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An Efficient Computational Solution Scheme of the Random Eigenvalue Problems

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Outline

- Introduction
- Random Eigenvalue Problem
- High Dimensional Model Representation (HDMR)
- Examples
- Conclusions

Sources of uncertainty

- (a) parametric uncertainty - e.g., uncertainty in geometric parameters, friction coefficient, strength of the materials involved;
- (b) model inadequacy - arising from the lack of scientific knowledge about the model which is a-priori unknown;
- (c) experimental error - uncertain and unknown error percolate into the model when they are calibrated against experimental results;
- (d) computational uncertainty - e.g, machine precession, error tolerance and the so called ‘h’ and ‘p’ refinements in finite element analysis,

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Random Eigenvalue Problem

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{\boldsymbol{x}}(t) + \boldsymbol{K}\boldsymbol{x}(t) = \boldsymbol{p}(t)$$

- Due to the presence of uncertainties, mass, damping and stiffness matrices are random matrices.
- The primary objectives are
  - To quantify the uncertainties in system matrices.
  - To estimate the variability of system responses.

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Random Eigenvalue Problem

- Random eigenvalue of linear structural system

$$K(X)\Phi(X) = \Lambda(X)M(X)\Phi(X)$$

- Main issues
  - To find probabilistic characteristics of eigenpair.
  - To find the joint statistics (moments, correlation).
  - Several approaches are available on random eigenvalue problem, which are based on
    - Perturbation method (Boyce, 1968; Zhang & Ellingwood, 1995)
    - Iteration method (Boyce, 1968)
    - Ritz method (Mehlhose, 1999)
    - Crossing theory (Grigorie, 1992)
    - Stochastic reduced basis (Nair & Keane, 2003)
    - Asymptotic method (Adhikari, 2006)

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Perturbation Method

Taylor series expansion of $\lambda_j(\mathbf{x})$ about $\mathbf{x} = \boldsymbol{\alpha}$

$$\lambda_j(\mathbf{x}) \approx \lambda_j(\boldsymbol{\alpha}) + \mathbf{d}^T_{\lambda_j}(\boldsymbol{\alpha})\,(\mathbf{x} - \boldsymbol{\alpha}) + \frac{1}{2}(\mathbf{x} - \boldsymbol{\alpha})^T\,\mathbf{D}_{\lambda_j}(\boldsymbol{\alpha})\,(\mathbf{x} - \boldsymbol{\alpha})$$

In the mean-centered approach α is the mean of x

Alternatively, α can be obtained such that the any moment of each eigenvalue is calculated most accurately

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Multidimensional Integrals

We want to evaluate an m-dimensional integral over the unbounded domain $\mathbb{R}^m$:

$$\mathcal{J} = \int_{\mathbb{R}^m} e^{-f(\mathbf{x})}\, d\mathbf{x}$$

- Assume $f(\mathbf{x})$ is smooth and at least twice differentiable
- The maximum contribution to this integral comes from the neighborhood where $f(\mathbf{x})$ reaches its global minimum, say $\boldsymbol{\theta} \in \mathbb{R}^m$

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Multidimensional Integrals

Therefore, at $\mathbf{x} = \boldsymbol{\theta}$

$$\frac{\partial f(\mathbf{x})}{\partial x_k} = 0, \forall k \quad \text{or} \quad \mathbf{d}_f(\boldsymbol{\theta}) = 0$$

Expand $f(\mathbf{x})$ in a Taylor series about $\boldsymbol{\theta}$:

$$\mathcal{J} = \int_{\mathbb{R}^m} e^{-\left\{f(\boldsymbol{\theta}) + \frac{1}{2}(\mathbf{x}-\boldsymbol{\theta})^T \mathbf{D}_f(\boldsymbol{\theta})(\mathbf{x}-\boldsymbol{\theta}) + \varepsilon(\mathbf{x},\boldsymbol{\theta})\right\}} d\mathbf{x}$$

$$= e^{-f(\boldsymbol{\theta})} \int_{\mathbb{R}^m} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\theta})^T \mathbf{D}_f(\boldsymbol{\theta})(\mathbf{x}-\boldsymbol{\theta}) - \varepsilon(\mathbf{x},\boldsymbol{\theta})} d\mathbf{x}$$

Multidimensional Integrals

- Use the coordinate transformation:

$$\xi = (\mathrm{x} - \boldsymbol{\theta})\, \mathrm{D}_f^{-1/2}(\boldsymbol{\theta})$$

- The Jacobian: $\|\mathrm{J}\| = \|\mathrm{D}_f(\boldsymbol{\theta})\|^{-1/2}$
- The integral becomes:

$$\mathcal{J} \approx e^{-f(\boldsymbol{\theta})} \int_{\mathbb{R}^m} \|\mathrm{D}_f(\boldsymbol{\theta})\|^{-1/2} e^{-\frac{1}{2}\left(\xi^T \xi\right)}\, d\xi$$

or

$$\mathcal{J} \approx (2\pi)^{m/2} e^{-f(\boldsymbol{\theta})} \|\mathrm{D}_f(\boldsymbol{\theta})\|^{-1/2}$$

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Moments of Eigenvalues

An arbitrary rth order moment of the eigenvalues can be obtained from

$$\mu_j^{(r)} = \mathrm{E}\left[\lambda_j^r(\mathbf{x})\right] = \int_{\mathbb{R}^m} \lambda_j^r(\mathbf{x}) p_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}$$

$$= \int_{\mathbb{R}^m} e^{-(L(\mathbf{x}) - r \ln \lambda_j(\mathbf{x}))}\, d\mathbf{x}, \quad r = 1, 2, 3 \cdots$$

- Previous result can be used by choosing $f(\mathbf{x}) = L(\mathbf{x}) - r \ln \lambda_j(\mathbf{x})$

Moments of Eigenvalues

After some simplifications

$$\mu_j^{(r)} \approx (2\pi)^{m/2} \lambda_j^r(\boldsymbol{\theta}) e^{-L(\boldsymbol{\theta})}$$

$$\left\| \mathrm{D}_L(\boldsymbol{\theta}) + \frac{1}{r}\mathrm{d}_L(\boldsymbol{\theta})\mathrm{d}_L(\boldsymbol{\theta})^T - \frac{r}{\lambda_j(\boldsymbol{\theta})}\mathrm{D}_{\lambda_j}(\boldsymbol{\theta}) \right\|^{-1/2}$$

$$r = 1, 2, 3, \cdots$$

θ is obtained from:

$$\mathrm{d}_{\lambda_j}(\boldsymbol{\theta}) r = \lambda_j(\boldsymbol{\theta}) \mathrm{d}_L(\boldsymbol{\theta})$$

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Moments of Eigenvalues

- Mean of the eigenvalues:

$$\widehat{\lambda}_j = \mu_j^{(1)} = \lambda_j(\boldsymbol{\theta}) e^{-L(\boldsymbol{\theta})} \left\| \mathbf{D}_L(\boldsymbol{\theta}) + \mathbf{d}_L(\boldsymbol{\theta})\mathbf{d}_L(\boldsymbol{\theta})^T - \mathbf{D}_{\lambda_j}(\boldsymbol{\theta})/\lambda_j(\boldsymbol{\theta}) \right\|^{-1/2}$$

- Central moments of the eigenvalues:

$$\mathrm{E}\left[\left(\lambda_j - \widehat{\lambda}_j\right)^r\right] = \sum_{k=0}^{r} \binom{r}{k} (-1)^{r-k} \mu_j^{(k)} \widehat{\lambda}_j^{r-k}$$

Multivariate Gaussian Case

$$L(\mathbf{x}) = \frac{m}{2}\ln(2\pi) + \frac{1}{2}\ln\|\boldsymbol{\Sigma}\| + \frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})$$

so $\mathrm{d}_L(\mathbf{x}) = \boldsymbol{\Sigma}^{-1}\mathbf{x}$ and $\mathrm{D}_L(\mathbf{x}) = \boldsymbol{\Sigma}^{-1}$. Therefore:

$$\mu_j^{(r)} \approx \lambda_j^r(\boldsymbol{\theta})e^{-\frac{1}{2}(\boldsymbol{\theta}-\boldsymbol{\mu})^T\boldsymbol{\Sigma}^{-1}(\boldsymbol{\theta}-\boldsymbol{\mu})} \left\|\mathbf{I} + \frac{1}{r}\boldsymbol{\theta}\boldsymbol{\theta}^T\boldsymbol{\Sigma}^{-1} - \frac{r}{\lambda_j(\boldsymbol{\theta})}\boldsymbol{\Sigma}\mathrm{D}_{\lambda_j}(\boldsymbol{\theta})\right\|^{-1/2}$$

where $\boldsymbol{\theta} = \frac{r}{\lambda_j(\boldsymbol{\theta})}\boldsymbol{\Sigma}\mathrm{d}_{\lambda_j}(\boldsymbol{\theta})$

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Maximum Entropy pdf

Constraints for $u \in [0, \infty]$:

$$\int_0^\infty p_{\lambda_j}(u)du = 1$$

$$\int_0^\infty u^r p_{\lambda_j}(u)du = \mu_j^{(r)}, \quad r = 1, 2, 3, \cdots, n$$

Maximizing Shannon's measure of entropy $\mathcal{S} = -\int_0^\infty p_{\lambda_j}(u)\ln p_{\lambda_j}(u)du$, the pdf of λ_j is

$$p_{\lambda_j}(u) = e^{-\left\{\rho_0 + \sum_{i=1}^n \rho_i u^i\right\}} = e^{-\rho_0} e^{-\sum_{i=1}^n \rho_i u^i}, \quad u \geq 0$$

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Maximum Entropy pdf

Taking first two moments, the resulting pdf is a truncated Gaussian density function

$$p_{\lambda_j}(u) = \frac{1}{\sqrt{2\pi}\sigma_j \, \Phi\left(\widehat{\lambda}_j/\sigma_j\right)} \exp\left\{ -\frac{\left(u - \widehat{\lambda}_j\right)^2}{2\sigma_j^2} \right\}$$

where $\sigma_j^2 = \mu_j^{(2)} - \widehat{\lambda}_j^2$

- Ensures that the probability of any eigenvalues becoming negative is zero

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Maximum Entropy pdf

- With three moments

Pdf of jth eigenvalue

$$p_{\lambda_j}(u) \approx \frac{1}{\gamma_j} p_{\chi^2_{\nu_j}}\left(\frac{u-\eta_j}{\gamma_j}\right) = \frac{(u-\eta_j)^{\nu_j/2-1} e^{-(u-\eta_j)/2\gamma_j}}{(2\gamma_j)^{\nu_j/2}\Gamma(\nu_j/2)}$$

The constants η_j, γ_j, and ν_j are such that the first three moments of λ_j are the same.

HDMR

Input $\boldsymbol{x} \in \mathbb{R}^N$ → SYSTEM → Output $y(\boldsymbol{x}) \in \mathbb{R}$

$$y(\boldsymbol{x}) = \underbrace{\underbrace{\underbrace{y_0 + \sum_{i=1}^{N} y_i(x_i)}_{=\hat{y}_1(\boldsymbol{x})} + \sum_{\substack{i_1,i_2=1 \\ i_1<i_2}}^{N} y_{i_1 i_2}(x_{i_1}, x_{i_2})}_{=\hat{y}_2(\boldsymbol{x})} + \cdots + \sum_{\substack{i_1,\cdots,i_S=1 \\ i_1<\cdots<i_S}}^{N} y_{i_1\cdots i_S}(x_{i_1}, \cdots, x_{i_S})}_{=\hat{y}_S(\boldsymbol{x})} + \cdots + y_{12\cdots N}(x_1, \cdots, x_N)$$

First-order → $=\hat{y}_1(\boldsymbol{x})$

$=\hat{y}_2(\boldsymbol{x})$ ← **Second-order (2D cooperative effects)**

S-order (SD cooperative effects)

Conjecture: **Component functions arising in proposed decomposition will exhibit insignificant *S*-order effects cooperatively when $S \rightarrow N$.**

(Rabitz & Alis, 1999; Alis & Rabitz, 2001)

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HDMR

- **Lower-order Approximations**

First-order Approximation

reference point

$$\hat{y}^I(\boldsymbol{x}) \equiv \hat{y}^I(x_1,\cdots,x_N) \equiv \sum_{i=1}^{N} \underbrace{y(c_1,\cdots,c_{i-1},x_i,c_{i+1},\cdots,c_N)}_{=y_i(x_i)} \underbrace{-(N-1)\,y(\boldsymbol{c})}_{=y_0}$$

Second-order Approximation

$$\hat{y}^{II}(\boldsymbol{x}) \equiv \hat{y}^{II}(x_1,\cdots,x_N) \equiv \sum_{\substack{i_1,i_2=1 \\ i_1<i_2}}^{N} \overbrace{y(c_1,\cdots,c_{i_1-1},x_{i_1},c_{i_1+1},\cdots,c_{i_2-1},x_{i_2},c_{i_2+1},\cdots,c_N)}^{=y_{i_1i_2}(x_{i_1},x_{i_2})}$$

$$+\sum_{i=1}^{N} \underbrace{-(N-2)\,y(c_1,\cdots,c_{i-1},x_i,c_{i+1},\cdots,c_N)}_{=y_i(x_i)} + \underbrace{\frac{(N-1)(N-2)}{2}\,y(\boldsymbol{c})}_{=y_0}$$

$$y_i(x_i) \cong \sum_{j=1}^{n} \phi_j(x_i)\, y(c_1,\cdots,c_{i-1},x_i^{(j)},c_{i+1},\cdots,c_N)$$

$$y_{i_1i_2}(x_{i_1},x_{i_2}) \cong \sum_{j_2=1}^{n}\sum_{j_1=1}^{n} \phi_{j_1j_2}(x_{i_1},x_{i_2})\, y(c_1,\cdots,c_{i_1-1},x_{i_1}^{(j_1)},c_{i_1+1},\cdots,c_{i_2-1},x_{i_2}^{(j_2)},c_{i_2+1},\cdots,c_N)$$

Interpolation function

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Convergence Issue

- Two-dimensional Taylor Series Expansion

$$y(x_1,x_2)=y(c_1,c_2)+\frac{\partial y(c_1,c_2)}{\partial x_1}(x_1-c_1)+\frac{\partial y(c_1,c_2)}{\partial x_2}(x_2-c_2)+\frac{\partial^2 y(c_1,c_2)}{\partial x_1^2}(x_1-c_1)^2$$
$$+\frac{\partial^2 y(c_1,c_2)}{\partial x_2^2}(x_2-c_2)^2+\frac{\partial^2 y(c_1,c_2)}{\partial x_1\partial x_2}(x_1-c_1)(x_2-c_2)+\cdots$$

Taylor expansion at $x_1 = c_1$ and $x_2 = c_2$

- One-dimensional Taylor Series Expansion

$$y(x_1,c_2)=y(c_1,c_2)+\frac{\partial y(c_1,c_2)}{\partial x_1}(x_1-c_1)+\frac{\partial^2 y(c_1,c_2)}{\partial x_1^2}(x_1-c_1)^2+\cdots$$

Taylor expansion at $x_1 = c_1$

$$y(c_1,x_2)=g(c_1,c_2)+\frac{\partial y(c_1,c_2)}{\partial x_2}(x_2-c_2)+\frac{\partial^2 y(c_1,c_2)}{\partial x_2^2}(x_2-c_2)^2+\cdots$$

Taylor expansion at $x_2 = c_2$

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(Li et.al., 2001)

Convergence Issue

- Two-dimensional Taylor Series Expansion

$$y(x_1,x_2)=y(c_1,c_2)+\frac{\partial y(c_1,c_2)}{\partial x_1}(x_1-c_1)+\frac{\partial y(c_1,c_2)}{\partial x_2}(x_2-c_2)+\frac{\partial^2 y(c_1,c_2)}{\partial x_1^2}(x_1-c_1)^2$$
$$+\frac{\partial^2 y(c_1,c_2)}{\partial x_2^2}(x_2-c_2)^2+\frac{\partial^2 y(c_1,c_2)}{\partial x_1\partial x_2}(x_1-c_1)(x_2-c_2)+\cdots$$

2D cooperative effect

- Sum of Two One-dimensional Taylor Series

$$y(x_1,c_2)+y(c_1,x_2)-y(c_1,c_2)=y(c_1,c_2)+\frac{\partial y(c_1,c_2)}{\partial x_1}(x_1-c_1)+\frac{\partial y(c_1,c_2)}{\partial x_2}(x_2-c_2)$$
$$+\frac{\partial^2 y(c_1,c_2)}{\partial x_1^2}(x_1-c_1)^2+\frac{\partial^2 y(c_1,c_2)}{\partial x_2^2}(x_2-c_2)^2+\cdots$$

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(Li et.al., 2001)

Errors in HDMR Approximation

- Residual Error

$$y(\boldsymbol{x}) - \hat{y}(\boldsymbol{x}) = \sum_{j_2}^{\infty}\sum_{j_1}^{\infty}\frac{1}{j_1!\,j_2!}\sum_{i_1<i_2}\frac{\partial^{j_1+j_2}y}{\partial x_{i_1}^{j_1}\partial x_{i_2}^{j_2}}(\boldsymbol{c})\left(x_{i_1}-c_{i_1}\right)^{j_1}\left(x_{i_2}-c_{i_2}\right)^{j_2}$$

Exact Approximate

- $\hat{y}(x)$ represents reduced dimensional approximation, because only N number of 1-dimensional model approximation are required, as opposed to one N-dimensional approximation in $y(x)$.
- If higher partial derivatives are negligibly small, $\hat{y}(x)$ provides a convenient approximation of $y(x)$
- First-order HDMR expansion is the sum of all Taylor series terms, which contains only variable x_i. Similarly, second-order HDMR expansion is the sum of all Taylor series terms, which contains only variable x_i and x_j. Therefore any truncated HDMR expansion provides better approximation of $y(x)$ than any truncated Taylor series (e.g., FORM/SORM).

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(Li et.al., 2001)

HDMR (Continued)

First-order Approximation

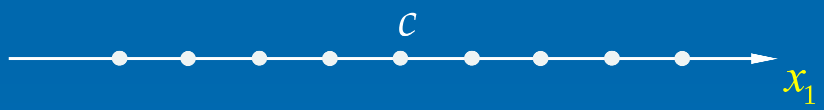
Reference point

$$\hat{y}_1(\boldsymbol{x}) \cong \sum_{i=1}^{N} \sum_{j=1}^{n} \phi_j(x_i)\, y(c_1,\cdots,c_{i-1},x_i^{(j)},c_{i+1},\cdots,c_N) - (N-1)\, y(\boldsymbol{c})$$

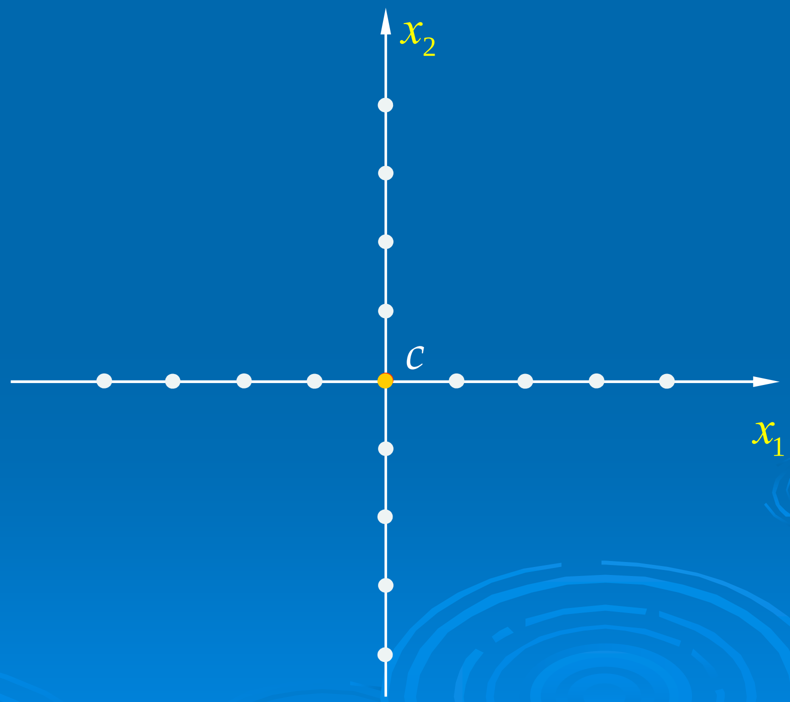
coefficients

coefficients

Interpolation function



One Variable



Two Variable

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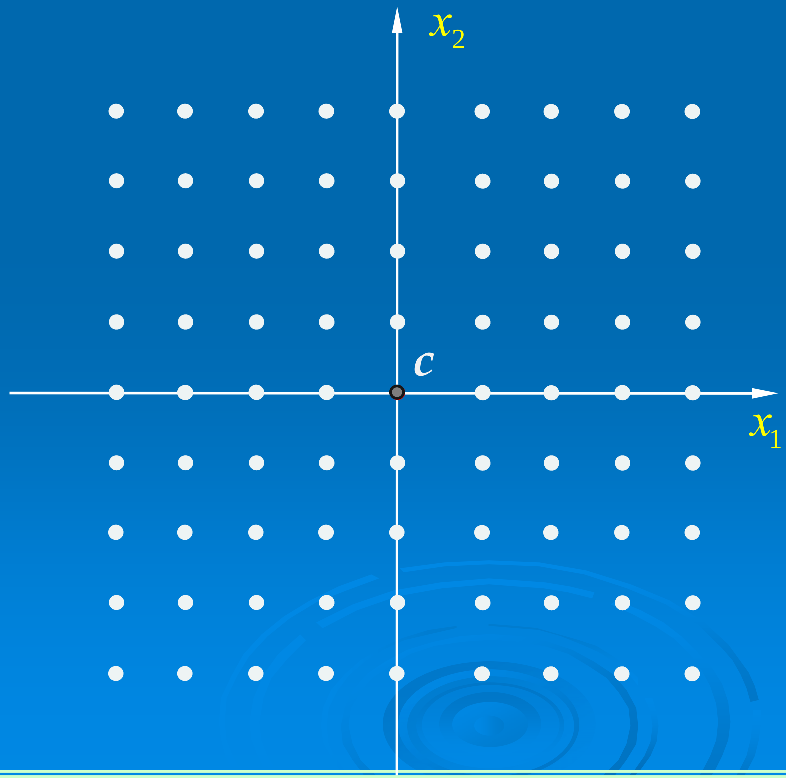
HDMR (Continued)

Second-order Approximation

$$\hat{y}_2(\boldsymbol{x}) \cong \sum_{\substack{i_1,i_2=1 \\ i_1<i_2}}^{N} \sum_{j_2=1}^{n}\sum_{j_1=1}^{n} \phi_{j_1 j_2}(x_{i_1}, x_{i_2}) \underbrace{y(c_1,\cdots,c_{i_1-1},x_{i_1}^{(j_1)},c_{i_1+1},\cdots,c_{i_2-1},x_{i_2}^{(j_2)},c_{i_2+1},\cdots,c_N)}_{\text{coefficients}}$$

$$- (N-2)\sum_{i=1}^{N}\sum_{j=1}^{n}\phi_j(x_i)\underbrace{y(c_1,\cdots,c_{i-1},x_i^{(j)},c_{i+1},\cdots,c_N)} + \frac{(N-1)(N-2)}{2}\underbrace{y(\boldsymbol{c})}$$



Two Variables

HDMR (Continued)

- **Computational Effort (Calculating Coefficients)**

No. of FEA for a linear/nonlinear problem,

$y(\boldsymbol{c})$ ⟶ 1 FEA

$$y(c_1, \cdots, c_{i-1}, x_i^{(j)}, c_{i+1}, \cdots, c_N)$$
$$(i = 1, \cdots, N;\ j = 1, \cdots, n)$$

⟶ nN FEA

$$y(c_1, \cdots, c_{i_1-1}, x_{i_1}^{(j_1)}, c_{i_1+1}, \cdots, c_{i_2-1}, x_{i_2}^{(j_2)}, c_{i_2+1}, \cdots, c_N)$$
$$(i_1, i_2 = 1, \cdots, N;\ j_1, j_2 = 1, \cdots, n)$$

⟶ $N(N\text{-}1)n^2/2$ FEA

First-order: $(n\text{-}1)N + 1$ (linear)

Second-order: $N(N\text{-}1)(n\text{-}1)^2/2 + (n\text{-}1)N + 1$ (quadratic)

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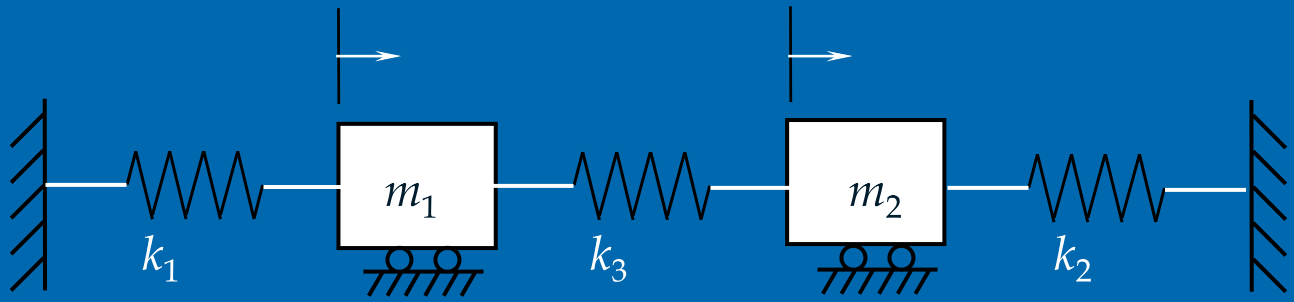
(Chowdhury & Rao, 2009)

Example 1: 2-DOF system



$$\boldsymbol{m} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\boldsymbol{k}(\boldsymbol{x}) = \begin{bmatrix} k_1(\boldsymbol{x}) + k_3 & -k_3 \\ -k_3 & k_2(\boldsymbol{x}) + k_3 \end{bmatrix}$$

$m_1 = 1.0\,\text{kg}$

$m_2 = 1.5\,\text{kg}$

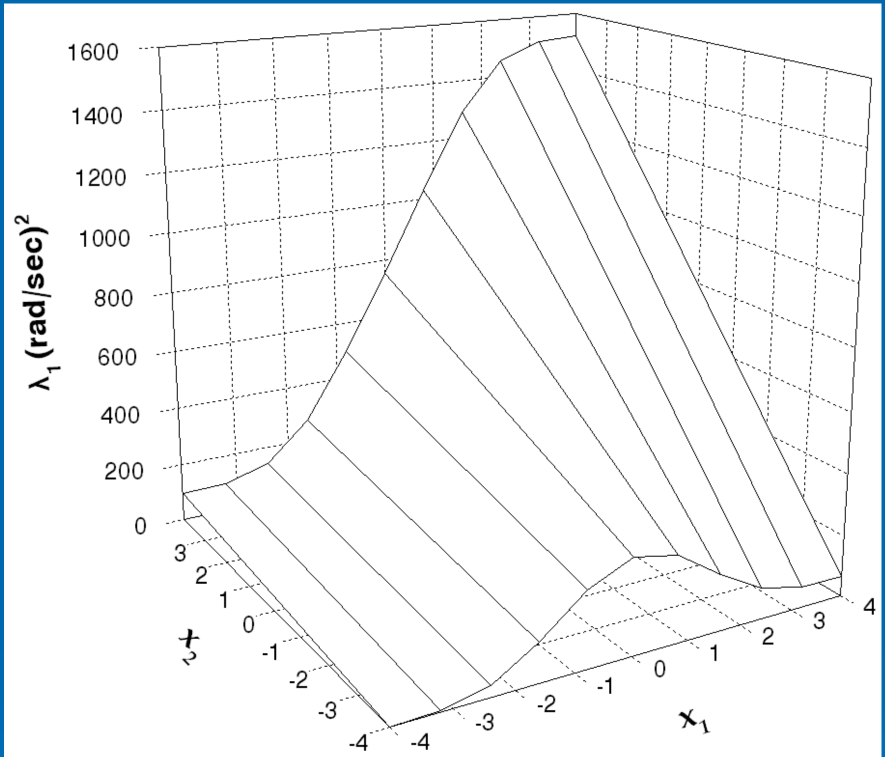
$k_1(\boldsymbol{x}) = 1000(1 + 0.25x_1)\,\text{N/m}$

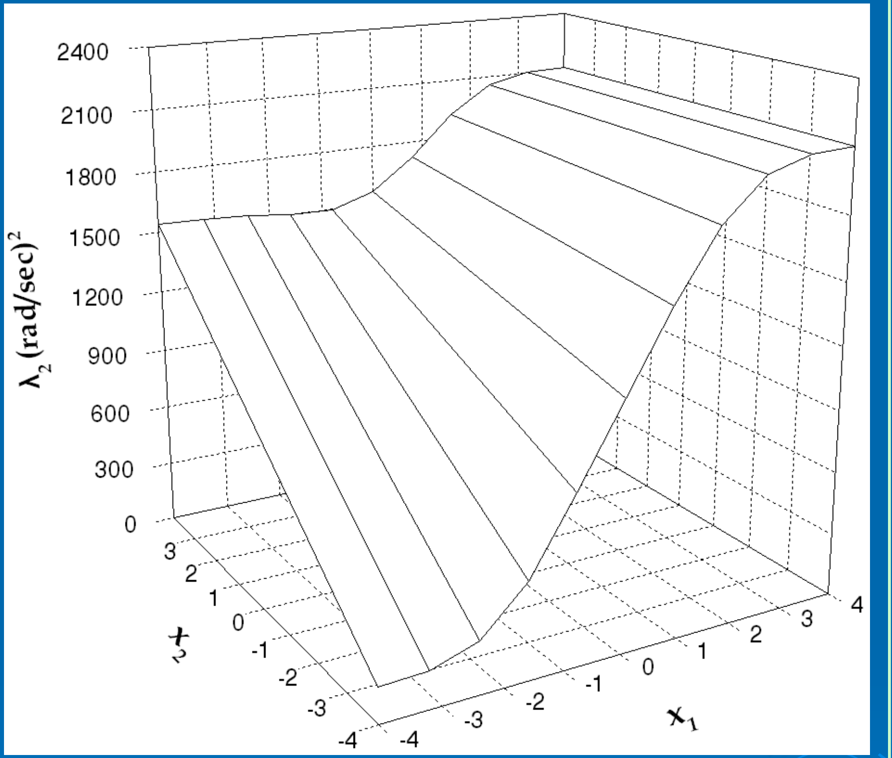
$k_2(\boldsymbol{x}) = 1100(1 + 0.25x_2)\,\text{N/m}$

$k_1(\boldsymbol{x}) = 100\,\text{N/m}$

$\boldsymbol{x} = \{x_1, x_2\}^T; \quad \boldsymbol{\mu}_x = \boldsymbol{0}, \boldsymbol{\Sigma}_x = \boldsymbol{I}$

Example 1: 2-DOF system





Exact eigenvalues for the 2-DOF system

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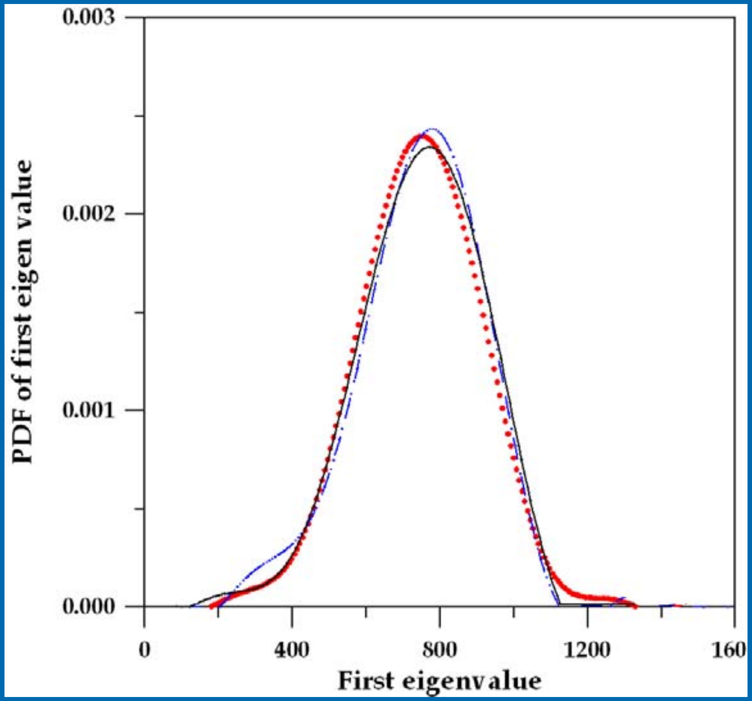
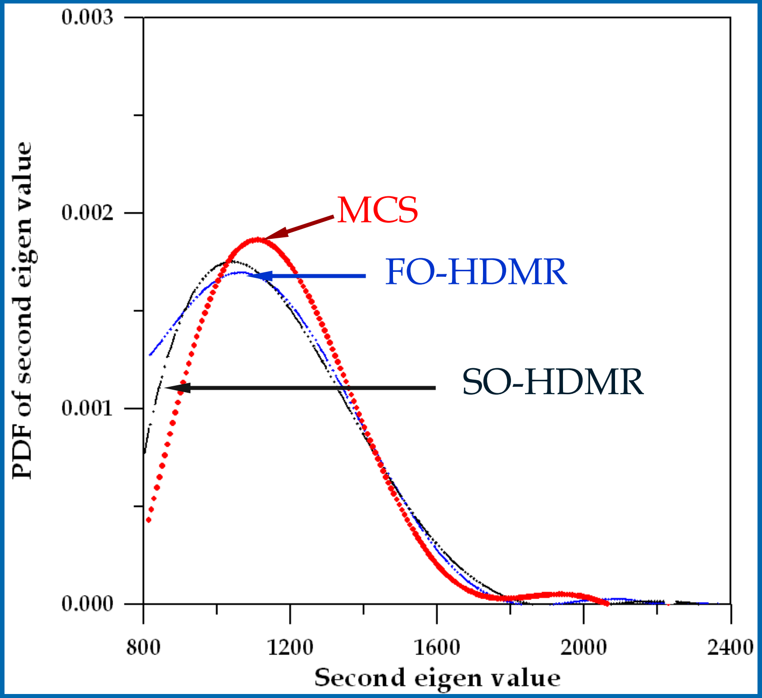
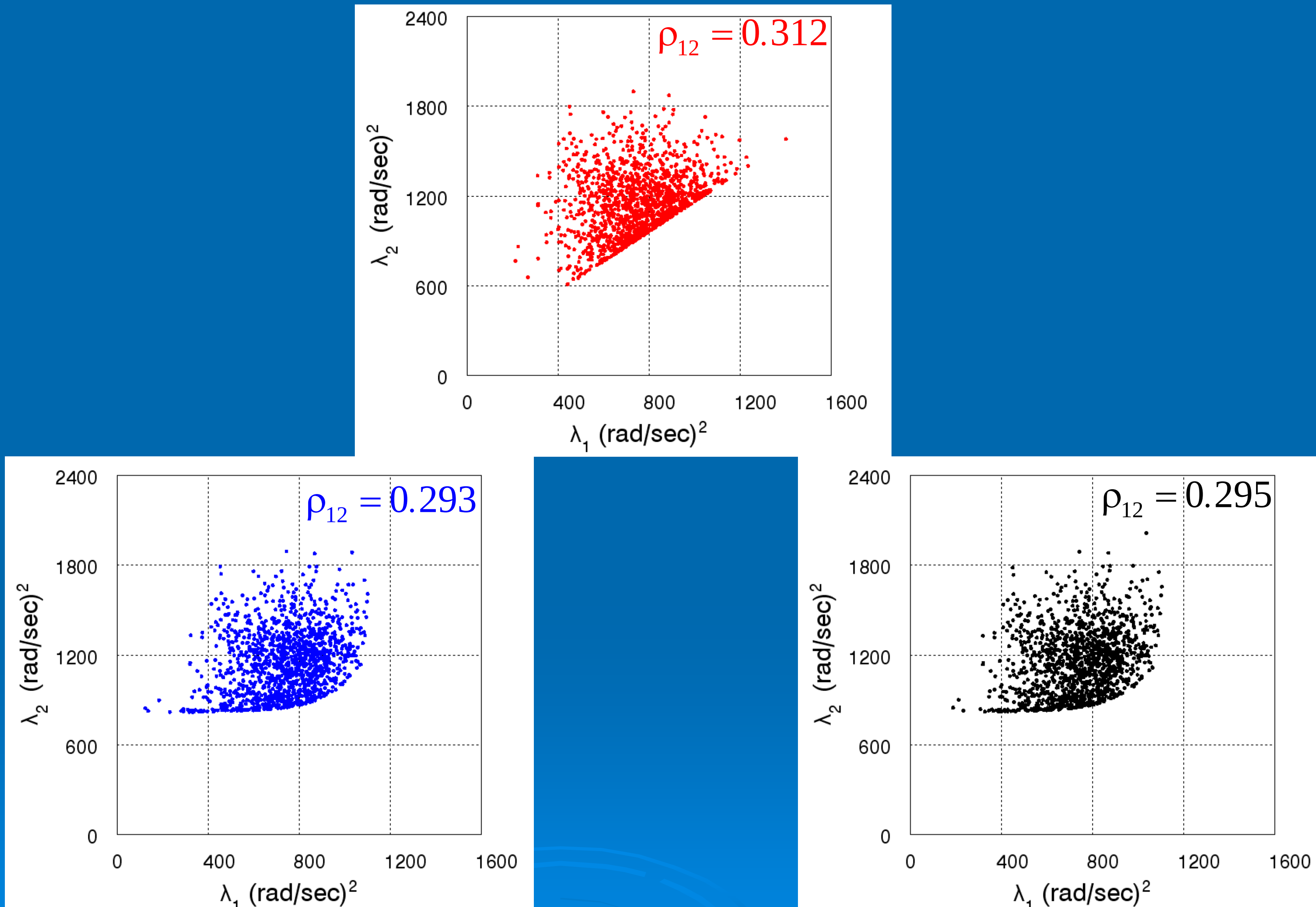
Example 1: 2-DOF system





Probability densities for the 2-DOF system

Example 1: 2-DOF system



Scattered plot of λ_1 & λ_2

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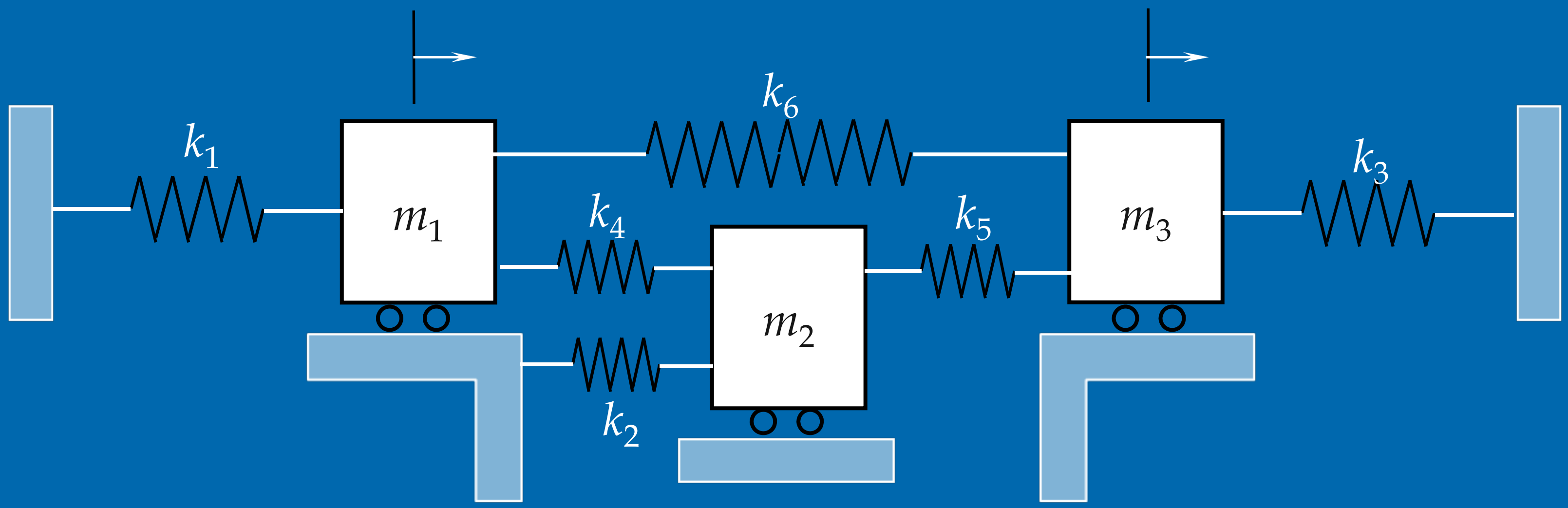
Example 2: 3-DOF system



Example 2: 3-DOF system

$$\boldsymbol{M}(\boldsymbol{x})=\begin{bmatrix} m_1(\boldsymbol{x}) & 0 & 0 \\ 0 & m_2(\boldsymbol{x}) & 0 \\ 0 & 0 & m_3(\boldsymbol{x}) \end{bmatrix}$$

$$k(\boldsymbol{x})=\begin{bmatrix} k_1(\boldsymbol{x})+k_4(\boldsymbol{x})+k_6(\boldsymbol{x}) & -k_4(\boldsymbol{x}) & -k_6(\boldsymbol{x}) \\ -k_4(\boldsymbol{x}) & k_4(\boldsymbol{x})+k_5(\boldsymbol{x})+k_2(\boldsymbol{x}) & -k_5(\boldsymbol{x}) \\ -k_6(\boldsymbol{x}) & -k_5(\boldsymbol{x}) & k_5(\boldsymbol{x})+k_3(\boldsymbol{x})+k_6(\boldsymbol{x}) \end{bmatrix}$$

$m_i(\boldsymbol{x})=\mu_i x_i;\ \ i=1,2,3$ with $\mu_i=1.0\,\text{kg};\qquad i=1,2,3$

$k_i(\boldsymbol{x})=\mu_{i+3}x_{i+3};\ \ i=1,\cdots,6$ with

$\mu_{i+3}=1.0\,\text{N/m};i=1,\cdots,5$

$\mu_9=3.0\,\text{N/m}(\text{Case1 \& Case 3});\mu_9=1.275\,\text{N/m}(\text{Case 2})$

$\boldsymbol{x}=\{x_1,\cdots,x_9\}^T;\qquad \boldsymbol{\mu}_{\boldsymbol{x}}=\boldsymbol{0},\Sigma_{\boldsymbol{x}}=\nu^2\boldsymbol{I}$

$\nu=0.15(\text{Case 1 \& Case 2});$

$\nu=0.30(\text{Case 3});$

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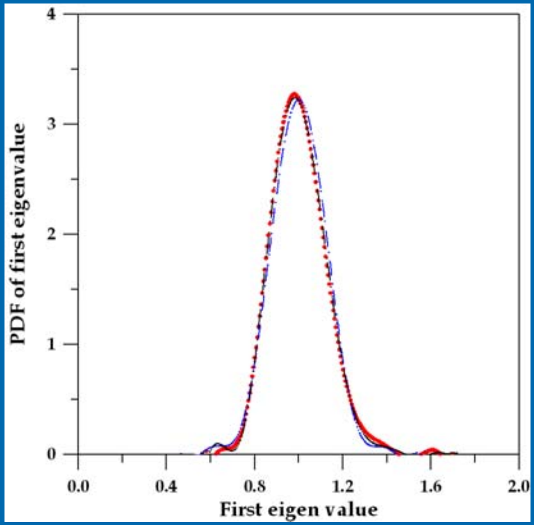
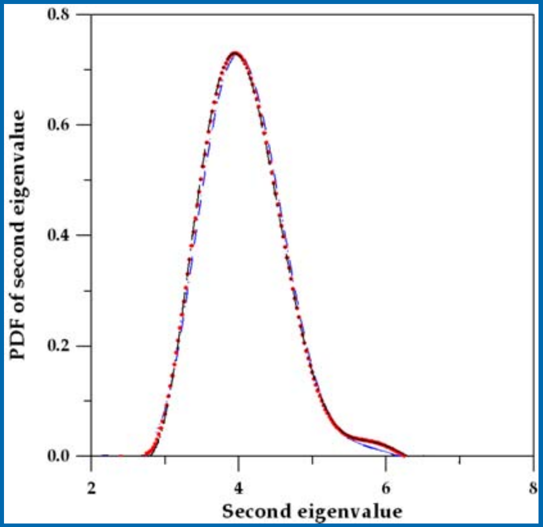
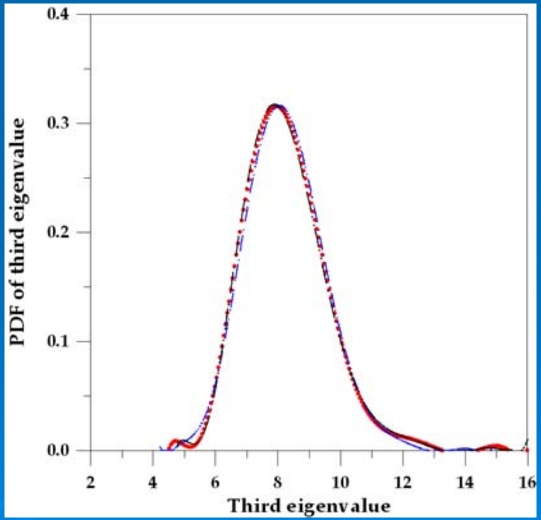
Case 1: Well separated engenvalues







Probability densities for the 3-DOF system

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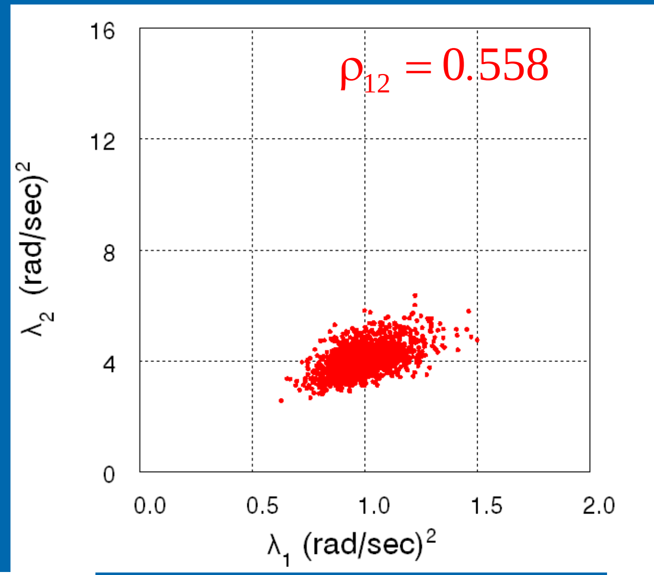
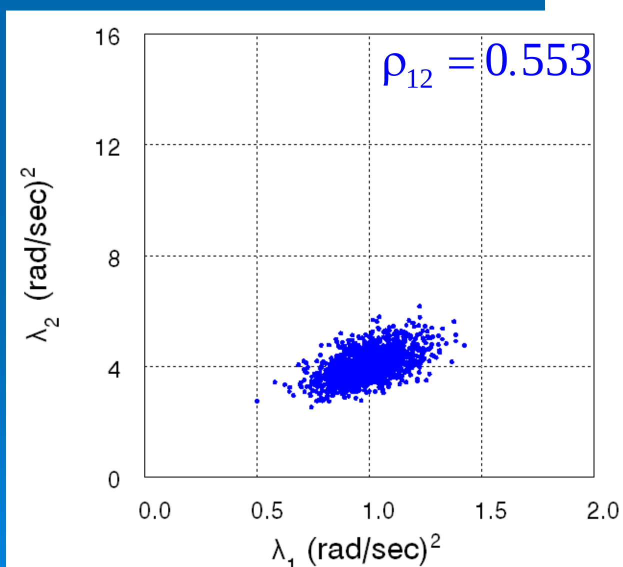
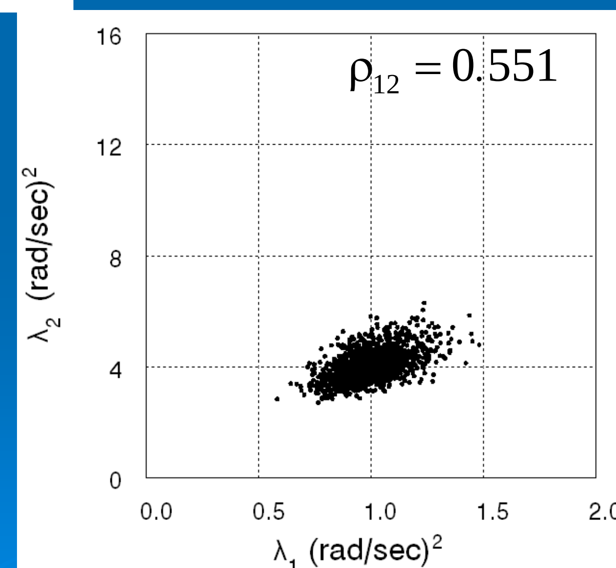
Case 1: Well separated engenvalues







Scattered plot of λ_1 & λ_2

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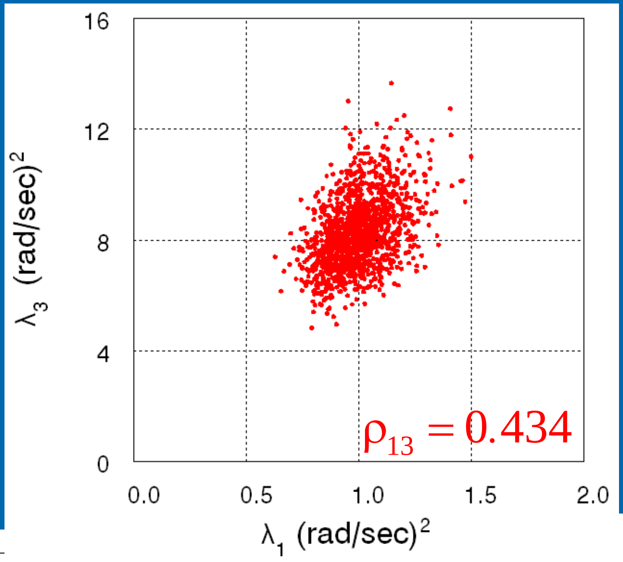
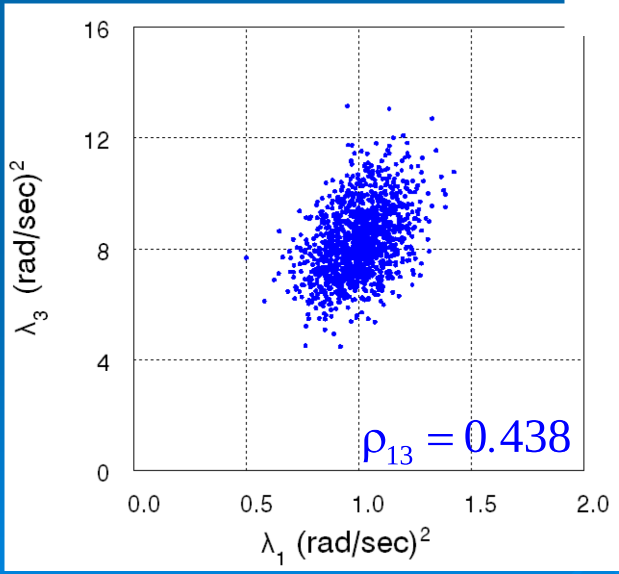
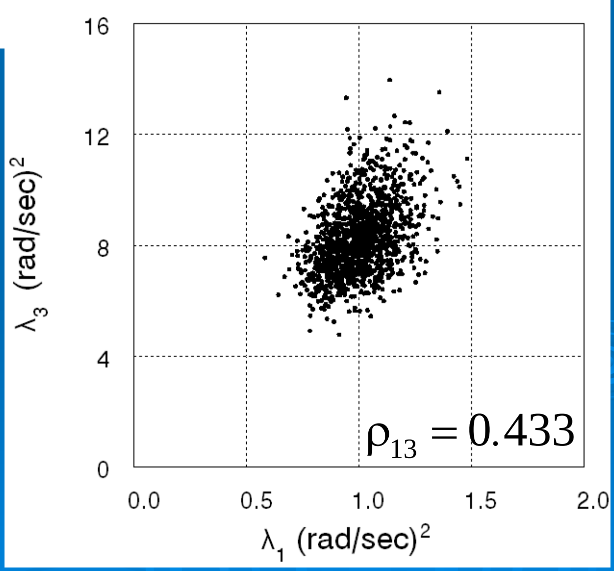
Case 1: Well separated engenvalues







Scattered plot of λ_1 & λ_3

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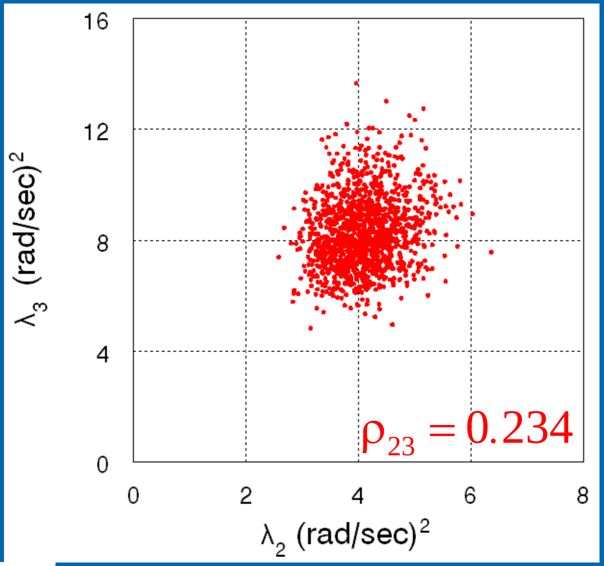
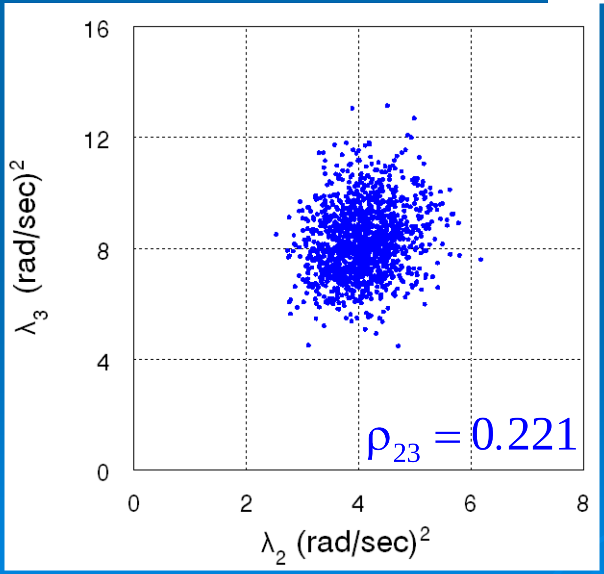
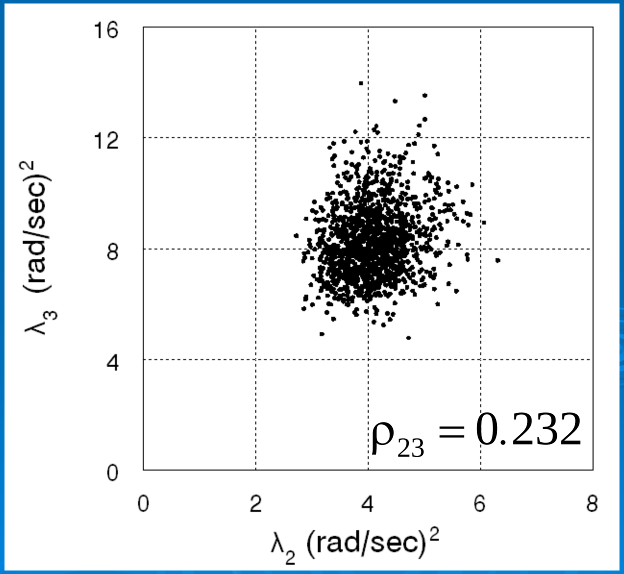
Case 1: Well separated engenvalues







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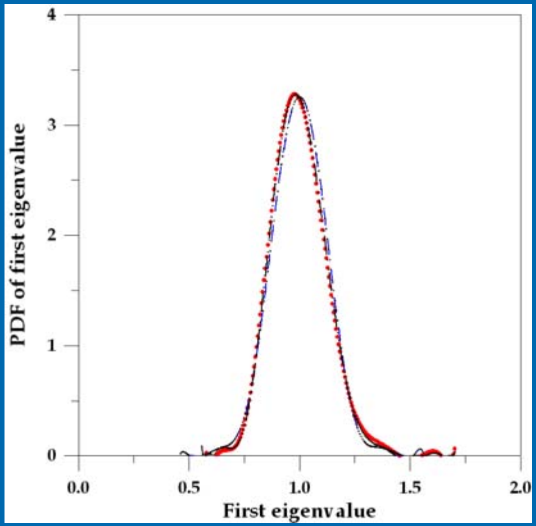
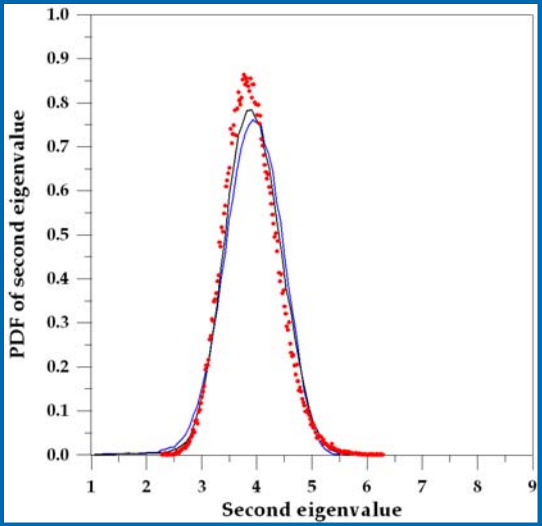
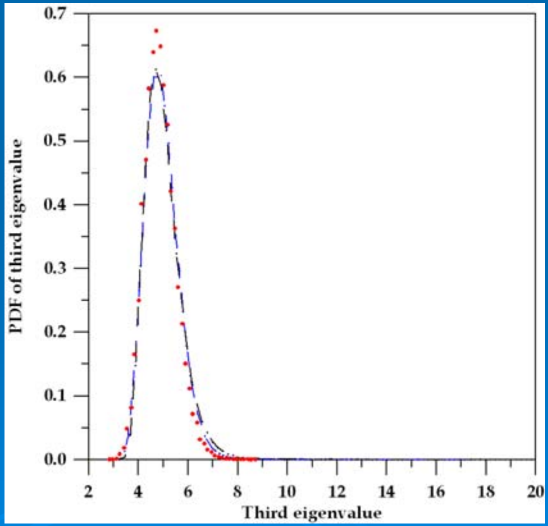
Scattered plot of λ_2 & λ_3

Case 2: Closely spaced engenvalues







Probability densities for the 3-DOF system

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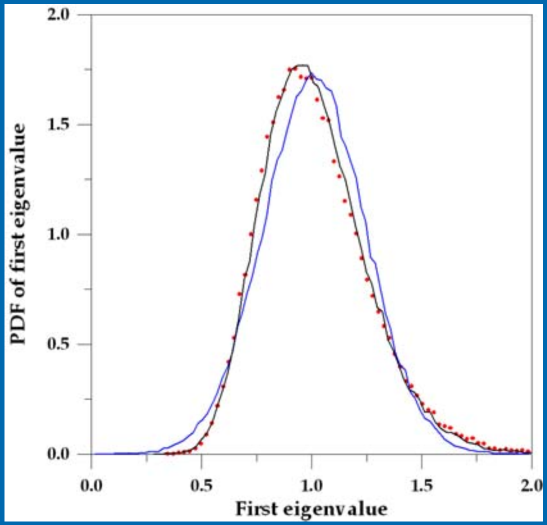
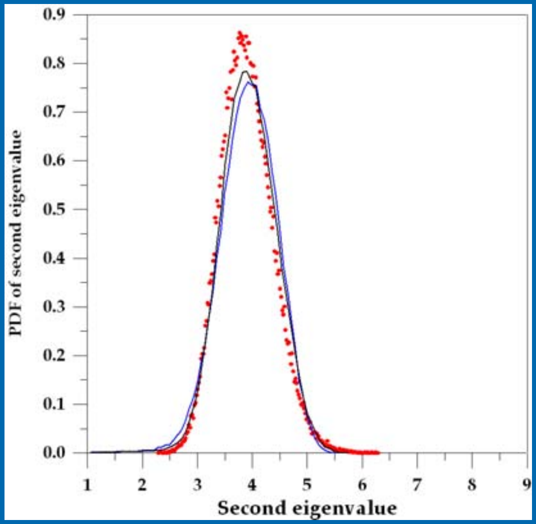
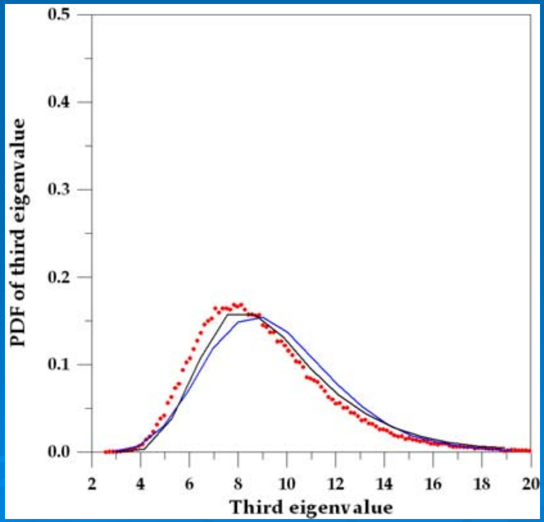
Case 3: Large statistical variation of input







Probability densities for the 3-DOF system

Conclusions

- The statistics of the eigenvalues of linear stochastic dynamic systems has been Considered
- HDMR approximation method has been developed for efficient scheme for random eigenvalue problems.
- Pdf of the eigenvalues are obtained using using the maximum entropy method
- Yields accurate and convergent solutions
- Future works will look into joint moments and pdf of the eigenvalues and eigenvectors

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Thank you

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