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SENSOR DESIGN FOR PIEZOELECTRIC CANTILEVER BEAM ENERGY HARVESTERS

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ABSTRACT

Energy harvesting for the purpose of powering low power electronic sensor systems has received explosive attention in the last few years. A common device uses the piezoelectric effect for a cantilever beams at resonance to harvest ambient vibration energy. However most of these devices have a rectangular piezoelectric patch covering all or part of the beam. This paper considers the optimum design of such a device, and in particular investigates the effect that the size and shape of piezoelectric sensor has on the harvested energy. It is shown that significant increases in harvested energy may be obtained by optimising the sensor design.

1. INTRODUCTION

The harvesting of ambient vibration energy for use in powering low energy electronic devices has formed the focus of much recent research [1–3]. Energy harvesting of ambient vibration has become important and new electronic devices are being developed that require very low power. Completely wireless sensor systems are desirable and this can only be accomplished by using batteries and/or harvested energy. Harvesting is attractive because harvested energy can be used directly or used to recharge batteries or other storage devices, which enhances battery life. Of the published results that focus on the piezoelectric effect as the transduction method, almost all have focused on harvesting using cantilever beams and on single frequency ambient energy, i.e., resonance based energy harvesting. Several authors (for example, [4–8]) have proposed methods to optimize the parameters of the system to maximize the harvested energy.

Erturk and Inman [9–11] developed the equations of motion for cantilever beam energy harvesters. Goldschmidtboeing and Woias [12] considered the effect of different beam shapes on the harvested energy. Schoeftner and Irschik [13] considered shaped transducers for increasing passive damping, which is closely related to energy harvesting. Kaal et al. [14] considered energy harvesters with piezoelectric transducers that only covered part of the beam.

This paper considers beam energy harvesters clamped at one end to a support that vibrates sinusoidally. The novel feature of this analysis is the incorporation of a piezoelectric transducer whose width can be varied along the beam length. Shaped piezoelectric sensors are well established for modal control [15] and applications are emerging in structural health monitoring [16]. The approach specifies the shape of the sensor by the width and the slope at the finite element nodes; this allows the shape to be specified easily using a small number of variables, and can lead to easy optimisation of the shape.

2. ELECTROMECHANICAL MODEL OF A CANTILEVER BEAM HARVESTER

The energy harvester consists of a cantilever beam whose support is subjected to motion from the host structure. The development in this paper will assume that the motion of the beam support is only translational, although the extension to allow rotation of the support is straightforward. The system is shown schematically in Figure 1, where the beam is not necessarily uniform, and discrete masses may be added to the beam, for example as shown at the end in the figure. The mechanical parts of this model, and the excitation from the support motion, are standard.

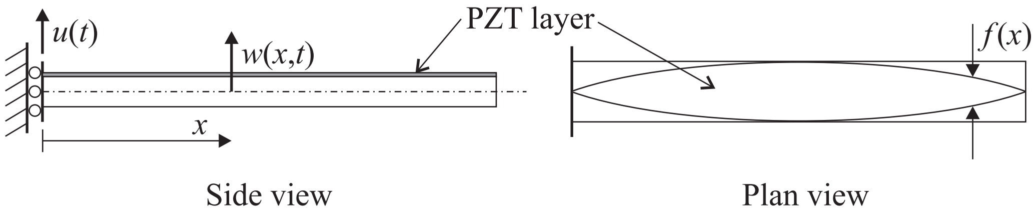


Figure 1. Schematic of the cantilever beam energy harvester excited at the support

We now concentrate on the electromechanical coupling. Suppose that the displacement of the beam is given by $w(x)$, where x is the position along the beam. This displacement is the elastic deformation, that is the displacement relative to the rigid body displacement due to the support excitation. The constant thickness piezoelectric material is placed along the beam, and the effective width is given by $f(x)$. The variable width would most likely be implemented by varying the width of the electrode while keeping the width of piezoelectric material constant. The current output is then given by

$$i_p = -e_{31} h \int_0^L f(x) \frac{\partial^3 w}{\partial x^2 \partial t} \mathrm{d}x \tag{1}$$

where h is the distance of the piezoelectric material from the beam centreline (assuming the beam is uniform and the piezoelectric layer is very thin) and e_{31} is the piezoelectric constant. This equation assumes that the material is operating in the 31 mode, which is the case for a thin layer of homogeneous piezoelectric material on the beam with the electrodes on the top and bottom surfaces; a similar equation may be developed for transducers with interdigitated electrodes.

If the system is modelled with finite element analysis, using standard Euler-Bernoulli beam elements, then the equations of motion are

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{Q}_p + \mathbf{B}_M \ddot{u} + \mathbf{B}_D \dot{u} + \mathbf{B}_K u \tag{2}$$

where $\mathbf{q}$ are the nodal displacements of the beam model, $\mathbf{Q}_p$ is the force arising from charge on the piezoelectric material, and the dot denotes the derivative with respect to time. The coefficients of the terms involving the support displacement u are obtained from the mass, damping and stiffness matrices for the free-free beam. Using the finite element displacement functions, the charge on the piezoelectric material may be estimated for each element and the total charge obtained by summing the charge over the elements. This is exactly the same as the development for shaped sensors for control or structural health monitoring [15, 16]. The result is that the current generated by the piezoelectric can be written in terms of the nodal displacements and rotations as

$$i_p = -\mathbf{f}^\top \mathbf{C}_{pq}\dot{\mathbf{q}} - \mathbf{f}^\top \mathbf{C}_{pu}\dot{u} \tag{3}$$

where $\mathbf{C}_{pq}$ is a matrix and $\mathbf{C}_{pu}$ is a vector that are easily assembled from element coupling matrices and $\mathbf{f}$ is a vector that defines the sensor shape [15, 16]. The last term in Equation (3) arises because of the imposed motion at the beam support. $\mathbf{f}$ actually gives the transducer width and slope at the nodes of the finite element model, and the shape within the elements is approximated using the beam shape functions.

The force on the beam from the piezoelectric material is given by

$$\mathbf{Q}_p = \mathbf{C}_{pq}^\top \mathbf{f} v_p \tag{4}$$

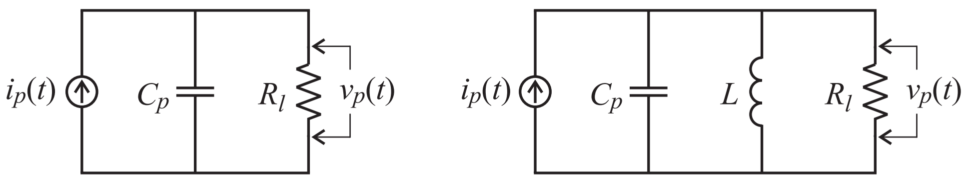
$v_p(t)$ is the voltage across the piezoceramic.

3. SIMPLE ELECTRICAL CIRCUITS FOR THE ENERGY HARVESTER

Figure 2 shows two simple electrical circuits that will be modelled to investigate the performance of the energy harvester. The governing equations are obtained using Kirchoff's laws, where the current generted the piezoelectric material is given by Equation (3). Thus, without the inductor (case (a)), the governing electrical equation is

$$\mathbf{f}^\top \mathbf{C}_{pq}\dot{\mathbf{q}} + \mathbf{f}^\top \mathbf{C}_{pu}\dot{u} + C_p \dot{v}_p + \frac{1}{R_l} v_p = 0 \tag{5}$$

where the load resistance is R_l, and the capacitance of the piezoceramic is C_p.



(a) without inductor (b) with inductor

Figure 2: The two electrical circuits for the harvester (a) without an inductor, (b) with an inductor.

When an inductor is included (case (b)) the governing equation is

$$\mathbf{f}^\top \mathbf{C}_{pq}\ddot{\mathbf{q}} + \mathbf{f}^\top \mathbf{C}_{pu}\ddot{u} + C_p \ddot{v}_p + \frac{1}{R_l} \dot{v}_p + \frac{1}{L} v_p = 0 \tag{6}$$

where L is the inductance of the circuit.

One important feature of the shaped transducer is that the capacitance of the piezoelectric device depends on the area of the electrode, and therefore the shape of the electrode. Thus

$$C_p = \frac{\epsilon_{33}}{t_p} \int_0^L f(x)\mathrm{d}x = \mathbf{f}^\top \mathbf{g} \tag{7}$$

where ϵ_{33} is a constant, t_p is the thickness of the piezoelectric material, and g is a vector calculated from the beam element shape functions.

4. TWO DEGREE OF FREEDOM ELECTROMECHANICAL MODEL

Often the energy harvester is designed to be resonant, as this increases the power output of the harvester for a fixed frequency excitation. Thus one of the natural frequencies of the harvester corresponds to the excitation frequency, and the deformation shape of the beam is dominated by a single mode. Suppose the rth mode is excited, then we may approximate the response using a single degree of freedom, namely the rth principal coordinate p_r, so that

$$\mathbf{q} = \boldsymbol{\phi}_r p_r \tag{8}$$

where $\boldsymbol{\phi}_r$ is the rth mode shape. Substituting this displacement into Equation (2), using Equation (4) and pre-multiplying by $\boldsymbol{\phi}_r^T$, gives the single degree of freedom mechanical model as

$$\ddot{p}_r + 2\zeta_r\omega_r\dot{p}_r + \omega_r^2 p_r - \frac{\theta_r}{m_r}v_p = B_M\ddot{u} + B_D\dot{u} + B_K u \tag{9}$$

where ζ_r and ω_r are the rth damping ratio and natural frequency, $m_r = \boldsymbol{\phi}_r^\top \mathbf{M}\boldsymbol{\phi}_r$ is the rth modal mass, and

$$\theta_r = \boldsymbol{\phi}_r^\top \mathbf{C}_{pq}^\top \mathbf{f}. \tag{10}$$

The base excitation coefficients are given by

$$B_M = \boldsymbol{\phi}_r^\top \mathbf{B}_M/m_r, \qquad B_D = \boldsymbol{\phi}_r^\top \mathbf{B}_D/m_r, \qquad B_K = \boldsymbol{\phi}_r^\top \mathbf{B}_K/m_r. \tag{11}$$

The equations governing the electrical circuit become,

$$\theta_r\dot{p}_r + C_p\dot{v} + \frac{1}{R_l}v_p = -\mathbf{f}^\top \mathbf{C}_{pu}\dot{u} \tag{12}$$

without the inductor, or

$$\theta_r\ddot{p}_r + C_p\ddot{v} + \frac{1}{R_l}\dot{v} + \frac{1}{L}v_p = -\mathbf{f}^\top \mathbf{C}_{pu}\ddot{u} \tag{13}$$

with the inductor.

5. POWER GENERATED FOR RESONANT HARVESTER UNDER RESONANT HARMONIC EXCITATION

Suppose now that the beam support is excited sinusoidally, so that

$$u(t) = u_0 \cos\omega t = \Re\left\{u_0 e^{\mathrm{i}\omega t}\right\} \tag{14}$$

where the excitation frequency, ω, is assumed to be constant and $\mathrm{i} = \sqrt{-1}$. Then Equation (9) becomes

$$\ddot{p}_r + 2\zeta_r\omega_r\dot{p}_r + \omega_r^2 p_r - \frac{\theta_r}{m_r}v_p = \Re\left\{u_0\left(-\omega^2 B_M + \mathrm{i}\omega B_D + B_K\right)e^{\mathrm{i}\omega t}\right\}. \tag{15}$$

Looking for solutions of the form,

$$p_r(t) = \Re\left\{P_r(\omega)e^{\mathrm{i}\omega t}\right\} \quad \text{and} \quad v_p(t) = \Re\left\{V_p(\omega)e^{\mathrm{i}\omega t}\right\} \tag{16}$$

where P_r and V_p are complex, gives

$$\left(-\omega^2 + 2\zeta_r\omega_r\mathrm{i}\omega + \omega_r^2\right)P_r - \frac{\theta_r}{m_r}V_p = \left(-\omega^2 B_M + \mathrm{i}\omega B_D + B_K\right)u_0. \tag{17}$$

Equation (12), for the electrical circuit without the inductor, becomes

$$\theta_r \mathrm{i}\omega P_r + \left(C_p \mathrm{i}\omega + \frac{1}{R_l} \right) V_p = -\mathbf{f}^\top \mathbf{C}_{pu} \mathrm{i}\omega u_0. \tag{18}$$

For the case with the inductor, Equation (18), becomes

$$-\theta_r \omega^2 P_r + \left(-C_p \omega^2 + \frac{\mathrm{i}\omega}{R_l} + \frac{1}{L} \right) V_p = \mathbf{f}^\top \mathbf{C}_{pu} \omega^2 u_0. \tag{19}$$

The voltage may be estimated by eliminating the displacement in Equation (17) and one of Equations (18) and (19), and the amplitude of the harvested power is obtained as

$$P_{\text{out}} = \frac{|V_p|^2}{R_l}. \tag{20}$$

6. CHOOSING PARAMETERS FOR THE ENERGY HARVESTER

Designing a harvester for a particular situation is difficult because all of the parameters interact. To gain some insight a number of simplifying assumptions will be made for the system without an inductor. Suppose the load impedance is low, so that the optimum is a resonant harvester, that is $\omega = \omega_r$ for some mode (i.e. some r). Further, suppose that the resonance condition, and other constraints such as maximum harvester mass and maximum dimensions, fixes the dimensions of the beam. The excitation term on the right side of Equation (18) will be neglected, as this will certainly be small if the lengths of the beam elements are small. Finally the inertia and damping excitation terms on the right side of Equation (15) will be neglected. The output voltage may then be written as

$$V_p = \frac{m_r \theta_r B_K u_0}{c_r \left(C_p \mathrm{i}\omega_r + \frac{1}{R_l} \right) + \theta_r^2} \tag{21}$$

where $c_r = 2\zeta_r \omega_r m_r$ is the rth modal damping parameter.

Since the beam parameters are assumed to be fixed, the output voltage, and hence power harvested, for a given output load and excitation frequency, is a function of the electromechanical coupling parameter, θ_r, and the capacitance of the piezoelectric material, C_p. Both of these parameters depend on the shape of the piezoelectric transducer; as the area of coverage of the electrode increases, both C_p and θ_r tend to increase. For a typical case, where $\omega_r = 20$Hz, $R_l = 20\Omega$, $B_K = 1$, $m_r = 1$kg, $\zeta_r = 0.05$, Figure 3 shows the power generated for a range of values of C_p and θ_r. It is clear that the electromechanical coupling, θ_r, should be maximised, and the piezoelectric capacitance, C_p, should be minimised. For rectangular transducers, these two requirements are conflicting, and hence a shaped transducer may be able to help. Of course increasing the thickness of the piezoelectric material is another way to reduce the capacitance, but may not be possible.

Of course if a mode has a vibration node, then it is important that the transducer only covers parts of the beam where the curvature has the same sign (or the transducer could be segmented).

7. NUMERICAL EXAMPLE

This example is designed to demonstrate that different shaped sensors can influence significantly the power generated by the harvester. Consider a steel beam of length 150mm, breadth 10mm and thickness 3mm, clamped at one end at a support that vibrates with amplitude 1mm at frequency of 110Hz. Note that this is very close to the first natural frequency of the cantilever

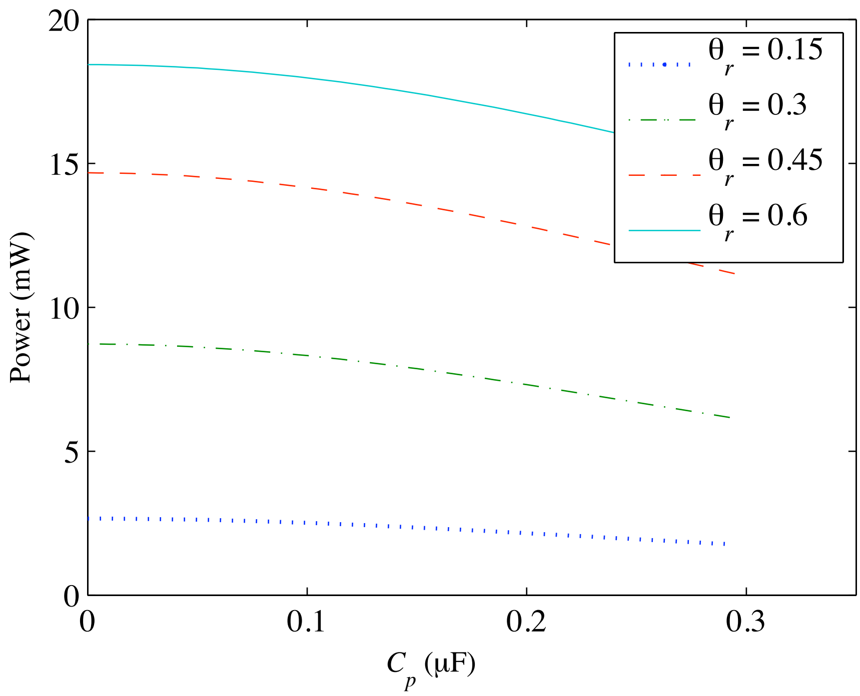


Figure 3: The effect of coupling coefficient and piezoelectric capacitance for the resonant harvester without an inductor.

beam at 111.4Hz. The damping ratio for the first mode is 1%. The piezoelectric material is 0.27mm thick, with permittivity given by $\epsilon_{33} = 1800\epsilon_0$, where the permittivity of free space is $\epsilon_0 = 8.854$pF/m, and the piezoelectric constant is given by $e_{31} = -12.54$C/m^2. The shaped sensors are tested, shown in Figure 4. Since the excitation frequency is very close to the first natural frequency the harvester is resonant and the two degree of freedom model is a good approximation. Table 1 gives the capacitance, coupling coefficient and power output for the three sensors, and shows that the sensor shape has a significant effect on the power, through the competing influences of the capacitance and the coupling coefficient. Furthermore covering the whole beam with piezoelectric material generates about one third of the power than the case where the material only covers part of the beam.

Sensor number	Description	Capacitance (pF)	Coupling	Power Output
1	Uniform	119.5	-0.00918	1.093
2	Triangular	59.8	-0.00667	2.125
3	Root segment	59.8	-0.00776	2.885

Table 1. The properties and performance of the three shaped sensors.

8. CONCLUSIONS

The paper has developed the equations to estimate the power output from a beam energy harvester with a piezoelectric transducer. In particular the possibility of changing the shape of the transducer has been included by specifying the width and slope of the transducer at the finite element nodes. The transducer shape alters both the capacitance of the piezoelectric material, and also the coupling factor; often these parameters give rise to opposing effects on the power output. Three example shapes have been evaluated to determine the power output obtained, and this clearly shows that the shape has a significant influence. The example was a cantilever beam

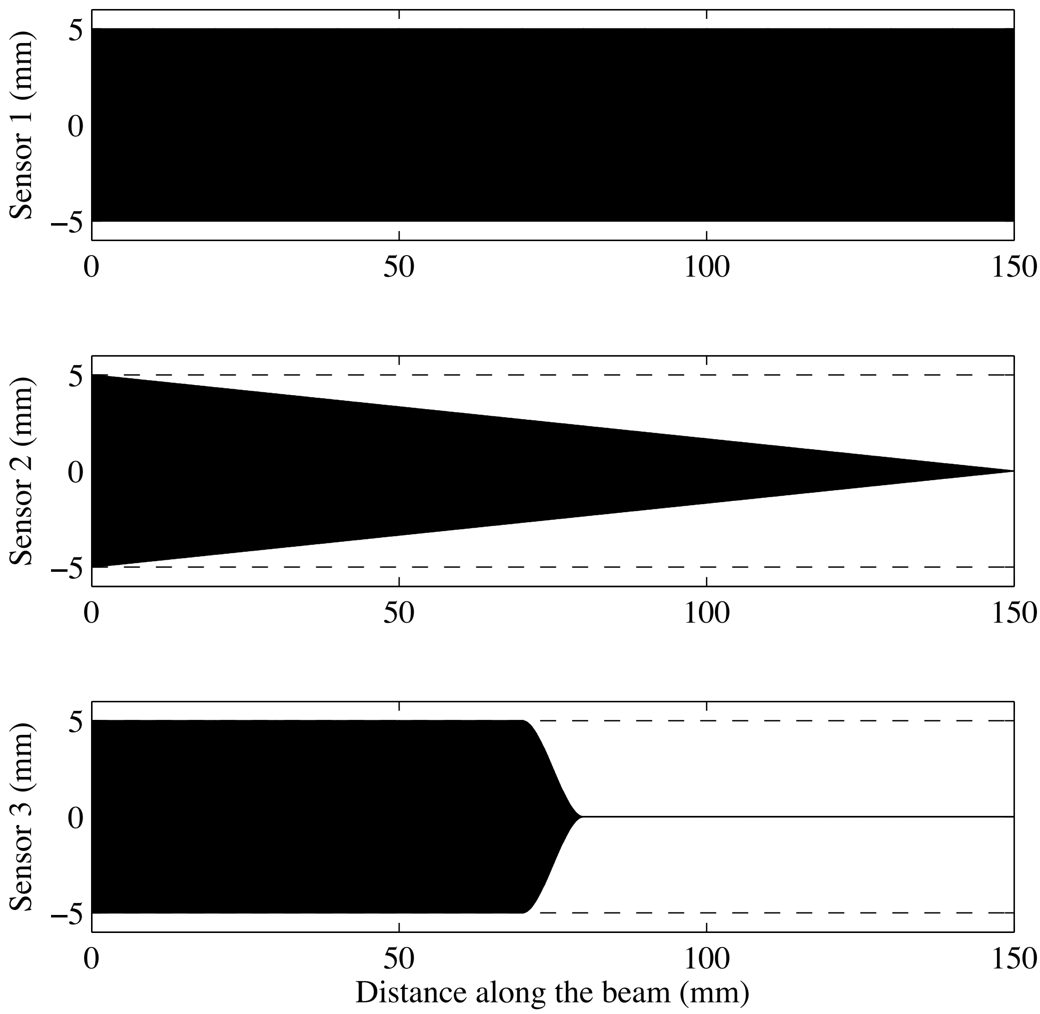


Figure 4: The shapes of the three candidate sensors. The dotted lines represent the extent of the steel beam.

where the support was excited; other systems, for example where a discrete mass is placed at the free end of the beam, may also be analysed. The manner in which the governing equations are developed, where the transducer shape is determined by a small number of variables, opens the opportunity to optimise the sensor shape to maximise the power output. This is the subject of ongoing research.

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