# Vibration suppression in MEMS devices using electrostatic forces

H. Khodaparast, H. Madinei, M.I. Friswell & **S. Adhikari**

College of Engineering, Swansea University, UK

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# Overview

- Introduction and motivation
- Theory of Incremental Non-linear Control Parameters (INCP)
- Application to vibration suppression in Microelectromechanical Systems (MEMS) device
- Conclusions and Future Works

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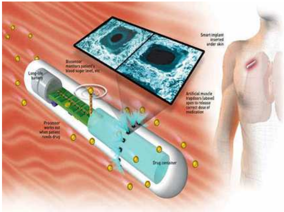
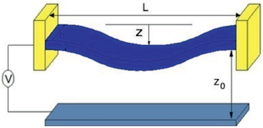
# Application of MEMS devices

- Automotive (MEMS pressure sensors)
- Biomedical (smart pills)
- Wireless and optical communications
- Optical displays
- Chemical



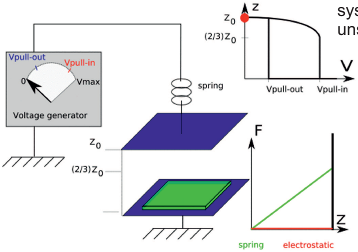
# Types of actuation mechanism in MEMS

- **Electrostatic**
- Thermal
- Pneumatic
- Piezoelectric




**Electrostatic force:** $$\frac{\varepsilon_0 b V^2}{2(z_0 - z)^2}$$

Pull-in: the voltage at which the system becomes unstable

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# Motivation- the objective of this work

- The objective of this study is:
  - to minimize the vibration amplitude of a MEMS device by controlling the resonance frequency of the system.
- To this end,
  - DC voltages are applied to the electrodes to change the resonance frequency of the system.
- Applying DC voltages to the system makes the system non-linear. To solve the non-linear system of equations,
  - the non-linearity is parametrised by a set of 'non-linear control parameters' such that the dynamic system is effectively linear for zero values of these parameters and non-linearity increases with increasing values of these parameters.
  - 'non-linear control parameters' are the applied DC voltages in this problem as when they are zeros, the system is linear.

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# Incremental non-linear control parameters (1)

- The idea is to develop an extended harmonic balance method for the steady-state solution of non-linear multiple-degree-of-freedom dynamic problems based on incremental non-linear control parameters.
- The method only requires the solution of linear equations for the non-linear problem
- It also provides the sensitivities of the solution with respect to non-linear control parameters.
- The non-linear control parameters are those with which the non-linearity in the model is triggered.

# Incremental non-linear control parameters (2)

- This property of the non-linear control parameters can be exploited in the solution of a non-linear problem.
- They are incremented from zero to one (note that the parameters are normalised so that the maximum values are unity) and a linear equation giving the sensitivities of the responses with respect to the parameters is obtained at each increment.
- Using these sensitivities, the solution at each step can be calculated through the solution at the previous increment.
- The method starts from the linear system and continues until all non-linear control parameters reach unity.

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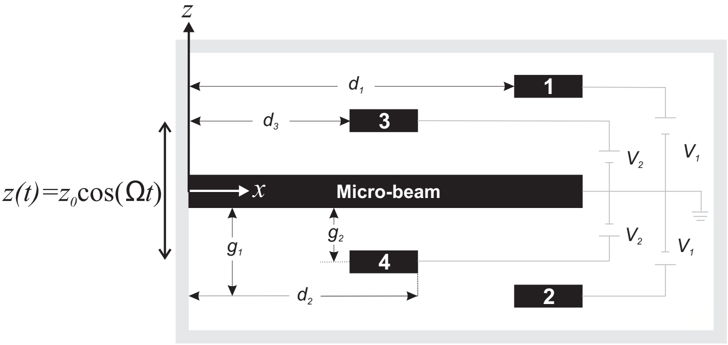
# Description of the method

- Consider the model of a MEMS cantilever beam with electrodes (shown in the figure below)

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## Mathematical model

- The equation of motion of the beam can be expressed as:

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + c_a\frac{\partial w(x,t)}{\partial t} + \rho A\frac{\partial^2 w(x,t)}{\partial t^2} = \frac{\epsilon_0 a H(x-d_1)}{2}\left(\frac{V_1^2}{(g_1 - w(x,t))^2}\right) - \frac{\epsilon_0 a H(x-d_1)}{2}\left(\frac{V_1^2}{(g_1 + w(x,t))^2}\right) + \frac{\epsilon_0 a (H(x-d_2) - H(x-d_3))}{2}\left(\frac{V_2^2}{(g_2 - w(x,t))^2}\right) - \frac{\epsilon_0 a (H(x-d_2) - H(x-d_3))}{2}\left(\frac{V_2^2}{(g_2 + w(x,t))^2}\right) - \rho A\frac{\partial^2 z(t)}{\partial t^2} \tag{1}$$

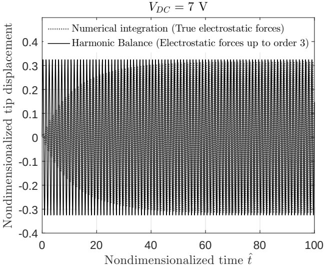
# The nondimensionalized equation of the micro-beam

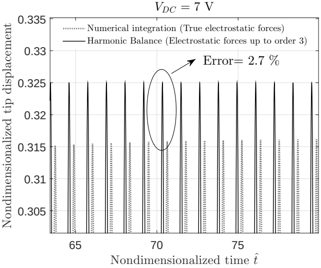
- The electrostatic force functions in Eq.(1) may be expressed in terms of its Taylor series.
- Therefore the nondimensionalised form of Eq.(1) with the truncated cubic terms of electrostatic force becomes

$$\frac{\partial^4 \hat{w}\left(\hat{x},\hat{t}\right)}{\partial \hat{x}^4}+c\frac{\partial \hat{w}\left(\hat{x},\hat{t}\right)}{\partial \hat{t}}+\frac{\partial^2 \hat{w}\left(\hat{x},\hat{t}\right)}{\partial \hat{t}^2}+\alpha_1\hat{w}+\alpha_3\hat{w}^3+\mathcal{O}\left(\hat{w}^5\right)=\gamma \exp\left(i\hat{\Omega}\hat{t}\right)+cc. \quad (2)$$

# Is cubic order accurate enough?

- This depends on the amplitude of excitation frequency and damping
- If the beam is excited at its first non-dimensionalized resonance frequency and $V_1 = V_2 = 7$ V and $z_0 = 0.1\ \mu$m, the electrostatic force can be estimated by its third-order Taylor series with a reasonable degree of accuracy. This is shown in the figure below



# How the method works

- When the voltages are zeros, the system is linear and therefore the solution of linear system can be assumed as

$$\hat{w}_0 = \sum_{j=1}^{N} Y_j(x) \left( Q_{0j} exp\left(i\hat{\Omega}\hat{t}\right) + cc. \right) \quad (3)$$

where $Q_{0j}$, the components of vector $\mathbf{q}_0 \in \mathbb{R}^N$, are obtained from the following equation

$$\mathbf{q}_0 = \left[ -\hat{\Omega}^2 \mathbf{M} + i\hat{\Omega}\mathbf{C} + \mathbf{K} \right]^{-1} \mathbf{F} \quad (4)$$

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# Linear system

- The non-linear control parameters are normalised in which $\theta_i = \frac{V_i}{V_{pi}}$ ($V_{pi}$ is pull-in voltage (the maximum voltage that can be applied to the system)
- If all the normalised non-linear parameters are perturbed by $\delta\theta$, the steady state solution of weakly non-linear system may be expressed by

$$\hat{w}_1 = \hat{w}_0 + \left( \frac{\partial \hat{w}_0}{\partial \theta_1} + \frac{\partial \hat{w}_0}{\partial \theta_2} \right) \delta\theta + \mathcal{O}\left( \delta\theta^{(2)} \right) \approx \hat{w}_0 + \acute{\hat{w}}_1 \delta\theta \quad (5)$$

where $\acute{\hat{w}}_1 = \left( \frac{\partial \hat{w}_0}{\partial \theta_1} + \frac{\partial \hat{w}_0}{\partial \theta_2} \right)$

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# Perturbation

- Substituting $\hat{w}_1 = \hat{w}_0 + \acute{\hat{w}}_1 \delta\theta$ into the governing equation of the beam and neglecting the higher order terms of $\delta\theta$ yield

$$\left( \frac{\partial^4 \acute{\hat{w}}_1}{\partial x^4} + c \frac{\partial \acute{\hat{w}}_1}{\partial \hat{t}} + \frac{\partial^2 \acute{\hat{w}}_1}{\partial \hat{t}^2} + \alpha_1 (\boldsymbol{\theta}) \acute{\hat{w}}_1 \right) \delta\theta + \alpha_1 (\boldsymbol{\theta}) \hat{w}_0 + \alpha_3 (\boldsymbol{\theta}) \left( \hat{w}_0{}^3 + 3 \hat{w}_0{}^2 \acute{\hat{w}}_1 \delta\theta \right) = 0 \quad (6)$$

- The above partial differential equation is a linear function in terms of $\acute{\hat{w}}_1$ and standard discretization methods (such as Galerkin) can be used to obtain the solution of $\acute{\hat{w}}_1$.

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# Non-linear solution (1)

- The steady state solution of $\acute{\hat{w}}_1$ includes primary and higher harmonics of the excitation frequency. One may ignore the higher harmonics and assume

$$\acute{\hat{w}}_1 = \sum_{j=1}^{m} Y_j(x)\left(\acute{Q}_{1j} exp\left(i\hat{\Omega}\hat{t}\right) + cc.\right) \tag{7}$$

- Balancing the harmonic terms and applying standard Galerkin projection gives

$$\mathbf{A}_1 \acute{\mathbf{q}}_1 = \mathbf{b}_1 \tag{8}$$

- where $\acute{\mathbf{q}}_1 = \left\{\acute{Q}_{1j}\right\} \in \mathbb{R}^N$

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# Non-linear solution (2)

- The same assumption, i.e. $\hat{w}_{k+1} = \hat{w}_k + \acute{\hat{w}}_k \delta\theta$, can be made for the following iterations and obtain the following recursive linear system of equations

$$\mathbf{A}_k \acute{\mathbf{q}}_k = \mathbf{b}_k \tag{9}$$

- where $\acute{\mathbf{q}}_1 = \left\{ \acute{Q}_{1j} \right\} \in \mathbb{R}^N$. This will continue until the non-dimensionalised non-linear control parameters reach unity.

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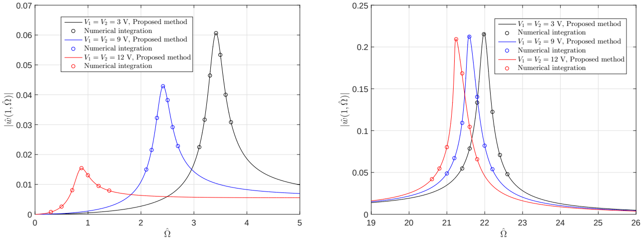
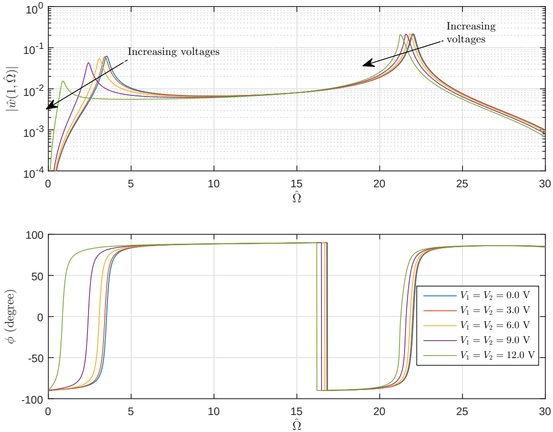
# Validation by numerical integration

- Validation of the results. Left figure: mode 1 and right figure: mode 2.



# Frequency Response function

- Frequency and phase responses of beam-tip displacement

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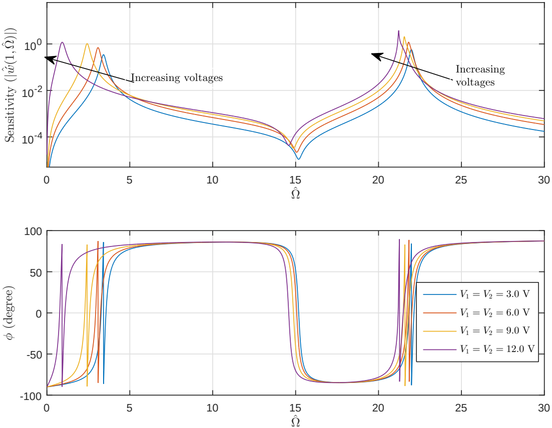
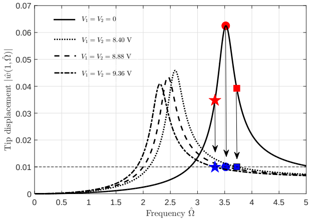
# Sensitivities

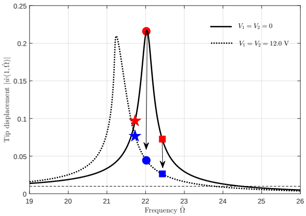
- Sum of sensitivities of frequency and phase responses of beam-tip displacement with respect to $V_1$ and $V_2$



# Vibration suppression of MEMS devices (1)

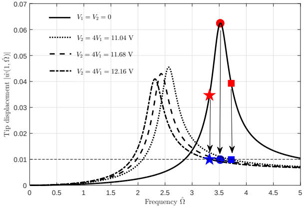
- Vibration suppression using the applied voltages in **case 1** when $V_1 = V_2$. Left figure: mode 1 and right figure: mode 2.

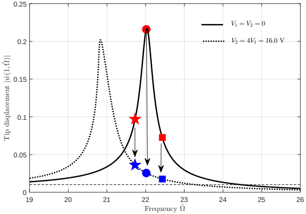


# Vibration suppression of MEMS devices (2)

- Vibration suppression using the applied voltages in **case 2** when $V_2 = 4V_1$ Left figure: mode 1 and right figure: mode 2.

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## Comparison between case 1 and 2

- In both cases, at frequency points about the first resonance, the proposed method is capable of reducing the vibration level of the MEMS device to acceptable level, i.e. 0.01 (non-dimensionalised displacement).
- However, the system requires 17% less total voltages in **case 2** to obtain this objective.
- the vibration level at frequency points around the second mode did not reach the acceptable vibration level in neither of the two cases, albeit it significantly reduced as shown in the table of next slide. Interestingly, the reductions in **case 2** are greater than **case 1**.
- The main observation is that the initial selection of $V_1/V_2$ can have significant effect on the efficiency of the system in terms of required voltages for vibration control.

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# Comparison between case 1 and 2

- Micro-beam tip Displacement (Disp.) at different excitation frequencies and the applied DC voltages

| Frequency | Tip Disp. at $V_1 = V_2 = 0$ | Tip Disp. at $V_1 = V_2 = V$ | Tip Disp. at $V_2 = 4V_1 = V$ |
|---|---|---|---|
| 3.32 | 0.035 | 0.01 at $V = 8.4$ V | 0.01 at $V = 11.04$ V |
| 3.52 | 0.062 | 0.01 at $V = 8.88$ V | 0.01 at $V = 11.68$ V |
| 3.72 | 0.039 | 0.01 at $V = 9.36$ V | 0.01 at $V = 12.16$ V |
| 21.73 | 0.097 | 0.077 at $V = 12$ V | 0.036 at $V = 16$ V |
| 22.02 | 0.22 | 0.044 at $V = 12$ V | 0.026 at $V = 16$ V |
| 22.43 | 0.073 | 0.026 at $V = 12$ V | 0.018 at $V = 16$ V |

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# Conclusions and Future Works

- A formulation for calculation of the steady state responses of non-linear dynamic systems and their sensitivities with respect to non-linear control parameters is shown.
- This formulation is exploited in vibration suppression of a MEMS device.
- It was observed that the performance of vibration control depends on the initial choice of the relation between voltage sources.
- This highlights the importance of performing an optimisation problem to achieve the best performance.
- Future work will be focused on the solution to an optimization problem that find the optimal relation between the voltages in which the total voltage required for control is minimized.

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