FRF-BASED PROBABILISTIC MODEL UPDATING IN STRUCTURAL DYNAMICS FOR UNCERTAINTY IDENTIFICATION AND QUANTIFICATION

Vikas Arora

University of Southern Denmark, Department of Technology and Innovation, Odense, Denmark
email: viar@iti.sdu.dk

Micheal Friswell

Swansea University, College of Engineering, Singleton Park, Swansea, UK

Sondipon Adhikari

Swansea University, College of Engineering, Singleton Park, Swansea, UK

In the recent years, probabilistic approaches have been developed to incorporate uncertainties in the dynamic systems. These uncertainties arise due to unknown experimental errors or variability in nominally identical dynamic systems. The majority of these probabilistic methods are based on modal data. These modal data based probabilistic methods do not employ damping matrices and hence cannot be used for accurate prediction of amplitudes of vibrations and complex frequency response functions (FRFs) and also these modal data based do not work well for the closed modes systems. In this paper, a new FRF-based parametric approach is presented which tackles the problem of incorporating damping and closed modes in uncertain dynamic systems. The advantages of using FRF data over modal data for probabilistic model updating are demonstrated. In the proposed FRF-based probabilistic updating approach, the finite element model is updated in such a way that the updated model reflects general damping in the experimental model by considering the updating parameters as complex. The effectiveness of the proposed finite element updating procedure is demonstrated by numerical examples. The results have shown that the proposed damped FRF-based probabilistic model updating procedure can be used to identify and quantify uncertainties in the dynamic systems.

Keywords: Probabilistic model updating, uncertain dynamic system, complex FRFs

1. Introduction

In various areas of computational modelling, it has been established over the past three decades that uncertainties should be taken into account for credible predictions. In the context of structural dynamics, such uncertainties can be broadly divided into two categories, namely, parametric and nonparametric uncertainties. Parametric uncertainty includes uncertainty in geometric parameters, friction coefficient, and stiffness of the materials involved. Nonparametric uncertainty, on the other hand, can arise due to lack of scientific knowledge about the model which is unknown a-priori. Although this distinction is often made, the origin of uncertainty in real-life system it is not always obvious [1]. Deterministic finite element model updating [2] is well established, both in terms of the development of the methods and in applications to industrial-scale structures. Early updating methods belonged to the nonparametric category [3]; although these methods are computationally cheaper and reproduce the measured modal data exactly, they violate structural connectivity and the

updated structural matrices are difficult to interpret. In contrast, parametric methods provide wide choices of updating parameters, structural connectivity can be easily maintained and corrections suggested in the selected parameters can be physically interpreted. The parametric methods are based on some kind of sensitivity analysis that minimizes the error between the predicted results and test data from a single physical structure. Model uncertainties should be located and parameterized sensitively to the predictions. Finally, the model should be validated by assessing the model quality within its range of operation and its robustness to modifications in the loading configuration, design changes, coupled structure analysis and different boundary conditions.

Collins et al. [4] developed a Bayesian approach to model updating using linearized sensitivities based on knowledge of the statistics of the unknown parameters and the vibration. In these approaches, the randomness arises only from the measurement noise and the updating parameters take unique values, to be found by iterative correction to the estimated means, whilst the variance is minimized [5]. These statistical approaches have been extended to update parameter distributions using measured response distributions from multiple measurements. These include Bayesian methods [6]. Hua et al. [7] considered an improved perturbation method where statistical correlations between the updating parameters were taken into account. McFarland et al. [8] used Gaussian process emulators to update linear systems with parametric updating. This approach is valid for both Gaussian and non-Gaussian random variables.

In the present paper, a new FRF-based stochastic model updating approach is presented which tackles the problem of incorporating damping and closely spaced modes in the dynamic system. The proposed method is able to identify and quantify the uncertainties in the dynamic system. It is shown that stochastic model updating methods based on modal data are related to resonance frequencies only and cannot be used for damping estimation and dynamic systems with close modes. The advantages of using FRF data over modal data to update the analytical model are demonstrated.

2. Frequency response functions (FRFs) with parametric uncertainty

In this section, equations of FRFs with parametric uncertainty are developed. We consider linear structural damped dynamic systems with parametric uncertainties. In the parametric approach, the uncertainties associated with the system parameters such as Young's modulus, structural damping, mass density, Poisson's ratio and geometric parameters are defined using statistical models. The parametric approach can be implemented either using random variables or using random fields. In the random variable approach, physical variables of a structure are directly modelled using random variables. The random variables in turn can be characterized by the mean vector and the covariance matrix. Problems of structural dynamics in which the uncertainty in the mass, damping and stiffness of a structure is modelled within a framework of random fields can be treated using the stochastic finite element method. Following either the random variable approach or the random field approach, the equation of motion of a structurally damped linear stochastic dynamical system in the frequency domain can be expressed as

$$\left([K(\theta)] - \omega^2[M(\theta)] + \Im[D(\theta)]\right)\{X(\omega)\} = \{F(\omega)\} \tag{1}$$

where $\Im$ represents the unit imaginary and $[M(\theta)] \in \Re^{n \times n}$, $[D(\theta)] \in \Re^{n \times n}$ and $[K(\theta)] \in \Re^{n \times n}$ are the random mass, structural damping and stiffness matrices based on the n degrees of freedom, $\{X(\omega)\} \in \Re^n$. The input excitation to the structure is given by the vector $\{F(\omega)\} \in \Re^n$. The parameter θ is used to denote the random nature of the system matrices. In general the global random stiffness, structural damping and mass matrices can be expressed as:

$$[K(\theta)] = [K_0] + [\Delta K(\xi\theta)] \tag{2}$$
$$[D(\theta)] = [D_0] + [\Delta D(\xi\theta)] \tag{3}$$
$$[M(\theta)] = [M_0] + [\Delta M(\xi\theta)] \tag{4}$$

Here the deterministic parts $[K_0], [D_0]$ and $[M_0]$ are the usual system matrices obtained from the conventional finite element method. The m dimensional random vector $\{\xi(\theta)\} \in \Re^m$ completely characterizes uncertainties in the system. The nature of the random parts $\Delta(\bullet)$ depends on the probabilistic approach used. For example, if the stochastic finite element approach is used, then the random part can be expanded in a linear series using the Karhunen-Loéve (KL) expansion [9]. As an example, for systems with Gaussian random fields, the structural damping matrix can be expressed as

$$\Delta D(\xi(\theta)) = \sum_{j=1}^{m} \xi_j(\theta) K_j \tag{5}$$

where $\xi_j(\theta)$ are uncorrelated standard Gaussian random variables and $[K_j] \in \Re^{m \times m}$ are constant matrices. It is assumed that the random vector $\xi(\theta)$ has a mean $\mu \in \Re^m$ and a covariance $\sum \in \Re^{m \times m}$.

In this paper, measured FRFs are considered as data for updating. The FRF matrix (α) of a random system can be obtained from the random dynamic system given in Eq. (1) as

$$[Z(\xi(\theta))]^{-1} = [\alpha(\xi(\theta))] = [[K(\xi(\theta))] - \omega^2 [M(\xi(\theta))] + \Im[D(\xi(\theta))]]^{-1} \tag{6}$$

$[Z(\xi(\theta))]$ represents the random dynamic stiffness matrix, which is the inverse of the random FRF matrix. Because of the presence of damping, the measured FRFs are complex. The FRFs will be functions of the random vector ξ because the system matrices are functions of these random variables.

3. Problems with conventional stochastic model updating methods

In this section, problems with conventional stochastic model updating methods are discussed..

The three degrees of freedom lumped mass system [10], shown in Fig. 1, is used to demonstrate the problems associated with using mean values of the FRF data for model updating. The modes for this example are well separated. The lumped masses, (m_1, m_2, m_3) are assumed to be fixed and each has a mass of 5 kg. The uncertain parameters in this model are taken to be all of the stiffness and damping parameters. The spring stiffnesses (k_1, k_2, k_3) are given by $k_i = \xi_{ki} \times 200$ kN/m, where the random variables ξ_{ki} are assembled into a vector $\{\xi_k\} = [\xi_{k1} \quad \xi_{k2} \quad \xi_{k3}]^T \in \Re^3$. For this example the random variable vector, $\{\xi_k\}$, is assumed to be Gaussian with mean and variance

$$\{\mu_k\} = \begin{Bmatrix} 1 \\ 1 \\ 1 \end{Bmatrix}, \Sigma_k = \begin{bmatrix} 0.04 & 0.01 & -0.009 \\ 0.01 & 0.04 & 0.005 \\ -0.009 & 0.005 & 0.04 \end{bmatrix} \tag{7}$$

In the conventional stochastic model updating approach using modal data, the mean of the modal data is calculated and subsequently used for deterministic updating to estimate the mean of the updated parameters. The success of this approach is based on the assumption that the mean modal data is approximately equal to the modal data for the mean parameters, which turns out to be a reasonable assumption for well separated modes. A similar approach could be proposed for model updating based on FRF data, where the mean of the random FRFs could be used for stochastic model updating. However, as will be demonstrated here, the mean FRF is very different from the FRF of the mean parameters.
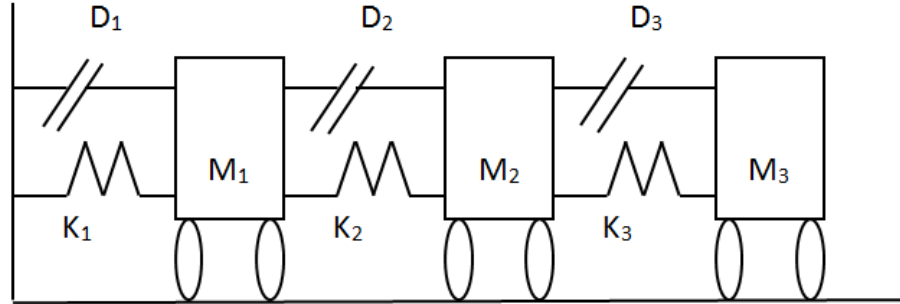

Figure 1: The three degree of freedom lumped mass system with well separated modes

The FRFs are generated using Monte Carlo simulation with 2000 samples of the stiffness and damping parameters. Hence, for this three dof system, 6000 FRFs are generated based on a single excitation location, and these generated FRFs are considered as the 'experimental' data. The mean of these FRFs are calculated and used for updating. First, consider this approach for the undamped system. Figure 2 (a) shows the random undamped FRFs, where the mean is given as the thick line. Clearly the shape of the mean FRF does not represent an actual FRF of a physical system. The resonance phenomenon for dynamic systems means that the relationship between the parameters and the FRF is highly nonlinear, and hence the mean FRF is very different from the FRF of the mean parameters. Suppose the mean FRFs are subsequently used for FRF-based model updating method using the method of Lin and Ewins [11]. The result is the updated FRF shown in Fig. 2(b), which is also compared to the mean FRF. Clearly the updated FRF does not match the mean FRF very well, as expected. The covariance matrix of the updating parameters is calculated using the expression given by [12] as

$$\Sigma = \left[S^T S\right]^{-1} S^T \Sigma_\theta S \left[S^T S\right] \tag{8}$$

where $[S]$ is the FRF sensitivity matrix and $[\Sigma_\theta]$ is the covariance matrix of the measured data. In this case, the calculated covariance matrix of the updated parameters is

$$\Sigma_k = \begin{bmatrix} 0.0223 & 0.0069 & -0.0077 \\ 0.0069 & 0.0128 & 0.0075 \\ -0.0077 & 0.0075 & 0.0355 \end{bmatrix} \tag{9}$$

which is very different to the initial covariance matrix given in Eq. (7).
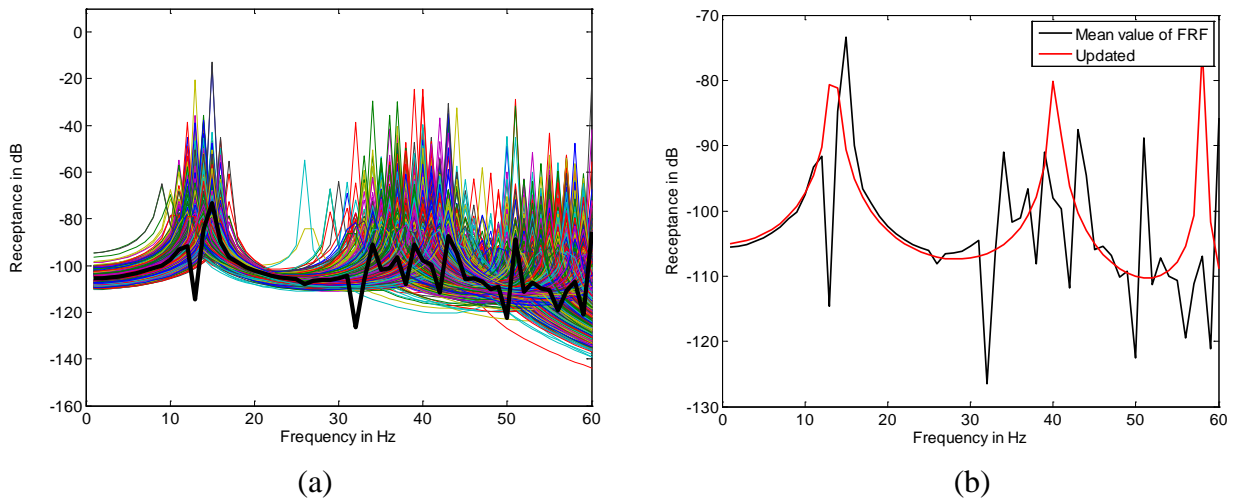
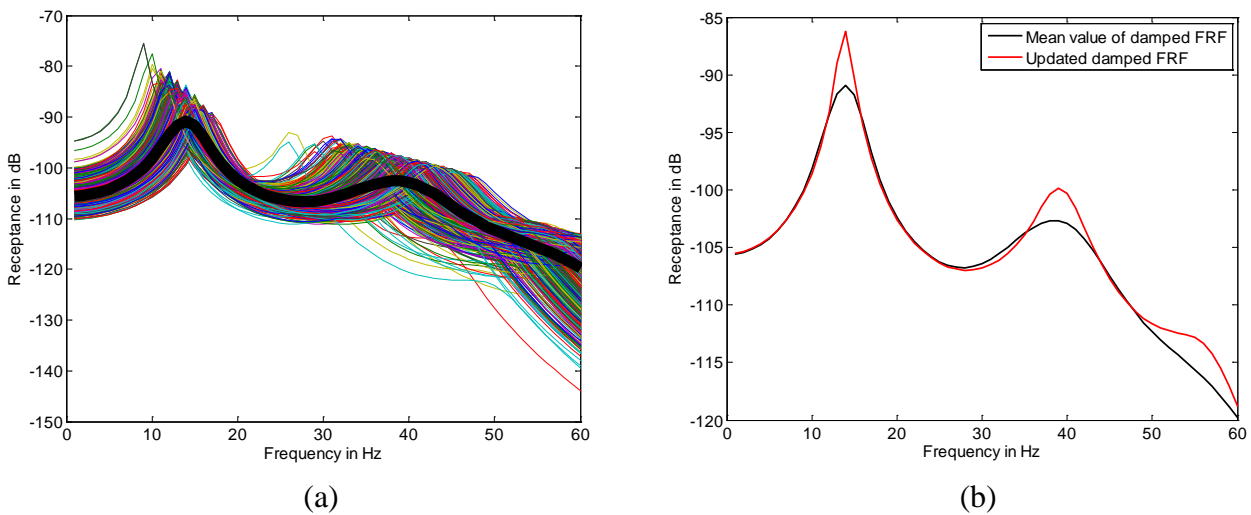Figure 2: Stochastic undamped FRFs and subsequent model updating

Figure 3: Stochastic damped FRFs and subsequent model updating

A similar study is performed for the damped system, with 2000 samples of parameter sets. The FRF based complex updating parameter method given by Arora et al. [13] is used for updating. The real parts of the identified parameters represent stiffness and whereas imaginary part of the updating parameter represent structural damping. Figure 3 (a) shows the randomly generated damped FRFs and also the mean damped FRF shown by the thick line. The mean FRF is smooth because of the presence of damping but it still does not represent the actual FRF of the system. Figure 3(b) compares the updated damped FRF and demonstrates that it does not match the mean FRF very well. The covariance matrices of the stiffness and damping parameters are:

$$\Sigma_k = \begin{bmatrix} 0.0419 & 0.0069 & -0.007 \\ 0.0069 & 0.0228 & 0.00759 \\ -0.0007 & 0.00759 & 0.0355 \end{bmatrix} \qquad (10)$$

$$\Sigma_d = \begin{bmatrix} 0.072 & 0.0057 & -0.0063 \\ 0.0057 & 0.059 & 0.0042 \\ -0.0063 & 0.0042 & 0.0621 \end{bmatrix} \tag{11}$$

which are significantly different to their initial values.

This example has clearly shown that updating methods based on the mean FRFs perform very poorly. Even for an idealized simulated case the estimated covariance matrices of the parameters have significant errors. The reason for these problems is that the mean FRF is not a good representation of an FRF for any set of parameters. Furthermore the mean FRF is very different from the FRF of the mean parameters. Increased damping gives smoother mean FRFs and improves the convergence of the updating methods, because the resonance peaks are not so sharp. However the updated model still has significant errors.

4. Uncertainty quantification of dynamic systems using FRF based stochastic model updating

The example in above section has shown that conventional stochastic model updating methods for damped dynamic systems do not work particularly well for damped systems with close modes. There is a need to develop procedure for stochastic damped model updating to quantify the uncertainties not only in stiffness and mass matrices but also in the damping. A new FRF based method is proposed to overcome the difficulties in updating damping matrix for systems with well-separated or close modes. This method is a development of the complex updating method given by Arora et al. [13], in which updating parameters are considered as complex; the real part of the updating parameter represents the stiffness and the imaginary part represents structural damping in the dynamic system. This method has been fully tested and demonstrated for the deterministic case with only a single set of measurements. Here the method is applied to each set of FRF data separately, yielding a set of identified parameters for each set of measurements. The statistics of the parameters (for example, mean and variance) are then obtained from these sampled identified parameters. Eq. (13) is the basic relationship of the deterministic model updating

$$\left[S\left(\xi(\theta,\omega) \right) \right]\left\{ u\left(\xi(\theta) \right) \right\} = \left\{ \Delta\alpha\left(\xi(\theta,\omega) \right) \right\} \tag{12}$$

Since every set of measured FRF data gives a set of parameters, these parameters may be regarded as samples from the probability distribution of parameters. Hence we may take the mean and variance of these updated parameters to solve our stochastic model updating problem.

5. Simulated case study

The three degrees of freedom lumped mass system with well separated modes as presented in previous section is considered. The values of mass, stiffness and damping are identical to above section including the simulated parameter uncertainty. In the proposed approach, each set of random FRF vectors are used to update the random variables. After updating all the random parameters, the mean and covariance matrix of the random variables are used to quantify the uncertainty in the random dynamic system. Monte Carlo simulations with 2000 sample parameter sets are used to generate the random stiffnesses and hence the 2000 FRF vectors.

Figure 4(a) shows the FRF obtained from the mean values of stiffnesses after updating the undamped system, and demonstrate that the mean stiffnesses are estimated accurately. The covariance matrix of the updated parameters is

$$\Sigma = \begin{bmatrix} 0.0399 & 0.0101 & -0.0089 \\ 0.0101 & 0.04 & 0.005 \\ -0.0089 & 0.005 & 0.04 \end{bmatrix} \tag{13}$$

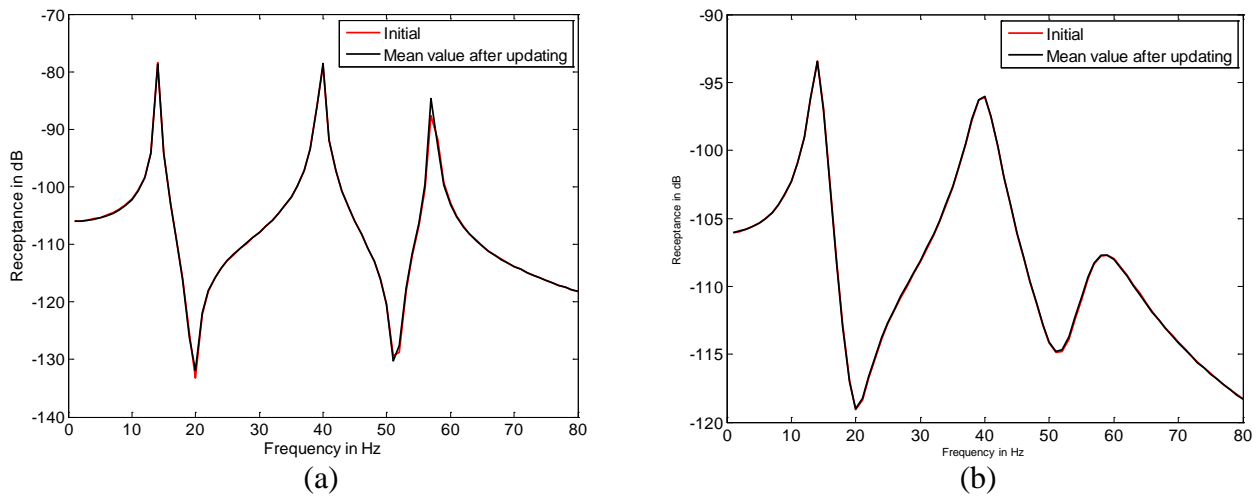which is very close to the variance used to generate the FRF data.

Figure 3: The updated using mean values FRF (a) undamped (b) damped system

A similar study was performed for the damped system, where six parameters are considered uncertain, namely three stiffness parameters and three damping parameters. Again 2000 samples are used to generate 2000 vectors of damped FRFs, which are used to the parameters. Figure 4(b) shows the FRF obtained from the mean values of the uncertain parameters after updating for the damped system, and demonstrates that mean parameters are accurately estimated. The estimated covariance matrices for the updated stiffness and damped parameters are close to the initial variances used for the simulations

$$\Sigma_k = \begin{bmatrix} 0.0398 & 0.01 & -0.0089 \\ 0.01 & 0.04 & 0.005 \\ -0.0089 & 0.005 & 0.04 \end{bmatrix} \tag{14}$$

$$\Sigma_d = \begin{bmatrix} 0.009 & 0.01 & -0.009 \\ 0.0101 & 0.09 & 0.005 \\ -0.009 & 0.005 & 0.09 \end{bmatrix} \tag{15}$$

Hence the proposed FRF based stochastic model updating approach is able to quantify the uncertainty in the dynamic system. The proposed approach is also able to estimate damping, in contrast to most of the stochastic model updating methods using modal data, which neglect damping.

6. Conclusions

A new FRF based stochastic damped model updating approach has been proposed in this paper. In order to overcome the problem of close modes and damping in random dynamic systems, a complex FRF based stochastic model updating method was proposed. The proposed probabilistic updating method is a parameter based method in which complex FRFs are used to update the parameters. These updated parameters are subsequently used quantify the uncertainties in the stochastic dynamic system. The effectiveness of the proposed updating procedure was demonstrated by numerical examples, using case studies with well separated modes and also with closely spaced modes. The results have shown that the proposed damped FRF-based probabilistic model updating procedure can be used for accurate quantification of uncertain parameters in these dynamic systems.

REFERENCES

1   Kiureghian, A. D. and Ditlevsen, O. Aleatory or epistemic? Does it matter?, *Structural Safety* **31**, 105-112, (2009).

2   Mottershead, J.E. and Friswell, M.I. Model Updating in Structural Dynamics: a Survey, *Journal of Sound and Vibration* **167**, 347-375, (1993).

3   Berman, A. and Nagy, E.J. Improvement of a large analytical model using test data, *AIAA Journal,* **21**, 927-928 (1983).

4   Collins, J.D., Hart, G.C., Hasselman, T.K. and Kennedy, B. Statistical identification of structures, *AIAA Journal*, **12**, 185-190, (1974).

5   M.I. Friswell, The adjustment of structural parameters using a minimum variance estimator, *Mechanical systems and signal processing,* **3**(2), 143-155, (1989).

6   Beck, J.L. and Katafygiotis, L.S. Updating models and their uncertainties II: Model identifiability, *Journal of Engineering Mechanics* **124**, 463-467, (1998).

7   Khodaparast, H.H., Mottershead, J.E. and Friswell, M.I. Perturbation methods for the estimation of parameter variability in stochastic model updating, *Mechanical systems and signal processing,* **22**(8), 1751-1773, (2008).

8   Hua, X., Ni, Y.Q., Chen, Z.Q. and J.M. Ko, An improved perturbation method for stochastic finite element model updating, *International Journal for Numerical methods in Engineering,* **73**, 1845-1864, (2008).

9   McFarland, J., Mahadevan, S., Romero, V. and Swiler, L. Calibration and uncertainty analysis for computer simulations with multivariable output, *AIAA Journal,* **46**, 1253-1265 (2008).

10 Adhikari, S. and Friswell, M.I. Distributed parameter model updating using Karhunen-Loéve expansion, *Mechanical systems and signal processing,* **24**, 326-339 (2010).

11 Arora, V. Structural damping identification using normal FRFs, *International Journal of Solids and Structures,* **51**, 133-143 (2014).

12 Lin, R.M. and Zhu, J. Model updating of damped structures using FRF data, *Mechanical Systems and Signal Processing,* **20**, 2200-2218, (2006).

13 Govers, Y. and Link, M. Stochastic model updating-Covariance matrix adjustment from uncertain experimental data, *Mechanical systems and signal processing,* **24**, 696-706 (2010).

14 Arora, V., Singh, S.P. and Kundra, T.K. Damped model updating using complex updating parameters, *Journal of Sound and Vibration,* **320**, 438-451 (2009).