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UNCERTAINTY IN STRUCTURAL DYNAMICS: EXPERIMENTAL CASE STUDIES ON BEAMS AND PLATES

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Abstract. *The consideration of uncertainties in numerical models to obtain the probabilistic descriptions of vibration response is becoming more desirable for industrial scale finite element models. Broadly speaking, there are two aspects to this problem. The first is the quantification of parametric and non-parametric uncertainties associated with the model and the second is the propagation of uncertainties through the model. While methods of uncertainty propagation have been extensively researched in the past two decades (e.g., the stochastic finite element method), only relatively recently has quantification been considered seriously. This paper considers uncertainty quantification with the aim of gaining more insight into the nature of uncertainties in medium and high frequency vibration problems. Results from two experimental studies that may be used for this purpose are described in detail. The first experiment uses a fixed-fixed beam with 12 masses placed at random locations. The total 'random mass' is about 2% of the total mass of the beam and this experiment simulates 'random errors' in the mass matrix. The second experiment uses a cantilever plate with 10 randomly placed spring-mass oscillators. This experiment simulates the problem of 'unmodelled dynamics', which in turn results in randomness in both the mass and stiffness matrices. For both experiments, 100 nominally identical dynamical systems are created and individually tested. The probabilistic characteristics of the frequency response functions are discussed in the low, medium and high frequency ranges. The data obtained in these experiments may be useful for the validation of uncertainty quantification and propagation methods in structural dynamics.*

1 INTRODUCTION

The steady development of powerful computational hardware in recent years has led to high-resolution finite element models of real-life engineering structural systems. However, for high-fidelity and credible numerical models, a high resolution in the numerical mesh is not enough. It is also required to quantify the uncertainties and robustness associated with a numerical model. As a result, the quantification of uncertainties plays a key role in establishing the credibility of a numerical model. Uncertainties can be broadly divided into two categories. The first type is due to the inherent variability in the system parameters, for example, different cars manufactured from a single production line are not exactly the same. This type of uncertainty is often referred to as *aleatoric uncertainty*. If enough samples are present, it is possible to characterize the variability using well established statistical methods and consequently the probably density functions (pdf) of the parameters can be obtained. The second type of uncertainty is due to the lack of knowledge regarding a system, often referred to as *epistemic uncertainty*. This kind of uncertainty generally arises in the modelling of complex systems, for example, in the modeling of cabin noise in helicopters. Due to its very nature, it is comparatively difficult to quantify or model this type of uncertainty. There are two broad approaches to quantify uncertainties in a model. The first is the *parametric approach* and the second is the *non-parametric approach*. In the parametric approach the uncertainties associated with the system parameters, such as Young's modulus, mass density, Poisson's ratio, damping coefficient and geometric parameters are quantified using statistical methods and propagated, for example, using the stochastic finite element method [1, 2, 3]. This type of approach is suitable to quantify aleatoric uncertainties. Epistemic uncertainty on the other hand does not explicitly depend on the system parameters. For example, there can be unquantified errors associated with the equation of motion (linear or non-linear), in the damping model (viscous or non-viscous), in the model of structural joints, and also in the numerical methods (e.g, discretisation of displacement fields, truncation and round-off errors, tolerances in the optimization and iterative algorithms, step-sizes in the time-integration methods). It is evident that the parametric approach is not suitable to quantify this type of uncertainty. As a result non-parametric approaches [4, 5] have been proposed for this purpose.

Uncertainties associated with a variable can be characterized using the probabilistic approach or possibilistic approaches based on interval algebra, convex sets or Fuzzy sets. In this paper the probabilistic approach has been adopted. The equation of motion of a damped n-degree-of-freedom linear structural dynamic system can be expressed as

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{1}$$

where $\mathbf{M} \in \mathbb{R}_{n,n}$, $\mathbf{C} \in \mathbb{R}_{n,n}$ and $\mathbf{K} \in \mathbb{R}_{n,n}$ are the mass, damping and stiffness matrices respectively. The importance of considering parametric and/or non-parametric uncertainty also depends on the frequency of excitation. For example, in high frequency vibration the wave lengths of the vibration modes become very small. As a result the vibration response can be very sensitive to the small details of the system. In such situations a non-parametric uncertainty model may be adequate. Overall, three *different* approaches are currently available to analyze stochastic structural dynamic systems across the frequency range:

- *Low frequency vibration problems*: Stochastic Finite Element Method [1, 2, 3] (SFEM) - considers parametric uncertainties in detail;
- *High frequency vibration problems*: Statistical Energy Analysis [6] (SEA) - does not consider parametric uncertainties in detail;

- *Mid-frequency vibration problems* [7, 8, 9]: both parametric and non-parametric uncertainties need to be considered.

This paper describes two experimental studies that may be used to test methods of uncertainty quantification across the frequency range. The difference between this data and previous experimental data is that the tests are closely controlled and the uncertainty can be considered to be *known* for all practical purposes. This allows one to model uncertainty, propagate it through dynamical models and compare the results with this experimentally obtained data.

The first experiment, described in Section 2, uses a fixed-fixed beam with 12 masses placed at random locations. The total *random mass* is about 2% of the total mass of the beam and this experiment simulates *random errors* in the mass matrix. The second experiment, described in Section 3, considers a cantilever plate with 10 randomly placed spring-mass oscillators. This experiment simulates *unmodelled dynamics*, where random errors occur in both the mass and stiffness matrices. For both experiments, 100 nominally identical dynamical systems are created and individually tested in the Bristol Laboratory for Advanced Dynamic Engineering (BLADE). The probabilistic characteristics of the frequency response functions are discussed in the low, medium and high frequency ranges. The data presented here will be available on the world wide web for research purposes. This data may be used to validate different uncertainty quantification and propagation methods in structural dynamics.

2 UNCERTAINTY QUANTIFICATION IN A FIXED-FIXED BEAM

A steel beam with uniform rectangular cross-section is used for the experiment. The beam is 40.06mm wide, 2.05mm thick, with a mass per unit length of 0.641kg/m and a bending rigidity (EI) of 5.752Nm^2. The beam is actually a 1.5m long ruler made of mild steel. The use of a ruler ensures that the masses may be easily placed at predetermined locations. The ruler is clamped between 0.05m and 1.25m so that the effective length of the vibrating beam is 1.2m, with a mass of 0.7687kg. The overall experimental setup is shown in Figure 1. The end clamps are bolted to two heavy steel blocks, which in turn are fixed to a rigid table with bolts.

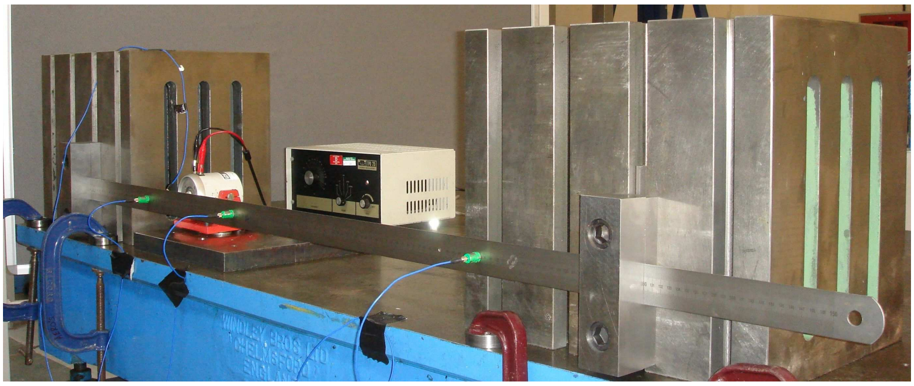


Figure 1: The test rig for the fixed-fixed beam.

Twelve equal masses are used to simulate a randomly varying mass distribution. The masses are actually magnets so that they can be easily attached at any location on the steel beam. These magnets are cylindrical in shape, with a length of 12.0mm and a diameter of 6.0mm. Some of the attached masses for a sample realization are shown in Figure 2.

Each mass weights 2g so that the total variable mass is 1.6% of the mass of the beam. The location of the 12 masses are assumed to be between 0.2m and 1.0m from the left end of the

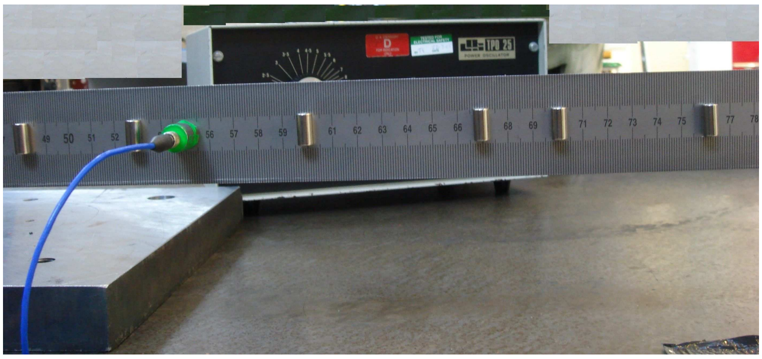


Figure 2: Attached masses (magnets) at random locations. In total 12 masses, each weighting 2g, are used.

beam. A uniform distribution with 100 samples is used to generate the mass locations. The mean and the standard deviations of the mass locations are given by

$$\bar{\mathbf{x}}_m = [0.2709, 0.3390, 0.3972, 0.4590, 0.5215, 0.5769, \\ 0.6398, 0.6979, 0.7544, 0.8140, 0.8757, 0.9387] \quad (2)$$

and

$$\sigma_{\mathbf{x}_m} = [0.0571, 0.0906, 0.1043, 0.1034, 0.1073, 0.1030, \\ 0.1029, 0.1021, 0.0917, 0.0837, 0.0699, 0.0530] . \quad (3)$$

The three main components of the implemented experiment technique are (a) excitation of the structure, (b) sensing of the response, and (c) data acquisition and processing. In this experiment a shaker was used to act as an impulse hammer. The usual manual impact hammer was not used because of the difficulty in ensuring the impact occurs at exactly at the same location with the same force for every sample run. The shaker generates impulses at a pulse interval of 20s and a pulse width of 0.01s. Using the shaker in this way eliminates, as far as possible, any uncertainties arising from the input forces. This innovative experimental technique is designed to ensure that the resulting uncertainty in the response arises purely due to the random locations of the attached masses. A small circular brass plate weighting 2g is attached to the beam to take the impact from the shaker. Figure 3 shows the arrangement of the shaker.

In this experiment three accelerometers are used as the response sensors. The locations of the three sensors are selected such that two of them are near the two ends of the beam and one is at the exciter location, near the middle of the beam, so that driving point frequency response function may be obtained. The exact locations are calculated such that the nodal lines of the first few bending modes are avoided. Small holes are drilled into the beam and all of the three accelerometers are attached by bolts through these holes.

The signals are measured and analyzed using LMS Test Lab, with a bandwidth of 8192Hz and 8192 spectral lines. The steel tip used in the experiment only gives clean data up to approximately 4.5kHz, and thus 4.2kHz is used as the upper limit of the frequency in the measured frequency response functions.

Figure 4 shows the amplitude of the frequency response function (FRF) at point 1 (23cm from the left end of the beam) without any masses (the baseline model). In the same figure 100 samples of the amplitude of the FRF are shown together with the ensemble mean, 5% and 95% probability lines.

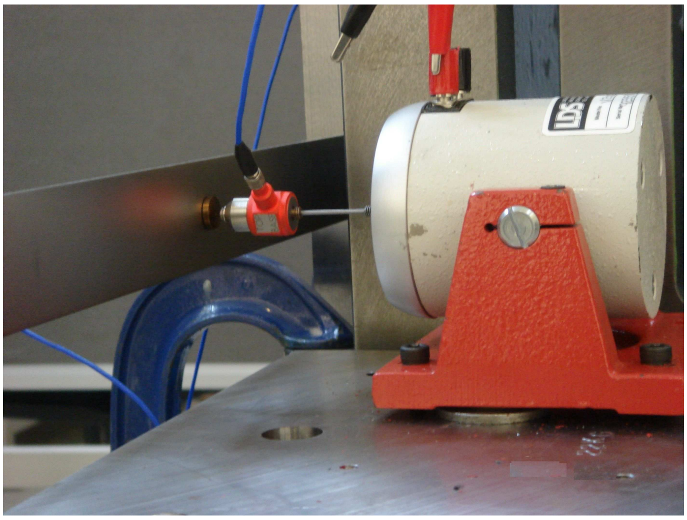


Figure 3: The shaker used as an impulse hammer. A hard steel tip was used.

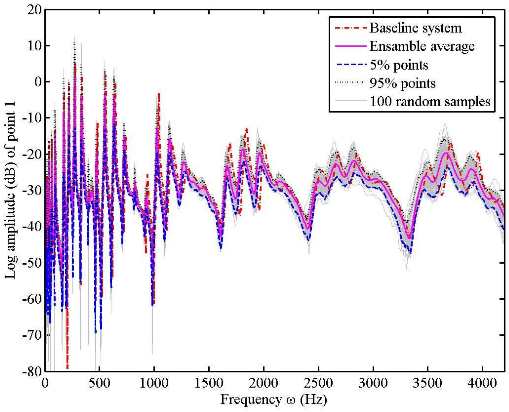
Figures 4(b)-(d) show the low, medium and high-frequency response separately, obtained by zooming around the appropriate frequency ranges in Figure 4(a). There are, of course, no fixed and definite boundaries between the low, medium and high-frequency ranges. Here we have selected $0 - 1.0$kHz as the low-frequency vibration, $1.0 - 2.5$kHz as the medium-frequency vibration and $2.5 - 4.2$kHz as the high-frequency vibration. These frequency boundaries are selected on the basis of the qualitative nature of the response and devised purely for the presentation of the results. The experimental approach discussed here is independent of these selections. The ensemble mean follows the result of the baseline system closely only in the low frequency range. The relative variance of the amplitude of the FRF remains more or less constant in the mid and high frequency ranges.

3 THE PLATE EXPERIMENT

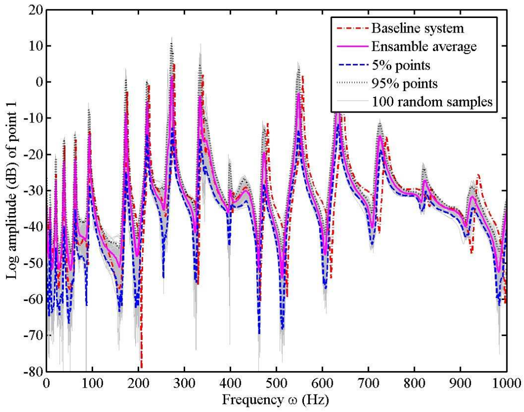
The aim of this experiment is to simulate uncertain unmodelled dynamics. The uncertain dynamics is realized by 10 spring-mass oscillators with randomly distributed stiffness properties attached at random locations. This test rig, like the previous experiment, has been designed for simplicity and ease of replication and modelling. The arrangement of the test-rig is shown in Figures 5 and 6.

A rectangular steel plate with uniform thickness is used for the experiment. The length is 998mm, the width is 530mm and the thickness is 3mm. The plate is clamped along one edge using a clamping device. The clamping device is attached on the top of a heavy concrete block and the whole assembly is placed on a steel table. The plate has a mass of approximately 12.28kg and special care has been taken to ensure its stability and to minimize vibration transmission. The plate is *divided* into 375 elements (25 along the length and 15 along the width). Taking one corner of the cantilevered edge as the origin, co-ordinates have been assigned to all of the nodes. Oscillators and accelerometers are attached to these nodes. This approach allows easy correlation to a finite element model, where the oscillators are attached, and the measurements are made, at nodes of the model. The bottom surface of the plate is marked with node numbers so that the oscillators can be hung at the nodal locations. This scheme also reduces the uncertainly arising from the measurement of the locations of the oscillators.

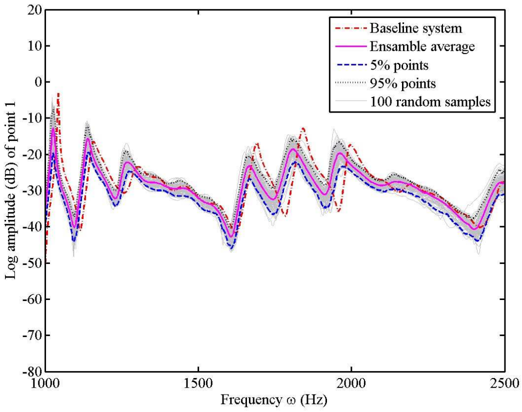
A discrete random number generator is used to generate the nodal locations for the attach-



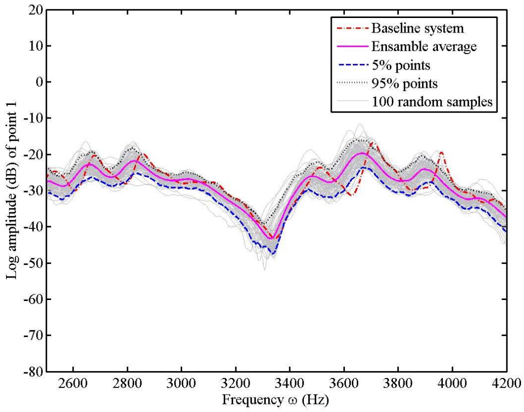
(a) Response across the frequency range



(b) Low-frequency response



(c) Medium-frequency response



(d) High-frequency response

Figure 4: Amplitude of the FRF of the beam at point 1 (23cm from the left end) with 12 randomly placed masses. 100 FRFs, together with the ensemble mean, 5% and 95% probability points are shown.

ment of the oscillators. In total 10 oscillators are used to simulate random unmodelled dynamics. The springs are adhesively bonded to a magnet at the top and a mass at the bottom (Figure 7). The magnet at the top of the assembly allows the oscillators to be easily attached to the bottom of the plate. The oscillating mass of each of the 10 oscillators is 121.4g, and hence the total oscillator mass is 1.214kg, which is 9.8% of the mass of the plate. The springs are attached to the plate at the pre-generated nodal locations using the small magnets located at the top the ocillator assembly. The small magnets (weighting 2g) are found to be strong enough to hold the 121.4g mass attached to the spring below over the frequency range considered.

It should be noted that the mass of the magnets would have the *fixed mass effect* considered in the previous case study on the beam. However, since the plate is much heavier than the magnets ($\approx$ 12kg), the effect of the fixed masses (20g in total) is negligible. The stiffnesses of the 10 springs used in the experiments are given in Table 1. This table also shows the natural frequency of the individual oscillators.

One hundred such realizations of the oscillators are created (by hanging the oscillators at random locations) and individually tested in this experiment.

The measurement system used is similar to that for the beam experiment. A shaker was used

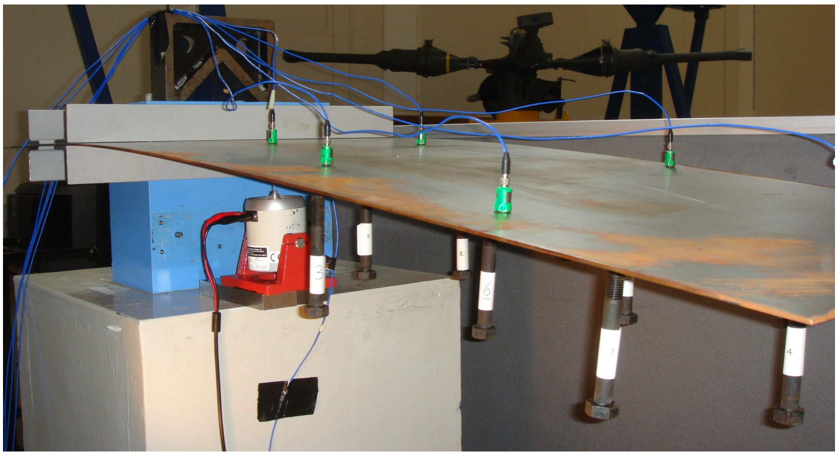


Figure 5: Front view of the experimental setup for the cantilever beam plate.

Oscillator number	Spring stiffness ($\times 10^4$ N/m)	Natural frequency (Hz)
1	1.6800	59.2060
2	0.9100	43.5744
3	1.7030	59.6099
4	2.4000	70.7647
5	1.5670	57.1801
6	2.2880	69.0938
7	1.7030	59.6099
8	2.2880	69.0938
9	2.1360	66.7592
10	1.9800	64.2752

Table 1: Stiffness of the springs and natural frequency of the oscillators used to simulate unmodelled dynamics (the mass of the each oscillator is 121.4g).

to provide the impulse excitation for the same reasons as the beam experiment. The shaker is placed so that it impacts at the $(4, 6)$ node of the plate (Figure 8). In this experiment six accelerometers are used as the response sensors. The locations of the six sensors are selected such that they cover a broad area of the plate. Small holes are drilled into the plate and all of the six accelerometers are attached by bolts through the holes. A bandwidth of 8192Hz with 8192 spectral lines was used. Five averages are taken for each FRF measurement. The steel tip used in the experiment only gives clean data up to approximately 4.5kHz. As a result 4.2kHz was used as the upper limit of the frequency in the measured frequency response functions.

Figure 9 shows the amplitude of the driving point frequency response function (FRF) of the plate without any oscillators (the baseline model). In the same figure 100 samples of the amplitude of the FRF are shown together with the ensemble mean, 5% and 95% probability lines.

In Figures 9(b)-(d) we have separately shown the low, medium and high frequency response, obtained by zooming around the appropriate frequency ranges in Figure 9(a). There are of course no fixed and definite boundaries between the low, medium and high-frequency ranges. Here we have selected $0 - 1.0$kHz as the low frequency vibration, $1.0 - 2.5$kHz as the medium frequency vibration and $2.5 - 4.2$kHz as the high frequency vibration. These frequency bound-

Figure 6: The test rig for the cantilever beam plate with attached oscillators at random locations.

aries are selected on the basis of the qualitative nature of the response and devised purely for the presentation of the results. The experimental approach discussed here is independent on these selections. The ensemble mean follows the result of the baseline system closely only in the low frequency range. The relative variance of the amplitude of the FRF remains more or less constant in the mid and high frequency range.

Note that most of the variability in the FRFs is concentrated at the low frequency region. This is because the frequency of the attached oscillators at random locations are below 70Hz.

4 CONCLUSIONS

This paper has described two experiments that may be used to study methods to quantify uncertainty in the dynamics of structures. The fixed-fixed beam is very easy to model and the results of a one hundred sample experiment with randomly placed masses are described in this paper. The cantilever plate experiment with randomly located oscillators with randomly varying stiffness is designed to simulate model uncertainty. Again, test results from one hundred nominally identical systems are presented. Special measures have been taken so that the uncertainty in the response only arises from the randomness in the mass and oscillator locations and the experiments are repeatable with minimum changes. Such measures include, but are not limited to, (a) the use of a shaker as an impact hammer to ensure a consistent force and location for all of the tests, (b) the use of a ruler in the beam experiment to minimize the error in measuring the mass locations, (c) the employment of a grid system and nodal points in the plate experiment to minimize the error in measuring the oscillator locations, and (d) the use of magnets. The statistics of the frequency response function measured at three points of the beam and six points of the plate are obtained for low, medium and high frequency ranges. For the beam experiment, there is more variability in the FRF at the high frequency range compared to the low frequency range. For the plate experiment, the opposite trend is observed as the natural frequencies of the randomly placed oscillators are quite low. This data may be used for the model validation and uncertainty quantification of dynamical systems.

Figure 7: Details of a typical oscillator used to simulate unmodelled dynamics. Fixed mass (magnet) 2g, oscillatory mass (the nut) 121.4g. The spring stiffness varies between 15 and 24 kN/m.

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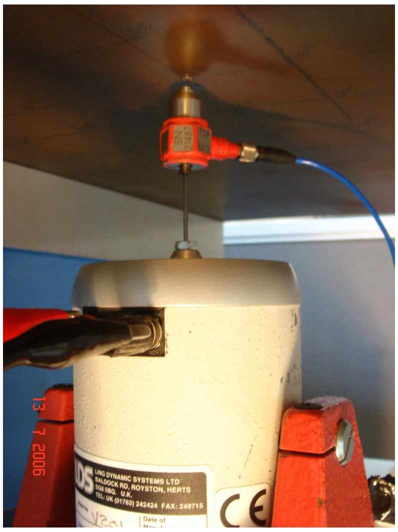
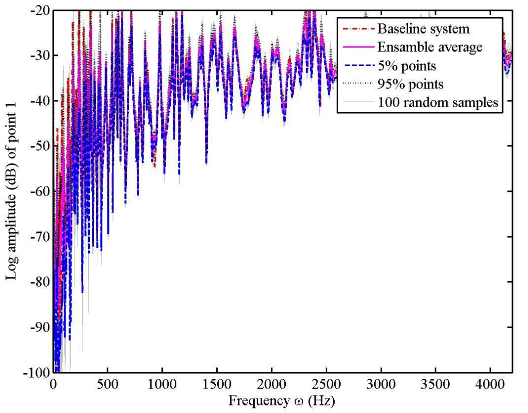
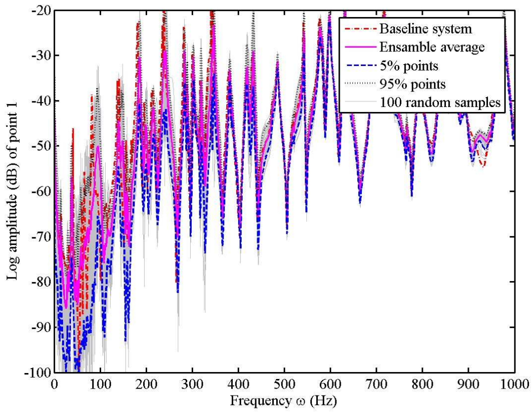


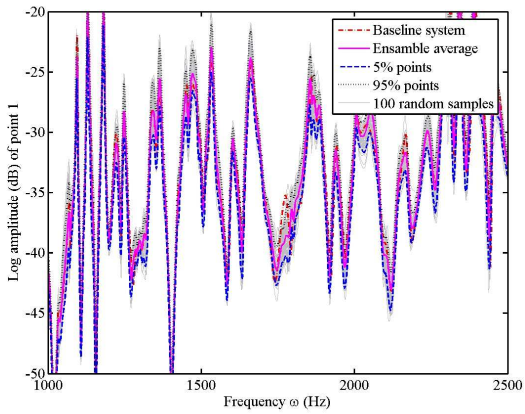
Figure 8: The shaker used to provide an impulse excitation. A hard steel tip was used and the shaker was placed at node (4,6).



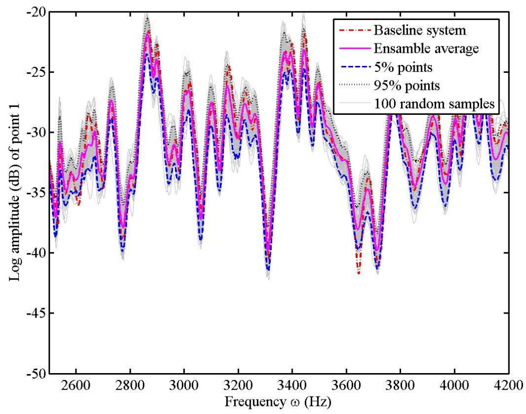
(a) Response across the frequency range



(b) Low-frequency response



(c) Medium-frequency response



(d) High-frequency response

Figure 9: Amplitude of the driving point FRF of the plate with 10 randomly placed oscillators. 100 FRFs, together with the ensemble mean, 5% and 95% probability points are shown.