Structural Health Monitoring of Beam Structures using Shaped Sensors

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Abstract: This paper is concerned with distributed sensors to measure the response of beam structures. The design of modal sensors for beam structures is well established. Most applications consider vibration control, but this paper is concerned with structural health monitoring, where the sensor output is made sensitive to changes in key stiffness parameters, for example in joints. The procedure is based on finite element models of beams, and thus the distributed transducers may be designed for arbitrary beam structures. A simulated beam excited by a rotating machine is used to demonstrate the approach and a Monte Carlo method is used to highlight the performance of the sensor in identifying damage.

Key words: Piezoelectric, Beams, SHM

1. INTRODUCTION

The idea of using modal sensors and actuators for beam and plate type structures has been a subject of intense interest for many years. Using modal sensors in active control reduces problems of spillover, where high frequency unmodelled modes affect the stability of the closed loop system. The sensors and actuators may be discrete or distributed, and are usually manufactured using piezoelectric material, such as polyvinylidene fluoride (PVDF) film. For example, a modal sensor for a beam type structure may be obtained by varying the sensor width along the length of the beam. If the sensor covers the whole beam the shape of the sensor may be derived using the mode shape orthogonality property [19, 5]. Modal sensors may be designed that cover only part of the beam [9], or are segmented sensors sensitive to multiple modes. The effect of geometric tolerances during manufacture on the quality of the sensors may be determined [9]. For beam structures the width of the sensor may be parameterized using the finite element method and the underlying shape functions used to approximate the transducer shape [10].

Modal sensors for two-dimensional structures have also been designed. The approach used for beams may be implemented by varying the thickness of the PVDF, although this is very difficult to achieve in practice. Sun *et al.* [24] replaced an actuator layer with variable thickness by many small segments of uniform thickness. Kim *et al.* [17, 18] developed two design methods for distributed modal transducers for composite plates, the first using multi-layered PVDF films with optimized electrode pattern, lamination angle, and poling direction, and the second using PVDF film segments and an interface circuit. Preumont *et al.* [22] introduced the porous electrode concept, which allows the gains to be introduced by changing the local effectiveness of the electrodes. The alternative is to design a distributed modal transducer by optimizing the continuous boundary shape of a constant thickness PVDF film by assuming a smooth boundary [15, 16, 25].

This paper uses the design methods for modal sensors for beam structures, but extends this approach to the structural health monitoring application.

The identification of the location and severity of cracks, loose bolts and other types of damage in structures using vibration data has received considerable attention [7]. Most of the approaches use the modal data of a structure before damage occurs as baseline data, and all subsequent tests are compared to it [4, 8, 12, 13]. Any deviation in the modal properties from this baseline data is used to estimate model parameters related to the damage severity and location. The advantage of using this baseline data is that some allowance is made for modelling errors. However changes in the structure not due to damage, for example due to environmental effects, will be difficult to distinguish from changes due to damage [11, 23]. The approach adopted in this paper is to use shaped transducers to reduce the sensitivity of the sensor output to the unmodelled parameter changes and environmental effects. For structural health monitoring this means that the response can be made sensitive to particular regions of interest, so that, for example, the sensor may be used to monitor the health of a single joint. The method is an extension of the selective sensitivity technique which was developed to design excitations that produce strong sensitivities to a subset of the

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parameters whilst causing the sensitivities to other parameters to vanish. This concept is modified to choose the shaped sensor, although the spatial and spectral properties of the expected in-service excitation are still an important aspect of the design.

The finite element model of the structure is of the form

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}\mathbf{u}, \qquad \mathbf{y} = \mathbf{C}\mathbf{q} \tag{1}$$

where $\mathbf{M}$, $\mathbf{D}$ and $\mathbf{K}$ are the mass, damping and stiffness matrices based on the degrees of freedom, $\mathbf{q}$. The inputs to the structure, $\mathbf{u}$, are applied via a matrix, $\mathbf{B}$, which determines the location and gain of the actuators (or the actuator shape for distributed actuators). Similarly the outputs, $\mathbf{y}$, are obtained via the output matrix $\mathbf{C}$ which is determined by the sensor shape. For parameter estimation and health monitoring applications the mass, damping and stiffness matrices depend on a set of physical parameters, θ.

2. DEFINING SHAPED SENSORS

For beam structures, whose response is predominantly in bending in a single plane, the transducer may be shaped by varying the width of the PVDF material. In all these cases the width or thickness is assumed to be a continuous function. However this function needs to be parameterized to enable the optimization of the sensor. Using the shape functions of the underlying finite element model is a convenient approach to approximate the width of the piezoelectric material [10]. In this way modal transducers may be designed for arbitrary beam type structures, and the sensor width (and often its slope also) will be continuous across the element boundaries. Furthermore modal transducers that only cover part of a structure may be designed. Most of the development will concern sensors, although actuators may be dealt with in a similar way.

For a beam element of length ℓ_e, the output (voltage or charge) from the part of the sensor covering element number e is [10]

$$y_e(t) = K_s \int_0^{\ell_e} f_e(\xi) \frac{\partial^2 w_e(\xi, t)}{\partial \xi^2} \mathrm{d}\xi \tag{2}$$

where the sensor effectiveness $f_e(\xi)$ allows for varying width. K_s is a constant relating to the piezoelectric and system properties, including the electrode effectiveness (including the distance from the beam neutral axis), thickness, piezoelectric coefficient and polarization direction. If any of these properties vary then this variation should be included in $f_e(\xi)$. ξ is the local element coordinate and w_e is the beam deflection within the element.

The objective is to approximate the continuous sensor effectiveness using a discrete set of parameters, based on the finite element shape functions. In finite element analysis the displacement is approximated as

$$w_e(\xi) = \mathbf{N}(\xi)\, \mathbf{q}_e \tag{3}$$

where $\mathbf{N}(\xi)$ are the shape functions and $\mathbf{q}_e$ are the nodal generalized displacements. Suppose that the sensor width is also approximated by the shape functions, so that

$$f_e(\xi) = \mathbf{N}(\xi)\, \mathbf{f}_{se} \tag{4}$$

where $\mathbf{f}_{se}$ is a vector of the same length as the nodal displacements, that defines the width of the sensor. In this way the continuous sensor thickness is approximated using a finite set of parameters in $\mathbf{f}_{se}$. Substituting into Eq. (2) gives

$$y_e(t) = \mathbf{f}_{se}^{\top} \mathbf{C}_{se} \mathbf{q}_e \tag{5}$$

where

$$\mathbf{C}_{se} = K_s \int_0^{\ell_e} \mathbf{N}(\xi)^{\top} \frac{\mathrm{d}^2 \mathbf{N}(\xi)}{\mathrm{d}\xi^2} \mathrm{d}\xi. \tag{6}$$

The total sensor output is obtained by summing the output from all of the elements, thus

$$y = \sum_e y_e = \mathbf{f}_s^{\top} \mathbf{C}_s \mathbf{q} \tag{7}$$

where $\mathbf{q}$ is the vector of generalized displacements of the full model, and $\mathbf{f}_s$ is a vector incorporating the sensor parameter vectors. In this way, the problem of a continuous width variation is changed into a discrete optimization problem for $\mathbf{f}_s$. The assembly process is exactly analogous to the assembly of mass and stiffness matrices in standard finite element analysis. The only slight difference is the incorporation of boundary conditions. For example a pinned boundary will require that some of the generalized displacements are set to zero, which reduces the number of degrees of freedom. Just because a generalized displacement at a given node is zero does not mean that the corresponding element of the sensor thickness parameters, $\mathbf{f}_s$, should be zero, as the sensor thickness may be non-zero at the boundary. In turn this means that $\mathbf{C}_s$ may be a rectangular matrix. Comparing Eqs. (1) and (7) it is clear that $\mathbf{C} = \mathbf{f}_s^{\top} \mathbf{C}_s$.

For a beam element with cubic shape functions [10],

$$\mathbf{C}_{se} = -\frac{K_s}{30\ell_e} \begin{bmatrix} 36 & 33\ell_e & -36 & 3\ell_e \\ 3\ell_e & 4\ell_e^2 & -3\ell_e & -\ell_e^2 \\ -36 & -3\ell_e & 36 & -33\ell_e \\ 3\ell_e & -\ell_e^2 & -3\ell_e & 4\ell_e^2 \end{bmatrix}. \tag{8}$$

3. HEALTH MONITORING AND SELECTIVE SENSITIVITY

A significant problem with any inverse problem, such as structural health monitoring, is ill-conditioning of the estimation equations. One major cause of this ill-conditioning is the large number of candidate parameters that may be regarded as uncertain. Applying excitations that produce strong sensitivities to a subset of the parameters whilst causing the sensitivities to other parameters to vanish is one way of reducing the number of parameters to estimate. For structural health monitoring this means that the response can be made sensitive to particular regions of interest, so that, for example, the sensor may be used to monitor the health of a single joint. One approach to local structural health monitoring is to use high frequency impedance measurements[20], although using shaped distributed actuators and sensors could also be applied to these impedance techniques.

The method of selective sensitivity considers the response predictions to a relatively large number of excitation forces. In order to provide further explanation, the work of Ben-Haim [2, 3, 6, 21] will be adapted to sensors rather than actuators. The selective sensitivity approach is best derived in the frequency domain. From Eq. (1)

$$\begin{aligned} \mathbf{y}(\omega,\theta) &= \mathbf{C}\left[-\omega^2\mathbf{M}+\mathrm{j}\omega\mathbf{D}+\mathbf{K}\right]^{-1}\mathbf{Bu}(\omega) \\ &= \mathbf{f}^\top\mathbf{C}_s\mathbf{H}(\omega,\theta)\,\mathbf{Bu}(\omega) \end{aligned} \tag{9}$$

where $\mathbf{H}(\omega,\theta)=\left[-\omega^2\mathbf{M}+\mathrm{j}\omega\mathbf{D}+\mathbf{K}\right]^{-1}$ is the frequency response function (receptance matrix).

At any frequency the sensitivity of the response to parameter θ_j is given by

$$S_j(\mathbf{f},\omega)=\left\|\frac{\partial y}{\partial\theta_j}\right\|^2. \tag{10}$$

Notice that only a single output has been assumed, so that y is a scalar, although the approach is easily extended to multiple outputs. Then

$$S_j(\mathbf{f},\omega)=\mathbf{f}^\top\mathbf{C}_s\frac{\partial\mathbf{H}}{\partial\theta_j}\mathbf{Buu}^\mathrm{H}\mathbf{B}^\mathrm{H}\frac{\partial\mathbf{H}^\mathrm{H}}{\partial\theta_j}\mathbf{C}_s^\top\mathbf{f}=\mathbf{f}^\top\mathbf{G}_j(\omega)\,\mathbf{f} \tag{11}$$

where $\mathbf{G}_j(\omega)$ may be calculated and the superscript H denotes the conjugate transpose. $\mathbf{G}_j(\omega)$ is usually complex, although the imaginary part of the matrix is skew-symmetric so that the sensitivity is real. Thus the imaginary part of $\mathbf{G}_j(\omega)$ does not affect the sensitivity and may be neglected. Note that for excitation at a single location, $\mathbf{G}_j(\omega)$ has rank 1. The derivative of $\mathbf{H}$ may be calculated as

$$\frac{\partial\mathbf{H}}{\partial\theta_j}=\mathbf{H}\left[-\omega^2\frac{\partial\mathbf{M}}{\partial\theta_j}+\mathrm{j}\omega\frac{\partial\mathbf{D}}{\partial\theta_j}+\frac{\partial\mathbf{K}}{\partial\theta_j}\right]\mathbf{H}. \tag{12}$$

The sensitivity defined in Eq. (11) varies with frequency. For the standard selective sensitivity method the force input is designed and so may be varied with both location and frequency. With a shaped sensor for health monitoring the excitation is likely to be due to either ambient forces or operational conditions. Furthermore, to use a simple threshold to determine damage the output required is a single response quantity. In these cases an integrated sensitivity may be defined as

$$\hat{S}_j(\mathbf{f})=\int_{\omega_1}^{\omega_2}W(\omega)\,S_j(\mathbf{f},\omega)\,\mathrm{d}\omega \tag{13}$$

for some frequency weighting function $W(\omega)$ (that can be designed along with the sensor shape) and frequency range (ω_1,ω_2). For a given force input

$$\hat{S}_j(\mathbf{f})=\mathbf{f}^\top\left[\int_{\omega_1}^{\omega_2}W(\omega)\,\mathbf{G}_j(\mathbf{f},\omega)\,\mathrm{d}\omega\right]\mathbf{f}=\mathbf{f}^\top\hat{\mathbf{G}}_j\mathbf{f}. \tag{14}$$

If the location and spectrum of the force is not known then this makes the design of a selectively sensitive sensor extremely difficult. The rest of the development will consider only a single frequency using $\mathbf{G}_j(\omega)$ defined in Eq. (11), although the methods are readily extended to the frequency weighted versions using $\hat{\mathbf{G}}_j$ defined in Eq. (14).

The selective sensitivity problem for parameter s is to design the sensor such that

$$S_j(\mathbf{f})=\begin{cases}\neq 0 & \text{if } j=s\\ =0 & \text{otherwise}\end{cases} \tag{15}$$

Since the sensitivities are all real and non-negative, the conditions in Eq. (15) may be written as

$$S_s(\mathbf{f})=\mathbf{f}^\top\mathbf{G}_s\mathbf{f}\neq 0 \tag{16}$$

and

$$\sum_{j\neq s}S_j(\mathbf{f})=\sum_{j\neq s}\mathbf{f}^\top\mathbf{G}_j\mathbf{f}=\mathbf{f}^\top\tilde{\mathbf{G}}_s\mathbf{f}=0, \tag{17}$$

where $\tilde{\mathbf{G}}_s=\sum_{j\neq s}\mathbf{G}_j$.

Solving the selective sensitivity requirements exactly is often not possible, in the sense that a solution may not exist, or may not be unique. One straight-forward approach is to employ an optimization scheme directly. For example the sensor shape $\mathbf{f}$ sensitive to parameter θ_s is obtained as the vector that minimizes the objective function

$$J(\mathbf{f})=\left\{\sum_{j\neq s}W_jS_j(\mathbf{f})\right\}/S_s(\mathbf{f}) \tag{18}$$

where W_j are weighting factors for the different parameters. The width of the sensor is then normalized to the maximum width required.

However this simple approach may lead to difficulties if the solution is not unique and hence there are many solutions for perfect sensors that satisfy Eq. (15). If the excitation is at a single degree of freedom then the complex matrix $\mathbf{G}_j(\omega)$ will have rank 1, although the real part alone will usually have rank 2. If there are p parameters, and the response should be insensitive to $p-1$ of them, then the dimension of the space of suitable sensor shape vectors, $\mathbf{f}$, based on Eq. (15), is $n-2p+2$, where n is the length of the vector $\mathbf{f}$. In this case one has to introduce other regularizing conditions into the optimization. One possibility is to reduce the number of elements that the sensor covers, and subset selection techniques could be used to determine the optimum choice of elements. Another possibility is to minimize the transducer curvature [10], which has the advantage to producing sensors that are easier to manufacture. For beam structures, minimizing transducer curvature for a sensor covering the whole beam was shown to be equivalent to minimizing $\mathbf{f}^\top\hat{\mathbf{K}}\mathbf{f}$ where $\hat{\mathbf{K}}$ is the stiffness matrix for the free-free beam. This is equivalent to minimizing a strain energy type expression, and motivates the optimization problem for the general system. For modal sensors for uncertain systems an equivalent approach where the matrix is defined based on the variance may be used to obtain robust sensors that minimise the effect of the uncertainties [1].

The best approach is to enforce the zero sensitivity to unwanted parameters before optimizing any other objectives. Thus a transformation, $\mathbf{T}$, is introduced such that

$$\mathbf{T}^{\top}\tilde{\mathbf{G}}_s\mathbf{T} = \mathbf{0}. \tag{19}$$

This transformation matrix may be found easily using the singular value decomposition of $\tilde{\mathbf{G}}_s$ (which is equivalent to the eigenvalue problem since the real part of $\tilde{\mathbf{G}}_s$ is symmetric), and will generally be of dimension $n \times (n - 2p + 2)$. Thus a reduced dimension sensor vector $\mathbf{f}_r$ may be defined such that $\mathbf{f} = \mathbf{T}\mathbf{f}_r$, and is chosen by minimizing

$$J(\mathbf{f}) = \mathbf{f}_r^{\top}\left[\mathbf{T}^{\top}\hat{\mathbf{K}}\mathbf{T}\right]\mathbf{f}_r \tag{20}$$

subject to

$$S_s(\mathbf{f}) = \mathbf{f}^{\top}\mathbf{G}_s\mathbf{f} = \mathbf{f}_r^{\top}\left[\mathbf{T}^{\top}\mathbf{G}_s\mathbf{T}\right]\mathbf{f}_r \neq 0. \tag{21}$$

This is easily solved by calculating the eigenvalues of $\mathbf{T}^{\top}\tilde{\mathbf{K}}\mathbf{T}$, and taking the eigenvector corresponding to the smallest eigenvalue, consistent with Eq. (21). An alternative is to maximize the sensitivity $S_s(\mathbf{f})$ by choosing $\mathbf{f}_r$ as the eigenvector corresponding to the largest eigenvalue of $\mathbf{T}^{\top}\mathbf{G}_s\mathbf{T}$.

4. A SIMULATED EXAMPLE

A clamped-clamped beam example inspired by Gawronski [14] will be used here to demonstrate the feasibility of the approach. The steel beam is 1.5 m long with cross-section 20×5 mm, and bending in the more flexible plane is modelled using 15 finite elements. The beam has three further supports along its length, modelled as springs at nodes 4, 8 and 11, where the node numbering convention means that node 1 is clamped, see Fig. 1. The damping is assumed to be proportional to stiffness with a factor of 10^{-4} s. The force is applied from a rotating machine operating at 40 Hz at node 6. The distributed sensor is to be designed for the whole length of the beam initially.

Figure 2(a) shows the sensor shape obtained by minimizing the objective function in Eq. (20) for the first support stiffness. Note that $\mathbf{T}^{\top}\tilde{\mathbf{K}}\mathbf{T}$ has two zero eigenvalues that produce zero sensitivity to the first parameter and are therefore not chosen. The sensor width is normalized so that the maximum width is 1. The result is sensitivity ratios of $S_1/S_2 = 1.2 \times 10^{15}$ and $S_1/S_3 = 8.7 \times 10^{12}$, demonstrating that the required sensitivity is obtained. Also shown is the frequency response between the force input location and the sensor output. The baseline (solid line) is based on the undamaged system, and the responses due to a 10% reduction in each of the three support stiffnesses in turn are also shown. Remember that the sensor has been designed to operate only at 40 Hz, and at this frequency the response is indeed insensitive to changes in support stiffnesses 2 and 3. Figure 2(b) shows the equivalent plots when the sensor is designed to be sensitive to support stiffness 2, also at 40 Hz. It is clear that the procedure has designed sensors with relatively simple shapes.

Suppose now that the sensor shape is chosen to maximize the sensitivity to the first parameter. The result is shown in Fig. 3(a), and highlights that the sensor shape is now more complicated. The sensitivity to parameter 1 is now over 600 times higher than that for the sensor in Fig. 2(a). Figure 3(b) shows the effect of restricting the region where the sensor is placed, by setting the elements of the parameter vector $\mathbf{f}$ corresponding to finite element nodes $1-4$ and $11-16$ to zero. The sensor is designed to ensure zero sensitivity to parameters 2 and 3 and the minimum curvature solution is chosen. There is clearly a compromise between the smaller region for the sensor and the complexity of the shape.

The examples thus far have considered the design of the sensors and have shown the frequency response function over a large frequency range. We now consider the performance of the sensor when there is noise and errors in the system. The three stiffness parameters are assumed to be uncertain with a Gaussian distribution with a 2% standard deviation based on the nominal values of the stiffness. Figure 4 shows the probability density function of the output of the sensor at 40Hz due to these uncertainties at the nominal values of stiffness (solid) and when the value of the first stiffness changes by 10%. Clearly the output is able to distinguish that the first support has changed significantly. The probability density was calculated using a Monte Carlo simulation with 10, 000 samples. Figure 4 shows the equivalent result for a 10% change to the second stiffness parameter and shows that the sensor is insensitive to large changes in this stiffness.

Suppose now that the stiffness of the *clamped* supports also varies. This is modelled by varying the stiffness of the elements nearest to the clamped ends. Figure 4 shows the sensor shape that is sensitive to support stiffness 1, and insensitive to support stiffnesses 2 and 3, and also the stiffnesses of the elements nearest the clamped ends. Also shown are the FRFs for the nominal values of the parameters, and for 10% reductions in all of the parameters. Interestingly the FRF is relatively insensitive to the element stiffnesses over the whole frequency range, although only the response at 40Hz is used in the design.

5. CONCLUSIONS

This paper has designed distributed sensors by varying the width (for beams) or thickness (for plates) for structural health monitoring applications. The finite element shape functions are used to define the width or thickness of the sensor and this allows much more flexibility in designing sensors that cover only part of the structure, and other constraints, such as the minimum curvature, may be easily included. It has been demonstrated that sensors may be designed that are sensitive to changes in a single parameter, and insensitive to changes in other parameters. Although much more analysis of these designs is required, the approach does allow the prospect of a simple distributed sensor to monitor a single

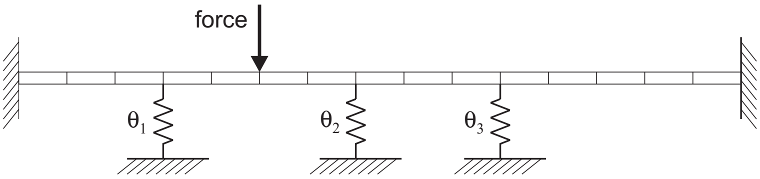

Figure 1: The beam with intermediate supports and harmonic excitation.

region, such as a joint, and be insensitive to other changes, such as those arising from environmental changes. The output from such a sensor could have a threshold alarm system, allowing a robust but simple distributed structural health monitoring system.

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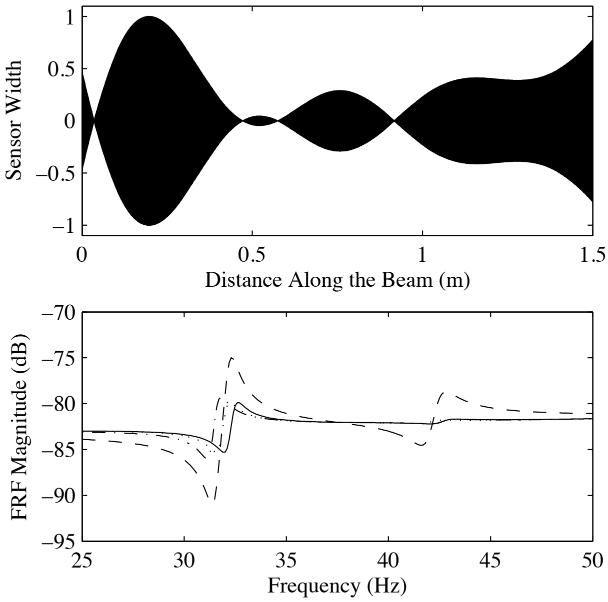
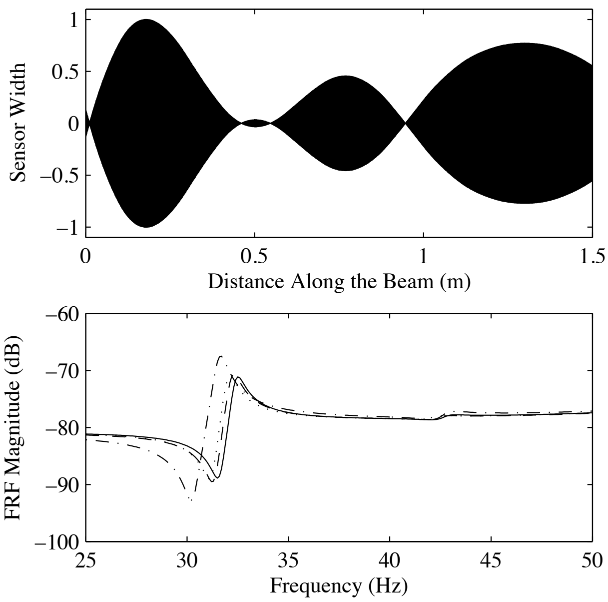
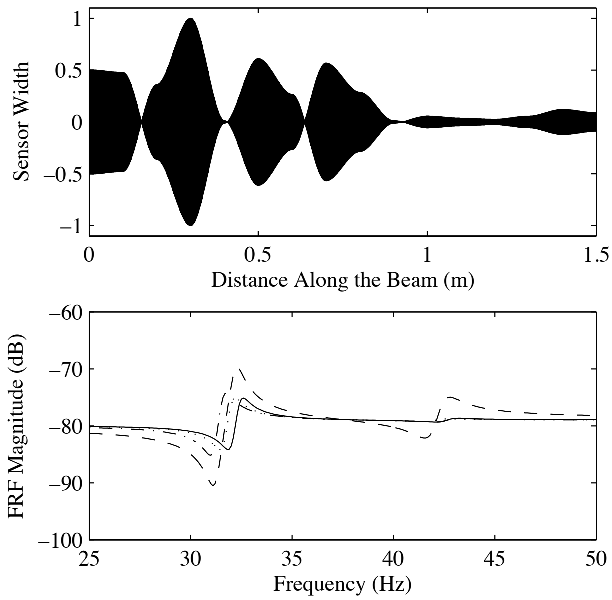


(a) Sensitive to support 1.

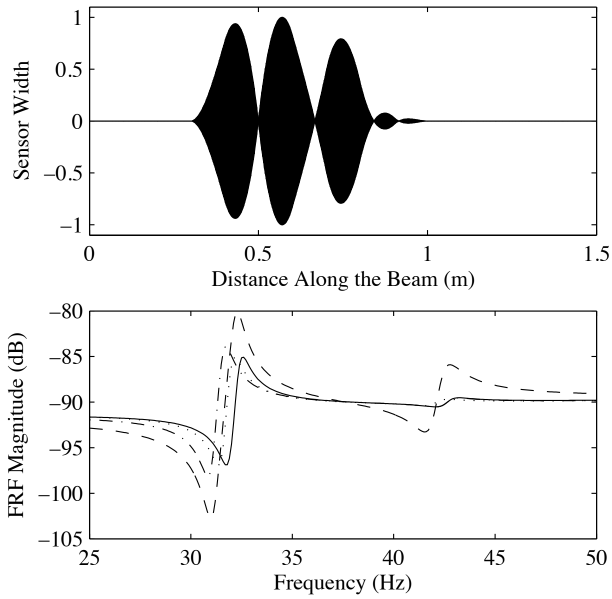


(b) Sensitive to support 2.

Figure 2: The distributed sensor shape designed to be sensitive to particular support stiffnesses at 40Hz for the beam example, and the associated receptances. The solid line is the baseline FRF, the dashed line is due to a 10% change in support stiffness 1, the dot-dashed line is due to a change in stiffness 2, and the dotted line is due to a change in stiffness 3.



(a) Maximize sensitivity to support 1.



(b) Restrict sensor region.

Figure 3: The distributed sensor shape designed to be sensitive to support stiffness 1 at 40Hz for the beam example, and the associated receptances. The solid line is the baseline FRF, the dashed line is due to a 10% change in support stiffness 1, the dot-dashed line is due to a change in stiffness 2, and the dotted line is due to a change in stiffness 3.

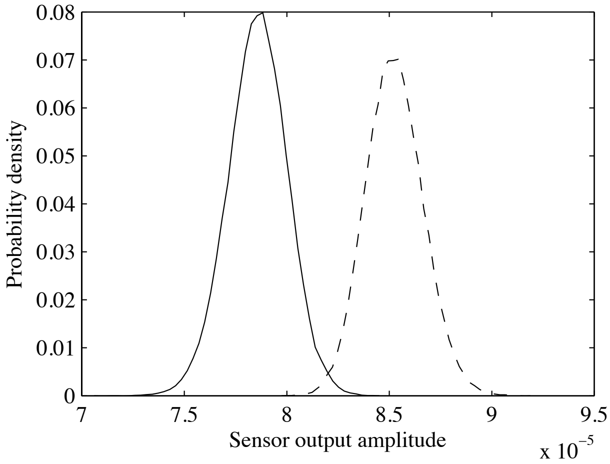


Figure 4: The probability density function of the output from the distributed sensor in Figure 2(a) to uncertainties in the support stiffnesses with 2% standard deviation. The solid line is for the nominal value of the stiffnesses and the dashed line due to a 10% increase in θ_1.

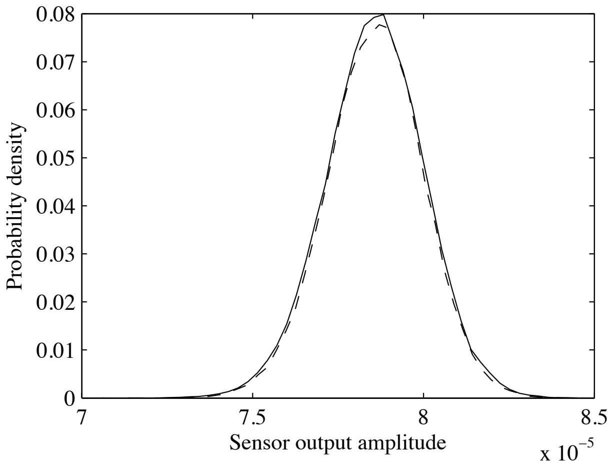


Figure 5: The probability density function of the output from the distributed sensor in Figure 2(a) to uncertainties in the support stiffnesses with 2% standard deviation. The solid line is for the nominal value of the stiffnesses and the dashed line due to a 10% increase in θ_2.

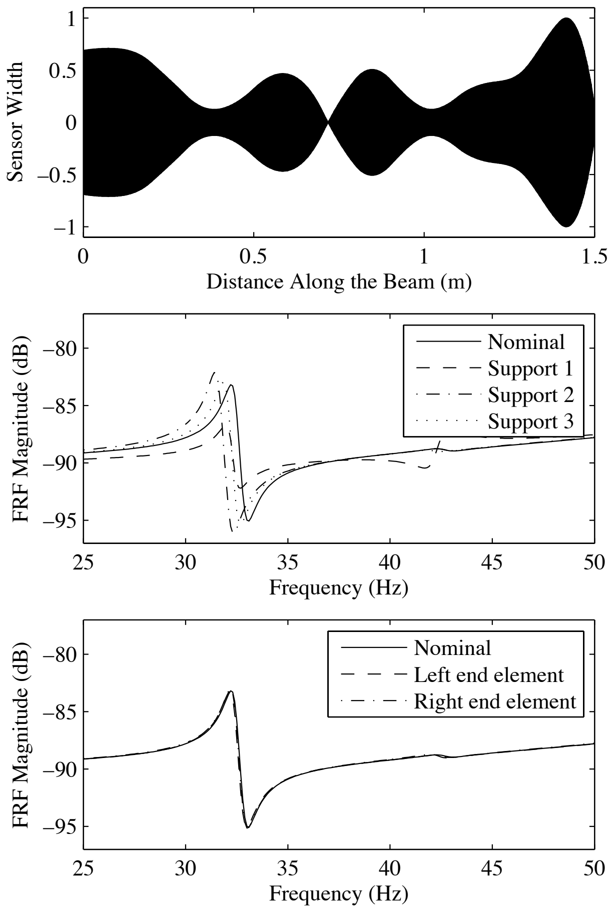


Figure 6: The distributed sensor shape designed to be sensitive to support stiffness 1 at 40Hz for the beam example, and the associated receptances. The solid line is the baseline FRF, and the other FRFs represent 10% changes to the support stiffnesses and the stiffness of the elements near the clamped ends.