Doubly Spectral Finite Element Method for Stochastic Field Problems in Structural Dynamics

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Outline of the presentation

- Motivation
- Spectral approach for dynamic systems
- Spectral Stochastic finite element method
- Unification of the two spectral approaches
- Numerical results
- Conclusions & future directions

Sources of uncertainty

(a) parametric uncertainty - e.g., uncertainty in geometric parameters, friction coefficient, strength of the materials involved;
(b) model inadequacy - arising from the lack of scientific knowledge about the model which is a-priori unknown;
(c) experimental error - uncertain and unknown error percolate into the model when they are calibrated against experimental results, and
(d) computational uncertainty - e.g, machine precession, error tolerance and the so called ‘h’ and ‘p’ refinements in finite element analysis

Motivation of the proposed approach

- Uncertainties in complex dynamical systems play an important role in the prediction of dynamic response in the mid and high frequency ranges.
- For distributed parameter systems, parametric uncertainties can be represented by random fields leading to stochastic partial differential equations.
- Over the past two decades spectral stochastic finite element method has been developed to discretise the random fields and solve such problems.
- On the other hand, for deterministic distributed parameter linear dynamical systems, spectral finite element method has been developed to efficiently solve the problem in the frequency domain.
- In spite of the fact that both approaches use spectral decomposition (one for the random fields and while the other for the dynamic displacement fields), there has been very little overlap between them in literature.
- In this paper these two spectral techniques have been unified with the aim that the unified approach would outperform any of the spectral methods considered on its own.

Rationale behind the proposed approach

- In the higher frequency ranges, as the wavelengths become smaller, very fine (static) mesh size is required to capture the dynamical behaviour.
- As a result, the deterministic analysis itself can pose significant computational challenges.
- One way to address this problem is to use a spectral approach in the frequency domain where the displacements within an element is expressed in terms of frequency dependent shape functions. The shape functions adapt themselves with increasing frequency and consequently displacements can be obtained accurately without fine remeshing.
- The spectral approach have the potential to be an efficient method for mid and high frequency vibration problems provided the random fields describing parametric uncertainties can be taken into account efficiently. Here the spectral decomposition of the random files are used in conjunction with the spectral decomposition of the displacements field.

Spectral method in the frequency domain

Spectral method for deterministic dynamical systems have been in use for more than three decades. This approach, or approaches very similar to this, is known by various names such as the dynamic stiffness method, spectral finite element method, exact element method and dynamic finite element method. Some of the notable features are:

- the mass distribution of the element is treated in an exact manner in deriving the element dynamic stiffness matrix;
- the dynamic stiffness matrix of one dimensional structural elements taking into account the effects of flexure, torsion, axial motion, shear deformation effects and damping are exactly determinable, which, in turn, enables the exact vibration analysis of skeletal structures by an inversion of the global dynamic stiffness matrix;
- the method does not employ eigenfunction expansions and, consequently, a major step of the traditional finite element analysis, namely, the determination of natural frequencies and mode shapes, is eliminated which automatically avoids the errors due to series truncation; this makes the method attractive for situations in which a large number of modes participate in vibration;

Spectral method in the frequency domain

- since the modal expansion is not employed, *ad hoc* assumptions concerning damping matrix being proportional to mass and/or stiffness is not necessary;
- the method is essentially a frequency domain approach suitable for steady state harmonic or stationary random excitation problems; generalization to other type of problems through the use of Laplace transforms is also available;
- the static stiffness matrix and the consistent mass matrix appear as the first two terms in the Taylor expansion of the dynamic stiffness matrix in the frequency parameter.

Stochastic Finite Element Method

Problems of structural dynamics in which the uncertainty in specifying mass and stiffness of the structure is modeled within the framework of random fields can be treated using the Stochastic Finite Element Method (SFEM). The application of SFEM in linear structural dynamics typically consists of the following key steps:

1. Selection of appropriate probabilistic models for parameter uncertainties and boundary conditions
2. Replacement of the element property random fields by an equivalent set of a finite number of random variables. This step, known as the 'discretisation of random fields' is a major step in the analysis.
3. Formulation of the equation of motion of the form $\mathbf{D}(\omega)\mathbf{u} = \mathbf{f}$ where $\mathbf{D}(\omega)$ is the random dynamic stiffness matrix, $\mathbf{u}$ is the vector of random nodal displacement and $\mathbf{f}$ is the applied forces. In general $\mathbf{D}(\omega)$ is a random symmetric complex matrix.
4. Calculation of the response statistics by either (a) solving the random eigenvalue problem, or (b) solving the set of complex random algebraic equations.

Spectral Decomposition of random fields

Suppose $H(\mathbf{r},\theta)$ is a random field with a covariance function $C_H(\mathbf{r}_1,\mathbf{r}_2)$ defined in a space Ω. Since the covariance function is finite, symmetric and positive definite it can be represented by a spectral decomposition. Using this spectral decomposition, the random process $H(\mathbf{r},\theta)$ can be expressed in a generalized fourier type of series as

$$H(\mathbf{r},\theta) = H_0(\mathbf{r}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i}\xi_i(\theta)\varphi_i(\mathbf{r}) \tag{1}$$

where $\xi_i(\theta)$ are uncorrelated random variables, λ_i and $\varphi_i(\mathbf{r})$ are eigenvalues and eigenfunctions satisfying the integral equation

$$\int_\Omega C_H(\mathbf{r}_1,\mathbf{r}_2)\varphi_i(\mathbf{r}_1)\mathrm{d}\mathbf{r}_1 = \lambda_i\varphi_i(\mathbf{r}_2), \quad \forall\ i = 1,2,\cdots \tag{2}$$

The spectral decomposition in equation (2) is known as the Karhunen-Loève (KL) expansion. The series in (2) can be ordered in a decreasing series so that it can be truncated after a finite number of terms with a desired accuracy.

Exponential autocorrelation function

The autocorrelation function:

$$C(x_1, x_2) = e^{-|x_1 - x_2|/b} \tag{3}$$

The underlying random process $H(x, \theta)$ can be expanded using the Karhunen-Loève expansion in the interval $-a \le x \le a$ as

$$H(x, \theta) = \sum_{j=1}^{\infty} \xi_j(\theta) \sqrt{\lambda_j} \varphi_j(x) \tag{4}$$

Using the notation $c = 1/b$, the corresponding eigenvalues and eigenfunctions for odd j are given by

$$\lambda_j = \frac{2c}{\omega_j^2 + c^2}, \qquad \varphi_j(x) = \frac{\cos(\omega_j x)}{\sqrt{a + \frac{\sin(2\omega_j a)}{2\omega_j}}}, \qquad \text{where} \quad \tan(\omega_j a) = \frac{c}{\omega_j}, \tag{5}$$

and for even j are given by

$$\lambda_j = \frac{2c}{{\omega_j}^2 + c^2}, \qquad \varphi_j(x) = \frac{\sin(\omega_j x)}{\sqrt{a - \frac{\sin(2\omega_j a)}{2\omega_j}}}, \qquad \text{where} \quad \tan(\omega_j a) = \frac{\omega_j}{-c}. \tag{6}$$

General Derivation of Doubly Spectral Element Matrices

- A linear damped distributed parameter dynamical system in which the displacement variable $U(\mathbf{r},t)$, where $\mathbf{r}$ is the spatial position vector and t is time, specified in some domain $\mathcal{D}$, is governed by a linear partial differential equation

$$\rho(\mathbf{r},\theta)\frac{\partial^2 U(\mathbf{r},t)}{\partial t^2} + L_1\frac{\partial U(\mathbf{r},t)}{\partial t} + L_2 U(\mathbf{r},t) = p(\mathbf{r},t); \quad \mathbf{r}\in\mathcal{D}, t\in[0,T] \tag{7}$$

with linear boundary-initial conditions of the form

$$M_{1j}\frac{\partial U(\mathbf{r},t)}{\partial t} = 0; \quad M_{2j}U(\mathbf{r},t) = 0; \quad \mathbf{r}\in\Gamma,\ t=t_0,\ j=1,2,\cdots \tag{8}$$

specified on some boundary surface Γ.

- In the above equation $\rho(\mathbf{r},\theta)$ is the random mass distribution of the system, $p(\mathbf{r},t)$ is the distributed time-varying forcing function, L_1 is the random spatial self-adjoint damping operator, L_2 is the random spatial self-adjoint stiffness operator and M_{1j} and M_{2j} are some linear operators defined on the boundary surface Γ. When parametric uncertainties are considered, the mass density $\rho(\mathbf{r},\theta)$ as well as the damping and stiffness operators involve random processes.

DSSFEM Formulation

- The equation of underlying homogeneous system without any forcing is given by

$$\rho_0 \frac{\partial^2 U(\mathbf{r},t)}{\partial t^2} + L_{10} \frac{\partial U(\mathbf{r},t)}{\partial t} + L_{20} U(\mathbf{r},t) = 0 \tag{9}$$

  together with suitable homogeneous boundary and initial conditions.

- Taking the Fourier transform and considering zero initial conditions

$$-\omega^2 \rho_0 u(\mathbf{r}) + \mathrm{i}\omega L_{10} \left\{u(\mathbf{r})\right\} + L_{20} \left\{u(\mathbf{r})\right\} = 0 \tag{10}$$

- Frequency-dependent displacement within an element is interpolated from the nodal displacements

$$u_e(\mathbf{r},\omega) = \mathbf{N}^T(\mathbf{r},\omega)\widehat{\mathbf{u}}_e(\omega) \tag{11}$$

  $\widehat{\mathbf{u}}_e(\omega) \in \mathbb{C}^n$ is the nodal displacement vector and $\mathbf{N}(\mathbf{r},\omega) \in \mathbb{C}^n$, the vector of frequency-dependent shape functions and n is the number of the nodal degrees-of-freedom.

DSSFEM Formulation

- Suppose the $s_j(\mathbf{r}, \omega) \in \mathbb{C}, j = 1, 2, \cdots m$ are the basis functions which exactly satisfy the baseline equation. Here m is the order of the ordinary differential equation. It can be shown that the shape function vector can be expresses as

$$\mathbf{N}(\mathbf{r}, \omega) = \mathbf{\Gamma}(\omega)\mathbf{s}(\mathbf{r}, \omega) \tag{12}$$

where the vector $\mathbf{s}(\mathbf{r}, \omega) = \{s_j(\mathbf{r}, \omega)\}^T, \forall j \in \mathbb{C}^m$ and the complex matrix $\mathbf{\Gamma}(\omega) \in \mathbb{C}^{nm}$ depends on the boundary conditions.

DSSFEM Formulation

- The frequency depended $n \times n$ complex random stiffness, mass and damping matrices can be obtained as

$$\mathbf{K}_e(\omega, \theta) = \int_{\mathcal{D}_e} k_s(\mathbf{r}, \theta) \mathcal{L}_2 \left\{ \mathbf{N}(\mathbf{r}, \omega) \right\} \mathcal{L}_2 \left\{ \mathbf{N}^T(\mathbf{r}, \omega) \right\} \mathrm{d}\mathbf{r} \tag{13}$$

$$\mathbf{M}_e(\omega, \theta) = \int_{\mathcal{D}_e} \rho(\mathbf{r}, \theta) \mathbf{N}(\mathbf{r}, \omega) \mathbf{N}^T(\mathbf{r}, \omega) \mathrm{d}\mathbf{r} \quad \text{and} \tag{14}$$

$$\mathbf{C}_e(\omega, \theta) = \int_{\mathcal{D}_e} c(\mathbf{r}, \theta) \mathcal{L}_1 \left\{ \mathbf{N}(\mathbf{r}, \omega) \right\} \mathcal{L}_1 \left\{ \mathbf{N}^T(\mathbf{r}, \omega) \right\} \mathrm{d}\mathbf{r} \tag{15}$$

$(\bullet)^T$ denotes matrix transpose, $k_s(\mathbf{r}, \theta)$ is the random distributed stiffness parameter, $\mathcal{L}_2\{\bullet\}$ is the strain energy operator, $c(\mathbf{r}, \theta)$ is the random distributed damping parameter and $\mathcal{L}_1\{\bullet\}$ is the energy dissipation operator.

DSSFEM Formulation

- The random fields $k_s(\mathbf{r},\theta)$, $\rho(\mathbf{r},\theta)$ and $c(\mathbf{r},\theta)$ are expanded using the Karhunen-Loève expansion. Using finite number of terms, each of the complex element matrices can be expanded in a spectral series as

$$\mathbf{K}_e(\omega,\theta) = \mathbf{K}_{0e}(\omega) + \sum_{j=1}^{N_\mathrm{K}} \xi_{\mathrm{K_j}}(\theta)\mathbf{K}_{je}(\omega) \tag{16}$$

$$\mathbf{M}_e(\omega,\theta) = \mathbf{M}_{0e}(\omega) + \sum_{j=1}^{N_\mathrm{M}} \xi_{\mathrm{M_j}}(\theta)\mathbf{M}_{je}(\omega) \tag{17}$$

and $$\mathbf{C}_e(\omega,\theta) = \mathbf{C}_{0e}(\omega) + \sum_{j=1}^{N_\mathrm{C}} \xi_{\mathrm{C_j}}(\theta)\mathbf{C}_{je}(\omega) \tag{18}$$

DSSFEM Formulation

- Here the complex deterministic symmetric matrices, for example in the case of the stiffness matrix, can be obtained as

$$\mathbf{K}_{0e}(\omega) = \int_{\mathcal{D}_e} k_{s_0}(\mathbf{r})\mathcal{L}_2\left\{\mathbf{N}(\mathbf{r},\omega)\right\}\mathcal{L}_2\left\{\mathbf{N}^T(\mathbf{r},\omega)\right\} \mathrm{d}\mathbf{r} \quad \text{and} \tag{19}$$

$$\mathbf{K}_{je}(\omega) = \sqrt{\lambda_{\mathrm{K_j}}} \int_{\mathcal{D}_e} \varphi_{\mathrm{K_j}}(\mathbf{r})\mathcal{L}_2\left\{\mathbf{N}(\mathbf{r},\omega)\right\}\mathcal{L}_2\left\{\mathbf{N}^T(\mathbf{r},\omega)\right\} \mathrm{d}\mathbf{r} \tag{20}$$

$$\forall j = 1, 2, \cdots, N_{\mathrm{K}}$$

The equivalent terms corresponding to the mass and damping matrices can be obtained in a similar manner. Substituting the shape function one obtains

$$\mathbf{K}_{0e}(\omega) = \mathbf{\Gamma}(\omega)\widetilde{\mathbf{K}}_{0e}(\omega)\mathbf{\Gamma}^T(\omega) \quad \text{and} \tag{21}$$

$$\mathbf{K}_{je}(\omega) = \sqrt{\lambda_{\mathrm{K_j}}}\mathbf{\Gamma}(\omega)\widetilde{\mathbf{K}}_{je}(\omega)\mathbf{\Gamma}^T(\omega); \quad \forall j = 1, 2, \cdots, N_{\mathrm{K}} \tag{22}$$

DSSFEM Formulation

- $$\widetilde{\mathbf{K}}_{0e}(\omega) = \int_{\mathcal{D}_e} k_{s_0}(\mathbf{r})\mathcal{L}_2\left\{\mathbf{s}(\mathbf{r},\omega)\right\}\mathcal{L}_2\left\{\mathbf{s}^T(\mathbf{r},\omega)\right\}\mathrm{d}\mathbf{r} \in \mathbb{C}^{mm} \quad \text{and} \tag{23}$$
  $$\widetilde{\mathbf{K}}_{je}(\omega) = \int_{\mathcal{D}_e} \varphi_{\mathrm{K_j}}(\mathbf{r})\mathcal{L}_2\left\{\mathbf{s}(\mathbf{r},\omega)\right\}\mathcal{L}_2\left\{\mathbf{s}^T(\mathbf{r},\omega)\right\}\mathrm{d}\mathbf{r} \in \mathbb{C}^{mm} \tag{24}$$
  $$\forall j = 1, 2, \cdots, N_{\mathrm{K}}$$
- The expressions of the eigenfunctions given in the previous section are valid within the specific domains defined before. One needs to change the coordinate in order to use them in the preceding equations. Once the element stiffness, mass and damping matrices are obtained in this manner, the global matrices can be calculated by summing the element matrices with suitable coordinate transformations as in the standard finite element method.

Global matrices

- Combining the element matrices one finally obtains

$$\left[-\omega^2\mathbf{M}(\omega,\theta) + \mathrm{i}\omega\mathbf{C}(\omega,\theta) + \mathbf{K}(\omega,\theta)\right]\mathbf{u}(\omega) = \mathbf{f}(\omega) \tag{25}$$

where

$$\mathbf{K}(\omega,\theta) = \mathbf{K}_0(\omega) + \sum_{j=1}^{N_\mathrm{K}} \xi_{\mathrm{K_j}}(\theta)\mathbf{K}_j(\omega) \tag{26}$$

$$\mathbf{M}(\omega,\theta) = \mathbf{M}_0(\omega) + \sum_{j=1}^{N_\mathrm{M}} \xi_{\mathrm{M_j}}(\theta)\mathbf{M}_j(\omega) \tag{27}$$

and $$\mathbf{C}(\omega,\theta) = \mathbf{C}_0(\omega) + \sum_{j=1}^{N_\mathrm{C}} \xi_{\mathrm{C_j}}(\theta)\mathbf{C}_j(\omega) \tag{28}$$

Axially vibrating rod

- The equation of motion of the stochastically inhomogeneous rod under axial motion is given by

$$\frac{\partial}{\partial x}\left[AE(x,\theta)\frac{\partial U}{\partial x}\right] + c_0\frac{\partial U}{\partial t} - m(x,\theta)\frac{\partial^2 U}{\partial t^2} = 0 \tag{29}$$

  Here the axial rigidity $AE(x)$ and the mass per unit length $m(x)$ is assumed to be random fields of the following form

$$AE(x,\theta) = A_0E_0\left[1 + \epsilon_{AE}H_{AE}(x,\theta)\right] \tag{30}$$

$$m(x) = m_0(1 + \epsilon_m H_m(x,\theta)) \tag{31}$$

  Here $H_{AE}(x,\theta)$ and $H_m(x,\theta)$ are assumed to homogeneous Gaussian random fields with zero mean and exponentially decaying autocorrelation function. The 'strength parameters' ϵ_{AE} and ϵ_m effectively quantify the amount of uncertainty in the axial rigidity and mass per unit length of the rod.

- The numerical values are taken as $\rho = 2700\,kg/m^3,\ E = 69\,GPa$ (corresponding to aluminium), length of the bar $L = 30\,m$ cross section $A = 1\,cm^2$, and $\epsilon_1 = \epsilon_2 = 0.1$.

Axially vibrating rod

- The constants $A_0 E_0$ and $m_0 = A_0 \rho_0$ are respectively the mass per unit length and axial rigidity of the underline baseline model. The equation of motion of the baseline model is given by

$$A_0 \rho_0 \frac{\partial^2 U(x,t)}{\partial t^2} + c_0 \frac{\partial U}{\partial t} - A_0 E_0 \frac{\partial^2 U}{\partial x^2} = 0 \,, \tag{32}$$

where ρ_0, E_0, A_0 and c_0 are the nominal value of the density, elastic stiffness, cross sectional area and damping factor within a domain $x \in [0, L]$. With spectral expansion of the axial displacement $U(x,t)$ in the frequency-wavenumber space, one has

$$U(x,t) = \left(\tilde{u}_1 e^{-ik_0 x/L} + \tilde{u}_2 e^{-ik_0 (1-x/L)} \right) e^{i\omega t} = u(x) e^{i\omega t} \tag{33}$$

where $i = \sqrt{-1}$ and k_0 is the non-dimensional wavenumber for the reference model:

$$k_0 = \omega L \sqrt{\frac{\rho_0}{E_0}} \sqrt{1 - \frac{ic_0}{\omega \rho_0}} \,. \tag{34}$$

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Axially vibrating rod

- The shape function:

$$\mathbf{N}(\mathbf{r},\omega)=\mathbf{\Gamma}(\omega)\mathbf{s}(\mathbf{r},\omega),\quad \text{where}\quad \mathbf{s}(\mathbf{r},\omega)=\begin{Bmatrix} e^{-ik_0x/L} \\ e^{ik_0x/L}\end{Bmatrix}$$
$$\text{and}\quad \mathbf{\Gamma}(\omega)=\frac{1}{1-e^{-i2k_0}}\begin{bmatrix} 1 & -e^{-2ik_0} \\ -e^{-ik_0} & e^{-ik_0}\end{bmatrix} \tag{35}$$

- The deterministic part of the stiffness matrix:

$$\widetilde{\mathbf{K}}_{0e}(\omega)=A_0E_0\int_0^L\left\{\frac{\partial\mathbf{s}(x,\omega)}{\partial x}\right\}\left\{\frac{\partial\mathbf{s}(x,\omega)}{\partial x}\right\}^T\mathrm{d}x \tag{36}$$

$$=\frac{A_0E_0k_0}{L}\begin{bmatrix} -1/2\,i\left(-1+\mathrm{e}^{-2\,ik_0}\right) & k_0 \\ k_0 & 1/2\,i\left(\mathrm{e}^{2\,ik_0}-1\right)\end{bmatrix} \tag{37}$$

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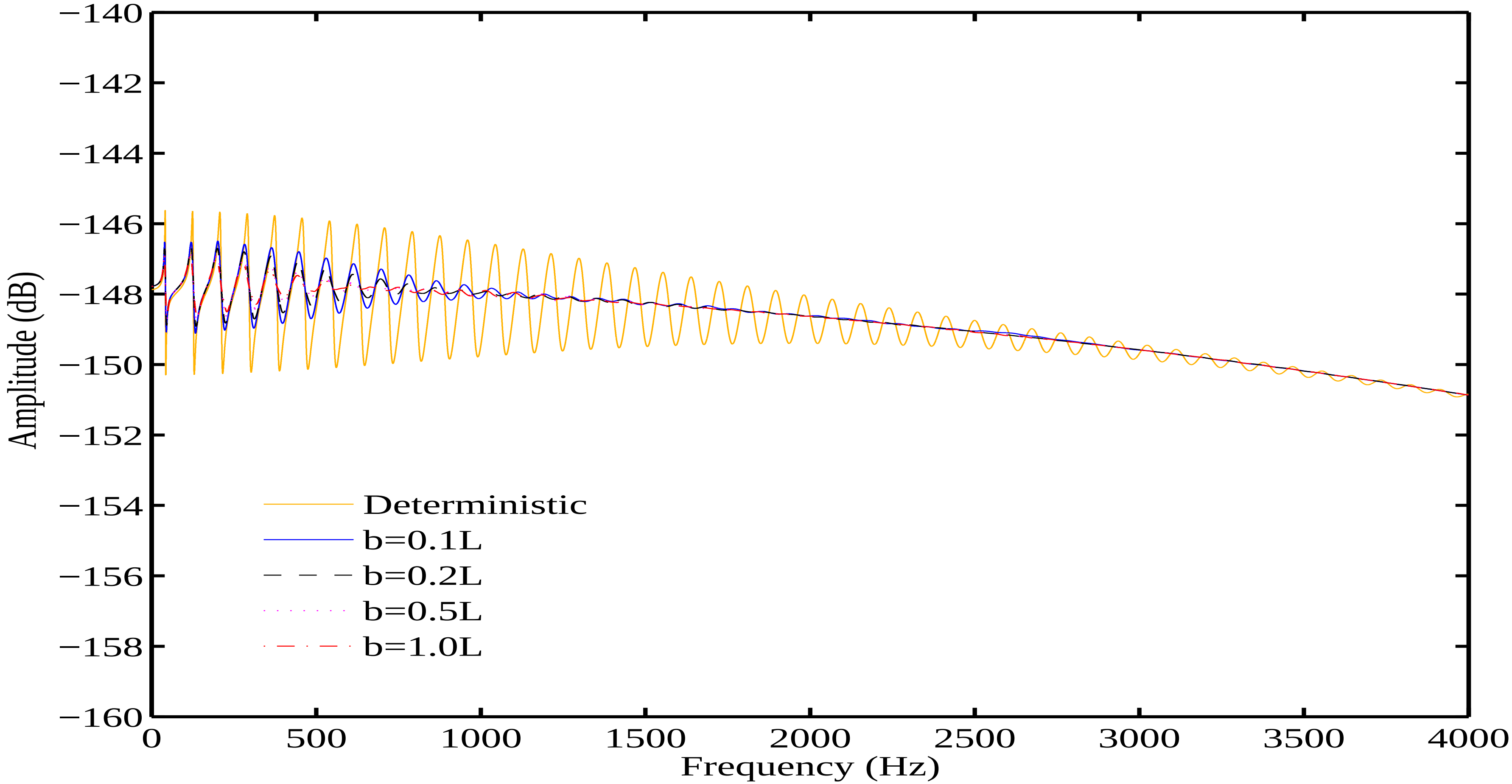
Axially vibrating rod

- The deterministic part of the mass matrix:

$$\widetilde{\mathbf{M}}_{0e}(\omega) = m_0 \int_0^L \mathbf{s}(x,\omega)\mathbf{s}^T(x,\omega)\mathrm{d}x \tag{38}$$

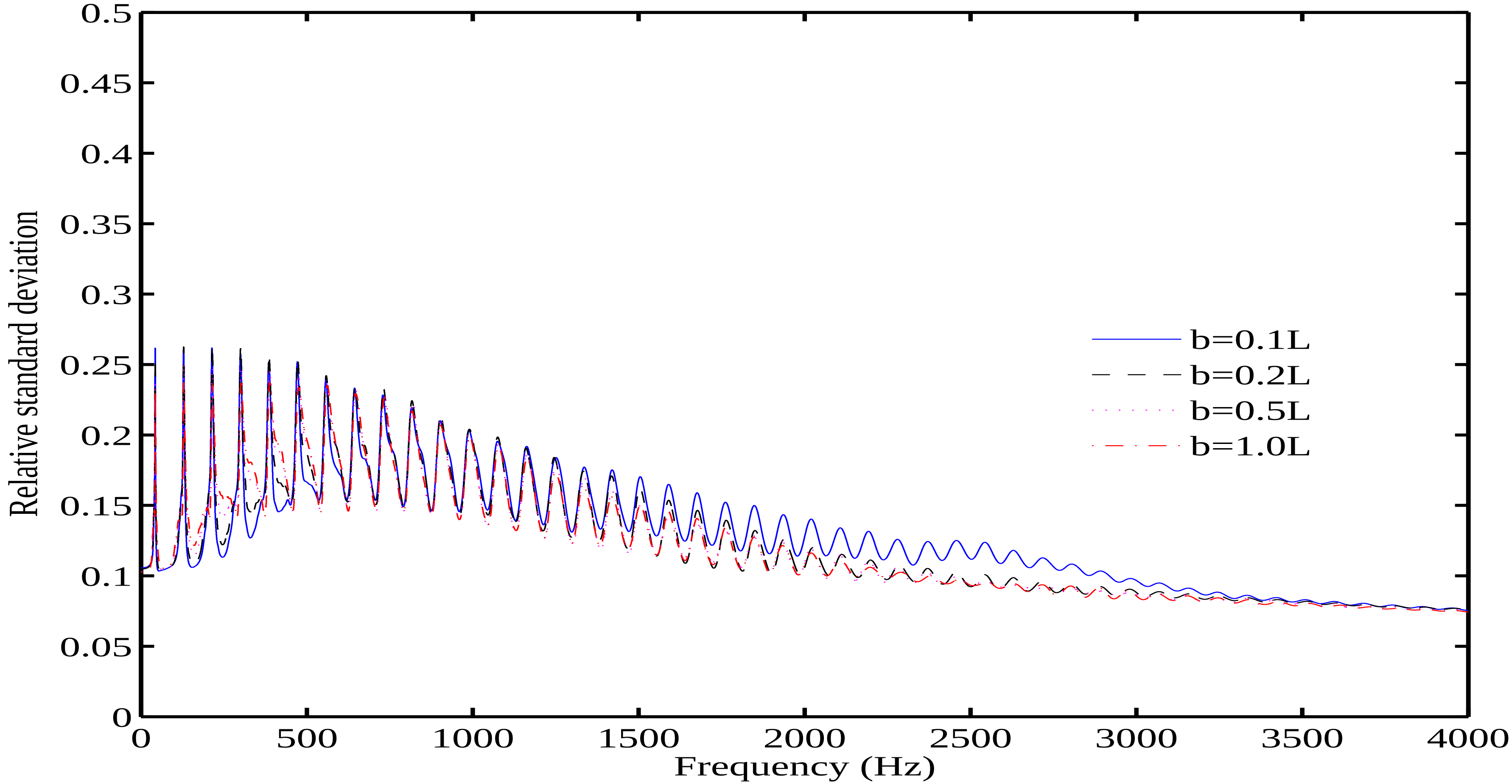
$$= m_0 L \begin{bmatrix} \frac{1/2\, i\left(-1+\mathrm{e}^{-2\, ik_0}\right)}{k_0} & 1 \\ 1 & \frac{-1/2\, i\left(\mathrm{e}^{2\, ik_0}-1\right)}{k_0} \end{bmatrix} \tag{39}$$

Mean of Driving-point FRF



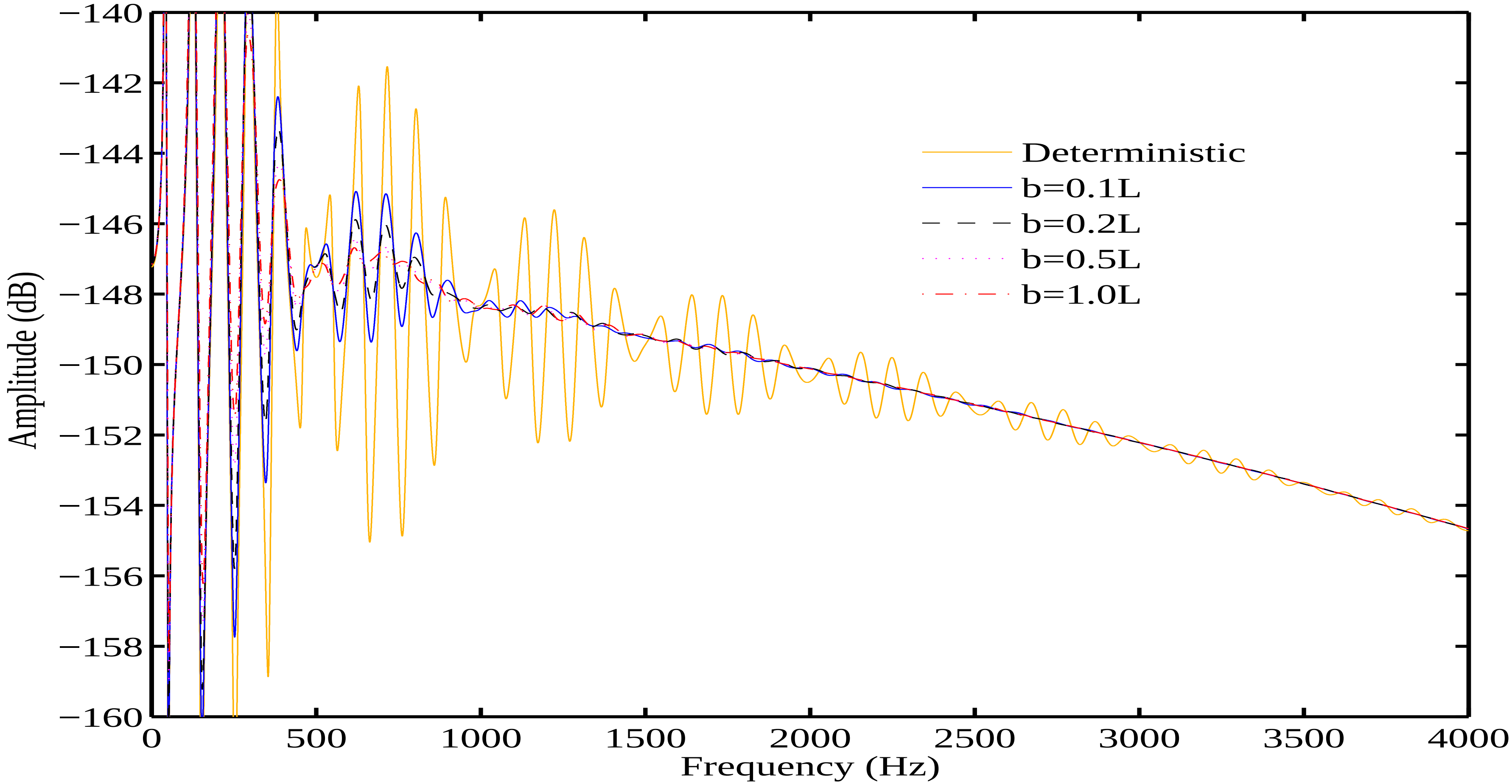
Mean of FRF obtained using DSSFEM for the bar with different correlation lengths.

Standard deviation of Driving-point FRF



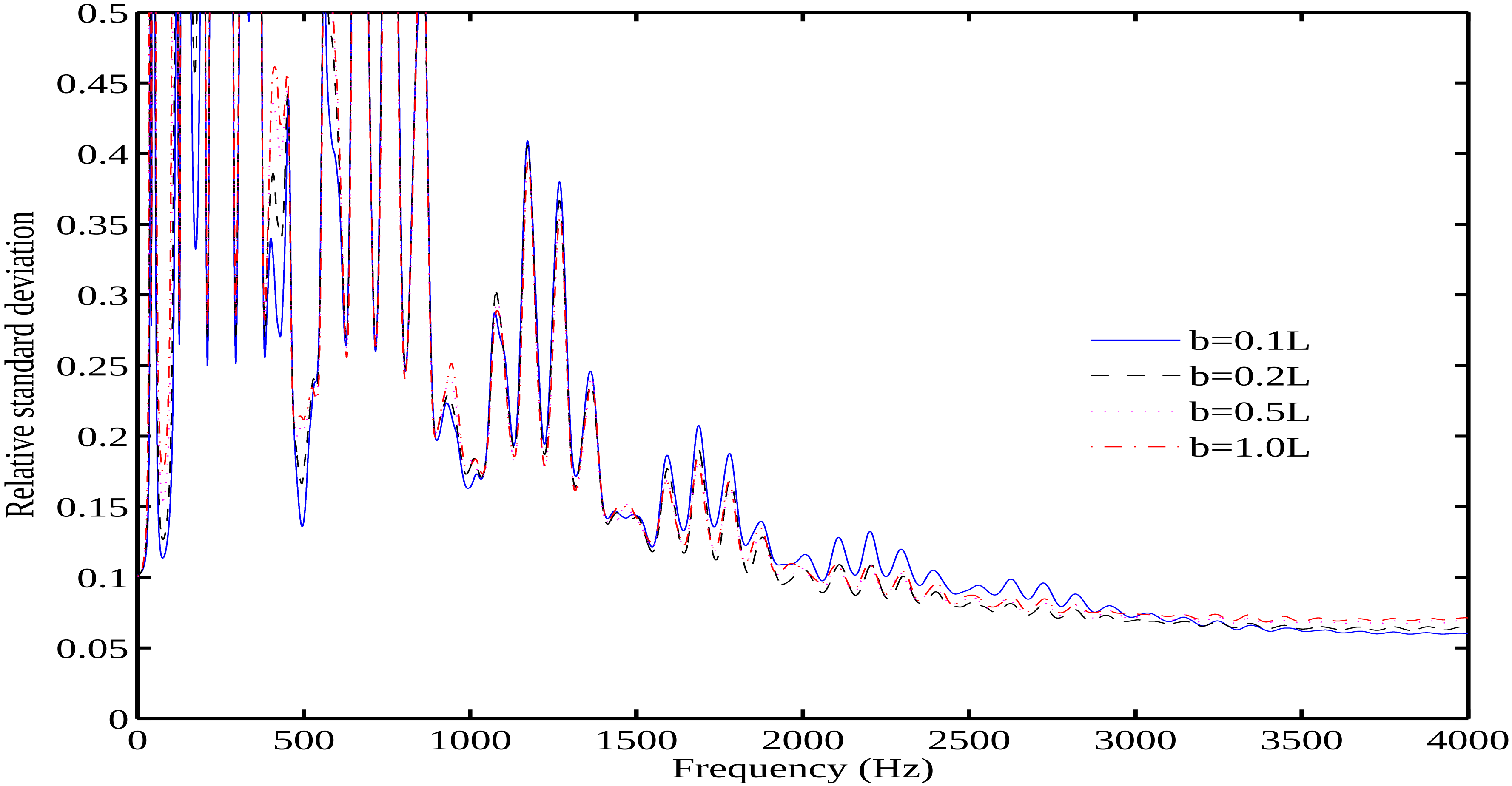
Relative standard deviation of FRF obtained using DSSFEM for the bar with different correlation lengths.

Mean of a Cross-FRF



Mean of FRF obtained using DSSFEM for the bar with different correlation lengths.

Standard deviation of a Cross-FRF



Relative standard deviation of FRF obtained using DSSFEM for the bar with different correlation lengths.

Euler-Bernoulli Beam

- The equation of motion of an undamped Euler-Bernoulli beam of length L with random bending stiffness and mass distribution can be expressed as

$$\frac{\partial^2}{\partial x^2}\left[EI(x,\theta)\frac{\partial^2 Y(x,t)}{\partial x^2}\right] + \rho A(x,\theta)\frac{\partial^2 Y(x,t)}{\partial t^2} = p(x,t). \quad (40)$$

  Here $Y(x,t)$ is the transverse flexural displacement, $EI(x)$ is the flexural rigidity, $\rho A(x)$= is the mass per unit length, and $p(x,t)$ is the applied forcing. It is assumed that the bending stiffness, EI, and mass per unit length, ρA, are random fields of the form

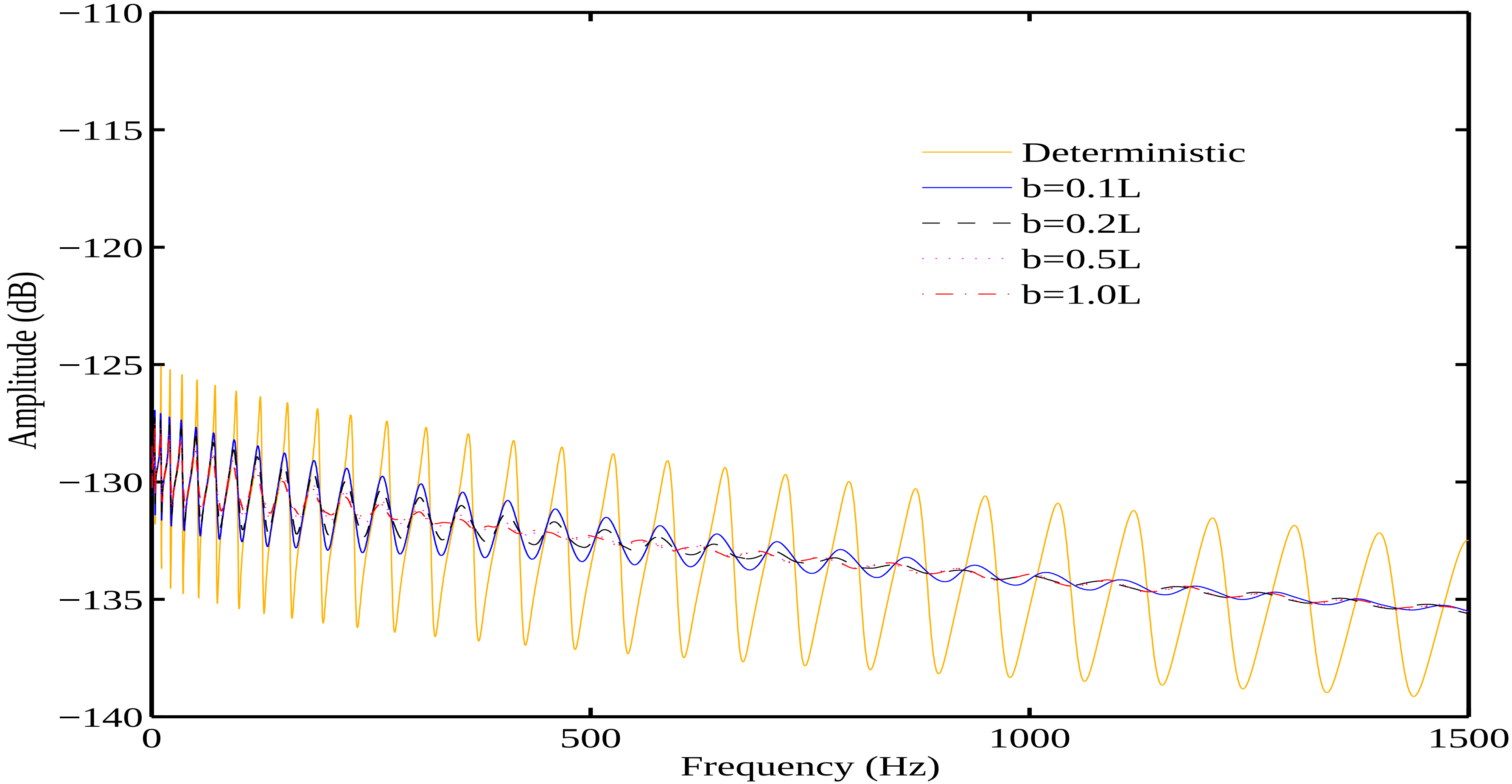
$$EI(x,\theta) = EI_0\left(1+\epsilon_1 F_1(x,\theta)\right) \quad (41)$$

$$\text{and} \quad \rho A(x,\theta) = \rho A_0\left(1+\epsilon_2 F_2(x,\theta)\right) \quad (42)$$

  The random fields $F_i(x,\theta)$ are taken to have zero mean, unit standard deviation and covariance $R_{ij}(\xi)$.

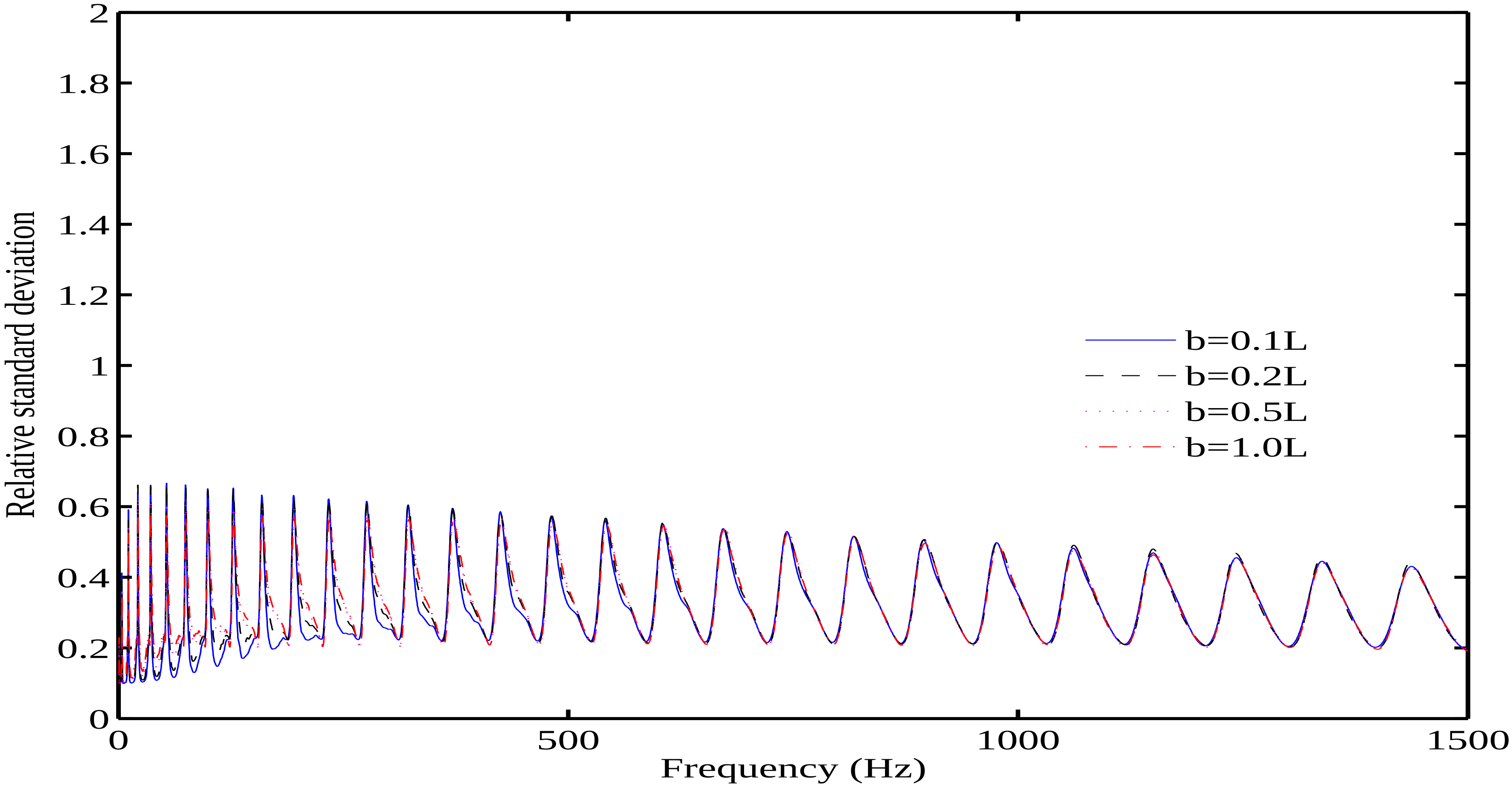
- The numerical values are taken as $\rho = 7800\,kg/m^3,\ E = 200\,GPa$ (corresponding to steel), length of the beam $L = 1.65\,m$, rectangular cross section with width $b = 40.06mm$ and thickness $2.05mm$, and $\epsilon_1 = \epsilon_2 = 0.1$.

Mean of Driving-point FRF



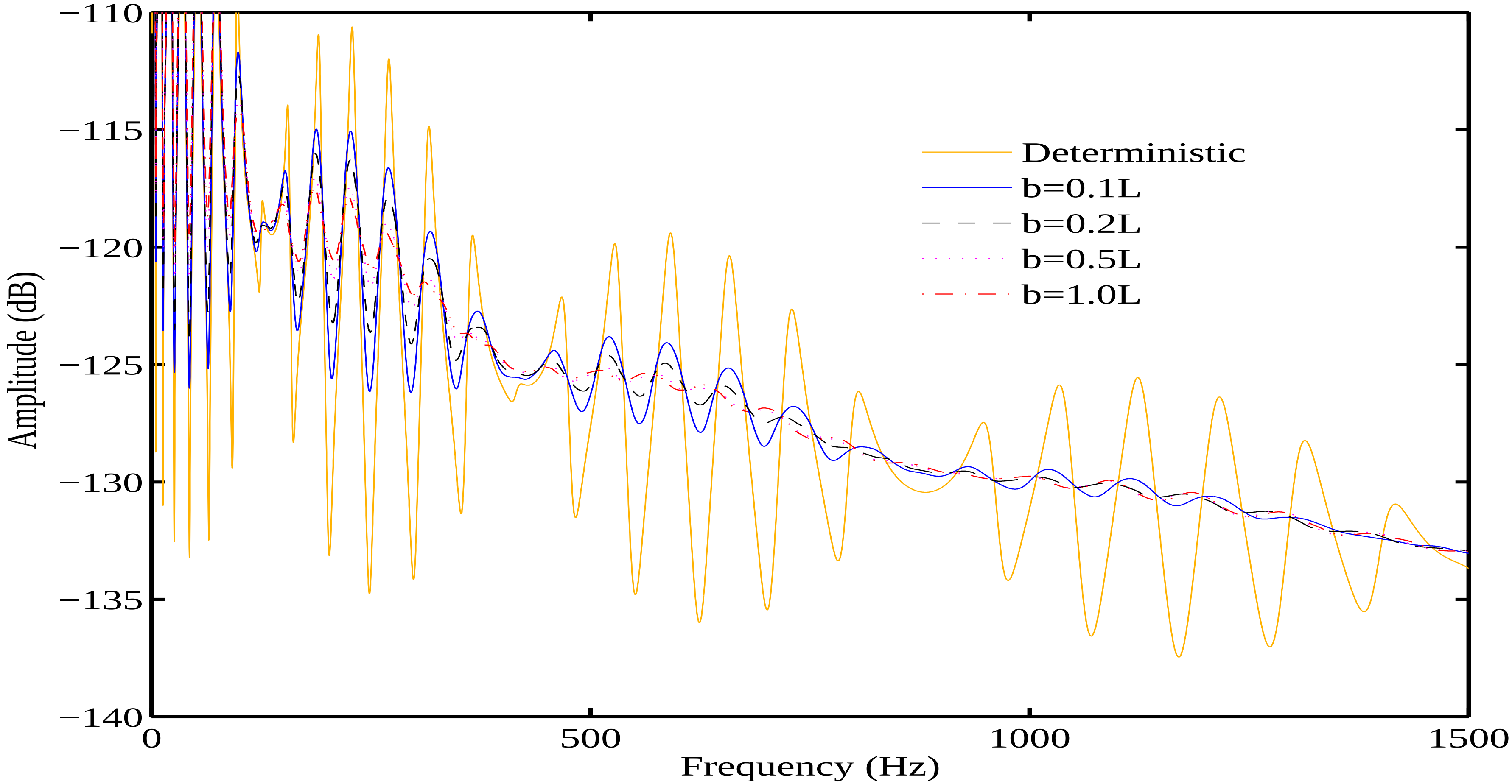
Mean of FRF obtained using DSSFEM for the beam with different correlation lengths.

Standard deviation of Driving-point FRF



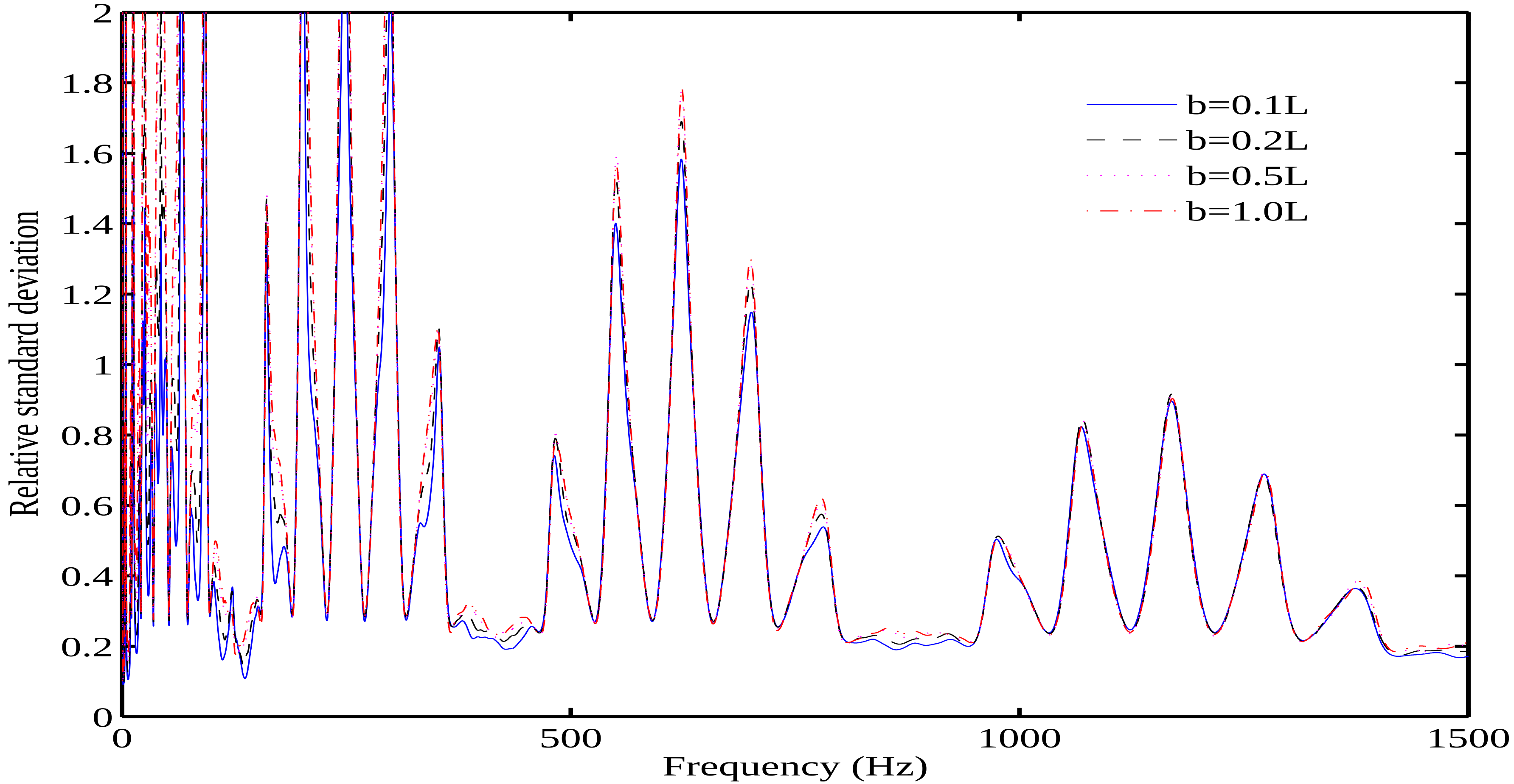
Relative standard deviation of FRF obtained using DSSFEM for the beam with different correlation lengths.

Mean of a Cross-FRF



Mean of FRF obtained using DSSFEM for the beam with different correlation lengths.

Standard deviation of a Cross-FRF



Relative standard deviation of FRF obtained using DSSFEM for the beam with different correlation lengths.

Conclusions

- The basic principles for Doubly Spectral Stochastic Finite Element Method (DSSFEM) for damped linear dynamical systems with distributed parametric uncertainty has been derived.
- This new approach simultaneously utilizes the spectral representations in the frequency and random domains. The spatial displacement fields are discretized using frequency-adaptive complex shape functions while the spatial random fields are discretized using the Karhunen-Loève expansion.
- In spite of the fact that these two spectral approaches existed for well over three decades, there has been very little overlap between them in literature. In this paper these two spectral techniques have been unified with the aim that the unified approach would outperform any of the spectral methods considered on its own.
- The resulting frequency depended random element matrices in general turn out to be complex symmetric matrices.
- The main computational advantage of the proposed approach is that the fine spatial discretisation will not be necessary for high and mid-frequency vibration analysis.
- Numerical examples have been given to illustrate the applicability of the proposed method.