Nonlocal Vibration of Coupled Double-Nanoplate-Systems

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ABSTRACT

Nonlocal vibration of a double-nanoplate-system is considered. Eringen's nonlocal elasticity is utilized for modelling the coupled nano-system. The nonlocal theory accounts for the small-scale effects arising at the nanoscale. The two nanoplates are assumed to be elastically connected by an enclosing elastic medium. The study highlights that the small-scale effects considerably influence the transverse vibration of double-nanoplate-system. The nonlocal frequencies are lower than the classical frequencies. The small-scale effects in double-nanoplate-system are higher with the increasing values of nonlocal parameter for the case of synchronous modes of vibration than in the asynchronous modes. The increase of the stiffness of the coupling springs in double-nanoplate-system reduces the small-scale effects during the asynchronous modes of vibration. Present work may provide an analytical scale-based nonlocal approach which could serve as the starting point for further investigation of more complex *n*-nanoplates systems arising in future generation graphene based nanocomposites.

Keywords: Double-nanoplate-systems, vibration, scale-effects, nonlocal elasticity, nanocomposites

1. INTRODUCTION

At the small-scale, the sizes of nanostructures often become prominent. Both experimental [**1**] and atomistic simulation [**2**] results have shown a significant 'size-effect' in the mechanical properties when the dimensions of these structures become small. Ignoring the size effects at small-scales and the atomic forces in sensitive nano-designing fields may cause completely incorrect solutions and hence erroneous designs. Atomistic methods such as molecular mechanics simulation [**3**] are able to capture the small-scale effects and atomic forces. However these approaches are computationally prohibitive for mechanical analyses of nanostructures with large numbers of atoms. Thus analyses have been generally carried out by using the classical mechanics [**4**]. Though classical mechanics delivers reasonable solutions, it is independent of small-scale effects and thus may not be always reliable for analysis of nanostructures.

Recently one such promising theory which contains information about the forces between atoms, and the internal length scale is the nonlocal elasticity theory [**5**]. Nonlocal elasticity theory has been applied in various structural studies of nanostructures. Nonlocal elasticity accounts for the small-scale effects arising at the nanoscale level. A large amount of investigations have been conducted on one dimensional nanostructure such as CNTs using nonlocal elasticity. However, compared to one-dimensional nonlocal nanostructures (nanobeams and nanorods), very limited number of studies have been reported on the nonlocal nanoplates (graphene sheets), even though nanoplates possesses many superior properties. Nanoplates such as graphene **[6]** would be one of the prominent new materials for the next generation nano-electronic devices and nanocomposites [**7**]. Reports related to its use as strain sensor, mass and pressure sensors, atomic dust detectors, enhancer of surface image

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resolution are observed. Studies on graphene sheets include vibration studies on single-walled and multi-walled graphene sheets.

It is reported that the study of transversal vibrations of an elastically connected double-plate system is important for both theoretical and practical reasons [**8**]. Many important structures can be modelled as composite structures. Similar to macro plates, an important technological extension of the concept of the single-nanoplate-system would be that of the complex-nanoplate-systems. Complex-nanoplate-systems may find applications in nanooptomechanical systems (NOMS). Vibration of double-nanobeam systems in NOMS is reported [**9-12**]. Vibration analysis of double-nanoplate systems with small aspect ratio is very relevant to NOMS. Elastically connected double-nanoplate system can also be used for the acoustic and vibration isolation.

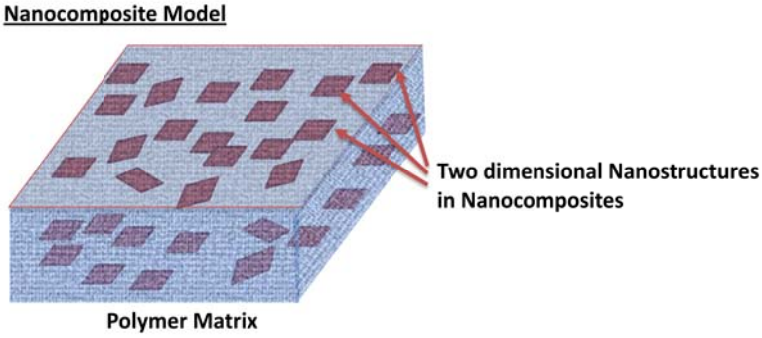

Figure. 1 Nanoplates dispersed in polymer in nanocomposites.

Further complex-nanoplate-systems can be important in nanosensors applications and in nanocomposites. Vibration characteristics of multiple nanoplates (graphene sheets) dispersed in nanocomposites can be important from the structural performance of nanocomposites. Vibration of double-nanoplate-system coupled by elastic medium is worth understanding in this respect. The different stiffness of elastic medium will impart different vibration characteristic of the system in nanocomposites. A schematic model of nanocomposites is shown in Fig. 1. Though complex-nanoplate-systems are important in nanodevices and nanocomposites, no works appear related to the study of its vibration characteristics.

Therefore, based on the above discussion there is a strong encouragement to gain an understanding of the vibration of complex-nanoplate-system and the mathematical modelling of such phenomena. In the present proceedings, study of scale-dependent vibration of double-nanoplates-systems is considered. The two nanoplates are elastically connected by enclosing elastic medium. The elastic medium is modelled by vertical springs. Expressions for free bending-vibration of double-nanoplate-system are established within the framework of Eringen's nonlocal elasticity. Two single-layered graphene sheets enclosed by a polymer matrix are considered for the study. The study highlights that the small-scale effects considerably influence the transverse vibration of NDNPS. The nonlocal natural frequencies of NDNPS are smaller when compared to the corresponding classical results. The small-scale effects in NDNPS are higher with increasing values of nonlocal parameter for the case of in-phase (synchronous) modes of vibration than in the out-of-phase (asynchronous) modes of vibration. The increase of the stiffness of the coupling springs in NDNPS reduces the small-scale effects during the out-of-phase modes of vibration. In summary, this work may provide an analytical scale-based nonlocal approach which could serve as the starting point for further investigation of more complex *n*-nanoplates systems arising in future generation graphene based nanocomposites.

2. NONLOCAL ELASTICITY THEORY

Here we provide a brief review of the nonlocal elasticity theory. In this theory the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain [**5**]**.** Nonlocal theory considers long-range inter-atomic interaction and yields results dependent on the size of a body. According to the nonlocal elasticity, the basic equations for an isotropic linear homogenous nonlocal elastic body neglecting the body force are given as [**5**]

$$\sigma_{ij,j} = 0, \qquad \sigma_{ij}(\mathbf{x}) = \int_{\mathbf{V}} \phi(|\mathbf{x}-\mathbf{x}'|,\alpha) t_{ij} d\mathbf{V}(\mathbf{x}'), \;\; \forall \mathbf{x} \in \mathbf{V}$$

$$t_{ij} = H_{ijkl}\varepsilon_{kl}, \qquad \varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{1}$$

The terms σ_{ij}, t_{ij}, ε_{kl}, H_{ijkl} are the nonlocal stress, classical stress, classical strain and fourth order elasticity tensors respectively. The volume integral is over the region **V** occupied by the body. The above equation (Eq. 1) couples the stress due to nonlocal elasticity and the stress due to classical elasticity. The kernel function $\phi(|\mathbf{x}-\mathbf{x}'|,\alpha)$ is the nonlocal modulus. The nonlocal modulus acts as an attenuation function incorporating into constitutive equations the nonlocal effects at the reference point $\mathbf{x}$ produced by local strain at the source $\mathbf{x}'$. The term $|\mathbf{x}-\mathbf{x}'|$ represents the distance in the Euclidean form and α is a material constant that depends on the internal (e.g. lattice parameter, granular size, distance between the C-C bonds) and external characteristics lengths (e.g. crack length, wave length). Material constant α is defined as

$$\alpha = e_0 a/\ell \tag{2}$$

Here e_0 is a constant for calibrating the model with experimental results and other validated models**.** The parameter e_0 is estimated such that the relations of the nonlocal elasticity model could provide satisfactory approximation to the atomic dispersion curves of the plane waves with those obtained from the atomistic lattice dynamics. The terms a and ℓ are the internal (e.g. lattice parameter, granular size, distance between C-C bonds) and external characteristics lengths (e.g. crack length, wave length) of the nanostructure.

3. NONLOCAL DOUBLE-NANOPLATE SYSTEM

Consider the nonlocal double nanoplate system (NDNPS) as shown in Fig. 2a. The two nanoplates of the NDNPS are referred to as nanoplate-1 and nanoplate-2. The two nanoplates are coupled by a polymer matrix. The polymer matrix is the elastic medium. For mathematical modelling it is assumed that vertically distributed identical Winkler springs couples the two nanoplates (Fig. 2b). In generality the springs may be used to substitute the elastic medium, forces due to nanooptomechanical effect [**9-12**], or Van der Waals forces between the two nanoplates. The springs are assumed to have stiffness *k*. Different values of *k* for different polymer matrix can be used for the study. The nanoplates are considered to be of length *L* and width *W*. Generally, the two nanoplates are different where the length, width, mass per unit length and bending rigidity of the ith plate are L_i, W_i, m_i and D_i (i=1, 2) respectively. These parameters are assumed to be constant along each nanoplate.

Figure 2. Two nanoplates coupled by elastic medium. (b) Mathematical representation of double-nanoplate-system

The bending displacements over the two nanoplates are denoted by $w_1(x,t)$ and $w_2(x,t)$, respectively (Fig. 2b). Using nonlocal plate equations, the individual governing equations for NDNPS can be derived as

(nanoplate -1)

$$D_1\nabla^4 w_1(x,y,t) + \rho_1 h_1 \ddot{w}_1(x,y,t) + k[w_1(x,y,t) - w_2(x,y,t)] - k(e_0a)^2\nabla^2[w_1(x,y,t) - w_2(x,y,t)] - (e_0a)^2\rho_1 h_1\nabla^2\ddot{w}_1(x,y,t) = f(x,y,t) \quad (3)$$

where the bending rigidity of nanoplate-1 can be expressed as $D_1 = E_1h_1^3/12(1-\upsilon_1^2)$.

The terms E, ρ, h and υ are the Young's modulus, mass density, thickness and Poisson's ratio of the naonoplate. Term $f(x,y,t)$ is the forcing function. Subscript 1 denotes the properties of the nanoplate-1.

(nanoplate -2)

$$D_2\nabla^4 w_2(x,y,t) + \rho_2 h_2 \ddot{w}_2(x,y,t) - k[w_1(x,y,t) - w_2(x,y,t)] + k(e_0a)^2\nabla^2[w_1(x,y,t) - w_2(x,y,t)] - (e_0a)^2\rho_2 h_2\nabla^2\ddot{w}_2(x,y,t) = 0 \quad (4)$$

where the bending rigidity of nanoplate-2 is expressed as $D_2 = E_2h_2^3/12(1-\upsilon_2^2)$.

It should be noted that when the nonlocal parameter is $(e_0a = 0)$, the equations reduce to that of classical plate equations. Further this model is different from multilayered-graphenes, where the coupling stiffness (Vander Waal forces) is constant.

4. BOUNDARY CONDITIONS IN DOUBLE-NANOPLATE -SYSTEM

Now we present the explicit mathematical expressions of the boundary conditions of the double-nanoplate-system. It is assumed that all the edges in the nanoplate system are simply supported. At each ends of the nanoplates in NDNPS, the displacement and the nonlocal moments are considered to be zero. They can be mathematically expressed as

(nanoplate-1)

Displacement condition

$$w_1(0,y,t)=0;\ w_1(L,y,t)=0;\ w_1(x,0,t)=0;\ w_1(x,W,t)=0; \quad (5)$$

Nonlocal moment condition

$$M_1(0,y,t)=0;\ M_1(L,y,t)=0;\ M_1(x,0,t)=0;\ M_1(x,W,t)=0\ ; \quad (6)$$

(nanoplate-2)

Displacement condition

$$w_2(0,y,t)=0;\ w_2(L,y,t)=0;\ w_2(x,0,t)=0;\ w_2(x,W,t)=0; \quad (7)$$

Nonlocal moment condition

$$M_2(0,y,t)=0;\ M_2(L,y,t)=0;\ M_2(x,0,t)=0;\ M_2(x,W,t)=0; \quad (8)$$

Here it should be noted that the moment is the nonlocal moment.

5. DISCUSSION

The nonlocal plate theory for NDNPS illustrated here is a generalised theory and can be applied for the bending-vibration analysis of coupled graphene sheets, gold nanoplates etc. The applicability of nonlocal elasticity theory in the analysis of nanostructures (nanotubes and graphene sheet) can be observed in various earlier works.

Let us assume that the two nanoplates are identical such that $D_1=D_2=D\equiv$ constant and $\rho_1 h_1=\rho_2 h_2=\rho h\equiv$ constant. Further free vibration is considered, i.e.. $f(x,y,t)=0$. We define the following parameters: $\Omega_{\mathrm{mn}}=\omega_{mn}L^2\sqrt{\frac{\rho h}{D}}$; $R=L/W$; (spring stiffness) $K=\frac{kL^2}{D}$. Length of graphene is taken as 10 nm. We use the conventional Navier solution method [**8**] to determine Ω_{mn}.

To see the influence of small-scale on the natural frequency of the coupled-graphene sheet-systems, curves have been plotted for frequency parameter and scale coefficient (nonlocal parameter,μ). The GSs are coupled by a polymer matrix of stiffness $K=100$. The Young's modulus of the GS is considered as $E=1.06$ TPa, the Poisson ratio $\upsilon=0.25$, and the mass density as $\rho=2250\ \mathrm{kg/m^3}$.The thickness of the GS is taken as $h=0.34$ nm. Length of graphene is taken as 10 nm. In the present study for generality, we take the scale coefficient μ or nonlocal parameter in the range as $\mu=0-1$.

To signify the small-scale effect we introduce a parameter frequency reduction percent (FRP). Frequency Reduction Percent (FRP) is defined as

$$\mathrm{FRP}=100\times\left(\frac{\Omega_{\mathrm{Local}}-\Omega_{\mathrm{Nonocal}}}{\Omega_{\mathrm{Local}}}\right) \quad (9)$$

Fig. 3 shows the variation of the frequency parameter with the scale coefficient for different cases of NDNPS (*m*=1, *n*=1). The results for the frequency parameter Ω are in the dimensionless form. The stiffness parameter of the springs is assumed to be constant (*K*=100). Aspect ratio is considered as *R*=1.Unless stated the frequency parameter would denote the parameter associated with the first natural frequencies (in-phase and out-of-phase type vibration). From the figure (Fig. 3) it can be observed that as the scale coefficient μ increases the FRP increases. This implies that for increasing scale coefficient the value of frequency parameter decreases. The reduction in frequency parameter is due to the assimilation of small-scale effects in the NDNPS in the material properties of the graphene sheets. The small-scale effect reduces the stiffness of the material and hence the comparative lower frequencies. Therefore by the nonlocal elastic model the size-effects are reflected in the NDNPS.

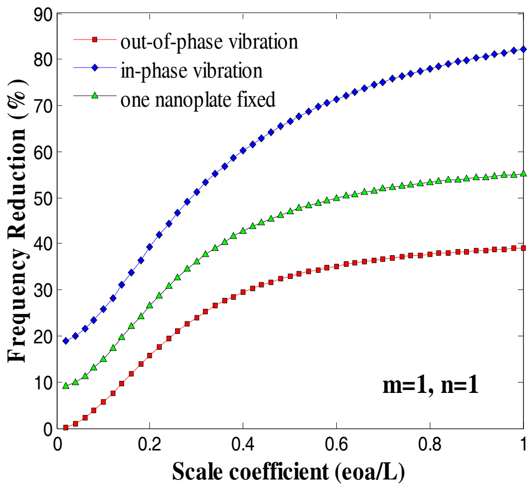

Figure. 3 Change of frequency reduction percent with scale coefficient for out-of-phase, in-phase and one nanoplate fixed in NDNPS

Three different cases of NDNPS are considered. Case 1, Case2 and Case 3 depicts the conditions (i) when both the GS vibrates in the out-of-phase sequence $(w_1 - w_2 \neq 0)$ (ii) when one of the GS in NDNPS is stationary $(w_2 = 0)$ (iii) when both the GS vibrates with in-phase sequence $(w_1 - w_2 = 0)$, respectively. Comparing the three cases of coupled-graphene-sheet-system, we observe that the FRP for case 3 (in-phase vibration) is larger than the FRP for case 1 (out of phase vibration) and case 2 (one-GS fixed). In other words, the scale coefficient significantly reduces the in-phase natural frequencies (thus higher FRP) compared to other cases considered. The relative higher FRP in in-phase vibration is due to the absence of coupling effect of the spring and the two nanoplates (GSs) and the whole NDNBS can be treated as a vibrating single GS. In general, it is worth noticing that the small-scale effects in NDNPS are higher with increasing nonlocal parameter in the in-phase vibration than in the out-of-phase vibration. This is because the stiffness of the springs in the out-of-phase (asynchronous) vibration reduces the nonlocal effects. In addition, it can be seen that the values of the FRP for case-2 (one-GS fixed) is larger than the values of the FRP for case-1 (out-of-phase vibration). For Case-2 the coupled-graphene-sheet-system becomes similar to the vibration characteristic of the single GS with the effect of elastic medium.

To illustrate the influence of stiffness of the springs (elastic medium) on the natural frequencies of the coupled-GS-systems, curves have been plotted for the FRP against the scale coefficient. Spring stiffness represents the stiffness of the enclosing elastic medium. Different values of stiffness parameter of the coupling springs are considered. Fig. 4 (a-d) depicts the stiffness of the springs on the FRP of coupled systems. The stiffness parameter of the coupling springs are taken as *K*=0, 10, 20, 50. Aspect ratio of coupled GS is taken as unity. As the stiffness parameter of the coupling springs increases the FRP decreases. Considering all values of the stiffness parameter; and comparing the three cases of coupled-GS system, it is noticed that the FRP for case 3 (in-phase vibration) is larger than the FRP for case 1 (out of phase vibration) and case 3 (one-GS fixed). These different changes of FRP with the increasing scale coefficient for the three different cases are more amplified as the stiffness parameter of the spring's increases. For case 1(out-of-phase vibration) and case 2 (one-GS-fixed), the FRP reduces with increasing values of stiffness parameter. This observation implies that case 1 (out-of-phase vibration) and case 2 (one GS fixed) are less affected by scale-effects. Comparing case 1 and case 2 it can be seen the FRP is lesser for out of phase vibration than for vibration in case 2. Thus the out-of-phase vibration is less affected by the small-scale or nonlocal effects. This out-of-phase vibration can be attributed to the fact that the coupling springs in the vibrating system dampens the nonlocal effects.

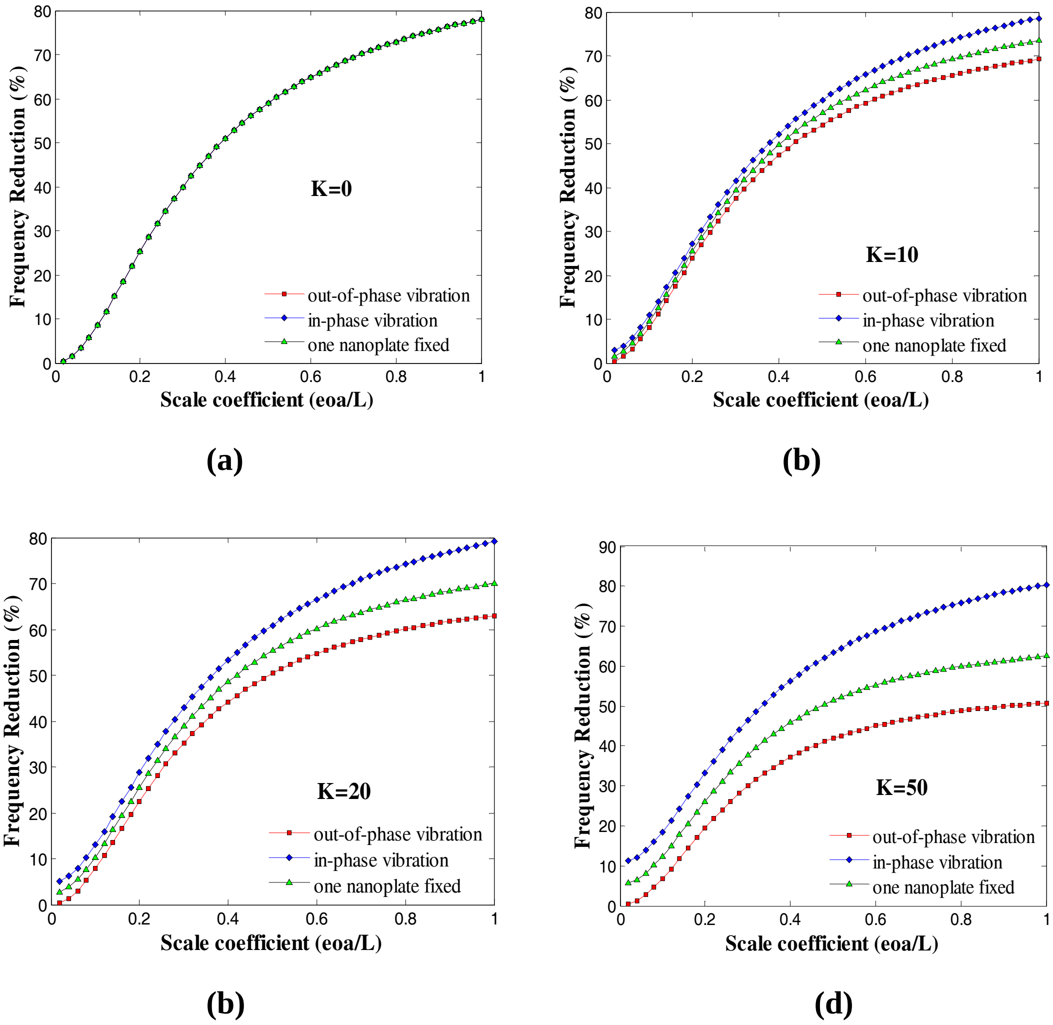

Figure 4 Change of frequency reduction percent with scale coefficient for different coupling springs in NDNPS; (a) K=0, (b) K=10, (c) K=20, (d) K=50.

In-phase vibration of coupled-system is unchangeable with increasing stiffness of springs. This is accounted due to the in-phase vibration mode of behaviour. For in-phase type of vibration the coupled system behaves as if a single SWGS without the effect of internal elastic medium. In other words the whole coupled system can be treated as a single nano element and the coupling internal structure is effect less. In summary, it should be noted that the in-phase vibration of coupled-system are more affected by small-scale effects compared to out-of-phase vibration.

Finally we say that the analytical scale-based nonlocal approach applied here can serve as the starting point for further investigation of more complex *n*-nanoplates systems arising in future generation graphene based nanocomposites.

6. CONCLUSION

In this paper, the expressions for free bending-vibration of double-nanoplate-system are discussed utilizing nonlocal elasticity. Two single-layered graphene sheets coupled by polymer matrix are considered for the study. The double-nanoplate-system executes two kinds of vibrations: the synchronous vibrations with lower frequencies and the asynchronous vibrations with higher frequencies. The study highlights that the small-scale effects considerably influence the transverse vibration of double-nanoplate-system. The small-scale effects in double-nanoplate-system are higher with increasing values of nonlocal parameter for the case of synchronous (in-phase) modes of vibration than in the asynchronous (out-of-phase) modes of vibration. The increase of the stiffness of the coupling springs in double-nanoplate-system reduces the small-scale effects during the asynchronous modes of vibration. The synchronous natural frequencies are not dependent on the stiffness parameter of the elastic medium. In this case, the double-nanoplate system oscillates as a single nanoplate with the same natural frequencies. The analytical scale-based nonlocal approach developed here can serve as the starting point for further investigation of more complex *n*-nanoplates systems arising in future generation graphene based nanocomposites.

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