Reliability and Uncertainty in Structural Dynamics

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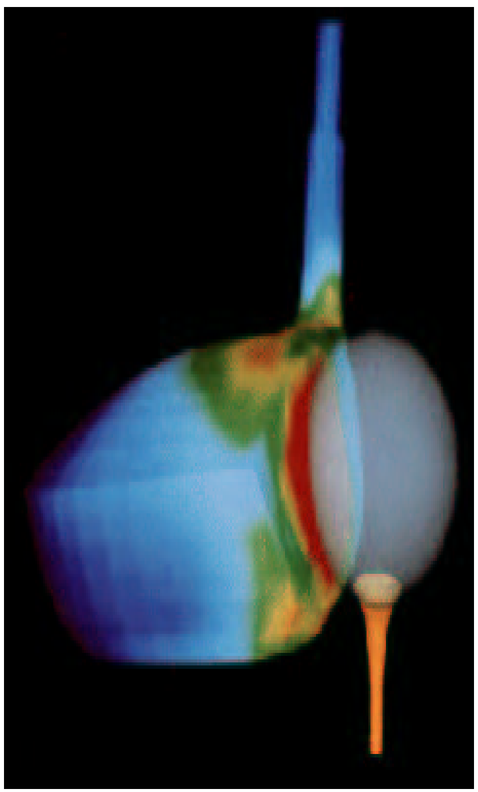
Outline of the Talk

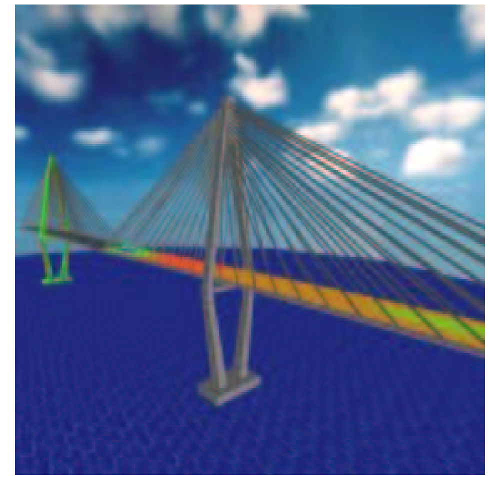
- Introduction: Research Interests
- Structural Reliability Analysis
- Reliability Analysis for Dynamics
- Conclusions

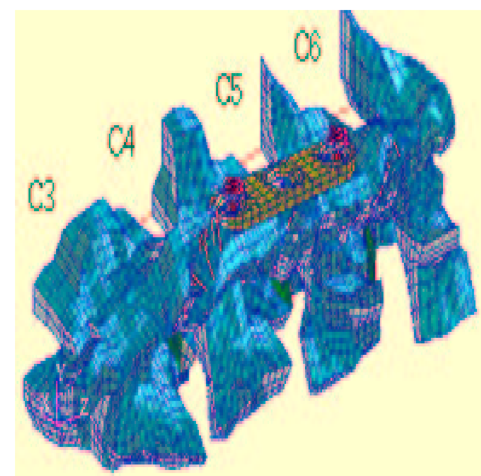
Research Areas

1. Identification of damping in vibrating structures
2. Deterministic and stochastic structural dynamics
3. Sensitivity analysis of damped structures
4. Statistical Energy Analysis (SEA)
5. Structural reliability analysis

Structural Reliability Analysis







The Fundamental Problem

Probability of failure:

$$P_f = \int_{G(\mathbf{y}) \leq 0} p(\mathbf{y}) d\mathbf{y} \tag{1}$$

- $\mathbf{y} \in \mathbb{R}^n$: vector describing the uncertainties in the structural parameters and applied loadings.
- $p(\mathbf{y})$: joint probability density function of $\mathbf{y}$
- $G(\mathbf{y})$: failure surface/limit-state function/safety margin/

Main Difficulties

- n is large
- $p(\mathbf{y})$ is non-Gaussian
- P_f is usually very small (in the order of 10^{-4} or smaller)
- $G(\mathbf{y})$ is a complicated nonlinear function of $\mathbf{y}$

Approximate Reliability Analyses

First-Order Reliability Method (FORM):

- Requires the random variables $\mathbf{y}$ to be Gaussian.
- Approximates the failure surface by a hyperplane.

Second-Order Reliability Method (SORM):

- Requires the random variables $\mathbf{y}$ to be Gaussian.
- Approximates the failure surface by a quadratic hypersurface.

FORM

- Original non-Gaussian random variables $\mathbf{y}$ are transformed to standardized gaussian random variables $\mathbf{x}$. This transforms $G(\mathbf{y})$ to $g(\mathbf{x})$.
- The probability of failure is given by

$$P_f = \Phi(-\beta) \quad \text{with} \quad \beta = (\mathbf{x}^{*^T}\mathbf{x}^*)^{1/2} \tag{2}$$

where $\mathbf{x}^*$, the design point is the solution of

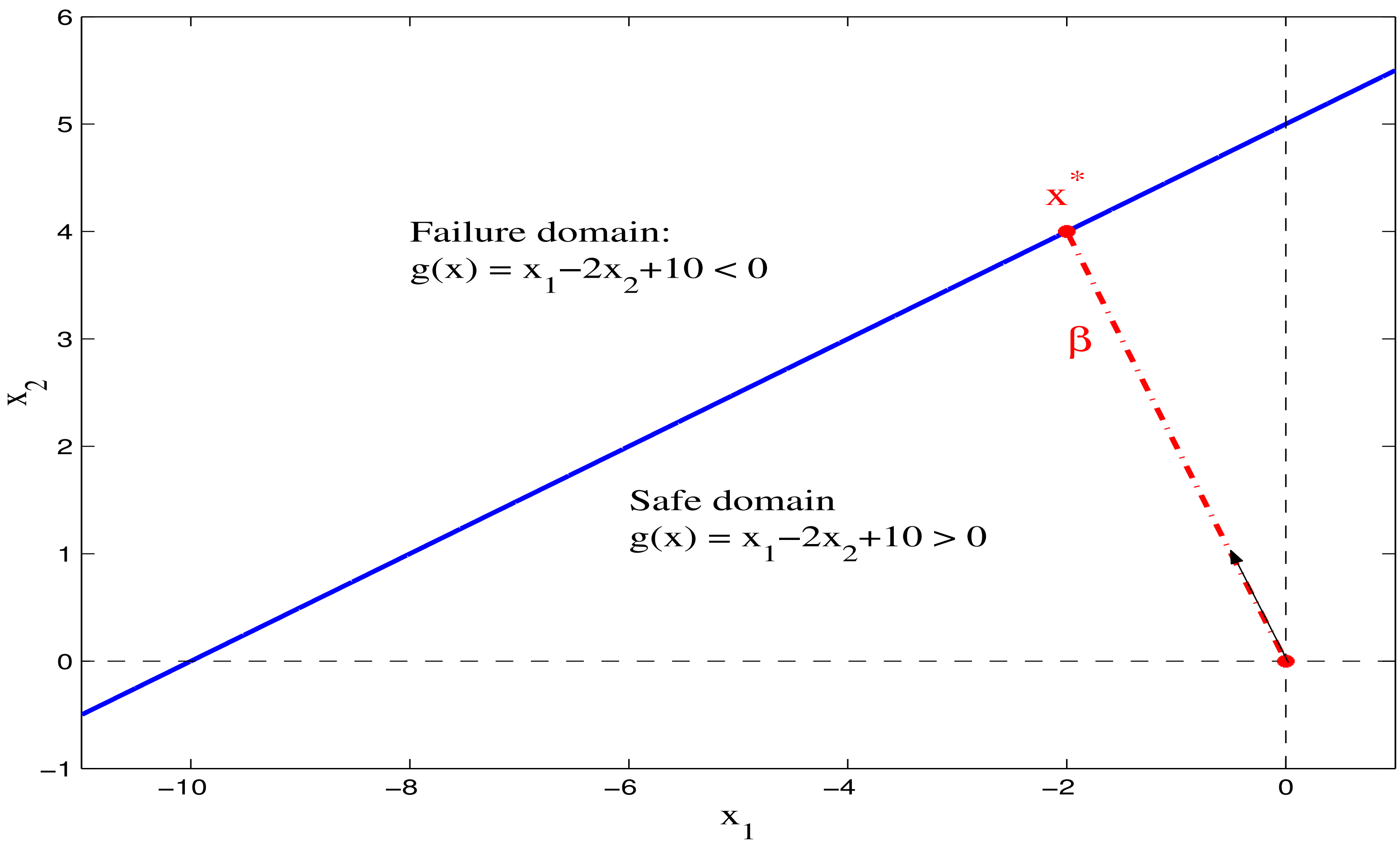
$$\min \left\{ (\mathbf{x}^T\mathbf{x})^{1/2} \right\} \quad \text{subject to} \quad g(\mathbf{x}) = 0. \tag{3}$$

Gradient Projection Method

- Uses the gradient of $g(\mathbf{x})$ noting that ∇g is independent of $\mathbf{x}$ for linear $g(\mathbf{x})$.
- For nonlinear $g(\mathbf{x})$, the design point is obtained by an iterative method.
- Reduces the number of variables to 1 in the constrained optimization problem.
- Is expected to work well when the failure surface is 'fairly' linear.

Example 1

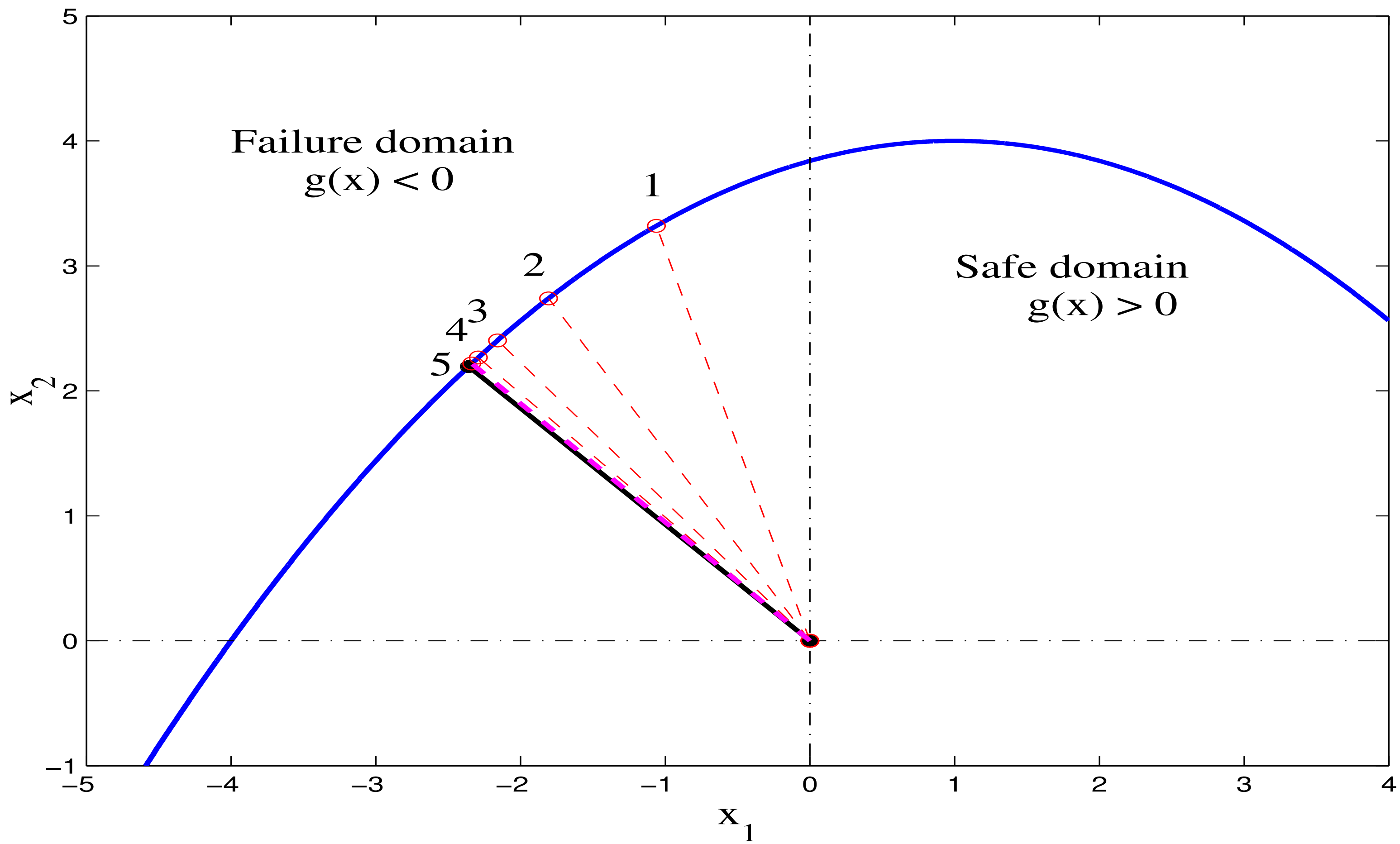
Linear failure surface in $\mathbb{R}^2$: $g(\mathbf{x}) = x_1 - 2x_2 + 10$



$\mathbf{x}^* = \{-2, 4\}^T$ and $\beta = 4.472$.

Example 2

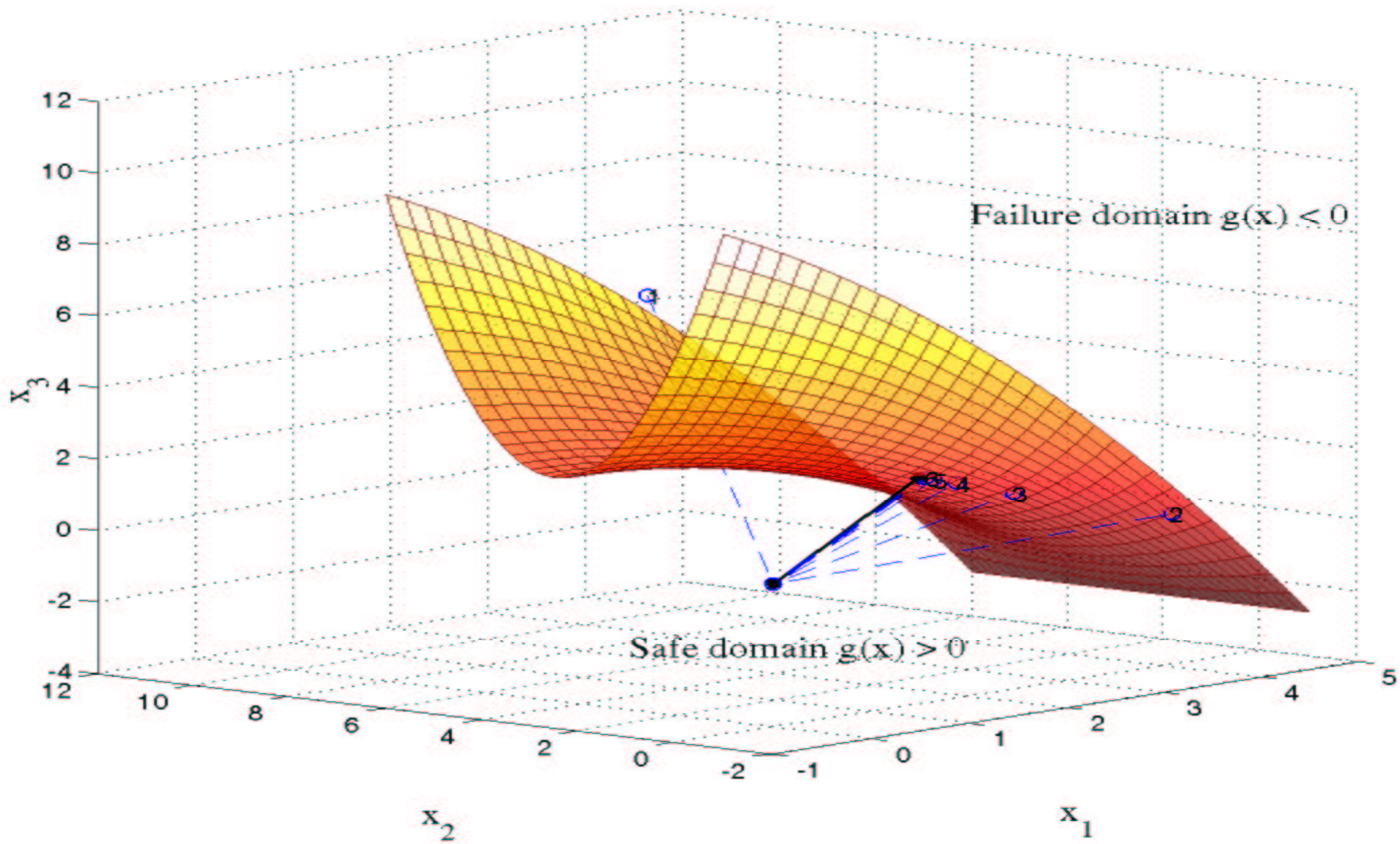
$$g(\mathbf{x}) = -\frac{4}{25}(x_1 - 1)^2 - x_2 + 4$$



$\mathbf{x}^* = \{-2.34, 2.21\}^T$ and $\beta = 3.22$.

Example 3

$$g(\mathbf{x}) = -\frac{4}{25}(x_1+1)^2 - \frac{(x_2-5/2)^2(x_1-5)}{10} - x_3 + 3$$

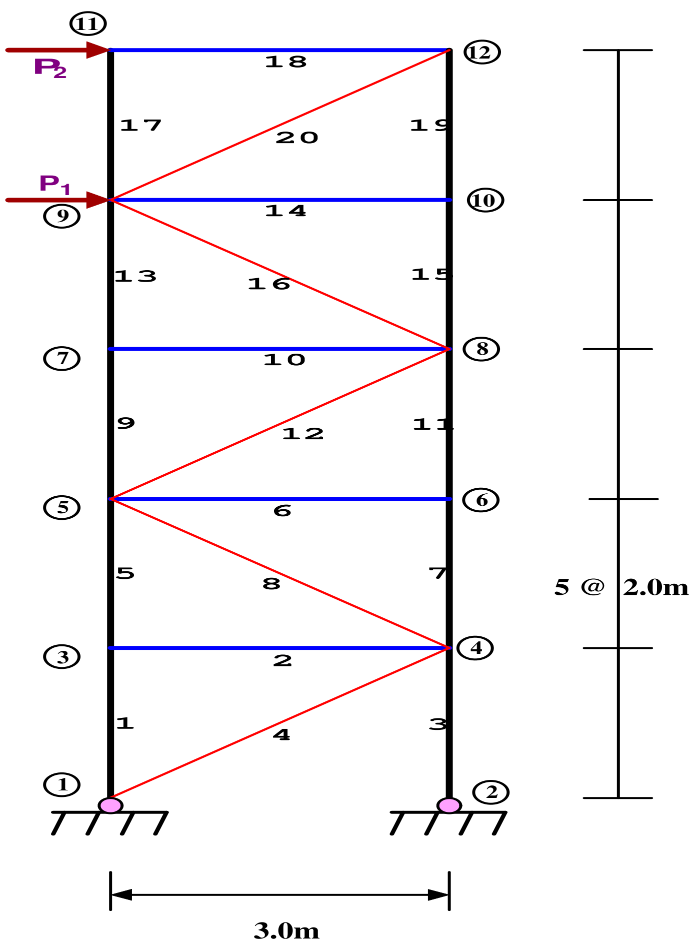


$\mathbf{x}^* = \{2.1286, 1.2895, 1.8547\}^T$ and $\beta = 3.104$.

Multistoried Portal Frame

$P_1 = 4.0e5$ KN, $P_2 = 5.0e5$ KN

Nel=20, Nnode=12



Random Variables:

Axial stiffness (EA) and the bending stiffness (EI) of each member are uncorrelated Gaussian random variables (Total $2 \times 20 = 40$ random variables: $\mathbf{x} \in \mathbb{R}^{40}$).

Element Type	EA (KN) Mean	EA (KN) Standard Deviation	EI (KNm^2) Mean	EI (KNm^2) Standard Deviation
1	5.0×10^9	7.0%	6.0×10^4	5.0%
2	3.0×10^9	3.0%	4.0×10^4	10.0%
3	1.0×10^9	10.0%	2.0×10^4	9.0%

Failure surface: $g(\mathbf{x}) = d_{max} - |\delta h_{11}(\mathbf{x})|$, δh_{11}: horizontal displacement at node 11, $d_{max} = 0.184 \times 10^{-2}$m

Multistoried Portal Frame

Results (with one iteration)

	Approximation ($n_{\text{reduced}} = 1$)	FORM $n = 40$	MCS[‡] (exact)
β	3.399	3.397	–
$P_f \times 10^3$	0.338	0.340	0.345

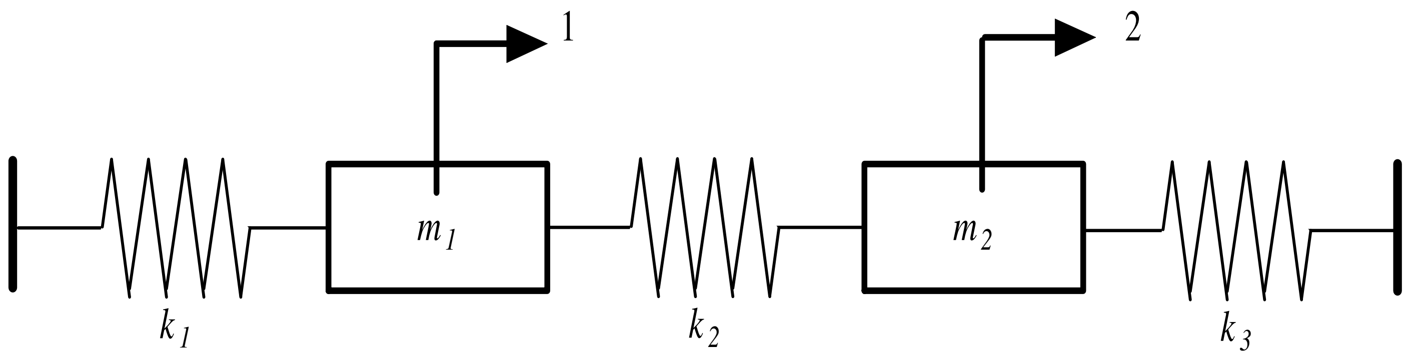
[‡] with 11600 samples (considered as benchmark)

Dynamic Reliability Problem

The central issues:

- The failure surface is discontinuous (hence not differentiable) and multiple-connected
- FORM and SORM, in its classical form, is not applicable

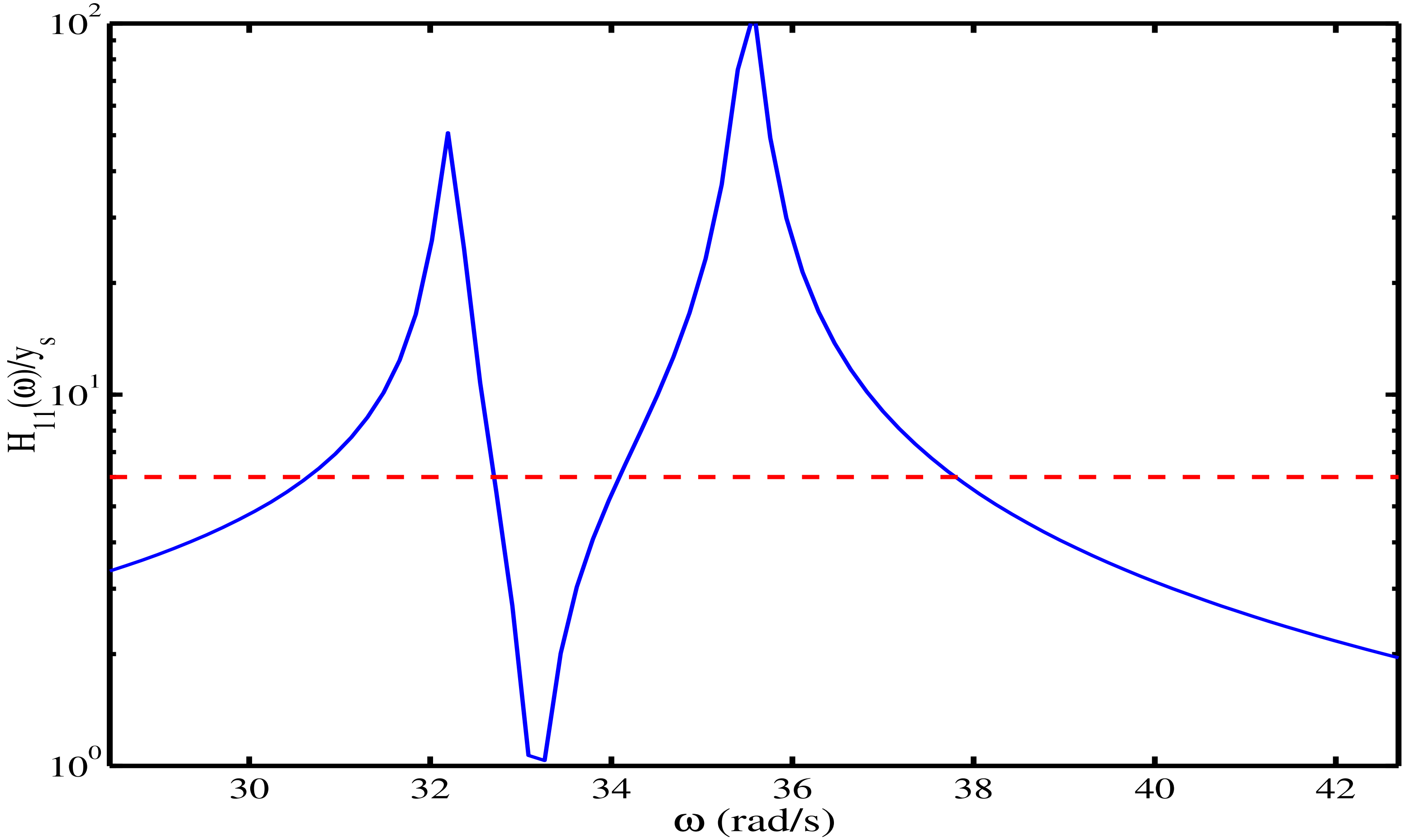
A 2 DOF System



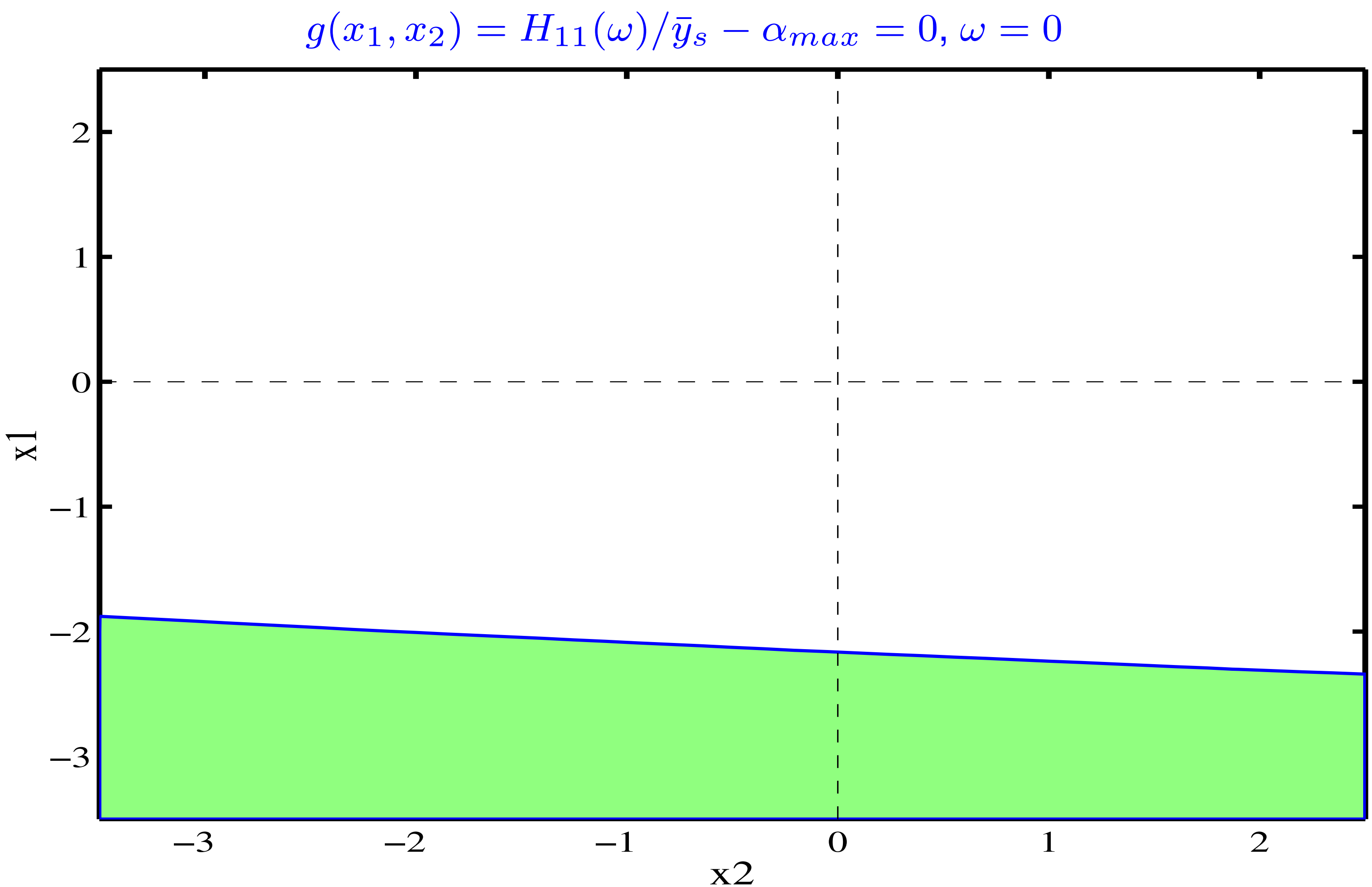
$k_1 = \bar{k}_1(1 + x_1/3)$, $k_2 = \bar{k}_2(1 + x_2/3)$,

$\omega_1 = 32.22$ and $\omega_2 = 35.52$

Transfer Function

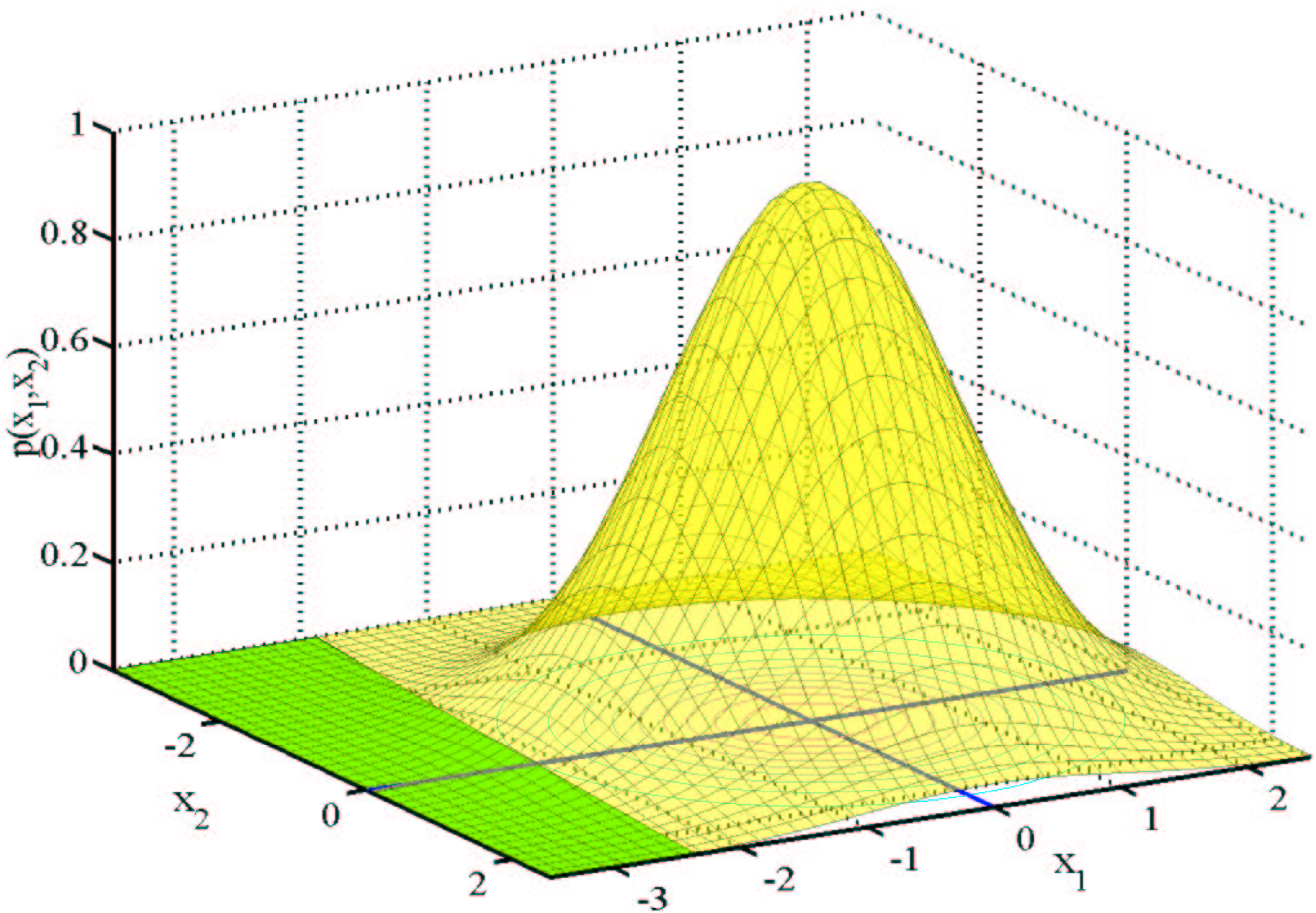


Static Failure Surface



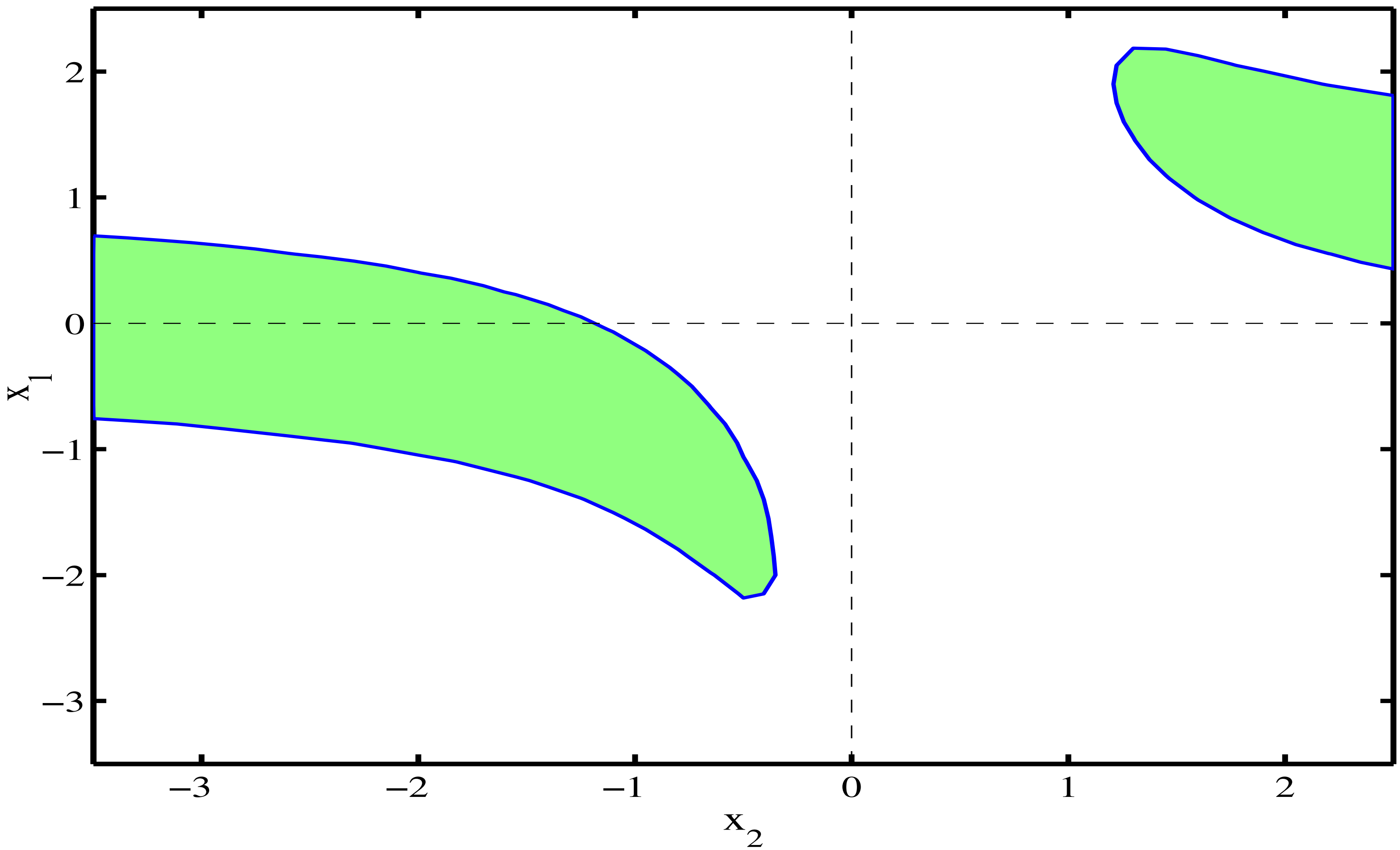
α_{max} : maximum allowable amplification=6

Static Reliability Integral



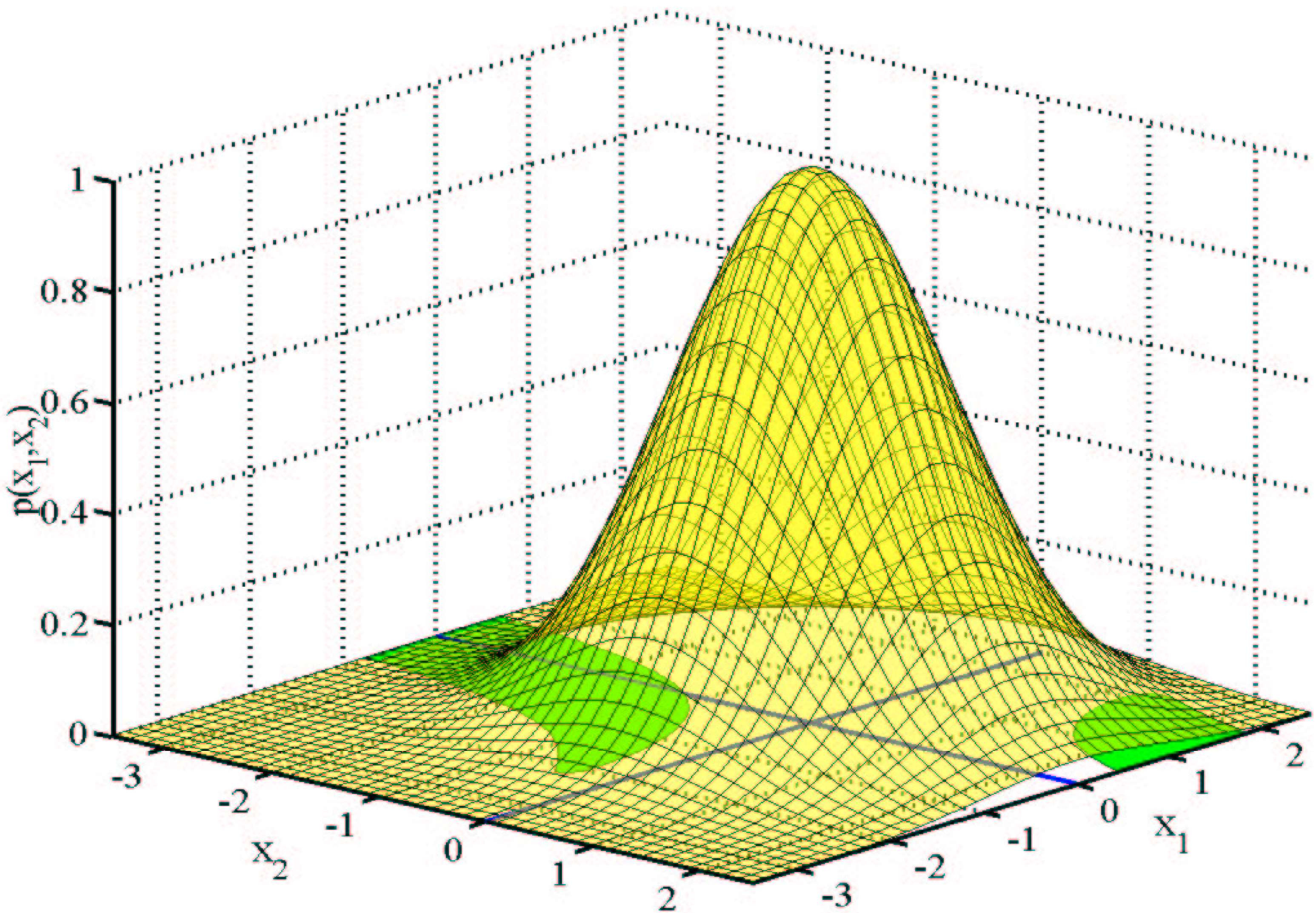
Dynamic Failure Surface

$g(x_1, x_2) = H_{11}(\omega)/\bar{y}_s - \alpha_{max} = 0$, $\omega = 33.26$ rad/s



α_{max} : maximum allowable amplification=6

Dynamic Reliability Integral



Conclusions & Future Research

- A gradient projection method based on the sensitivity vector of the failure surface is developed to reduce the number of random variables in structural reliability problems involving a large number of random variables.
- Current methods work well when the failure surface is close to linear (static problems).
- For dynamic problems the failure surface becomes highly non-linear, discontinuous and multiple-connected.

Conclusions & Future Research

- Further research is needed to develop non-classical methods for solving dynamic reliability problems.