development of simplified models for the dynamics of complex nanotechnological systems is thus necessary. This is because for many cases, fully atomistic simulations (MD) would be computationally expensive and prohibitive. Eringen's nonlocal elasticity theory[10] is one which holds promise to model the dynamics of nanostructures as it accounts for the small-scale effects in nanostructures. Nonlocal elasticity theory gives a good prediction for modeling the behavior of nanostructures. Literature[11–17] shows that the theory of nonlocal elasticity is receiving increasing attention for efficient analysis of nanostructures which include nanobeams, nanoplates, nanorings, carbon nanotubes, graphenes, and microtubules.

Nonlocal elasticity can account for the small-scale effects necessary for nanostructures. At the nanometer scale, size effects often become prominent. Both experimental[18,19] and atomistic simulation results[20] have shown a significant "size-effect" in mechanical properties when the dimensions of these structures become small. In this theory, the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain. Nonlocal theory considers long-range interatomic interaction and yields results dependent on the size of a body. Drawbacks of the classical continuum theory could be efficiently

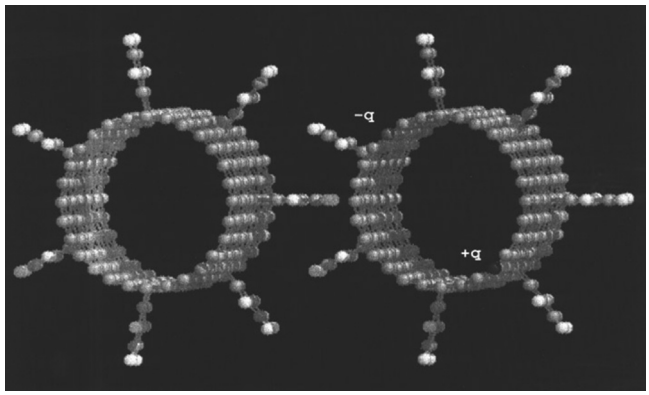


FIG. 1. Molecular diagram of carbon nanotube gears (Ref. 6).

[a]Electronic mail: t.murmu@swansea.ac.uk.

 

avoided and size-dependent phenomena can be satisfactorily explained by the nonlocal elasticity theory. Further from Chen *et al.*,[21] the nonlocal continuum based models are physically reasonable from the atomistic viewpoint of lattice dynamics and MD simulations.

Further, it is observed that during the fabrication of nanostructures, the residual stresses can be developed within the structures. This initial residual stresses could significantly modify the mechanical and electrical properties of microelectromechanical systems (MEMS) or nanoelectromechanical systems (NEMS) devices. Strains are usually developed during the material growth and temperature relaxation. For a suspended structure, this process-induced strain may cause the axial residual stress within the structure. This calls for a deep understanding of its influence on the performance of the devices for the optimum design. Works highlighting the effect of initial axial stresses in vibration of nanobeams and carbon nanotubes can be seen in Lu (Ref. 22) and Heireche *et al.*,[23] respectively.

In perspective of the above discussion, in this paper, we study the nonlocal bending-vibration of an initially prestressed single-walled carbon nanotube (SWCNT). The carbon nanotube is assumed to be attached to a molecular hub and is undergoing rotation. The other end of carbon nanotube is assumed to be having a tip and simply supported. Study of prestressed rotating carbon nanotubes is being studied for the first time in the framework of nonlocal elasticity. Nonlocal Euler–Bernoulli beam theory is employed to formulate the governing equations. Differential quadrature method (DQM) is utilized to determine the nonlocal bending frequencies of the rotating system. The effects of the initial preload on vibration characteristics of rotating carbon nanotube are examined. Influence of (a) nonlocal parameters, (b) angular velocities, (c) hub radii, and (d) higher mode frequencies are studied. It is shown that bending vibration of a rotating carbon nanotube is significantly influenced by the existence of a preload, angular velocity and the nonlocal parameter.

## II. NONLOCAL ELASTICITY

In the nonlocal elasticity theory, the stress field at a reference point $x$ in an elastic medium is considered to be dependent not only on the strain at that point, but also on the strains at all other points in the domain.[10] This is accounted from the atomic theory of lattice dynamics and experimental observations on phonon dispersion. The basic equations for a linear homogenous nonlocal elastic body neglecting the body force are given as[11–17]

$$\sigma_{ij,j} = 0,$$

$$\sigma_{ij}(x) = \int \phi(|x - x'|, \alpha) H_{ijkl} \varepsilon_{kl}(x') dV(x'), \quad \forall\, x \in V,$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{1}$$

The terms $\sigma_{ij}$, $\varepsilon_{kl}$, $H_{ijkl}$ are the stress, strain, and fourth order elasticity tensors, respectively. $\phi(|x-x'|, \alpha)$ is the nonlocal modulus or attenuation function incorporating into constitutive equations the nonlocal effects at the reference point $x$ produced by local strain at the source $x'$. $|x-x'|$ represents the Euclidean distance and $\alpha$ is a material constant that depends on the internal (e.g., lattice parameter, granular size, distance between C–C bonds) and external characteristics lengths (e.g., crack length, wavelength). Material constant $\alpha$ is defined as $\alpha = e_0 a / \ell$. $e_0$ is a constant for calibrating the model with experimental results.[10] $e_0$ is estimated such that the relations based on nonlocal elasticity model could provide satisfactory approximation of atomic dispersion curves and atomic lattice dynamics results.

Equation (1) is in partial-integral form and generally difficult to solve analytically. Thus, a differential form of nonlocal elasticity equation is often used.[11–17] According to Eringen,[10] the expression of nonlocal modulus can be given as

$$\phi(|x|, \alpha) = (2\pi \ell^2 \alpha^2)^{-1} K_0(\sqrt{x \cdot x}/\ell\alpha), \tag{2}$$

where $K_0$ is the modified Bessel function.

The equation of motion in terms of nonlocal elasticity can be expressed as[10]

$$\sigma_{ij,j} + f_i = \rho \ddot{u}_i, \tag{3}$$

where $f_i$, $\rho$, and $u_i$ are the components of the body forces, mass density, and the displacement vector, respectively. The terms $i$, $j$ take up the symbols $x$, $y$, and $z$.

Assuming the kernel function $\phi$ as the Green function, Eringen[10] proposed a differential form of the nonlocal constitutive relation as

$$\sigma_{ij,j} + \mathcal{L}(f_i - \rho \ddot{u}_i) = 0, \tag{4}$$

where

$$\mathcal{L} = [1 - (e_0 a)^2 \nabla^2] \tag{5}$$

and $\nabla$ is the Laplacian operator.

Using Eq. (5) the nonlocal constitutive stress-strain relation can be simplified as

$$(1 - \alpha^2 \ell^2 \nabla^2) \sigma_{ij} = t_{ij}. \tag{6}$$

## III. ROTATING CARBON NANOTUBE-SYSTEM BY NONLOCAL MODEL

Consider a carbon nanotube of length $L$ which is fixed at point $O$ to a rigid molecular hub. The molecular hub has radius $r$ and rotates in a counter clockwise direction at a constant rotational speed $\Omega$ [Fig. 2(a)]. The free end of the carbon nanotube has tip which is simply supported. The carbon nanotube is prestressed in the axial direction. The carbon nanotube is idealized as a nonlocal Euler–Bernoulli beam. The mathematical idealization of the nonlocal Euler–Bernoulli beam is shown in Fig. 2(b).

In one-dimensional form, the constitutive relation (6) can be reduced to as

$$\sigma(x) - (e_0 a)^2 \frac{\partial^2 \sigma(x)}{\partial x^2} = E\varepsilon(x), \tag{7}$$

where $E$ is the conventional Young's modulus of the nanobeam. For understanding the transformation of integropartial

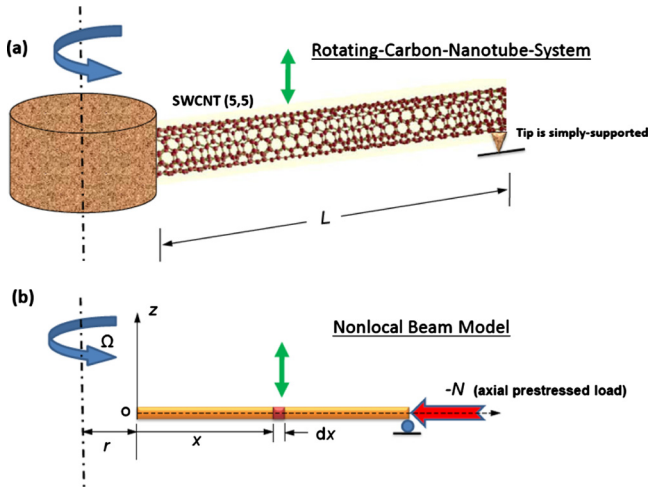


FIG. 2. (Color online) (a) A configuration of rotating carbon nanotubes-system, (b) mathematical idealization of the initially prestressed rotating system by the nonlocal beam model.

equation [Eq. (1)] to differential form, one may see Ref. 10.

Let the carbon nanotube is subjected to initial axial stress $\sigma^0$. Then the initial axial force is given as

$$N_x = A\sigma_x^0. \tag{8}$$

The Euler Lagrange equation associated with the nonlocal Euler–Bernoulli beam theory for an preloaded rotating carbon nanotube can be expressed as

$$\frac{\partial^2 M}{\partial x^2} + \frac{\partial}{\partial x}\left(T\frac{\partial w}{\partial x}\right) - N_x\frac{\partial^2 w}{\partial x^2} - \rho A\frac{\partial^2 w}{\partial t^2} = 0. \tag{9}$$

Here, $N_x$ is the preload on the rotating carbon nanotube, $A$ is the cross sectional area of the beam, and $T$ is the centrifugal tension which can be expressed as

$$T(x) = \int_x^L \rho A\Omega^2(r+x)dx. \tag{10}$$

Here, $\Omega$ denotes the angular velocity of the carbon nanotube, $r$ is known as the hub radius and is the distance from the origin of rotation to the inner end of the carbon nanotube (Fig. 1).

Using the nonlocal elastic approach and Eqs. (9) and (10), the equation of motion for the in-plane loaded rotating carbon nanotube can be expressed as

$$\begin{aligned} &EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} - (e_0a)^2\rho A\frac{\partial^4 w}{\partial x^2\,\partial t^2} + N_x\frac{\partial^2 w}{\partial x^2} \\ &\quad - (e_0a)^2 N_x\frac{\partial^4 w}{\partial x^4} - \rho A\Omega^2\left[(-r-x)\frac{\partial w}{\partial x} + \left(rL + \frac{L^2}{2}\right.\right. \\ &\quad \left.\left. - rx - \frac{x^2}{2}\right)\frac{\partial^2 w}{\partial x^2}\right] + (e_0a)^2\rho A\Omega^2\left[-3\frac{\partial^2 w}{\partial x^2} + 3(-r\right. \\ &\quad \left. - x)\frac{\partial^3 w}{\partial x^3} + \left(rL + \frac{L^2}{2} - rx - \frac{x^2}{2}\right)\frac{\partial^4 w}{\partial x^4}\right] = 0. \end{aligned} \tag{11}$$

## IV. NONLOCAL BOUNDARY CONDITION OF ROTATING SYSTEM

Now we present the nonlocal boundary conditions for the rotating system. The nonlocal boundary conditions of the rotating system at the left hand side (LHS) is clamped, i.e., $x=0$ (LHS of rotating carbon nanotube, clamped)

$$w = \frac{dw}{dx} = 0. \tag{12}$$

And at the right hand side (RHS) of the rotating system is simply supported, i.e., at $x=L$ (RHS of rotating carbon nanotube, propped)

$$\begin{aligned} w = 0; \quad M = &-EI\frac{\partial^2 w}{\partial x^2} + (e_0a)^2\left[-\frac{\partial}{\partial x}\left(T\frac{\partial w}{\partial x}\right) + N_x\frac{\partial^2 w}{\partial x^2}\right. \\ &\left. + \rho A\frac{\partial^2 w}{\partial t}\right] = 0. \end{aligned} \tag{13}$$

We assume the solution of Eq. (11) as

$$w(x,t) = W(x)e^{i\omega t}. \tag{14}$$

So moment at $x=L$ reduces to

$$\begin{aligned} M = &-EI\frac{d^2 W}{dx^2} + (e_0a)^2\left[-\frac{d}{dx}\left(T\frac{dW}{dx}\right) + N_x\frac{d^2 W}{dx^2}\right. \\ &\left. + \rho AW\omega^2\right] = 0. \end{aligned} \tag{15}$$

Now it should be noted that at $x=L$, $W=0$ so we get

$$M = -EI\frac{d^2 W}{dx^2} + (e_0a)^2\left[-\frac{d}{dx}\left(T\frac{dW}{dx}\right) + N_x\frac{d^2 W}{dx^2}\right] = 0. \tag{16}$$

As $T$ is constant at $x=L$ the Eq. (16) reduces to

$$M = (-EI - T + N_x)\frac{d^2 W}{dx^2} = 0. \tag{17}$$

This implies that at $x=L$ second boundary condition from Eq. (15) is

$$\frac{d^2 W}{dx^2} = 0. \tag{18}$$

Thus the local and nonlocal boundary conditions are equivalent for the present analysis.

## V. SOLUTION METHODOLOGY

In order to solve the governing equations [Eq. (11)], we use the DQM.[24] In the DQM, derivative of a function $F$ is approximated as a weighted linear sum of all functional values within the computational domain. This is expressed as

$$\left.\frac{d^n F}{dx^n}\right|_{x=x_i} = \sum_{j=1}^{gp} C_{ij}^{(n)} F(x_j), \tag{19}$$

where $C_{ij}^{(n)}$ are the weighting coefficients of the $n$th order and $gp$ is the number of grid points. The procedure for the determination of the weighting coefficients and the distribution of the grid points are presented in Ref. 24.

Weighting coefficients of the DQM is given by

$$C_{ij}^{(1)} = \frac{\pi(x_i)}{(x_i - x_j)\pi(x_j)}; \quad i,j = 1,2,\ldots,gp; \quad i \neq j, \tag{20}$$

where $\pi(x_i)$ is defined as

$$\pi(x_i) = \Pi_{j=1}^{gp}(x_i - x_j); \quad i \neq j. \tag{21}$$

And when $i=j$

$$C_{ij}^{(1)} = C_{ii}^{(1)} = -\Sigma_{k=1}^{N} C_{ik}^{(1)} \quad i = 1,2,\ldots,gp; \quad i \neq k; \quad i = j. \tag{22}$$

The weighting coefficients for the second, third, and fourth derivatives are determined via matrix multiplication

$$C_{ij}^{(n)} = \Sigma_{k=1}^{gp} C_{ij}^{(1)} C_{ij}^{(n-1)} = \Sigma_{k=1}^{gp} C_{ij}^{(n-1)} C_{ij}^{(1)}, \quad i,j = 1,2,\ldots gp; \quad n > 1. \tag{23}$$

In the present analysis, the grid points are chosen according to Gauss–Chebeshev–Lobatto points

$$\xi_i = \frac{1}{2}\left[1 - \cos\frac{(i-1)}{(gp-1)}\pi\right], \quad i = 1,2,\ldots,gp. \tag{24}$$

To generalize and simplify the solution of the governing equation [Eq. (11)] we introduce the following parameters:

$$\xi = \frac{x}{L}, \quad \delta = \frac{r}{L}, \quad \gamma^2 = \frac{\rho A \Omega^2 L^4}{EI}, \quad \lambda^2 = \frac{\rho A \omega^2 L^4}{EI},$$

$$\bar{N} = \frac{N_x L^2}{EI}, \quad \psi = \frac{e_0 a}{L}. \tag{25}$$

The boundary condition for the propped rotating carbon nanotube at the free support using the above parameters are expressed as

$$W = \frac{dW}{d\xi} = 0 \text{ at } \xi = 0; \quad W = \frac{d^2W}{d\xi^2} = 0 \text{ at } \xi = 1. \tag{26}$$

Using Eqs. (19)–(25), the governing equation for prestressed rotating nanotube in DQ analogous form can be expressed as

$$\begin{aligned}&\Sigma_{j=2}^{gp-1} C_{ik}^{(4)} W_j + \lambda^2 W_i - \psi^2 \lambda^2 \Sigma_{j=2}^{gp-1} C_{ik}^{(2)} W_j + \bar{N} \Sigma_{j=2}^{gp-1} C_{ik}^{(2)} W_j \\ &- \psi^2 \bar{N} \Sigma_{j=2}^{gp-1} C_{ik}^{(4)} W_j - \gamma^2[(-\delta - \xi)\Sigma_{j=2}^{gp-1} C_{ik}^{(1)} W_j \\ &+ (\delta + 0.5 - \delta\xi - 0.5\xi^2)\Sigma_{j=2}^{gp-1} C_{ik}^{(2)} W_j] \\ &+ \psi^2 \gamma^2[-3\Sigma_{j=2}^{gp-1} C_{ik}^{(2)} W_j + 3(-\delta - \xi)\Sigma_{j=2}^{gp-1} C_{ik}^{(3)} W_j \\ &+ (\delta + 0.5 - \delta\xi - 0.5\xi^2)\Sigma_{j=2}^{gp-1} C_{ik}^{(4)} W_j] = 0.\end{aligned} \tag{27}$$

The above equation can be easily transformed into an eigenvalue problem

$$[K]_{\text{total}}\{W\} = \lambda^2\{W\}. \tag{28}$$

$[K]_{\text{total}}$ is the cumulative DQ matrix. Using Eq. (28), one can easily compute the eigenvalues numerically and obtain the natural frequency of the rotating carbon nanotube. The nonlocal boundary conditions are incorporated within the formulation during the determination of weighting coefficients [modified weighting coefficient matrix (MWCM) approach]. Detail procedure can be found in Pradhan and Murmu.[25]

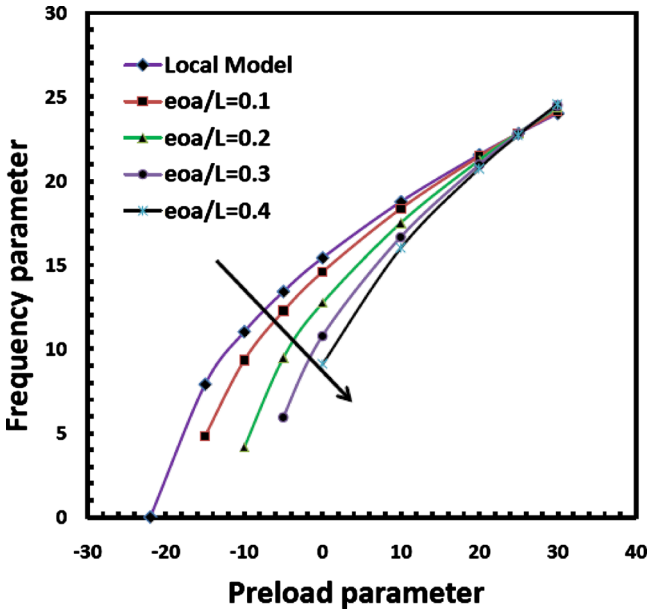


FIG. 3. (Color online) Change in frequency parameter against preload parameter for various nonlocal parameter.

## VI. VIBRATION OF ROTATING SYSTEM

The nonlocal governing equation [Eq. (11)] is solved utilizing the DQ approach.[24] For the present DQ analysis, sufficient numbers of grid points were considered in the computation. The number of grid points are assumed as $gp = 15$. The computer package MATLAB is used to write code based on DQ approach for the expression given by Eq. (27). Using the developed code, the effects of the initial preload on the natural frequencies of the rotating carbon nanotube (Fig. 2) for various (a) small-scale parameter or nonlocal parameter, (b) angular velocity, (c) hub radius, and (d) higher modes are investigated and the related graphs are plotted.

For the present study, the properties of the nonlocal nanobeams are considered that of a SWCNT. An armchair SWCNT with chirality (5, 5) is considered [Fig. 2(a)]. The radius of each individual SWCNT is assumed as 0.34 nm. Young's modulus, $E$, is taken as 0.971 TPa. The density $\rho$ is taken as 2300 $\text{kg/m}^3$. The length is taken as 100 nm. The analysis of SWCNT is computed based on nonlocal beam theory. The frequency results of the NDNBS are presented in terms of the frequency parameters [Eq. (25)]. Angular velocity of rotating carbon nanotube is expressed in rotation parameter $\gamma$. It should be noted that the results are depicted in normalized form. These parameters are used in the computation to keep the versatility as other different geometrical and material properties of nanotubes can be used.

### A. Effect of prestress load on the frequencies of rotating carbon nanotubes

Figure 3 shows the variation in fundamental frequency parameter (first mode) $\lambda$ with preload parameter $\bar{N}$ using both local and nonlocal elastic models. For the case of local model, the nonlocal parameter ($\psi = e_0 a/L$) is assumed to be zero; and is independent of scale effects. While for nonlocal model, the nonlocal parameters $\psi$ are assumed to be 0.1, 0.2,

0.3, and 0.4. Higher values of nonlocal parameter depict higher scale effects. Beyond this range of nonlocal parameters considered when dealing with vibration for higher modes, real solutions cannot be obtained. Analysis with higher values of nonlocal parameter is not performed here because at higher values of $\psi$ real values of frequency parameter $\lambda$ could not be obtained by DQ method. For the present numerical study, the preload parameter $\bar{N}$ is assumed in the range $-30$ to 30. Both the angular velocity parameter $\gamma$ of the carbon nanotube and hub radius $\delta$ is assumed to be unity.

From Fig. 3, it is seen that the fundamental frequencies obtained for a given prestress load of nanotube by nonlocal elastic models are lower than that obtained by local models. Furthermore as the value of nonlocal parameter $\psi$ increases, the value of frequency parameter, $\lambda$ decreases. When prestress loads are neglected, the present observations (Fig. 3) are in line with those of Reddy.[26] Further (Fig. 3) it is observed that as the preload parameter $\bar{N}$ increases the frequency parameter $\lambda$ also increases. With the increase in compressive load, $(-\bar{N})$ the frequencies of the rotating nanotube (nanobeam) decreases. As the frequency reaches zero, the nanotube is supposed to have reached the critical buckling state. With increasing tensile preload $\bar{N}$, the frequencies of the rotating nanotube increases. This is attributed to the fact that as the nanotube is put into tension it becomes stiffer and consequently the frequency increases. Further it can be noted from the Fig. 3 that with increasing compressive preload, the vibrating beam tends to buckle. The buckling mode of the rotating nanotube is enhanced by the increase in nonlocal parameter (Fig. 3). On application of compressive preload, the rotating nanotube with nonlocal parameter ($\psi$=0.4) is more prone to buckle than rotating nanotube analyzed with nonlocal model ($\psi$=0.0). This implies that rotating small-size nanostructures are more affected by nonlocal parameter. This shows the importance of small-scale parameter in the analysis of rotating nanotube. This observation is in-line with Refs. 22 and 23 when the rotation of the nanostructures was not considered.

### B. Effect of rotation on the frequencies of carbon nanotubes

In this section, we see the effect of rotation on the frequencies of carbon nanotubes. Figures 4 show the variation in frequency parameter $\lambda$ with preload parameter $\bar{N}$ for various values of angular velocities $\gamma$. Nonlocal elastic model is used here. We use a value of nonlocal parameter as ($\psi$=0.1). The angular velocities assumed in the present study are chosen as $\gamma$=1, 2, 3, and 4. The hub radius for the nanotube is assumed as $\delta$=1. From Fig. 4, it is observed that the natural fundamental frequency decreases and increases with increasing compressive and tensile preloads, respectively. This trend is noticed for all the values of frequency parameter considered. Further as the angular velocity $\gamma$ of the nanotube increase, the fundamental frequency increases for all given axial preload. This is accounted from the fact that the angular rotation makes the nanotube become stiffer due to centrifugal force which is directly proportional to the square of the angular velocity and hence the frequency rise. It is also important to notice that nonrotating nanotubes are more prone to reach the buckling state faster than the rotating ones.

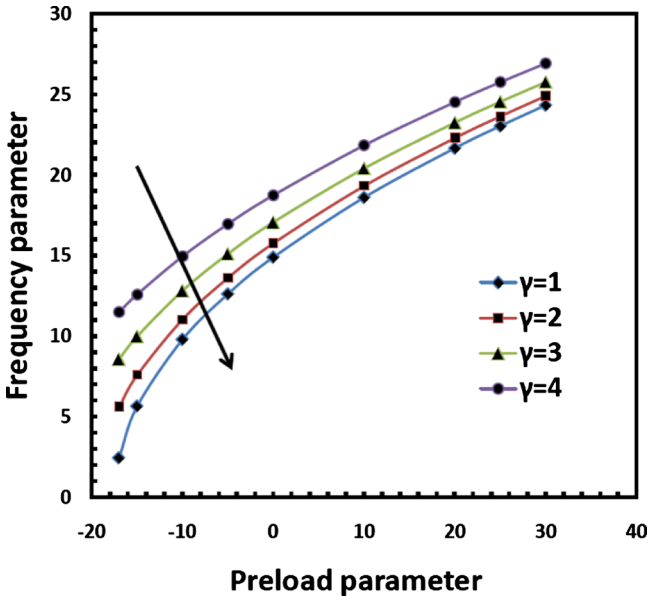


FIG. 4. (Color online) Change in frequency parameter against preload parameter for various angular rotation speed.

### C. Effect of hub radius on the vibration in rotating nanotube

In this section, we see the effect of hub radius on the frequencies of carbon nanotubes. Figures 5 show the variation in frequency parameter $\lambda$ with preload parameter $\bar{N}$ for varies values of hub radius. Nonlocal elastic model is used here with nonlocal parameter assumed ($\psi$=0.1). The angular velocities assumed in the present study are $\gamma$=1. The hub radii of the nanotube are assumed as $\delta$=0.5, 1.0, 1.5, and 2.0.

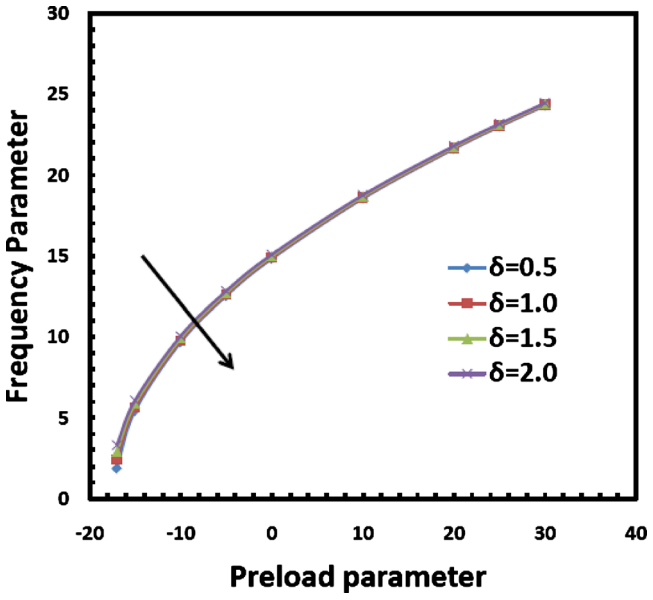


FIG. 5. (Color online) Change in frequency parameter against preload parameter for various hub radii parameter.

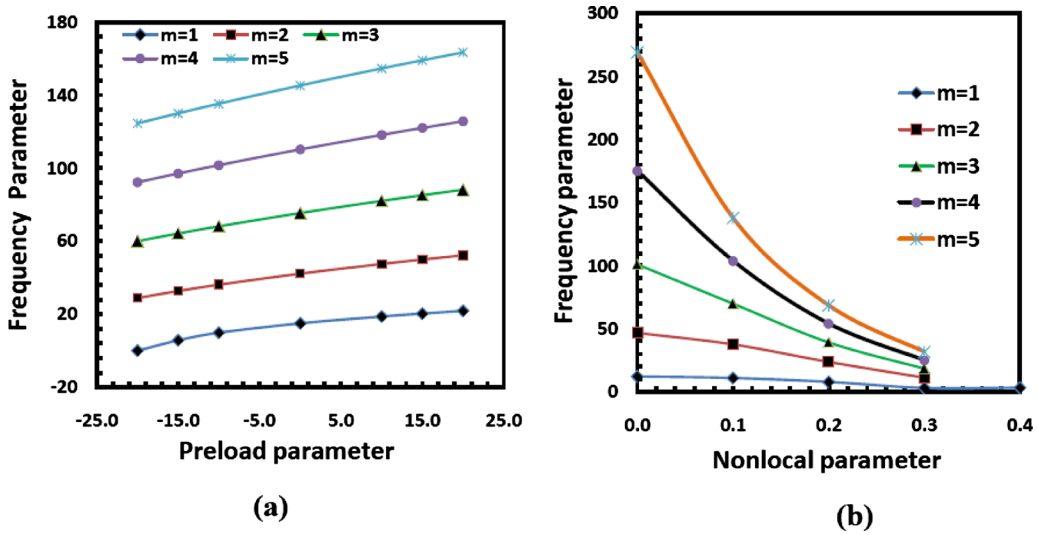


FIG. 6. (Color online) (a) Change in frequency parameter against preload parameter for higher modes (b) change in frequency parameter against nonlocal parameter for higher modes of vibration.

From Fig. 5, it is observed that the natural fundamental frequency decreases and increases with increasing compressive and tensile preload for all the hub radii, respectively. However, not much variation between the curves with different hub radii is noticed. Though the frequency with higher hub radius $\delta=2.0$ are marginally higher than the frequency with $\delta=0.5$.

### D. Analysis of higher modes on frequencies of rotating carbon nanotubes

Here, we analyze the scale effects in higher modes of vibration. Figure 6(a) shows the variation in frequency parameter $\lambda$ with preload parameter $\bar{N}$ for different modes of vibration. Five modes of vibration are analyzed ($m=1$, 2, 3, 4, and 5). The hub radius $\delta$, angular velocity $\gamma$, and the nonlocal parameter $\psi$ assumed as 1, 1, and 0.1, respectively. From Fig. 6(a), it is observed that the natural fundamental frequency decreases and increases with increasing compressive and tensile preload for all the modes considered, respectively. The trend is similar for all the modes. However, at higher compressive preloads, the frequencies with different modes are closer. While at higher tensile loads the frequencies with different modes are slightly farther.

For a preloaded rotating nanotube, Fig. 6(b) shows the variation in frequency parameter $\lambda$ with nonlocal parameter for different modes of vibration. Five modes of vibration are analyzed in the article ($m=1$, 2, 3, 4, and 5). Both the hub radius $\delta$ and angular velocity $\gamma$ are assumed as unity, respectively. An axial compressive preload parameter of ($\bar{N}=-10$) is assumed for the computation. For the rotating nanotube, it is observed that as the nonlocal parameter increases, the frequency decrease. The rate of drop of frequency with nonlocal parameter is magnified for higher modes ($m=3$, 4, and 5).

## VII. CONCLUSIONS

A nonlocal elastic beam model is applied to study the vibration response of rotating carbon nanotubes. DQM is utilized and numerical solutions of frequencies are obtained. Frequencies are presented in a dimensionless form for generality. The effects of the axial preload on nonlocal vibration behavior are examined and discussed. Frequency analyses are carried out for different (a) nonlocal parameters, (b) angular velocities, (c) hub radii, and (d) modes shapes. It is found that that as tensile axial preload increases the fundamental frequency parameter of nanotube also increases. Further with the increase in compressive preload, the frequencies of the rotating nanotube decreases. The buckling state mode of the rotating nanotube is enhanced by the increase in nonlocal parameter. Further as the angular velocity of the nanotube increase, the fundamental frequency increases for all given axial preload. The effect of hub radius is not prominent in the present nonlocal study. The higher modes frequencies of a given preloaded rotating nanotubes are much affected by nonlocal parameter. The present work illustrated here can be extended to the vibration analysis of rotating graphenes. This would be helpful in understanding the vibration characteristic of future rotating nanoturbine. In summary, this study would be useful in perspective to simple mathematical modeling and understanding the scale effects in rotating nanostructures.

[1] K. E. Drexler, *Nanosystems: Molecular Machinery, Manufacturing, and Computation* (Wiley, New York, 1992).
[2] P. V. M. Rao, M. Singh, and V. Jain, Nanotechnology **2**, 250 (2005).
[3] D. H. Robertson, B. L. Dunlap, D. W. Brenner, J. W. Mintmire, and C. T. White, *Fullerene/Tubule Based Hollow Carbon Nano-Gears*, MRS Symposia Proceedings No. 349 (Materials Research Society, Pittsburgh, 1994), p. 283.
[4] J. Han, A. Globus, R. Jaffe, and G. Deardorff, Nanotechnology **8**, 95 (1997).
[5] R. Jaffe, J. Han, and A. Globus, NAS Technical Report No. NAS-97-006, 1997.
[6] D. Srivastava, Nanotechnology **8**, 186 (1997).
[7] A. Lohrasebi and H. Rafii-Tabar, J. Mol. Graphics Modell. **27**, 116 (2008).
[8] S. Zhang, W. K. Liu, and R. S. Ruoff, Nano Lett. **4**, 293 (2004).
[9] A. M. Fennimore, T. D. Yuzvinsky, W.-Q. Han, M. S. Fuhrer, J. Cumings, and A. Zettl, Nature (London) **424**, 408 (2003).
[10] A. C. Eringen, J. Appl. Phys. **54**, 4703 (1983).
[11] Y. Yan, W. Q. Wang, and L. X. Zhang, Appl. Math. Model. **34**, 3422 (2010).
[12] F. Khademolhosseini, R. K. N. D. Rajapakse, and A. Nojeh, Comput.

Mater. Sci. **48**, 736 (2010).
[13]M. Aydogdu, Physica E (Amsterdam) **41**, 1651 (2009).
[14]J. Yang, L. L. Ke, and S. Kitipornchai, Physica E (Amsterdam) **42**, 1727 (2010).
[15]H. Heireche, A. Tounsi, A. Benzair, M. Maachou, and E. A. Adda Bedia, Physica E (Amsterdam) **40**, 2791 (2008).
[16]C. M. Wang, Y. Y. Zhang, S. S. Ramesh, and S. Kitipornchai, J. Phys. D **39**, 3904 (2006).
[17]C. M. Wang and W. H. Duan, J. Appl. Phys. **104**, 014303 (2008).
[18]E. W. Wong, P. E. Sheehan, and C. M. Lieber, Science **277**, 1971 (1997).
[19]J. Gaspar, V. Chu, and J. P. Conde, Appl. Phys. Lett. **84**, 622 (2004).
[20]C. T. Sun and H. Zhang, J. Appl. Phys. **93**, 1212 (2003).
[21]Y. Chen, J. D. Lee, and A. Eskandarian, Int. J. Solids Struct. **41**, 2085 (2004).
[22]P Lu, J. Appl. Phys. **101**, 073504 (2007).
[23]H. Heireche, A. Tounsi, A. Benzair, and I. Mechab, J. Appl. Phys. **104**, 014301 (2008).
[24]C. Shu, *Differential Quadrature and its Application in Engineering* (Springer, New York, 2000).
[25]S.C. Pradhan and T. Murmu, J. Sound Vib. **321**, 342 (2009).
[26]J. N. Reddy, Int. J. Eng. Sci. **45**, 288 (2007).