17th AIAA Non-Deterministic Approaches Conference on January 5-9, 2015 at Kissimmee , FL, USA

Free vibration analysis of angle-ply composite plates with uncertain properties

S. Dey, T. Mukhopadhyay, S. Adhikari

Swansea University
Prifysgol Abertawe

Swansea University, Singleton Park, Swansea SA2 8PP, United Kingdom

Outline

- Introduction
- Laminated Composites - Plate Model / Shell Model
- Governing Equations
- Uncertainty Propagation using surrogate models
  - Random Sampling - High Dimensional Model Representation (RS-HDMR) model
  - Polynomial Regression Model using D-Optimal design
  - Kriging Model
- Results and Discussion
- Conclusions

Introduction

Swansea University
Prifysgol Abertawe

- Suppose *f(x)* is a computationaly intensive multidimensional nonlinear (smooth) function of a vector of parameters x.
- We are interested in the statistical properties of *y=f(x)*, given the statistical properties of x.
- The statistical properties include, mean, standard deviation, probability density functions and bounds.
- This work considers computational methods for dynamics of composite structures with uncertain parameters.
- General categories of uncertainties are:
  - **Aleatoric: Due to variability in the system parameters**
  - **Epistemic: Due to lack of knowledge of the system**
  - **Prejudicial: Due to absence of variability characterization**
- To start with Bottom up approach to quantify the uncertainty from its sources of origin for the composite laminate.

Introduction

Swansea University
Prifysgol Abertawe

- Prime sources of uncertainties are:
  - **Material and Geometric uncertainties**
  - **Manufacturing uncertainties**
  - **Environmental uncertainties**

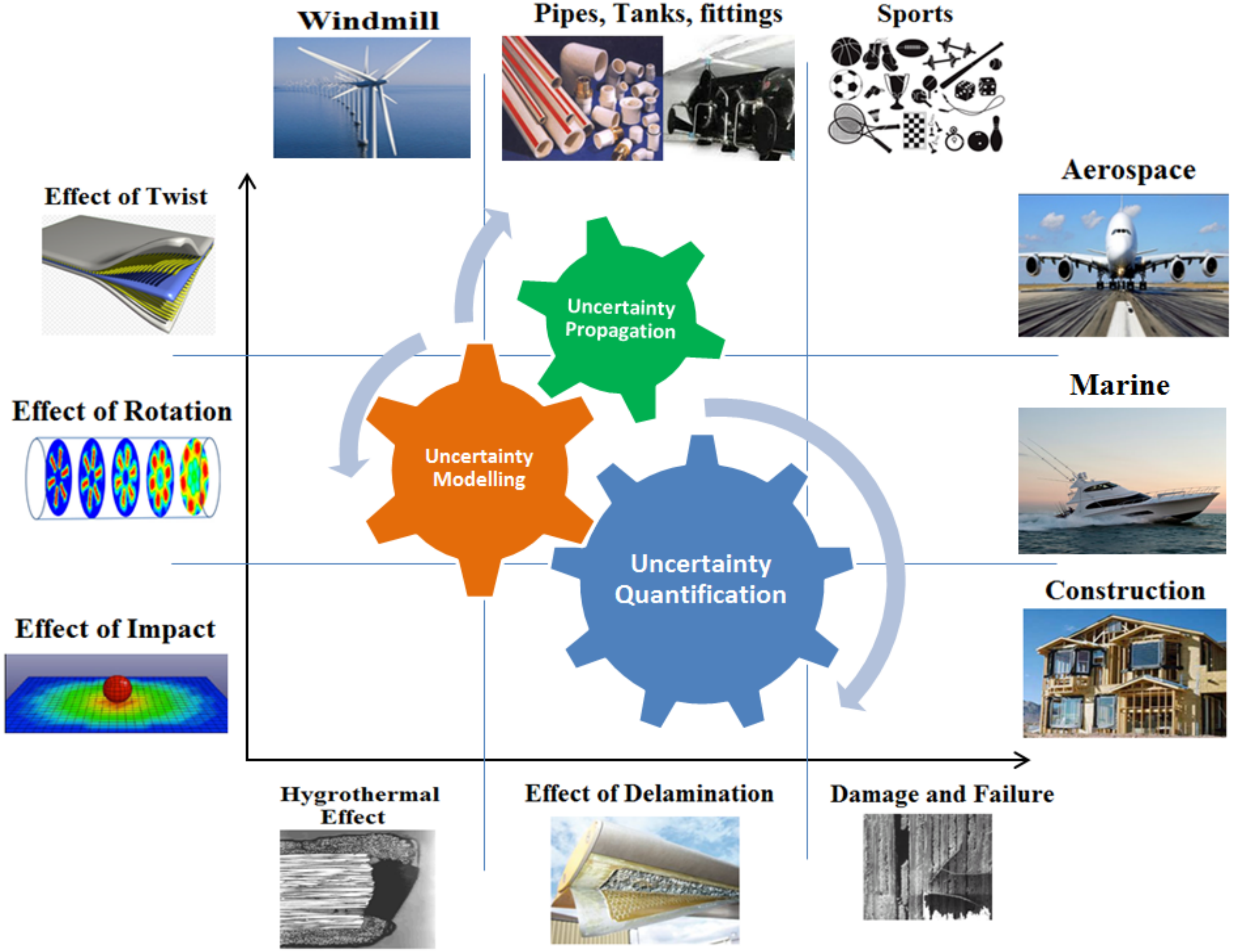
- Monte Carlo Simulation is an accurate but expensive computational method for Uncertainty Quantification.

- If the model is computationally expensive, this cost has a cascading effect on the increasing cost of computation.

- To save sampling time and hence the computational cost, the following three surrogate modelling methods are employed:
  - **Random Sampling-High Dimensional Model Representation**
  - **Polynomial Regression Model using D-Optimal**
  - **Kriging Model**

- These metamodels can be employed to general stochastic problems.

Factors affecting uncertainty in composites



Laminated Composites – Plate / Shell Model

Swansea University
Prifysgol Abertawe

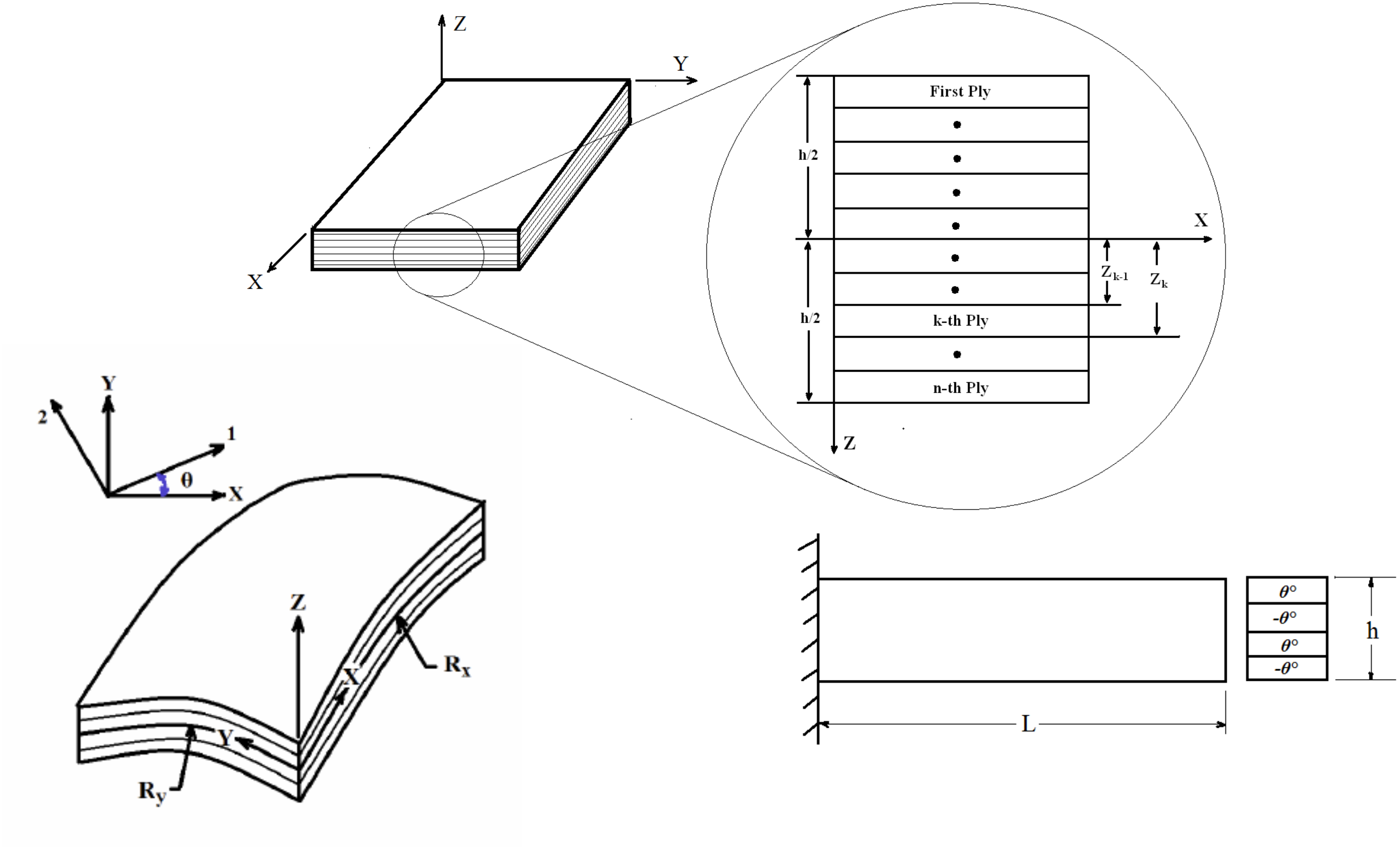


Figure : Laminated composite cantilever plate.

Governing Equations

Swansea University
Prifysgol Abertawe

- If mid-plane forms x-y plane of the reference plane, the displacements can be computed as

$$u(x,y,z)=u^0(x,y)-z\theta_x(x,y)\ ,\quad v(x,y,z)=v^0(x,y)-z\theta_y(x,y)\ ,\quad w(x,y,z)=w^0(x,y)=w(x,y)$$

- The strain-displacement relationships for small deformations can be expressed as

$$\varepsilon_{xx}=\varepsilon_x^0+zk_x \qquad \varepsilon_{yy}=\varepsilon_y^0+zk_y \qquad \gamma_{xy}=\gamma_{xy}^0+zk_{yy} \qquad \gamma_{xz}=w_{,x}^0-\theta_x \qquad \gamma_{yz}=w_{,y}^0-\theta_y$$

where $\varepsilon_x^0=u_{,x}^0\ ,\quad \varepsilon_y^0=u_{,y}^0\ ,\quad \gamma_{xy}^0=u_{,y}^0+v_{,x}^0$

$$k_x=-\theta_{x,x}=-w_{,xx}+\gamma_{xz,x} \qquad k_y=-\theta_{y,y}=-w_{,yy}+\gamma_{yz,y} \qquad k_{xy}=-(\theta_{x,y}+\theta_{y,x})=-2w_{,xy}+\gamma_{xz,y}+\gamma_{yz,x}$$

- The strains in the *k*-th lamina: $\{\varepsilon\}^k=\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+z\begin{Bmatrix}k_x^0\\ k_y^0\\ k_{xy}^0\end{Bmatrix}=\{\varepsilon^0\}+z\{k\}$ and $\{\gamma\}^k=\begin{Bmatrix}\gamma_{yz}\\ \gamma_{xz}\end{Bmatrix}=\{\gamma\}$

- In-plane stress resultant {N}, the moment resultant {M}, and the transverse shear resultants {Q} can be expressed

$$\{N\}=[A]\{\varepsilon^0\}+[B]\{k\}\ ,\ \{M\}=[B]\{\varepsilon^0\}+[D]\{k\}\ ,\ \{Q\}=[A^*]\{\gamma\}$$

$$\text{where}\quad [\overset{*}{A}_{ij}]=\int_{-h/2}^{h/2}\bar{Q}_{ij}\ dz \quad \textit{where}\ \ i,j=4,5$$

Bottom Up Approach

Swansea University
Prifysgol Abertawe

All cases consider an eight noded isoparametric quadratic element with five degrees of freedom for graphite-epoxy composite plate / shells

Material properties (Graphite-Epoxy)**: E_1=138.0 GPa, E_2=8.96GPa, G_{12}=7.1GPa, G_{13}=7.1 GPa, G_{23}=2.84 GPa, v=0.3

$$[\overline{Q}_{ij}(\overline{\omega})] = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & (m^4+n^4) & -4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2) \\ m^3n & mn^3 & (mn^3-m^3n) & 2(mn^3-m^3n) \\ mn^3 & m^3n & (m^3n-mn^3) & 2(m^3n-mn^3) \end{bmatrix} [Q_{ij}]$$

$$\{N\} = [A]\{\varepsilon^0\} + [B]\{k\}$$

$$\{M\} = [B]\{\varepsilon^0\} + [D]\{k\}$$

$$m = Sin\,\theta(\overline{\omega}) \qquad n = Cos\,\theta(\overline{\omega})$$

$\theta(\overline{\omega})$ = Random ply orientation angle

$$[D'(\overline{\omega})] = \begin{bmatrix} A_{ij}(\overline{\omega}) & B_{ij}(\overline{\omega}) & 0 \\ B_{ij}(\overline{\omega}) & D_{ij}(\overline{\omega}) & 0 \\ 0 & 0 & S_{ij}(\overline{\omega}) \end{bmatrix}$$

$$[A_{ij}(\overline{\omega}),\ B_{ij}(\overline{\omega}),\ D_{ij}(\overline{\omega})] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}(\overline{\omega})]_k\ [1,\ z,\ z^2]\, dz \qquad i,j = 1,2,6$$

$$[S_{ij}(\overline{\omega})] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha_s\ [\overline{Q}_{ij}(\overline{\omega})]_k\, dz \qquad i,j = 4,5$$

Eigenvalue Problem

Swansea University
Prifysgol Abertawe

- From Hamilton's principle: $\delta H = \int_{t_i}^{t_f} [\delta T - \delta U - \delta W]\,dt = 0$
- Potential strain energy: $U = U_1 + U_2 = \frac{1}{2}\{\delta_e\}^T[K_e(\varpi)]\{\delta_e\} + \frac{1}{2}\{\delta_e\}^T[K_{\sigma e}(\varpi)]\{\delta_e\}$
- Kinetic energy: $T = \frac{1}{2}\{\dot{\delta}_e\}^T[M_e(\varpi)] + [C_e(\varpi)]\{\delta_e\}$
- Mass matrix: $[M(\varpi)] = \int_{Vol}[N][P(\varpi)][N]\,d(vol)$ where $P(\varpi) = \sum_{k=1}^{n}\int_{z_{k-1}}^{z_k}\rho(\varpi)\,dz$
- Stiffness matrix: $[K(\varpi)] = \int_{-1}^{1}\int_{-1}^{1}[B(\varpi)]^T\,[D(\varpi)]\,[B(\varpi)]\,d\xi\,d\eta$
- Dynamic Equation: $[M(\varpi)]\ddot{\delta}(t) + [C]\dot{\delta}(t) + [K(\varpi)]\delta(t) = f(t)$

For free vibration, the random natural frequencies are determined from the standard eigenvalue problem, solved by the QR iteration algorithm

$$[A(\varpi)]\{\delta\} = \lambda(\varpi)\{\delta\} \quad \text{where} \quad [A(\varpi)] = ([K_e(\varpi)] + [K_{\sigma e}(\varpi)])^{-1}\,[M(\varpi)]$$

$$\lambda(\varpi) = \frac{1}{\{\omega_n(\varpi)\}^2}$$

Modal Analysis

- The eigenvalues and eigenvectors satisfy the orthogonality relationship

$$\mathbf{x}_l^T\,[\mathbf{M}(\varpi)]\,\mathbf{x}_j = \lambda_{lj} \text{ and } \mathbf{x}_l^T\,[\mathbf{K}(\varpi)]\,\mathbf{x}_j = \omega_j^2\,\lambda_{lj} \quad \text{where } l, j = 1,2\ldots\ldots n$$

$$\mathbf{X}^T\,[\mathbf{M}(\varpi)]\,\mathbf{X} = \mathbf{I} \qquad \text{and} \qquad \mathbf{X}^T\,[\mathbf{K}(\varpi)]\,\mathbf{X} = \Omega^2$$

- Using modal transformation, pre-multiplying by $\mathbf{X}^T$ and using orthogonality relationships, equation of motion of a damped system in the modal coordinates is obtained as $\ddot{y}(t) + X^T C X \dot{y}(t) + \Omega^2 y(t) = \tilde{f}(t)$

- The damping matrix in the modal coordinate: $\mathbf{C}' = \mathbf{X}^T\,[\mathbf{C}]\,\mathbf{X}$

- The generalized proportional damping model expresses the damping matrix as a linear combination of the mass and stiffness matrices

$$C(\varpi) = \alpha_1\, M(\varpi)\, f\,(M^{-1}(\varpi)\;K(\varpi)) \quad \text{where } \alpha_1 = 0.005 \text{ is constant damping factor}$$

- Transfer function matrix

$$H(i\omega)\,(\varpi) = X[-\omega^2 I + 2i\omega\zeta\Omega + \Omega^2]^{-1}\;X^T = \sum_{j=1}^{n} \frac{X_j X_j^T}{-\omega^2 + 2i\omega\zeta_j\omega_j + \omega_j^2}$$

- The dynamic response in the frequency domain with zero initial conditions:

$$\bar{\delta}(i\omega)\,(\varpi) = H(i\omega)\,\bar{f}(i\omega) = \sum_{j=1}^{n} \frac{X_j^T\,\bar{f}(i\omega)}{-\omega^2 + 2i\omega\zeta_j\omega_j + \omega_j^2} X_j$$

Swansea University
Prifysgol Abertawe

Random Sampling – High Dimensional Model Representation (RS-HDMR)

Random Sampling – High Dimensional Model Representation (RS-HDMR) Model

Swansea University
Prifysgol Abertawe

The mapping between the input variables $x_1, x_2, \ldots, x_n$ and the output $f(X) = f(x_1, x_2, \ldots, x_n)$ in the domain R^n can expressed in the following form:

$$f(X) = \underbrace{f_o}_{\text{constant term}} + \sum_{i=1}^{n} f_i(x_i) + \sum_{1 \le i \le j \le n} f_{ij}(x_i, x_j) + \ldots + f_{12\ldots n}(x_1, x_2, \ldots, x_n)$$

constant term
first order
second order
n^{th} order

- Use of orthonormal polynomial for the computation of RS-HDMR component functions:

$$f_i(x_i) \approx \sum_{r=1}^{k} \alpha_r^i \varphi_r(x_i)$$

$$f_{ij}(x_i, x_j) \approx \sum_{p=1}^{l} \sum_{q=1}^{l'} \beta_{pq}^{ij} \varphi_p(x_i) \varphi_q(x_j)$$

- **Check for Coefficient of determination (*R^2*) and Relative Error (*RE*):**

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T} \; (0 \le R^2 \le 1)$$

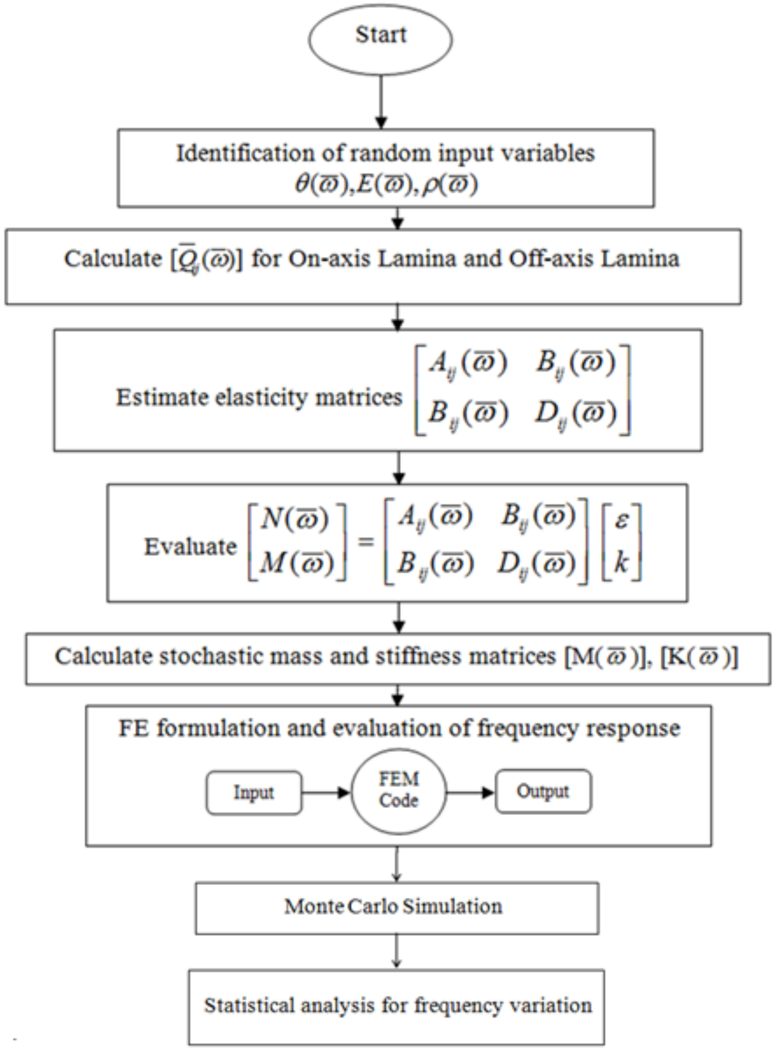
$$RE(\%) = \frac{|F - F'|}{F} \text{x}100$$

where, $SS_T = SS_E + SS_R$

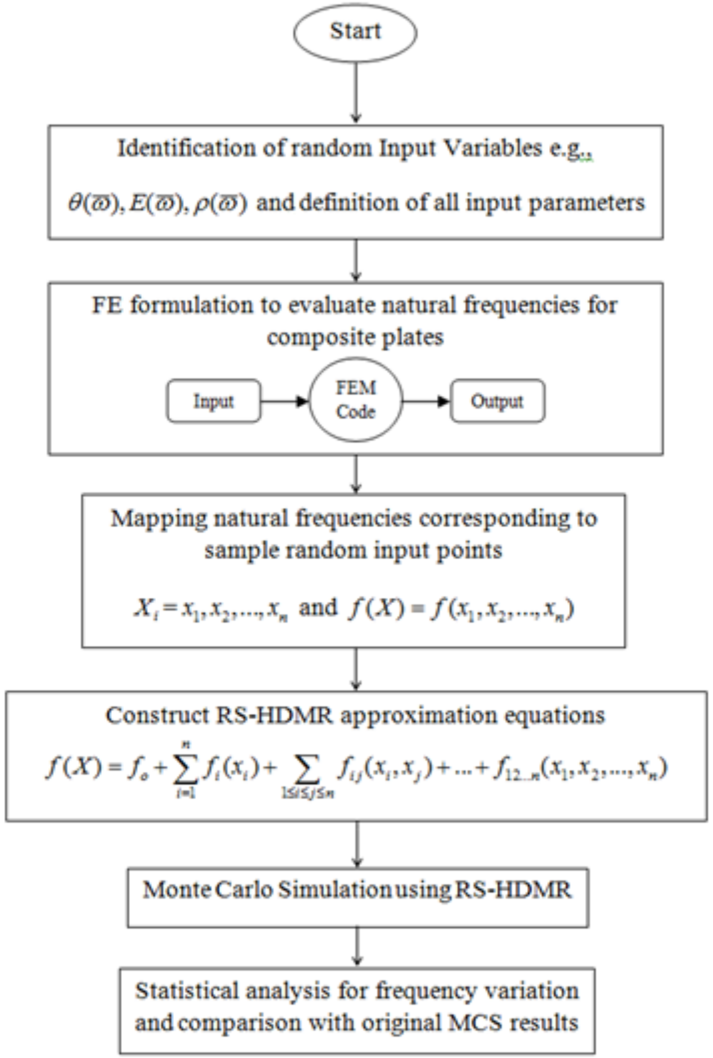
Uncertainty Quantification - Flow chart

Swansea University
Prifysgol Abertawe

Monte Carlo Simulation



Random Sampling – High Dimensional Model Representation (RS-HDMR) Model



Global Sensitivity Analysis based on RS-HDMR

Swansea University
Prifysgol Abertawe

- The orthogonal relationship between the component functions of **Random Sampling – High Dimensional Model Representation (RS-HDMR)** expression implies that the component functions are independent and contribute their effects independently to the overall output response.

- Sensitivity Index

$$\text{Sensitivity index an input parameter } (S_i) = \frac{\text{partial variance of the input parameter}}{\text{total variance}}$$

$$\text{Such that, } \sum_{i=1}^{n} S_i + \sum_{1 \le i < j \le n}^{n} S_{ij} + \ldots + S_{1,2,\ldots n} = 1$$

Input uncertainty model

- Ply angles are varied +/- 5 degrees in each layer of the laminate
- All material properties are varied +/- 10%

Validation – Random Sampling – High Dimensional Model Representation (RS-HDMR) Model

Figure : Probability distribution function (PDF) with respect to model response of first three natural frequencies for variation of ply-orientation angle of graphite-epoxy angle-ply (45°/-45°/45°) composite cantilever plate, considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 Kg/m^3, t=0.004 m, v=0.3

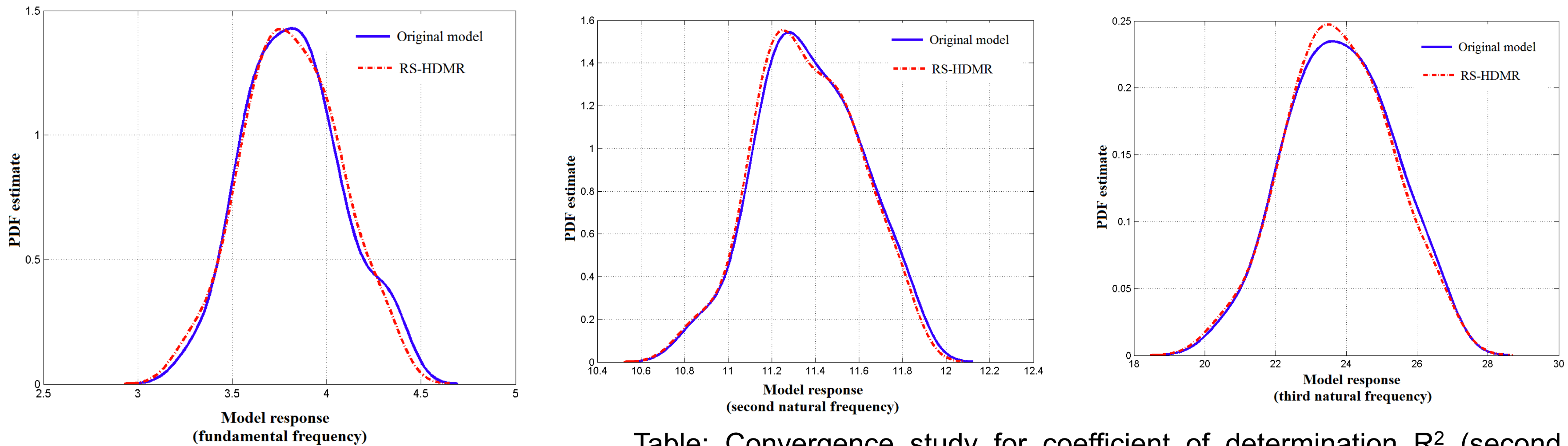


Figure: Scatter plot for fundamental frequencies for variation of ply-orientation angle of angle-ply (45°/-45°/45°) composite cantilever plate

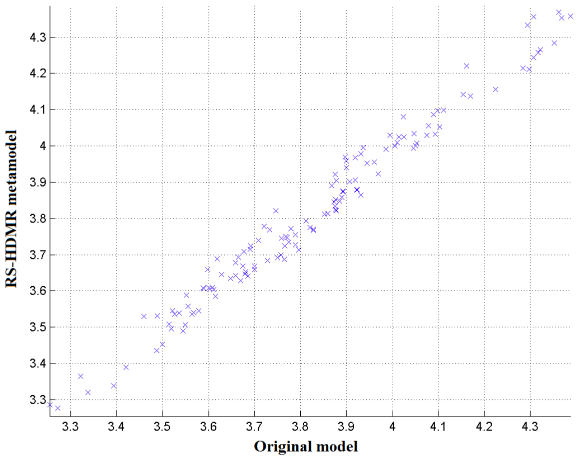


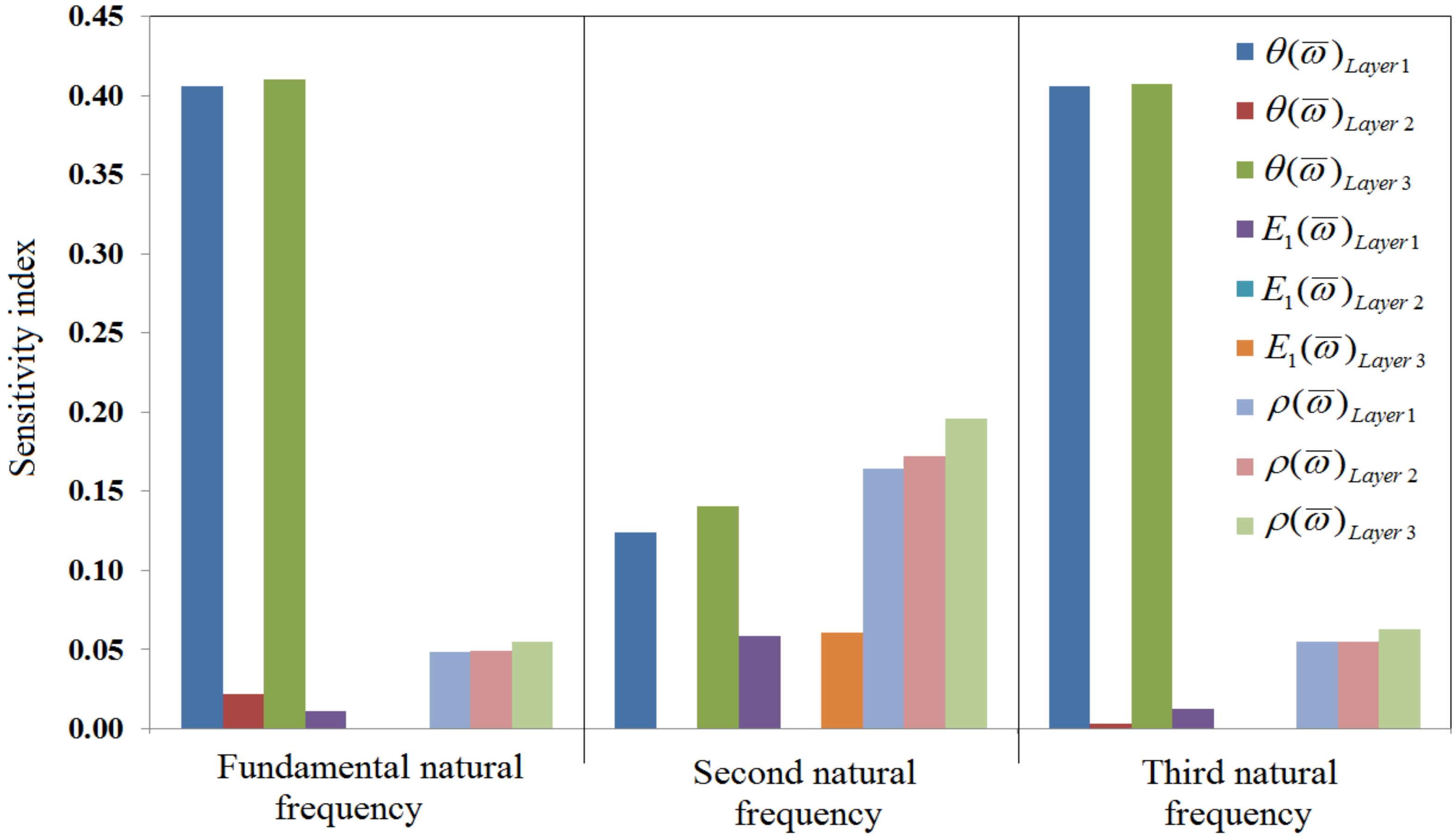
Table: Convergence study for coefficient of determination R^2 (second order) of the RS-HDMR expansions with different sample sizes for variation of only ply-orientation angle of graphite-epoxy angle-ply (45°/-45°/45°) composite cantilever plate, considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, t=0.004 m, v=0.3

Frequency	**Sample Size**				
	32	**64**	**128**	**256**	**512**
1st	**65.68**	**93.48**	**99.60**	**99.95**	**99.96**
2nd	**69.67**	**93.74**	**99.47**	**96.38**	**97.81**
3rd	**66.44**	**97.85**	**99.40**	**98.86**	**99.61**

Sensitivity Analysis

Swansea University
Prifysgol Abertawe

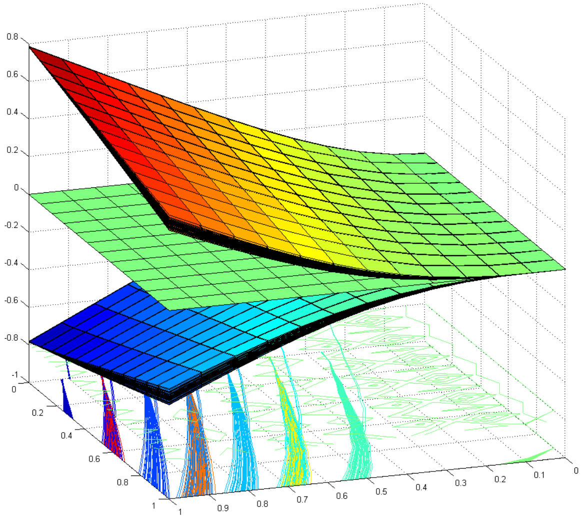
Figure: Sensitivity index for combined variation (10,000 samples) of ply-orientation angle, elastic modulus and mass density for graphite-epoxy angle-ply (45°/-45°/45°) composite cantilever plate, considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 Kg/m^3, t=0.004 m, v=0.3



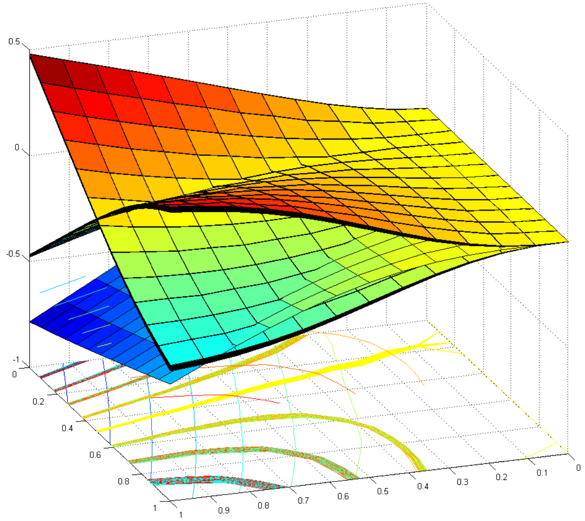
Modes Shapes : Random Sampling – High Dimensional Model Representation (RS-HDMR) Model

Swansea University
Prifysgol Abertawe

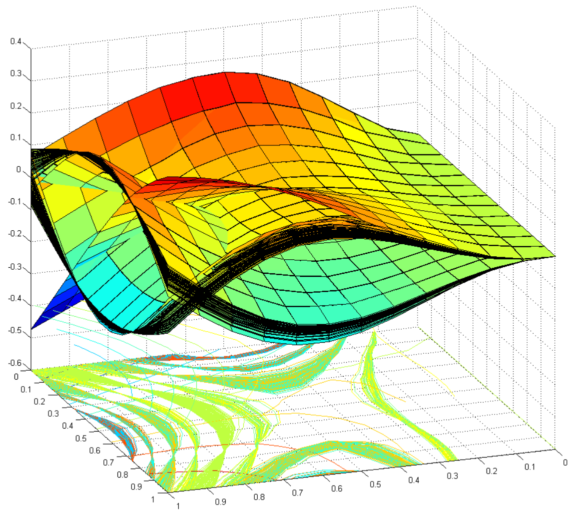
Figure: **Mode shapes by RS-HDMR** of first three modes due to combined stochasticity in ply-orientation angle, elastic modulus and mass density for three layered graphite-epoxy angle-ply (45°/-45°/45°) composite cantilever plate considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 Kg/m^3, t=0.004 m, v=0.3



Mode 1



Mode 2



Mode 3

Random Sampling – High Dimensional Model Representation (RS-HDMR) Model

Swansea University
Prifysgol Abertawe

Table Comparative study between MCS and RS-HDMR for maximum values, minimum values and percentage of difference for fundamental natural frequency obtained due to individual stochasticity in ply-orientation angle for graphite-epoxy angle-ply (45°/ - 45°/ 45°/ - 45°) composite cantilever plate considering E1=138 GPa, E2=8.9 GPa, G12=G13=7.1 GPa, G23=2.84 GPa, ρ=3202 Kg/m3, h=0.004 m, v=0.3.

Analysis	No of FE simulation	Max	Min	Mean	Standard Deviation
MCS	10,000	4.827352	3.828085	4.299911	0.178235
RS-HDMR	128	4.737744	3.838453	4.290139	0.174103
Difference (%)		1.86%	-0.27%	0.23%	2.32%

Swansea University
Prifysgol Abertawe

Polynomial Regression Model using D-Optimal Design

Polynomial Regression Model using D-Optimal Design

Swansea University
Prifysgol Abertawe

On the basis of statistical and mathematical analysis RSM gives an approximate equation which relates the input features x and output features y for a particular system.

$$y = f(x_1, x_2, \ldots, x_k) + \varepsilon$$

where ε is the statistical error term.

$Y = X\beta + \varepsilon$, X denotes the design matrix

where, $\beta = (X^T X)^{-1} X^T Y$

D-optimality is achieved if the determinant of $(X^T X)^{-1}$ is minimal

Check for quality of constructed model: R^2, R^2_{adj}, R^2_{pred}

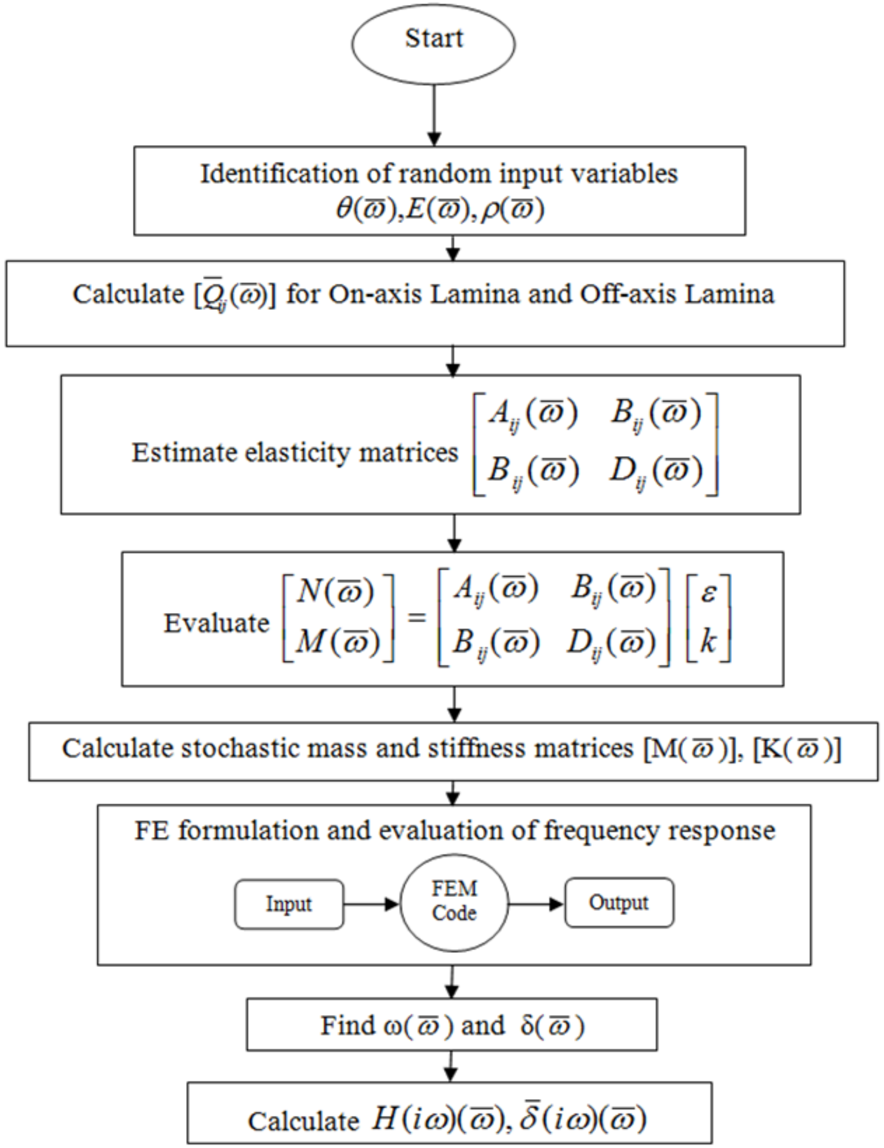
$$R^2 = \left(\frac{SS_R}{SS_T}\right) = 1 - \left(\frac{SS_E}{SS_T}\right) \qquad \text{where } 0 \le R^2 \le 1$$

$$R^2_{adj} = 1 - \frac{\left(\frac{SS_E}{n-k-1}\right)}{\left(\frac{SS_T}{n-1}\right)} = 1 - \left(\frac{n-1}{n-k-1}\right)\left(1 - R^2\right) \qquad \text{where } 0 \le R^2_{adj} \le 1$$

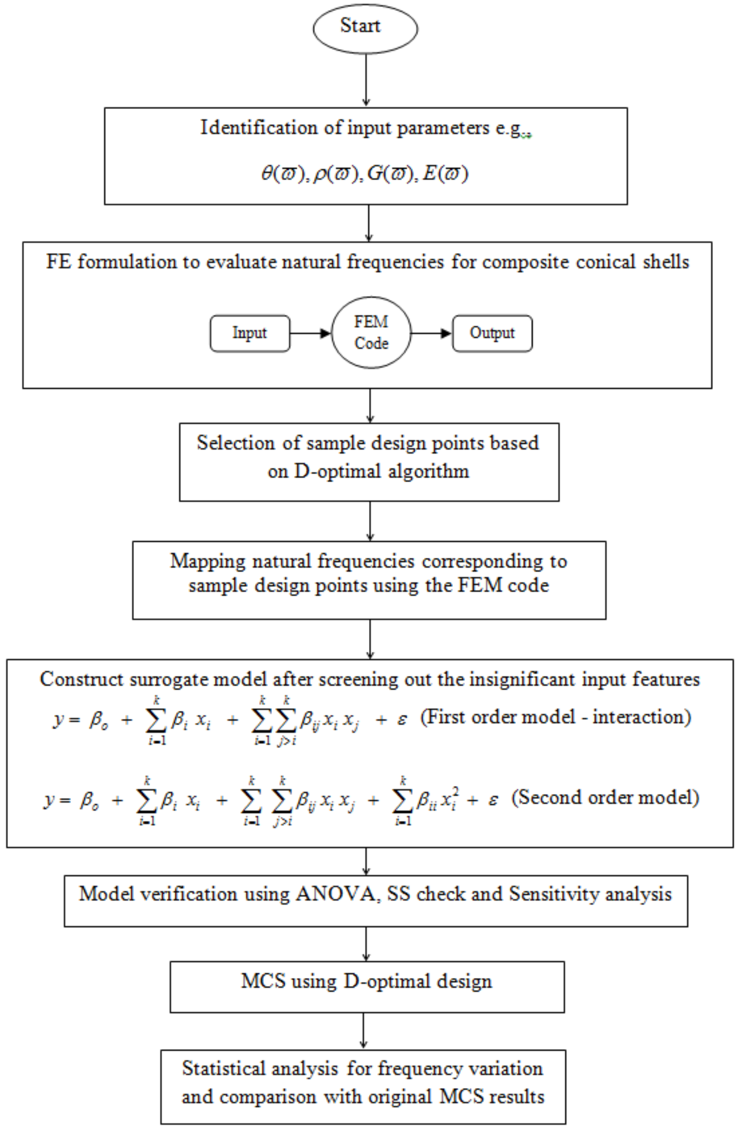
$$R^2_{pred} = 1 - \left(\frac{PRESS}{SS_T}\right) \qquad \text{where } 0 \le R^2_{pred} \le 1$$

Swansea University
Prifysgol Abertawe

MCS Model

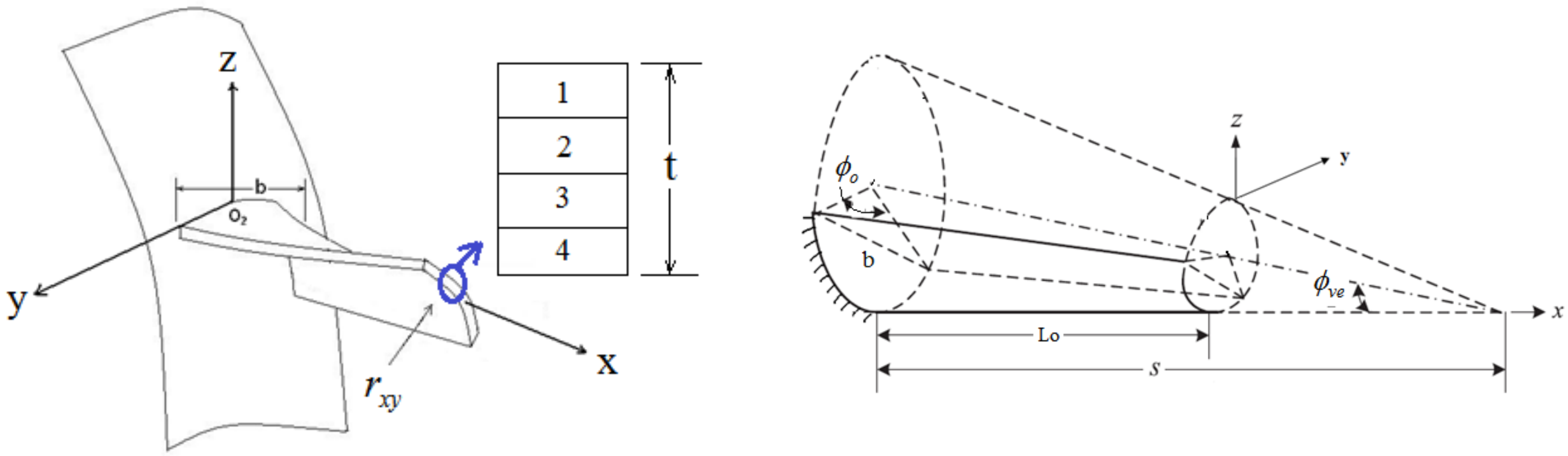


D-Optimal Design Model



Conical Shell Model

Swansea University
Prifysgol Abertawe



- Non-dimensional coordinate system given by

$$\xi = \frac{x}{L} \quad and \quad \eta = \frac{y}{b_o}$$

(where b=reference width)

- The varying radius of curvature $R_y(\xi,\eta) = \dfrac{\beta_o}{f(\xi,\eta)}$

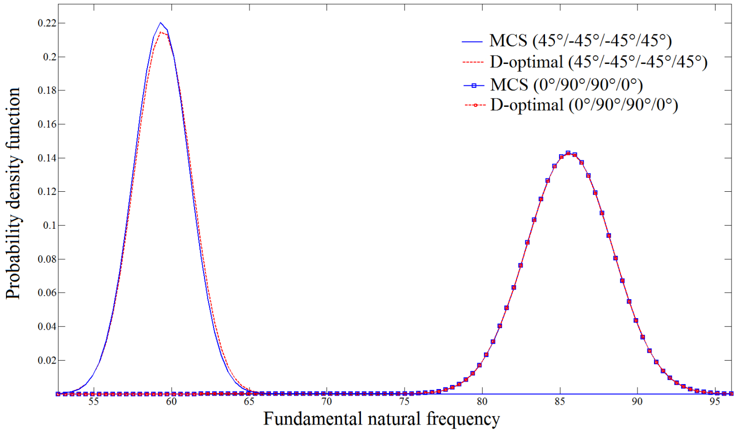
- The function expressed from the geometry of conical shell given by

$$f(\xi,\eta) = \tan(\phi_{ve}/2)\frac{s}{R_y(\xi,\eta)}$$

Validation – D-optimal

Swansea University
Prifysgol Abertawe

Figure: Probability density function obtained by original MCS and D-optimal design with respect to first three natural frequencies indicating for combined variation of mass density, longitudinal shear modulus, Transverse shear modulus and longitudinal elastic modulus for graphite-epoxy composite conical shells, considering sample size=10,000, E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.002 m, v=0.3, Lo/s=0.7, = 45º, = 20º



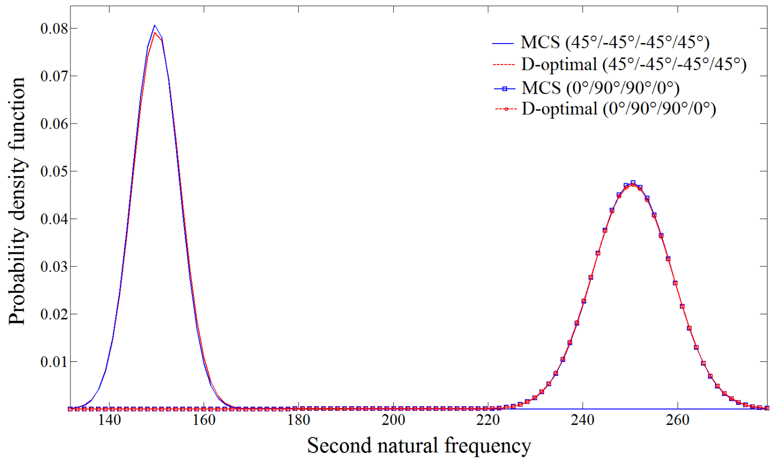
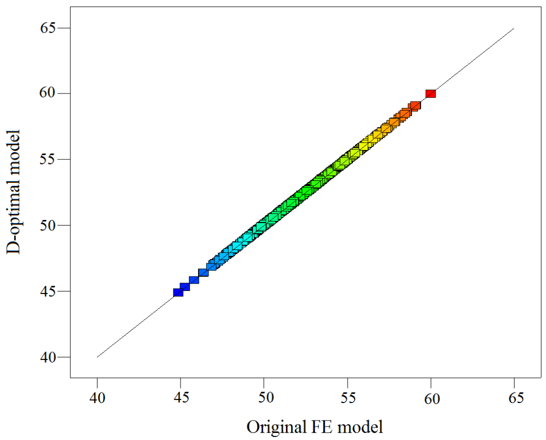
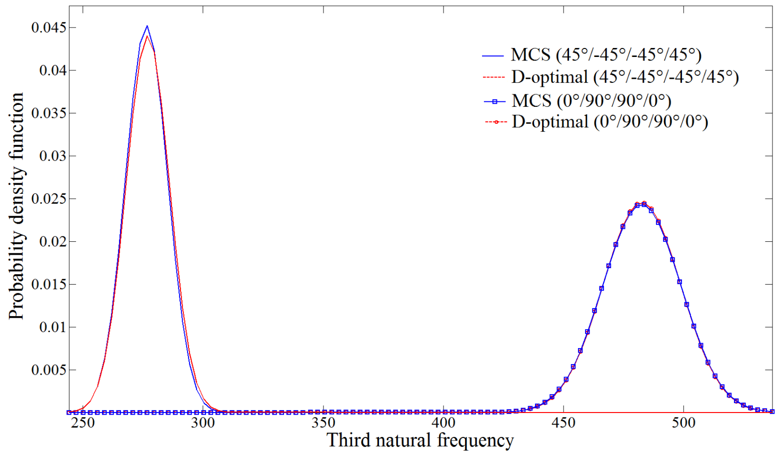


Figure: D-optimal design model with respect to original FE model of fundamental natural frequencies for variation of only ply-orientation angle of angle-ply (45°/-45°/-45°/45°) composite cantilever conical shells

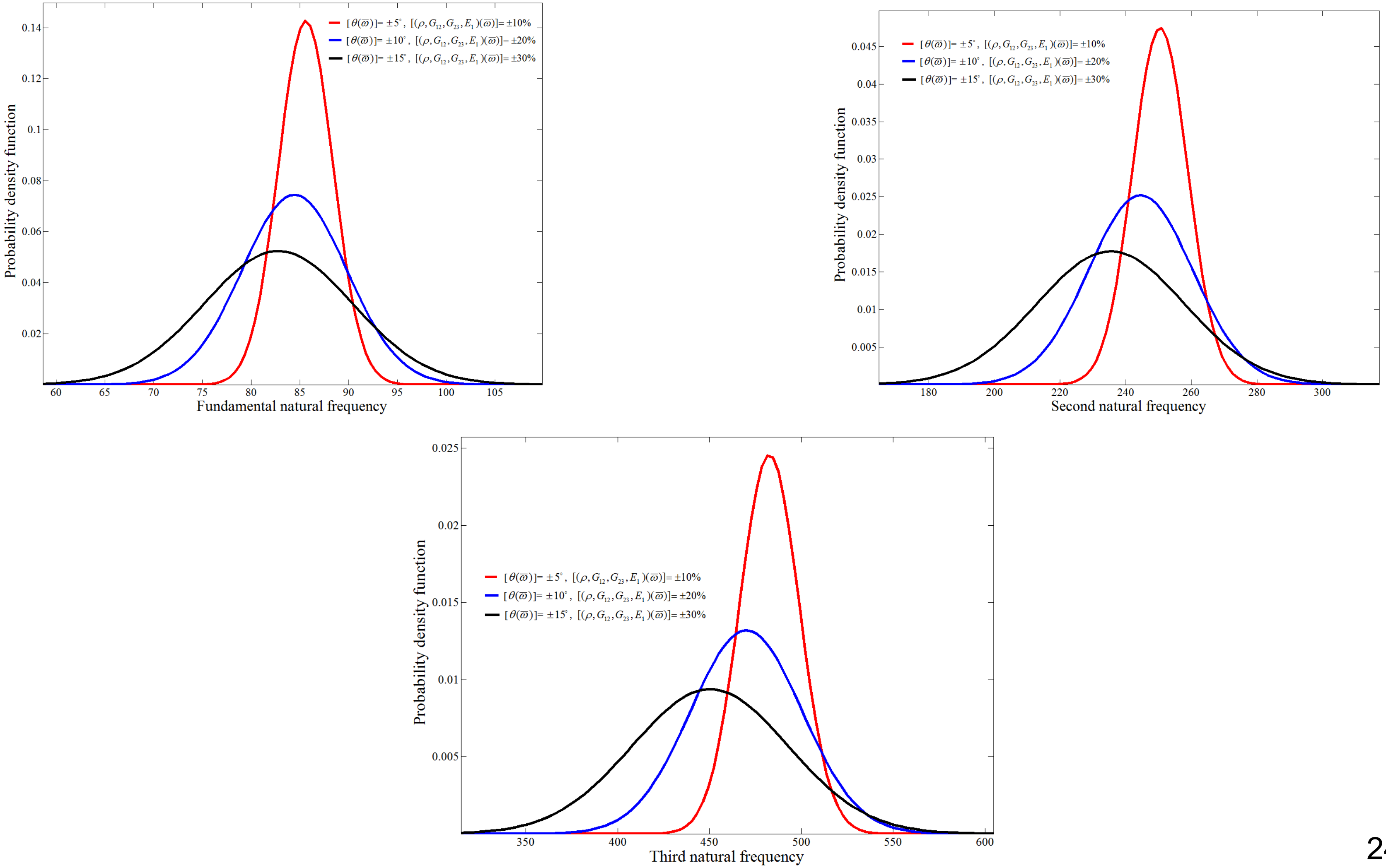




Combined Variation

Swansea University
Prifysgol Abertawe

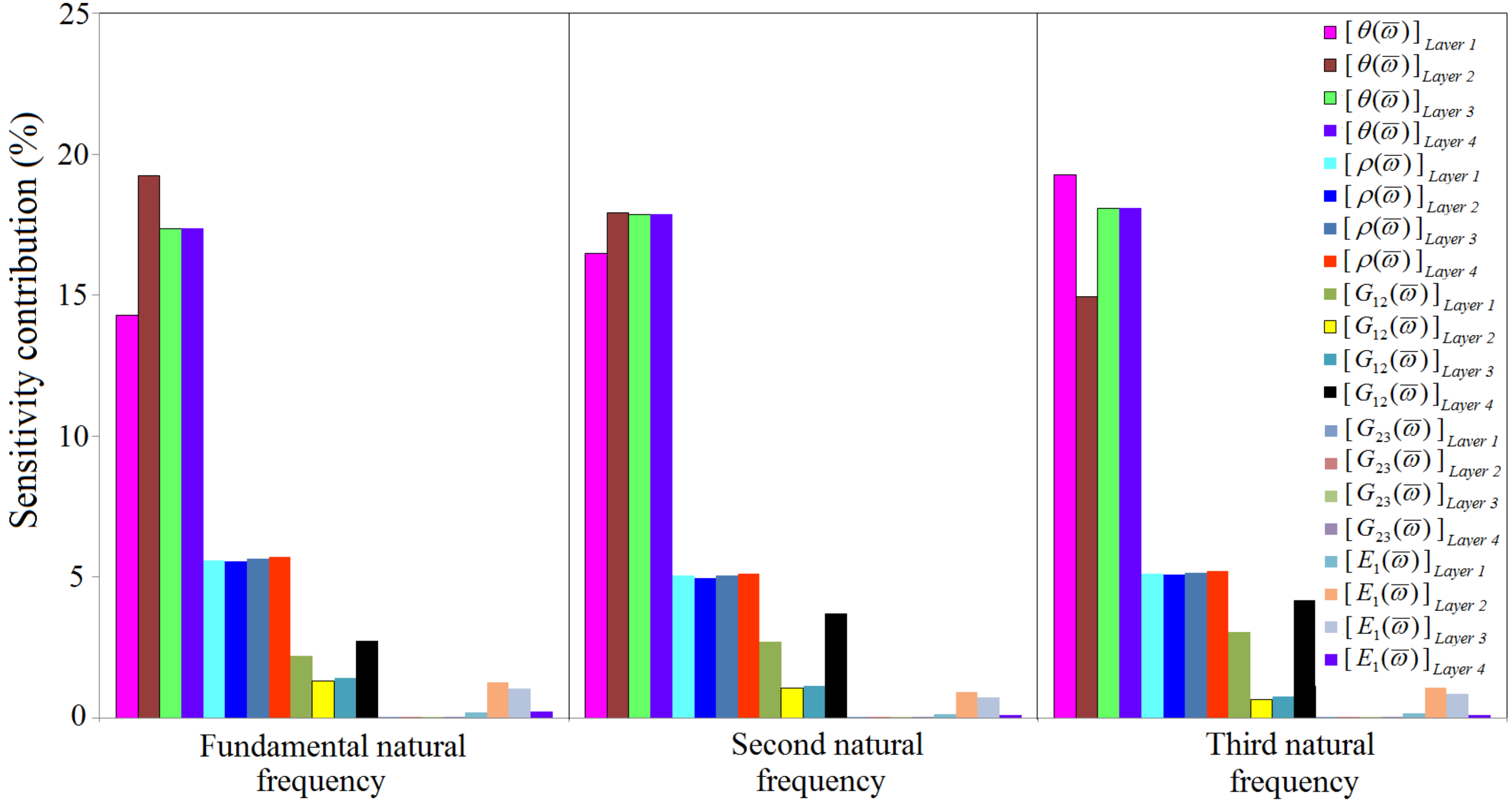
Figure: Probability density function with respect to first three natural frequencies due to combined variation for cross-ply (0°/ 90°/90°/0°) conical shells considering sample size=261, E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.002 m, v=0.3, Lo/s=0.7, = 45º, = 20º.



Sensitivity – Angle-ply

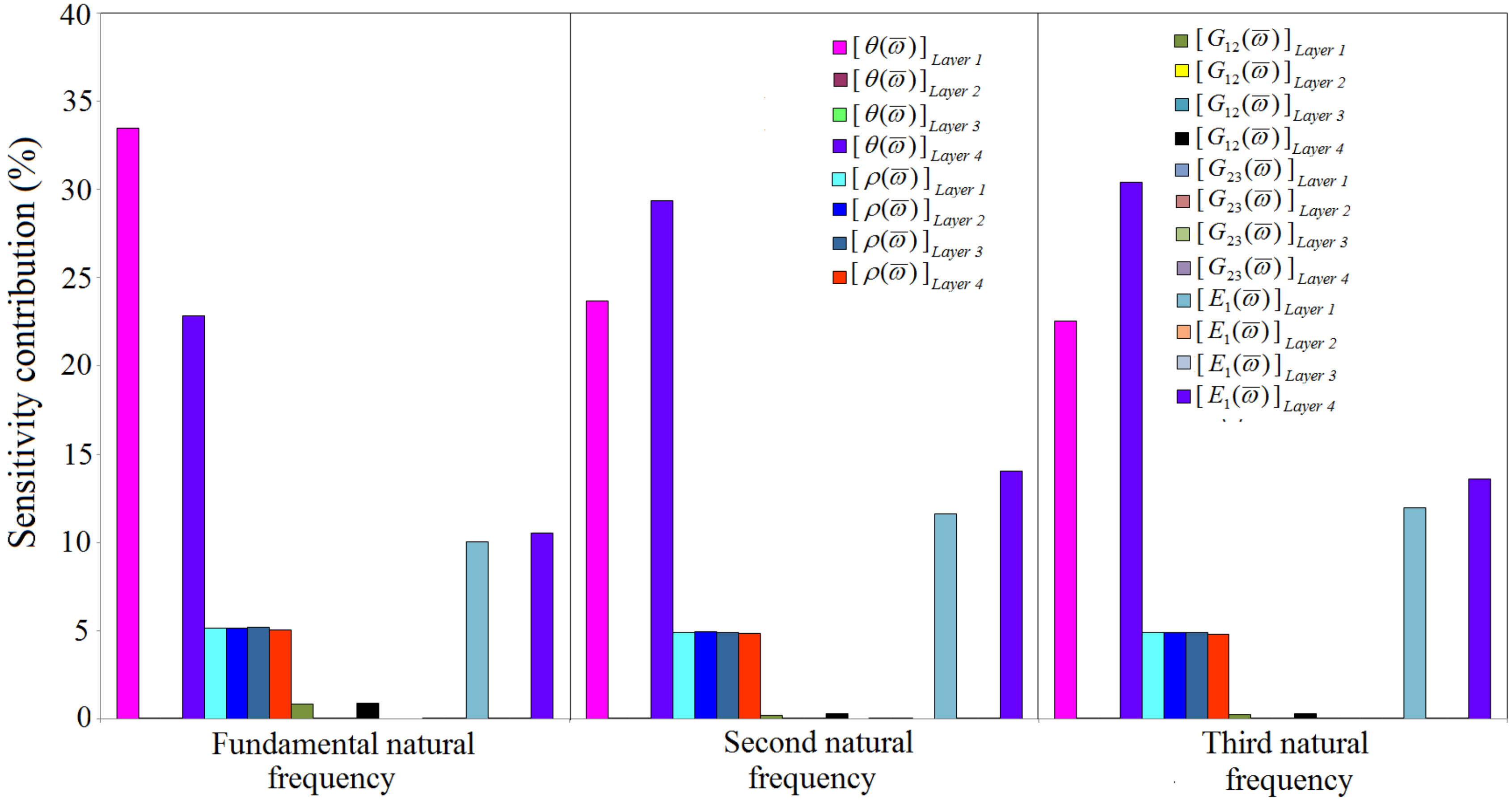
Swansea University
Prifysgol Abertawe

Sensitivity contribution in percentage for **combined variation in ply orientation angle, mass density, longitudinal shear modulus, Transverse shear modulus and longitudinal elastic modulus** for four layered graphite-epoxy **angle-ply (45°/-45°/-45°/45°)** composite conical shells, considering sample size=261, E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.002 m, ν=0.3, Lo/s=0.7, = 45º, = 20º



Sensitivity – Cross-ply

Sensitivity contribution in percentage for **combined variation in ply orientation angle, mass density, longitudinal shear modulus, Transverse shear modulus and longitudinal elastic modulus** for four layered graphite-epoxy **cross-ply (0°/90°/90°/0°)** composite conical shells, considering sample size=261, E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.002 m, ν=0.3, Lo/s=0.7, = 45º, = 20º



Polynomial Regression Model using D-Optimal Design

Swansea University Prifysgol Abertawe

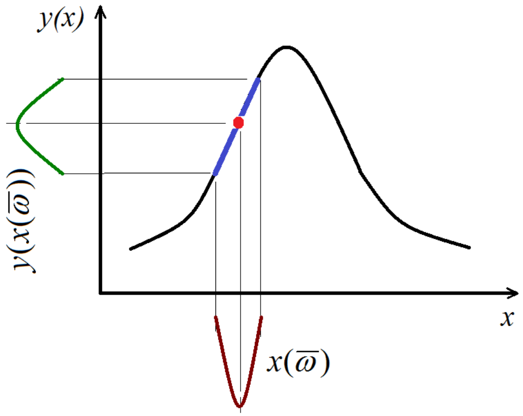
Table Comparative study between MCS and RS-HDMR for maximum values, minimum values and percentage of difference
for fundamental natural frequency obtained due to individual stochasticity in ply-orientation angle for graphite-epoxy angle-ply (45°/ - 45°/ 45°/ - 45°) composite cantilever conical shells considering E1=138 GPa, E2=8.9 GPa, G12=G13=7.1 GPa, G23=2.84 GPa, ρ=3202 Kg/m3, h=0.004 m, ν=0.3.

Analysis	No of FE simulation	Max	Min	Mean	Standard Deviation
MCS	10,000	41.06	37.01	39.07	0.68
Polynomial Regression by D-Optimal Design	32	41.19	36.98	39.08	0.67
Difference (%)		-0.32%	0.08%	-0.03%	1.47%

Swansea University
Prifysgol Abertawe

Kriging Model

Kriging Model



- Kriging model for simulation of required output $$y(x) = y_0(x) + Z(x)$$

Where *y(x)* is the unknown function of interest, *x* is an *m* dimensional vector (*m* design variables), $y_0(x)$ is the known approximation (usually polynomial) function and *Z(x)* represents is the realization of a stochastic process with mean zero, variance, and nonzero covariance.

- Kriging predictor: $\hat{y}(x) = \hat{\beta} + r^T(x) \; R^{-1} \; [y - f\hat{\beta}]$

$$\hat{\beta} = (f^T R^{-1} f)^{-1} \; f^T R^{-1} \; y$$

where *y* is the column vector of length *p* that contains the sample values of the frequency responses and *f* is a column vector of length *p* that is filled with ones when $y_0(x)$ is taken as constant. $r^T(x)$ is the correlation vector of length *p* between the random *x* and the sample data points $\{x^1, x^2, \ldots\ldots\ldots\ldots x^p\}$ and R is correlation matrix.

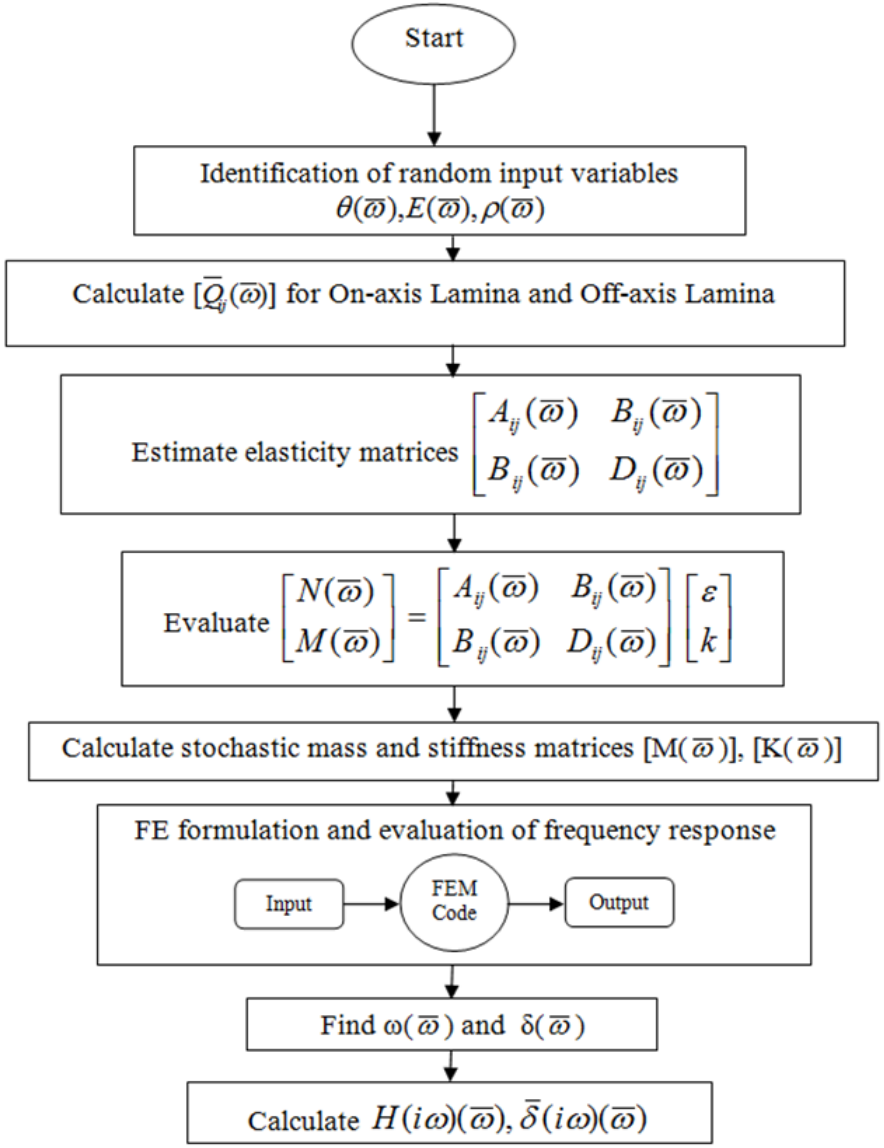
- **Check for maximum error (ME) and maximum mean square error (MMSE):**

$$ME\ (\%) = Max\left[\frac{y_{i,MCS} - y_{i,Kriging}}{Y_{i,MCS}}\right] \qquad MMSE = Max\left[\frac{1}{k}\sum_{i=1}^{k}(\bar{y}_i - y_i)^2\right]$$

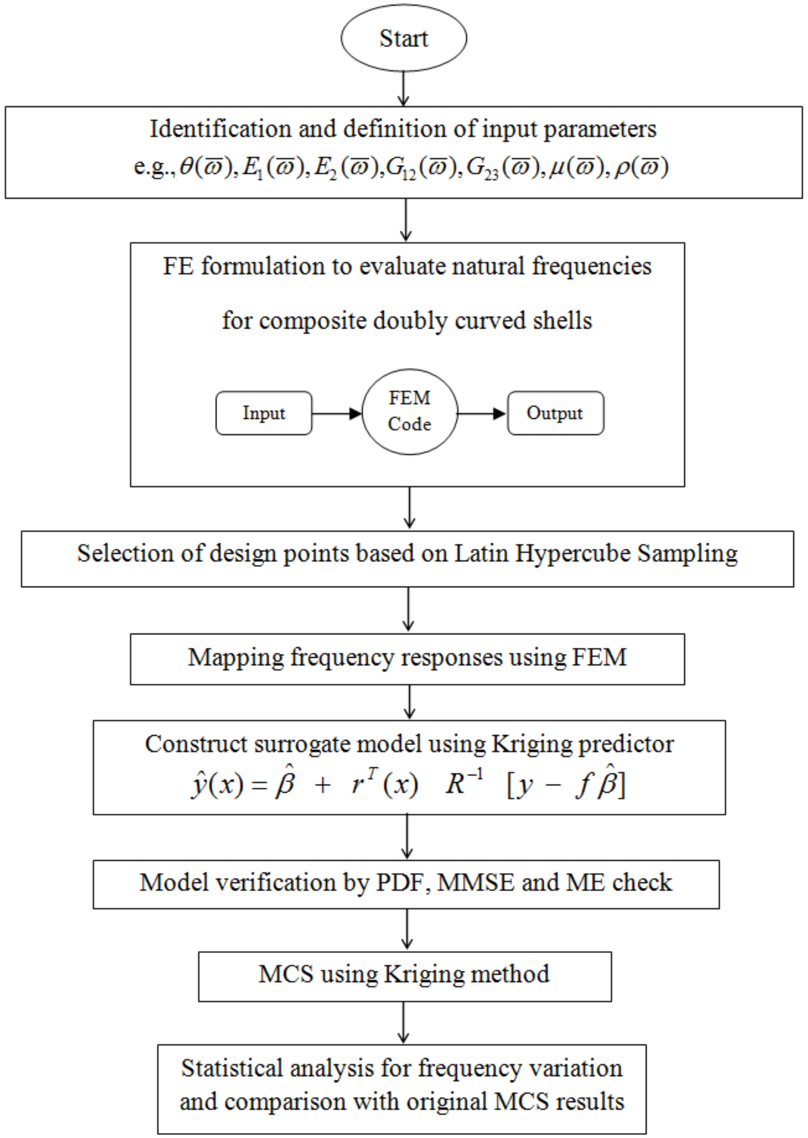
where y_i and $\bar{y}_i$ are the vector of the true values and the vector corresponding to *i*-th prediction, respectively.

Swansea University
Prifysgol Abertawe

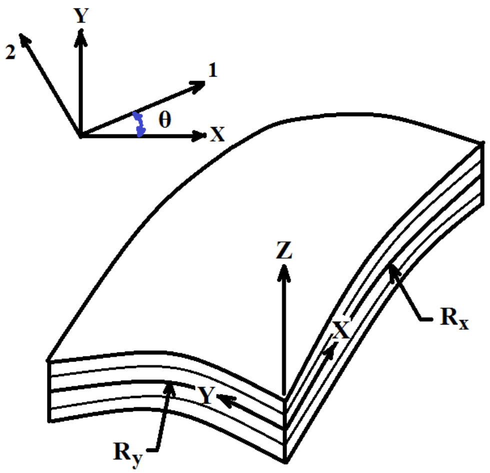
MCS Model

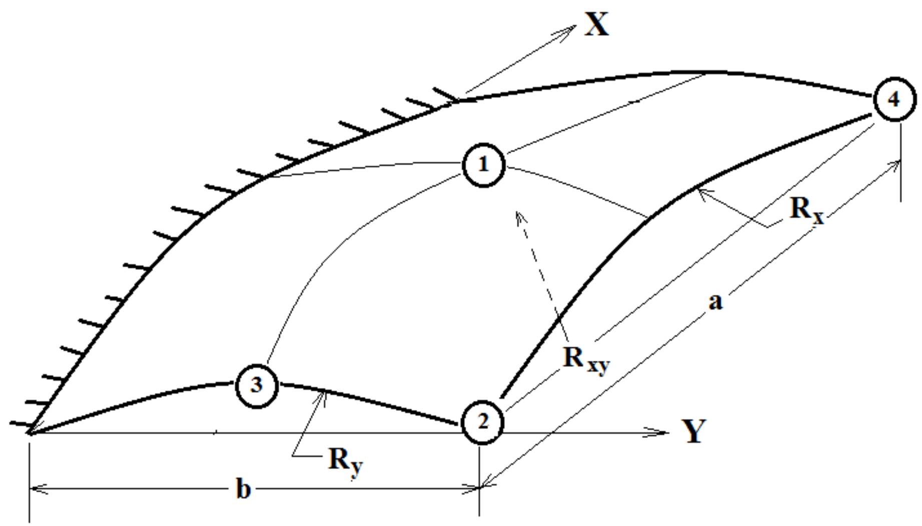


Kriging Model



Doubly Curved Composite Shell Model





- Spherical shell : Rx = Ry = R and Rxy=∞,
- Hyperbolic paraboloid shell : Rx = - Ry = R and Rxy=∞,
- Elliptic paraboloid shell : Rx ≠ Ry and Rxy=∞

Validation – Kriging Model

Swansea University Prifysgol Abertawe

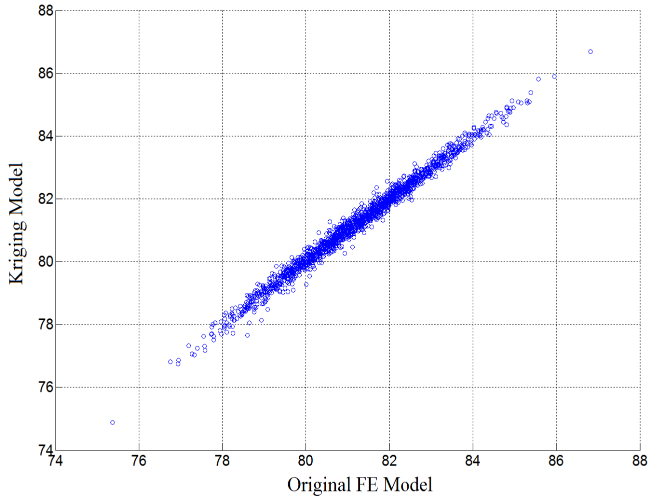
Table: Non-dimensional fundamental frequencies [$\omega=\omega_n a^2 \sqrt{(12 \rho (1-\mu^2) / E_1 t^2]}$ of isotropic, corner point-supported spherical and hyperbolic paraboloidal shells considering a/b=1, a′/a=1, a/t = 100, a/R = 0.5, μ = 0.3

R_x/R_y	Shell Type	Present FEM	Leissa and Narita [48]	Chakravorty et al. [39]
1	Spherical	50.74	50.68	50.76
-1	Hyperbolic paraboloid	17.22	17.16	17.25

Table: Convergence study for maximum mean square error (MMSE) and maximum error (in percentage) using Kriging model compared to original MCS with different sample sizes for combined variation of 28 nos. input parameters of graphite-epoxy angle-ply (45°/-45°/-45°/45°) composite cantilever spherical shells, considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, t=0.005 m, μ=0.3

Sample size	Parameter	Fundamental frequency	Second natural frequency	Third natural frequency
450	MMSE	0.0289	0.1968	0.2312
	Max Error (%)	2.4804	7.6361	6.5505
500	MMSE	0.0178	0.1466	0.2320
	Max Error (%)	1.6045	2.6552	3.0361
550	MMSE	0.0213	0.1460	0.2400
	Max Error (%)	1.2345	2.0287	1.8922
575	MMSE	0.0207	0.1233	0.2262
	Max Error (%)	1.1470	1.8461	1.7785
600	MMSE	0.0177	0.1035	0.2071
	Max Error (%)	1.1360	1.7208	1.7820
625	MMSE	0.0158	0.0986	0.1801
	Max Error (%)	1.0530	1.7301	1.6121
650	MMSE	0.0153	0.0966	0.1755
	Max Error (%)	0.9965	1.8332	1.6475

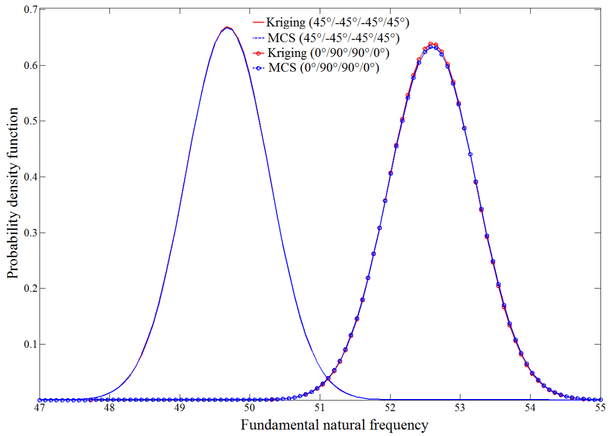
Figure: Scatter plot for Kriging model for combined variation of ply orientation angle, longitudinal elastic modulus, transverse elastic modulus, longitudinal shear modulus, Transverse shear modulus, Poisson's ratio and mass density for composite cantilevered spherical shells

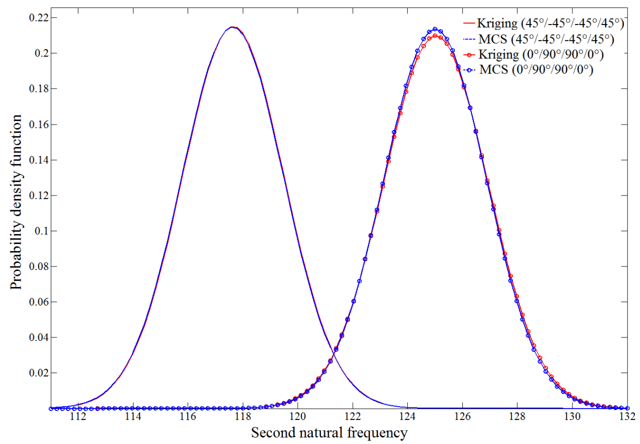


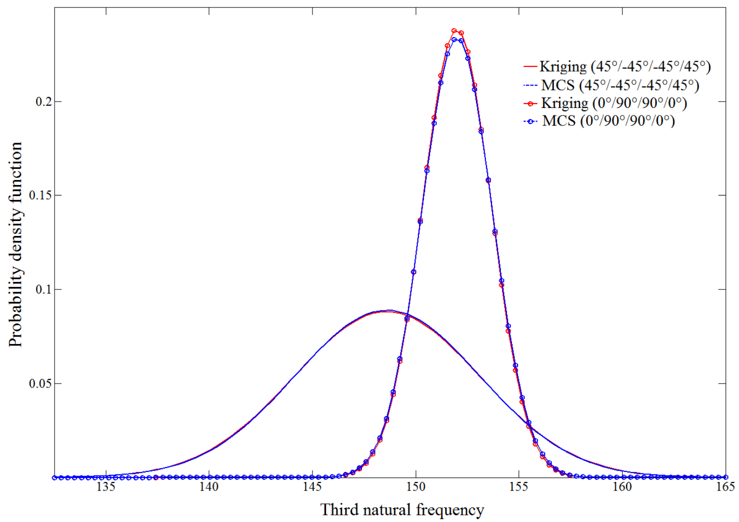
Individual variation : Kriging Model

Swansea University
Prifysgol Abertawe

Figure: Probability density function obtained by original MCS and Kriging model with respect to first three natural frequencies for **individual variation of ply orientation angle** for composite elliptical paraboloid shells, considering sample size=10,000, R_x R_y, R_{xy}=α, E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.005 m, μ=0.3



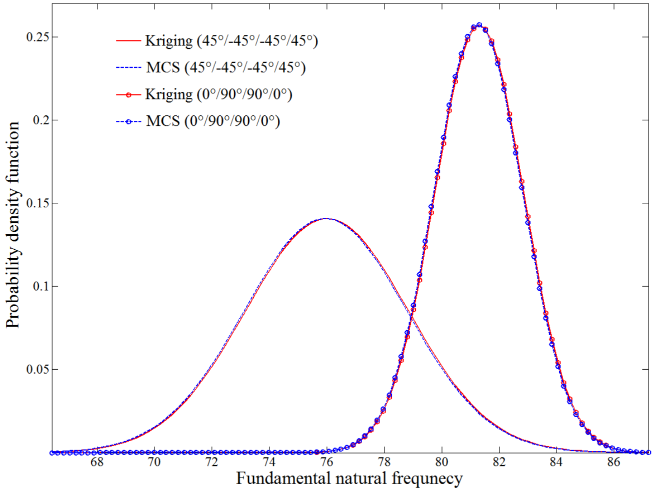


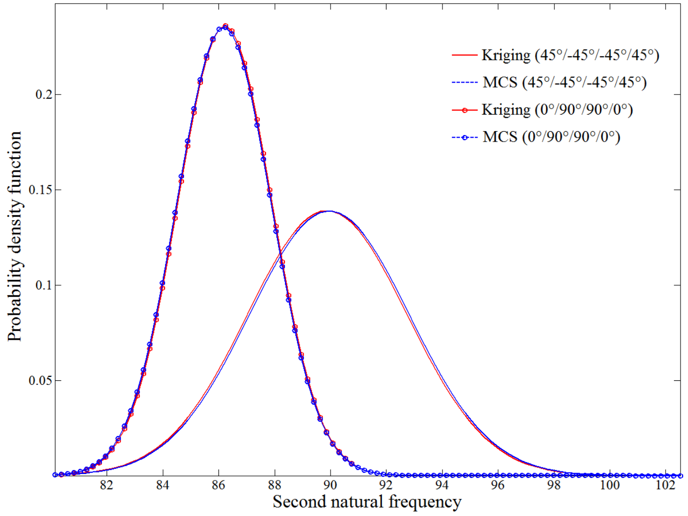


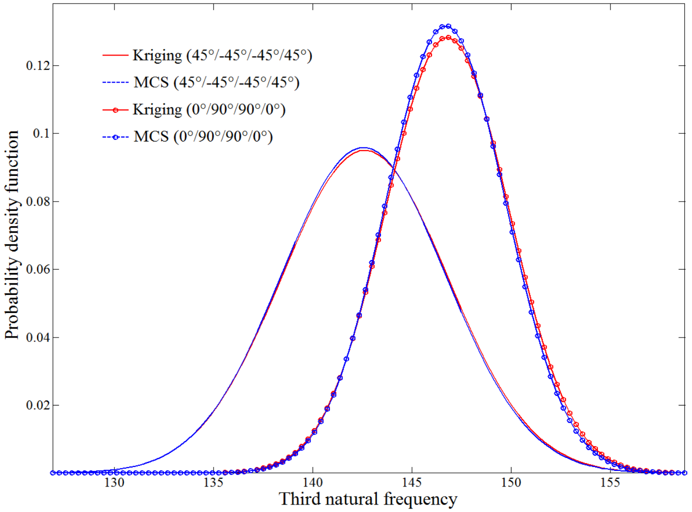
Combined Variation : Kriging Model

Swansea University Prifysgol Abertawe

Figure: Probability density function obtained by original MCS and Kriging model with respect to first three natural frequencies for **combined variation of ply orientation angle, elastic modulus (longitudinal and transverse), shear modulus (longitudinal and transverse), poisson's ratio and mass density** for composite elliptical paraboloid shells, considering sample size=10,000, R_x R_y, R_{xy}=α, E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.005 m, μ=0.3



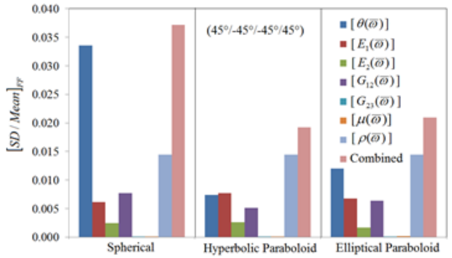




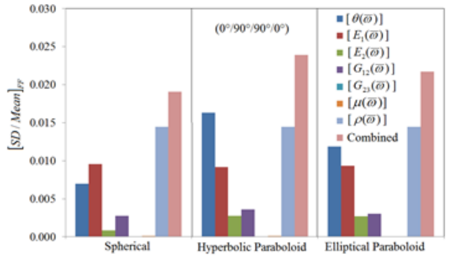
Comparative Sensitivity – Angle-ply Vs Cross-ply

Swansea University
Prifysgol Abertawe

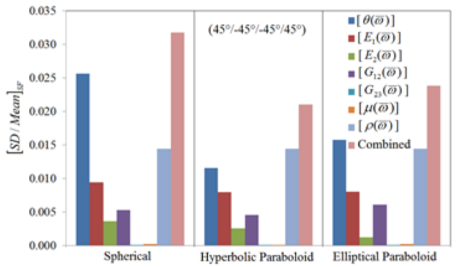
Figure: [SD/Mean] of first three natural frequencies for individual variation of input parameters and combined variation for angle-ply (45°/-45°/-45°/45°) and cross-ply (0°/90°/90°/0°) composite shallow doubly curved shells (spherical, hyperboilic paraboloid and elliptical paraboloid), considering deterministic values as E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.005 m, μ=0.3



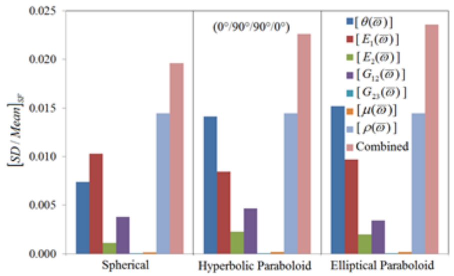
(a)



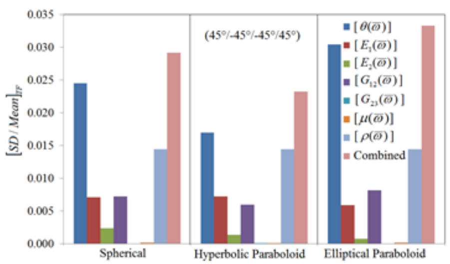
(b)



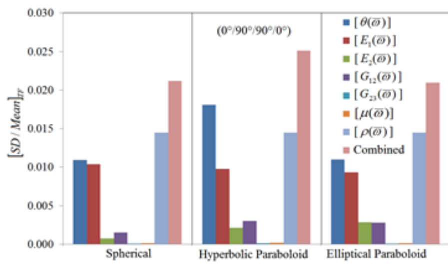
(c)



(d)



(e)

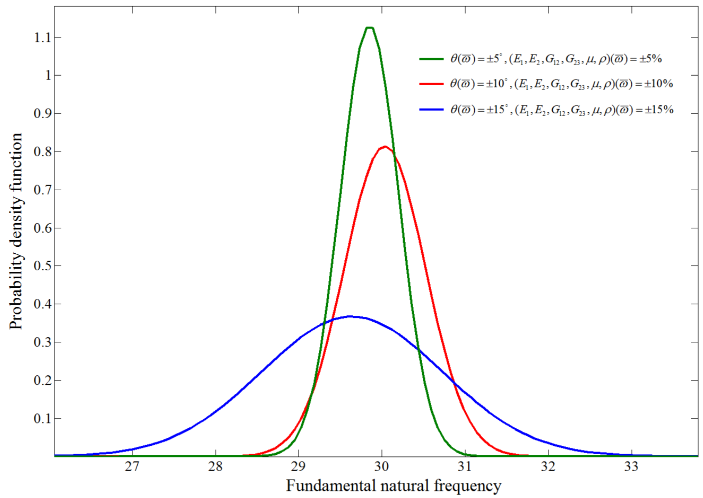


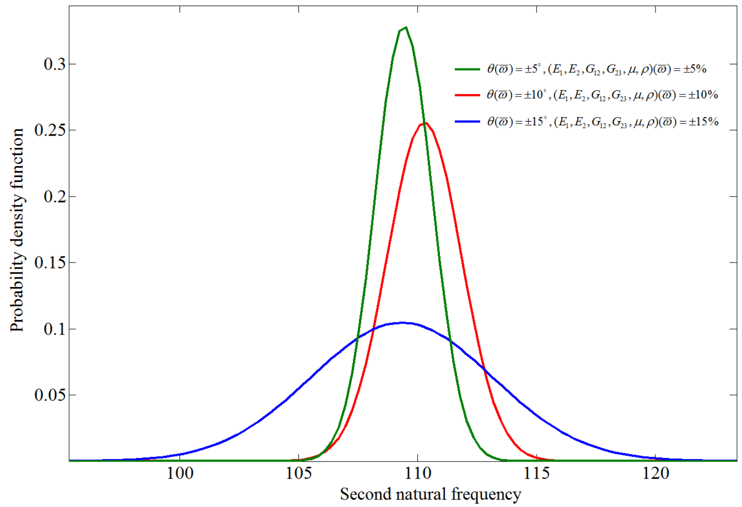
(f)

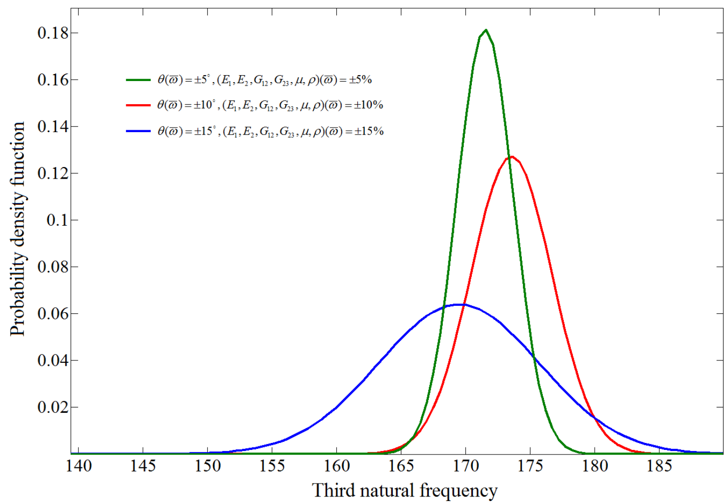
Combined Variation - Kriging Model

Swansea University
Prifysgol Abertawe

Probability density function with respect to first three natural frequencies with different combined variation for cross-ply (0°/90°/90°/0°) composite hyperbolic paraboloid shallow doubly curved shells considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 kg/m^3, t=0.005 m, μ=0.3

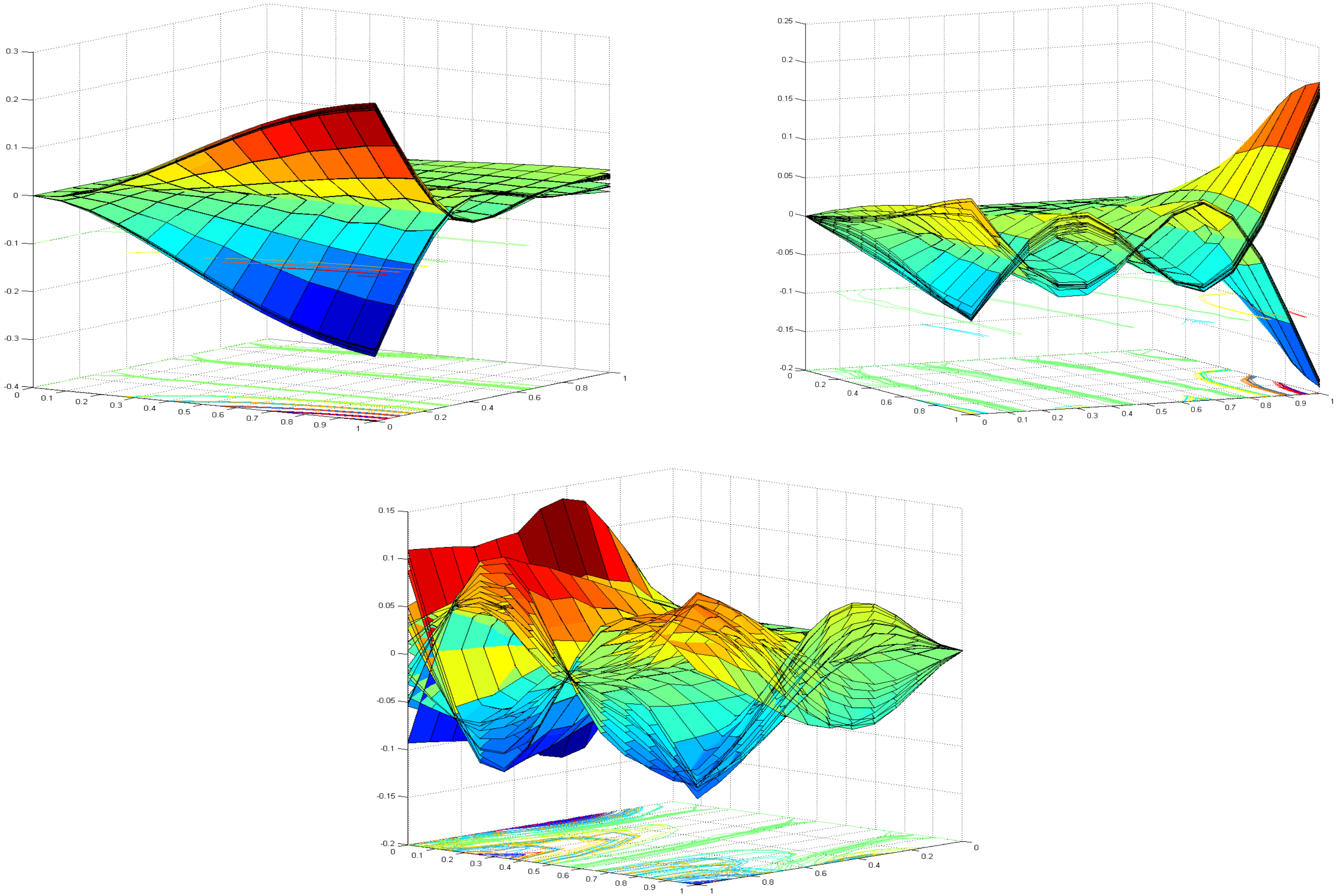






Mode Shapes - Kriging Model

Figure: Mode shapes by Kriging model of first three modes due to combined stochaticity for four layered angle-ply (45°/-45°/-45°/45°) composite cantilever elliptical paraboloid shells considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 Kg/m^3, t=0.005 m, μ=0.3

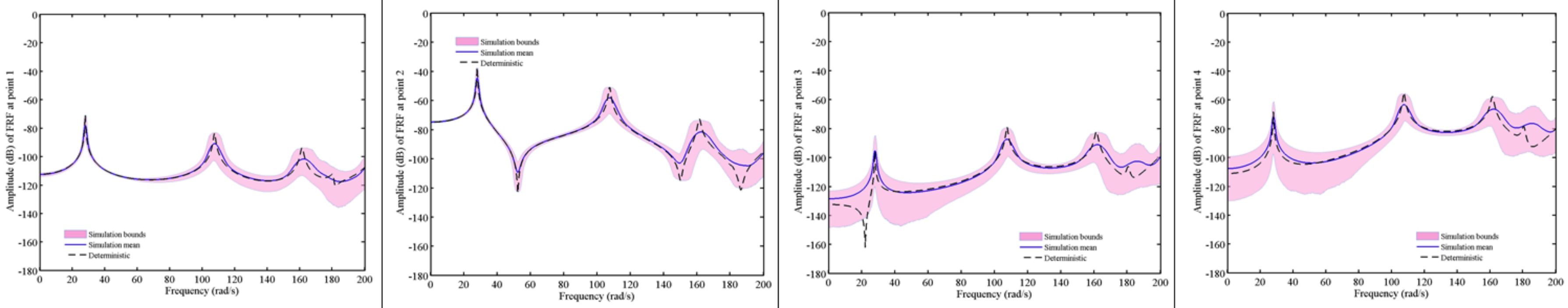


Frequency Response Function - Kriging Model

Swansea University
Prifysgol Abertawe

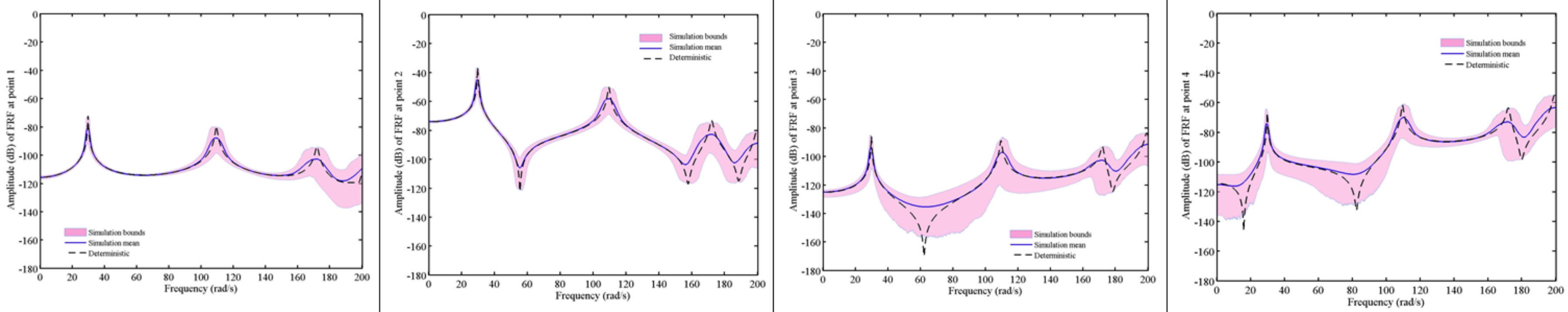
Figure: Frequency response function (FRF) plot of simulation bounds, simulation mean and deterministic mean for combined stochasticity with four layered graphite epoxy composite cantilever elliptical paraboloid shells considering E_1=138 GPa, E_2=8.9 GPa, G_{12}=G_{13}=7.1 GPa, G_{23}=2.84 GPa, ρ=3202 Kg/m^3, t=0.005 m, μ=0.3

Angle-ply (45°/-45°/-45°/45°)



(a) (b) (c) (d)

Cross-ply (0°/90°/90°/0°)



(e) (f) (g) (h)

Kriging Model

Swansea University
Prifysgol Abertawe

Table Comparative study between MCS and Kriging for maximum values, minimum values and percentage of difference for fundamental natural frequency obtained due to individual stochasticity in ply-orientation angle for graphite-epoxy angle-ply (45°/ - 45°/ 45°/ - 45°) composite cantilever Hyperbolic Paraboloid Shells considering E1=138 GPa, E2=8.9 GPa, G12=G13=7.1 GPa, G23=2.84 GPa, ρ=3202 Kg/m3, h=0.004 m, v=0.3.

Analysis	No of FE simulation	Max	Min	Mean	Standard Deviation
MCS	10,000	29.18	27.66	28.33	0.22
Kriging Model	64	29.28	27.79	28.40	0.21
Difference (%)		-0.34%	-0.47%	-0.25%	4.55%

Comparative Study: All three approaches

Swansea University
Prifysgol Abertawe

Number of input parameter = 4

Method	Number of samples	Normalised Iteration time
MCS	10,000	1
RS-HDMR Model (Plate)	128	1/80
Polynomial Regression Model using D-Optimal Design (Conical Shell)	**32**	**(1/312)**
Kriging Model (Hyperbolic Paraboloid Shell)	64	(1/156)

Polynomial Regression Model with D-Optimal Design is found comparatively efficient and cost-effective.

Comparative Study for layerwise stochasticity in Ply orientation angle (45°/ - 45°/ 45°/ - 45°)

Swansea University
Prifysgol Abertawe

Method	Max	Min	Mean	SD
MCS	**4.82**	**3.82**	**4.30**	**0.18**
RS-HDMR Model (Plate)	**4.74**	**3.83**	**4.29**	**0.17**
Difference (%)	**1.66%**	**-0.26%**	**0.23%**	**5.56%**
MCS	**41.06**	**37.01**	**39.07**	**0.68**
Polynomial Regression using D-Optimal Design (Conical Shell)	**41.19**	**36.98**	**39.08**	**0.67**
Difference (%)	**-0.32%**	**0.08%**	**-0.03%**	**1.47%**
MCS	**29.18**	**27.66**	**28.33**	**0.22**
Kriging Model (Hyperbolic Paraboloid Shell)	**29.28**	**27.79**	**28.40**	**0.21**
Difference (%)	**-0.34%**	**-0.47%**	**-0.25%**	**4.55%**

Polynomial Regression Model with D-Optimal Design is found comparatively more accurate with MCS.

Conclusions

Swansea University
Prifysgol Abertawe

- Three approaches to investigate the effect of random variation in input parameters on the dynamics of laminated composite plates / shells were discussed: (1) Random Sampling - High Dimensional Model Representation (RS-HDMR), (2) Polynomial Regression Model using D-Optimal design, and (3) Kriging

- The focus has been of the efficient generation of a surrogate model with limited use of the computational intensive finite element computations.

- The developed uncertainty propagation approaches are validated against results from direct Monte Carlo Simulations.

- Polynomial Regression Model using D-Optimal Design turns out to be most efficient for this problem (natural frequency statistics)

- The techniques have been integrated with ANSYS general purpose finite element software to solve complex problems.