Random Eigenvalue Problems in Structural Dynamics: An Experimental Investigation

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Outline of the presentation

- A Brief Overview of Random Eigenvlaue Problems
- Random Eigenvalues of a Fixed-Fixed Beam
- Random Eigenvalues of a cantilever plate
- System Model and Experimental Setup
- Experimental methodology
- Eigenvalue Statistics
  - Experimental results
  - Monte Carlo simulation
- Conclusions & future directions

Ensembles of structural dynamical systems

Many structural dynamic systems are manufactured in a production line (nominally identical systems)

A complex structural dynamical system

Complex aerospace system can have millions of degrees of freedom and significant 'errors' and/or 'lack of knowledge' in its numerical (Finite Element) model

Sources of uncertainty

(a) parametric uncertainty - e.g., uncertainty in geometric parameters, friction coefficient, strength of the materials involved;
(b) model inadequacy - arising from the lack of scientific knowledge about the model which is a-priori unknown;
(c) experimental error - uncertain and unknown error percolate into the model when they are calibrated against experimental results;
(d) computational uncertainty - e.g, machine precession, error tolerance and the so called ‘h’ and ‘p’ refinements in finite element analysis, and
(e) model uncertainty - genuine randomness in the model such as uncertainty in the position and velocity in quantum mechanics, deterministic chaos.

Overview of Random Eigenvalue Problems

- EVP of Undamped or proportionally damped systems:

$$\mathbf{K}\boldsymbol{\phi}_j = \lambda_j \mathbf{M}\boldsymbol{\phi}_j \tag{1}$$

  λ_j: Eigenvalue (natural frequency squared)
  $\boldsymbol{\phi}_j$: Eigenvector (modeshape)
  $\mathbf{M}$ &$\mathbf{K}$ are symmetric and P.D random matrices $\Rightarrow \lambda_j$ real and positive.

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$$\mathbf{M} = \overline{\mathbf{M}} + \delta\mathbf{M} \quad \text{and} \quad \mathbf{K} = \overline{\mathbf{K}} + \delta\mathbf{K}. \tag{2}$$

  $\overline{(\bullet)}$: Nominal (deterministic) of of $(\bullet)$
  $\delta(\bullet)$: Random parts of $(\bullet)$.

Randomness

$$\mathbf{M} = \overline{\mathbf{M}} + \delta\mathbf{M} \quad \text{and} \quad \mathbf{K} = \overline{\mathbf{K}} + \delta\mathbf{K}.$$

- $\delta\mathbf{M}$ and $\delta\mathbf{K}$ are zero-mean random matrices.
- Small randomness assumption that preserve symmetry and P.D of $\mathbf{M}$ and $\mathbf{M}$.
- No assumptions on the *type* of randomness: need not be Gaussian, for example
- Fixed-Fixed beam with random placement of equal masses gives $\delta\mathbf{M} \neq \mathbf{0}$ $\delta\mathbf{K} = \mathbf{0}$
- Cantilever plate with random placement of random oscillators gives $\delta\mathbf{M} \neq \mathbf{0}$ $\delta\mathbf{K} \neq \mathbf{0}$

Fixed-Fixed Beam: Experiments

The test rig for the fixed-fixed beam
Actuator: Shaker, Sensors: Accelerometers

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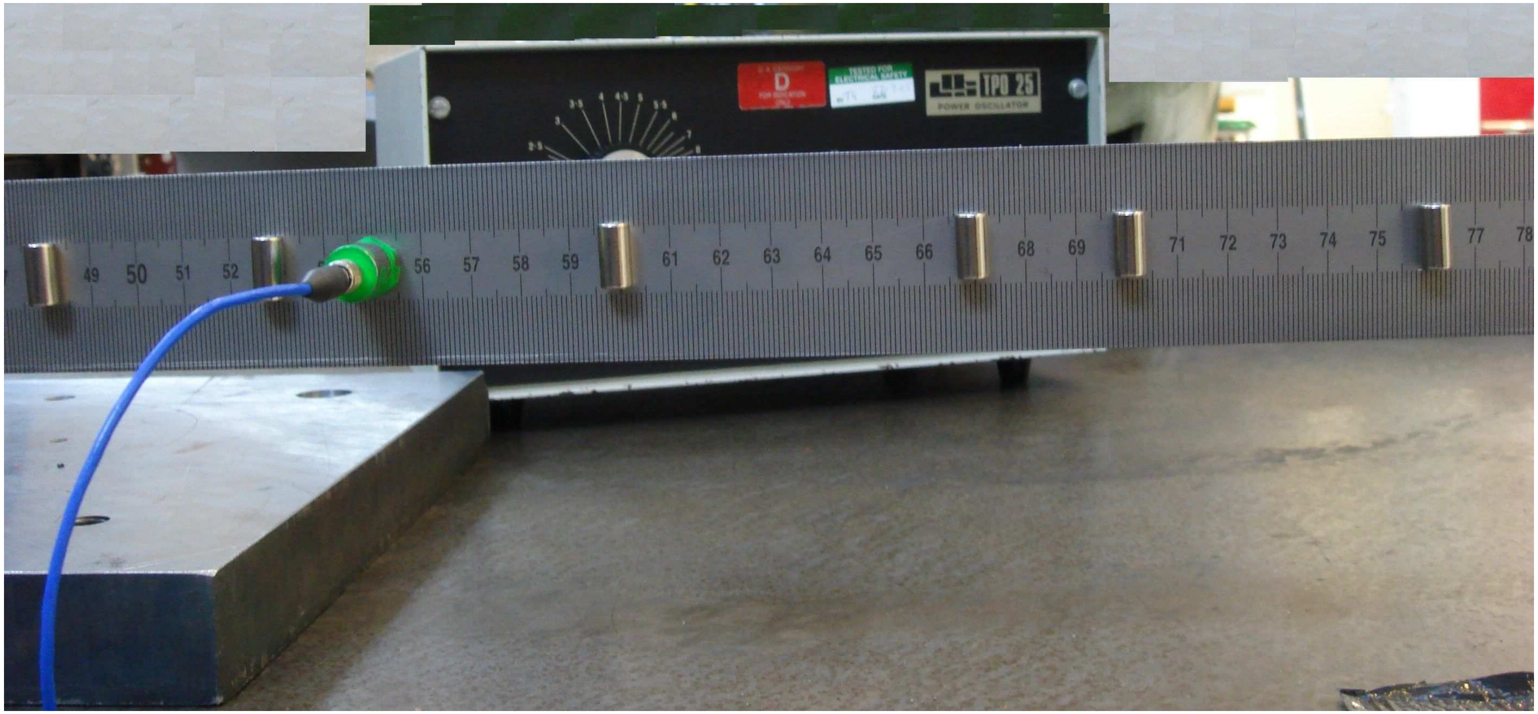
Fixed-Fixed Beam: Experiments



Attached masses (magnets) at random locations.
12 masses, each weighting 2g, are used.

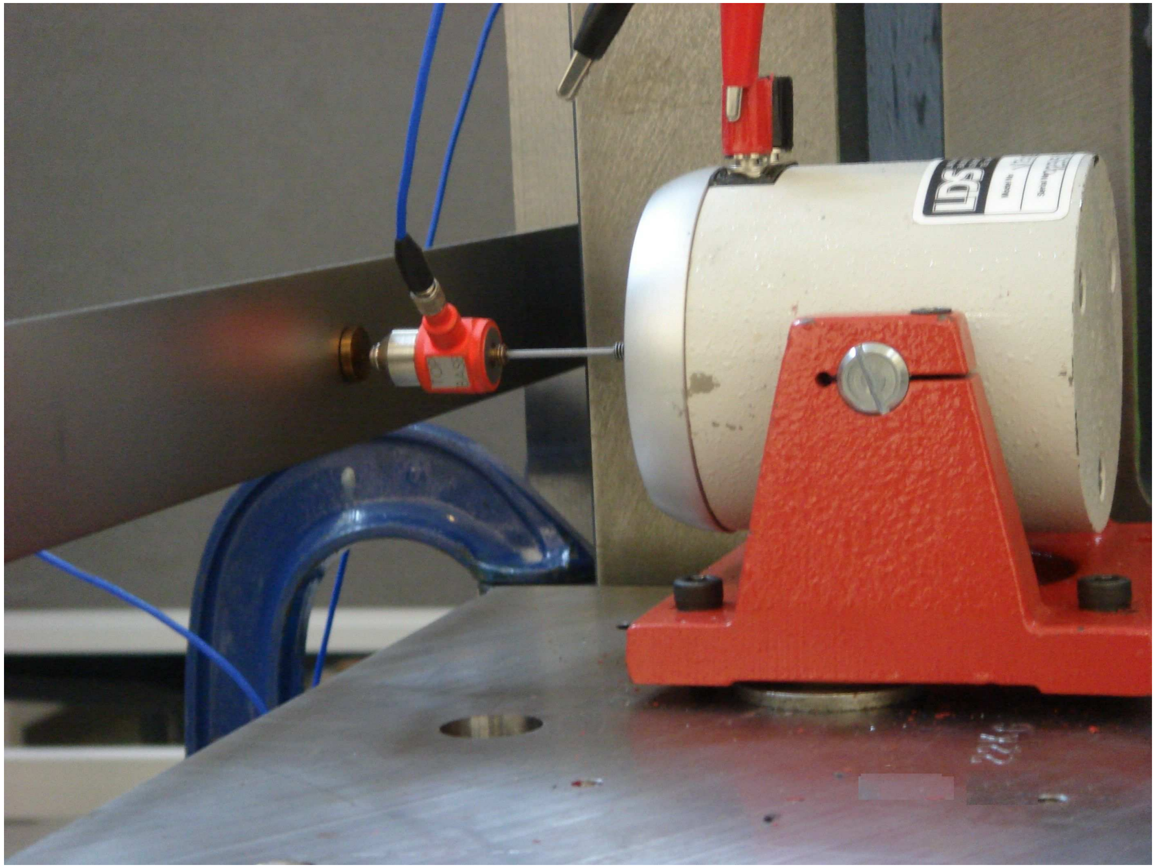
Fixed-Fixed Beam: Properties

Beam Properties	Numerical values
Length (L)	1200 mm
Width (b)	40.06 mm
Thickness (t_h)	2.05 mm
Mass density (ρ)	7800 Kg/m^3
Young's modulus (E)	2.0×10^5 MPa
Total weight	0.7687 Kg

Material and geometric properties of the beam.

Shaker as an Impulse Hammer

pulse rate: 20s & pulse width: 0.01s. Eliminate input uncertainties.

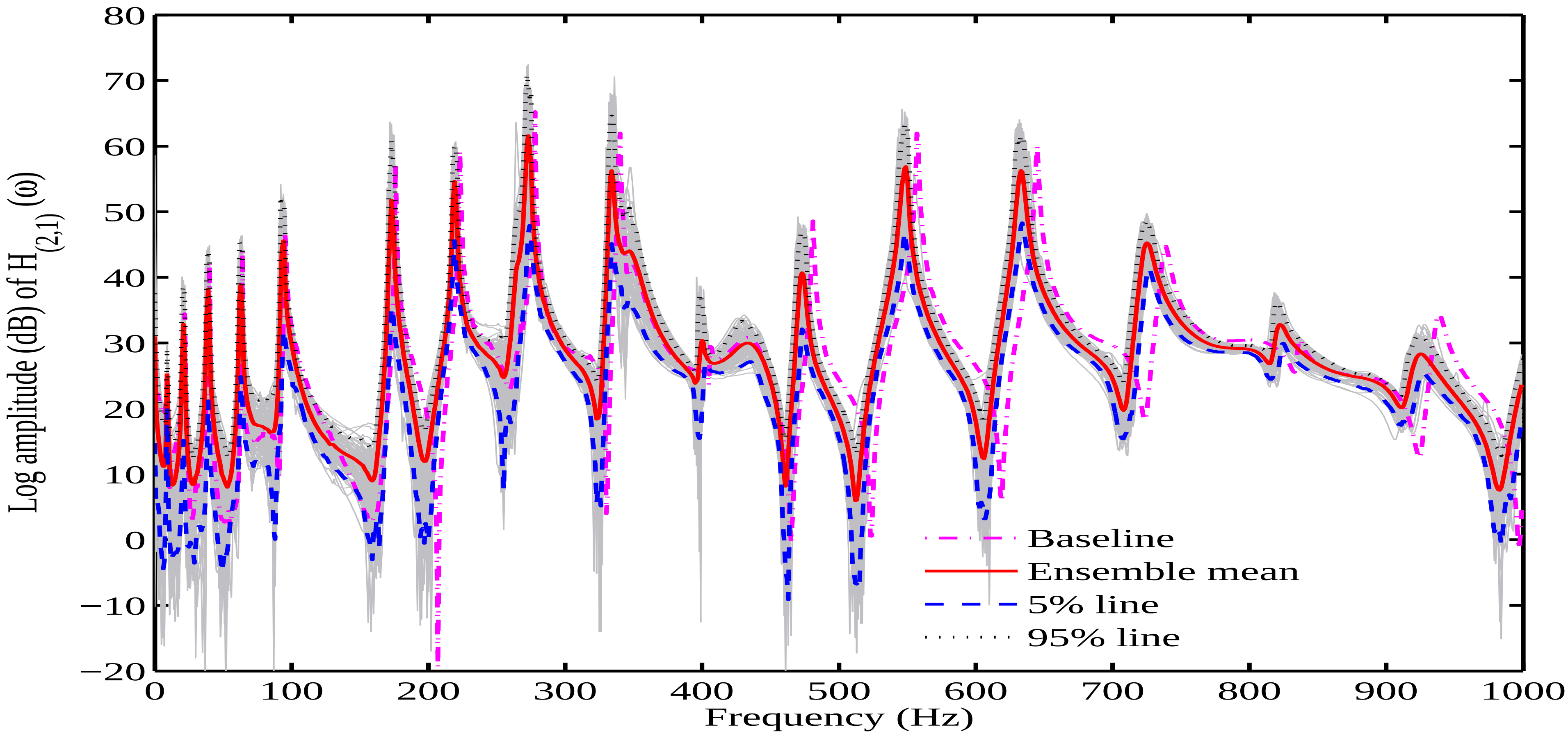


brass plate (2g) takes impact.

Experiments: Protocol

- Arrange the masses along the beam at random locations (computer generated)
- Measure impulse response at: 23 cm (Point1) 50 cm (Point2, also the actuation point) and 102 cm (Point3) from the left end of the beam in a 32 channel LMS™ system
- Transform to frequency domain to estimate frequency response function (FRF).
- Curvefit the FRF to estimate the natural frequencies ω_n and damping factors Q_n
  - Rational Fraction Polynomial (RFP) method
  - Nonlinear Leastsquares method
- Calculate the statistics of natural frequencies

Experiments: FRF at Point 1 (23 cm from the left end)



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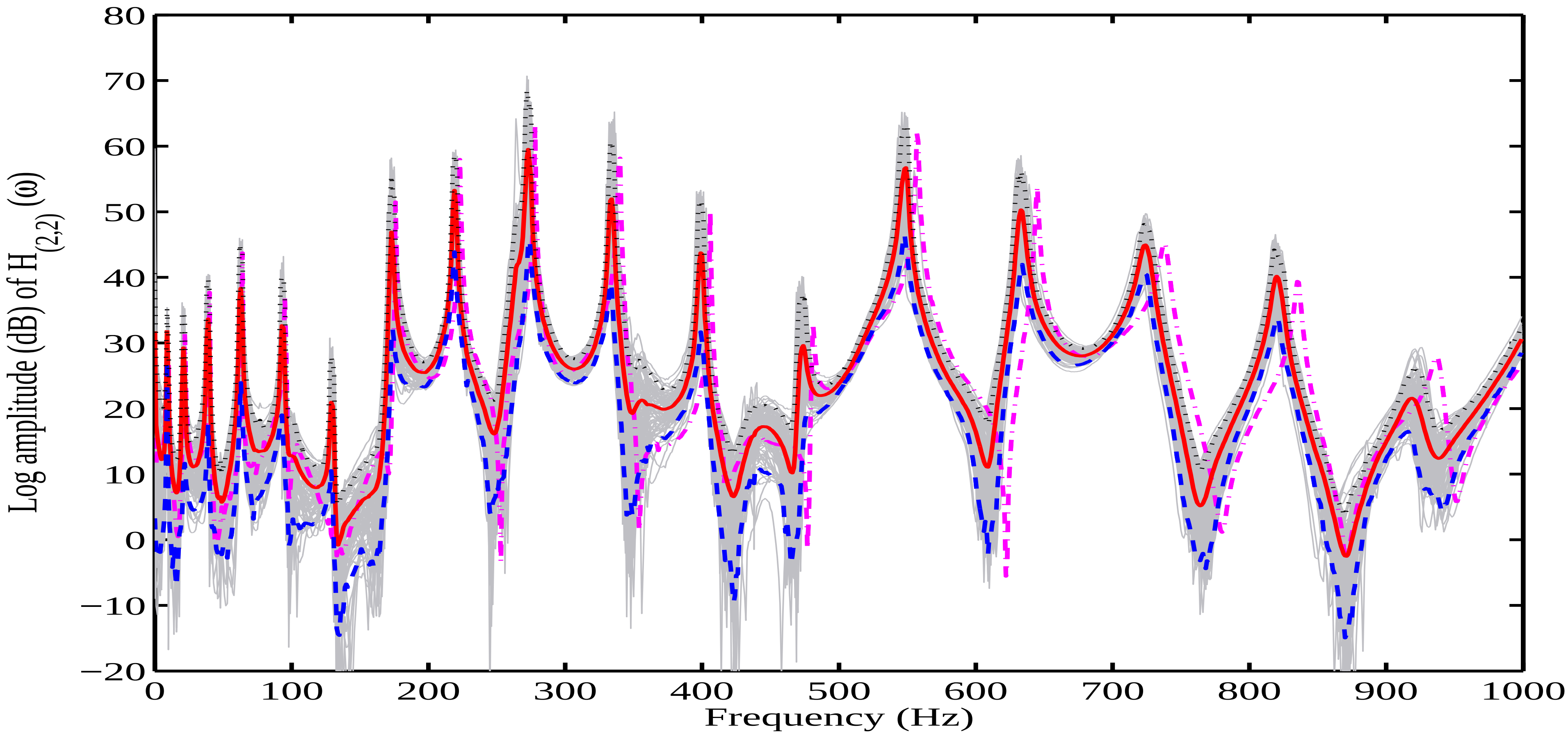
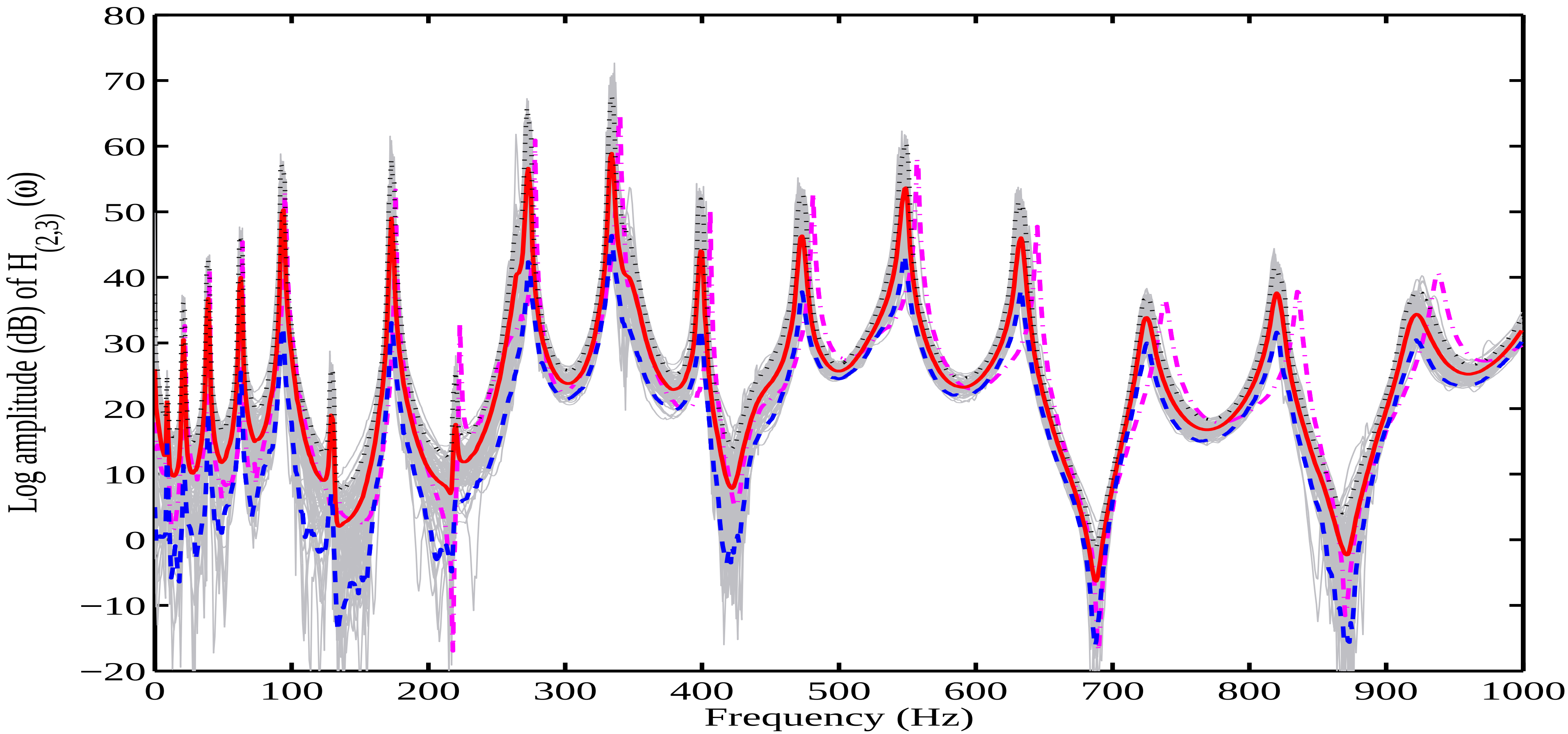
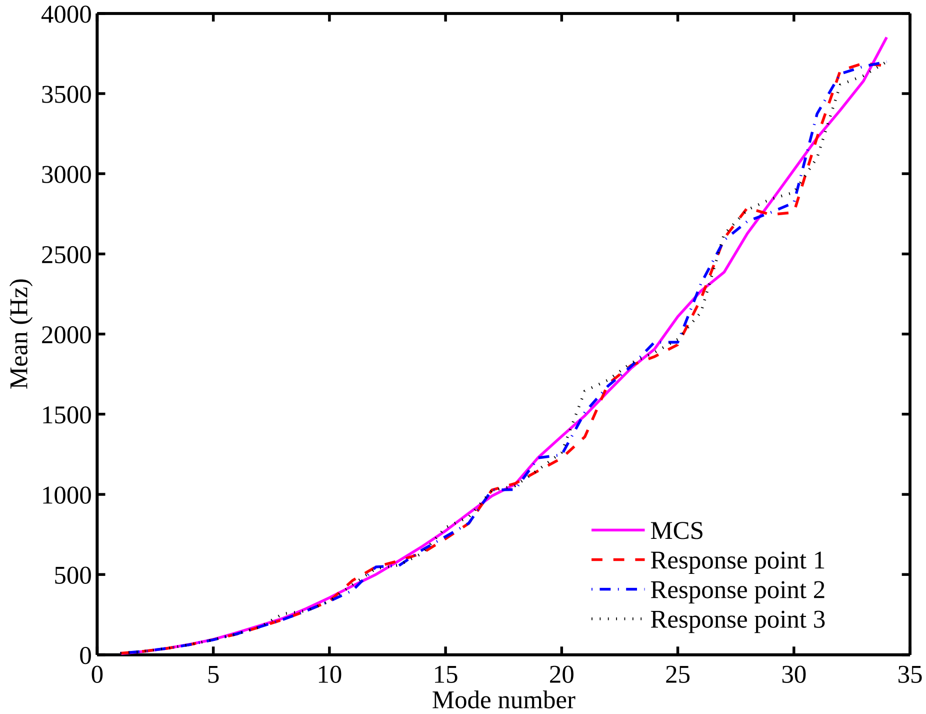
Experiments: FRF at point 2 (the driving point FRF, 50 cm from the left end)

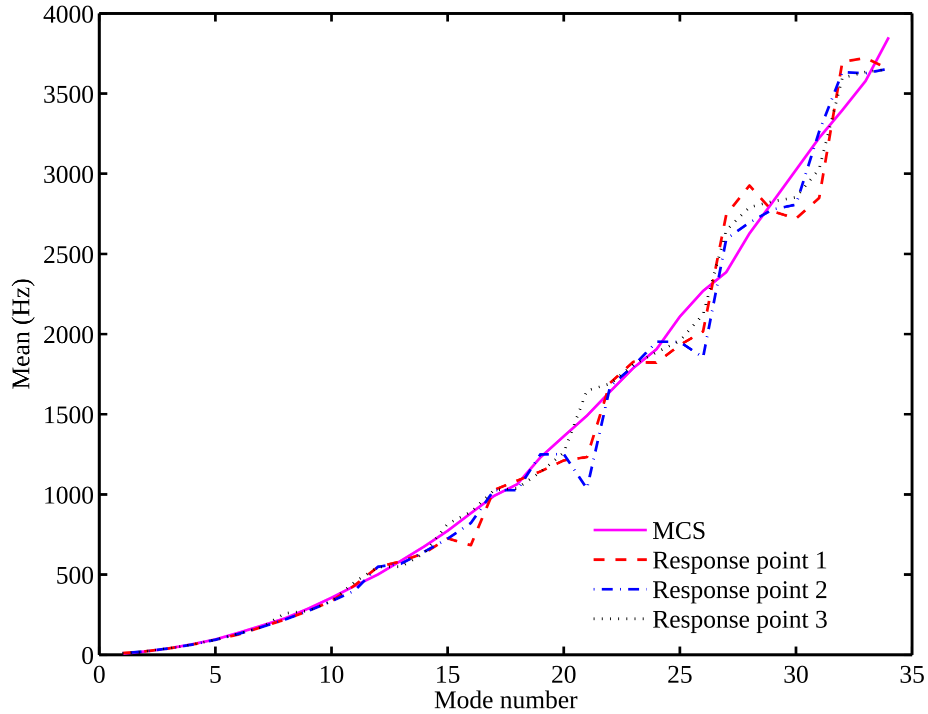


Experiments: FRF at point 3 (102 cm from the left end)



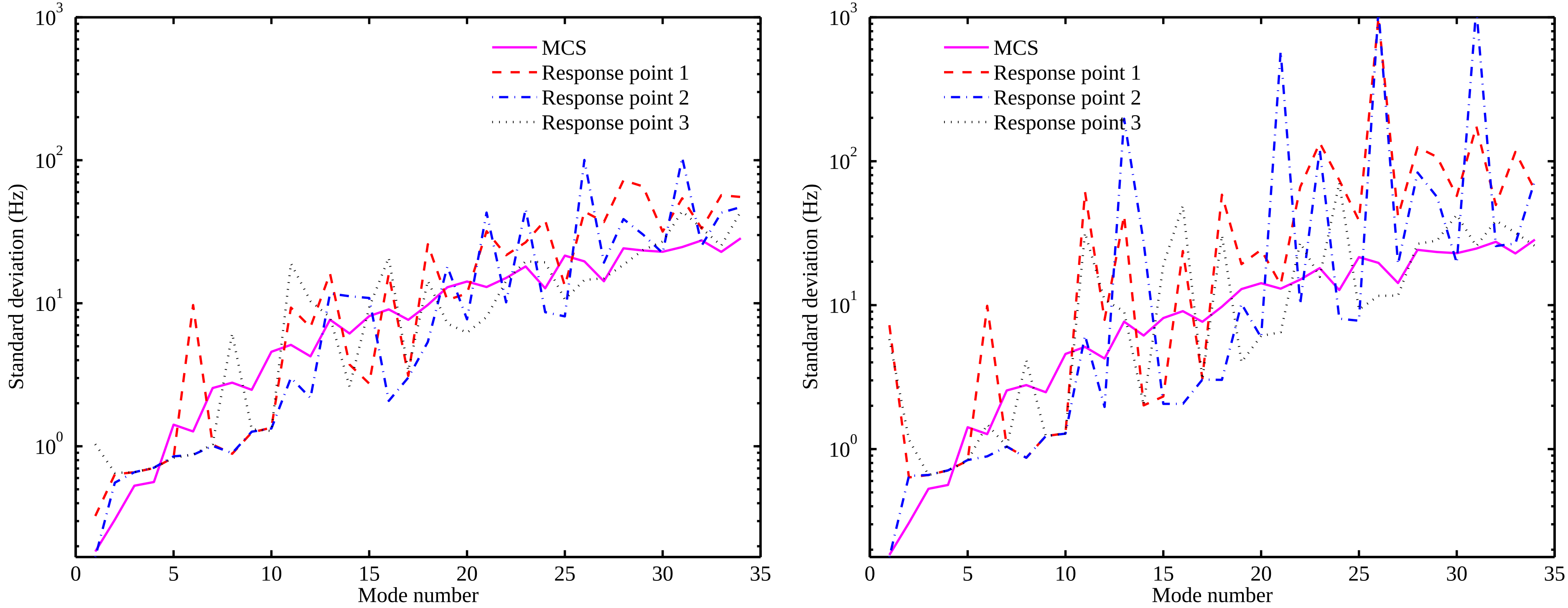
Ensemble Mean





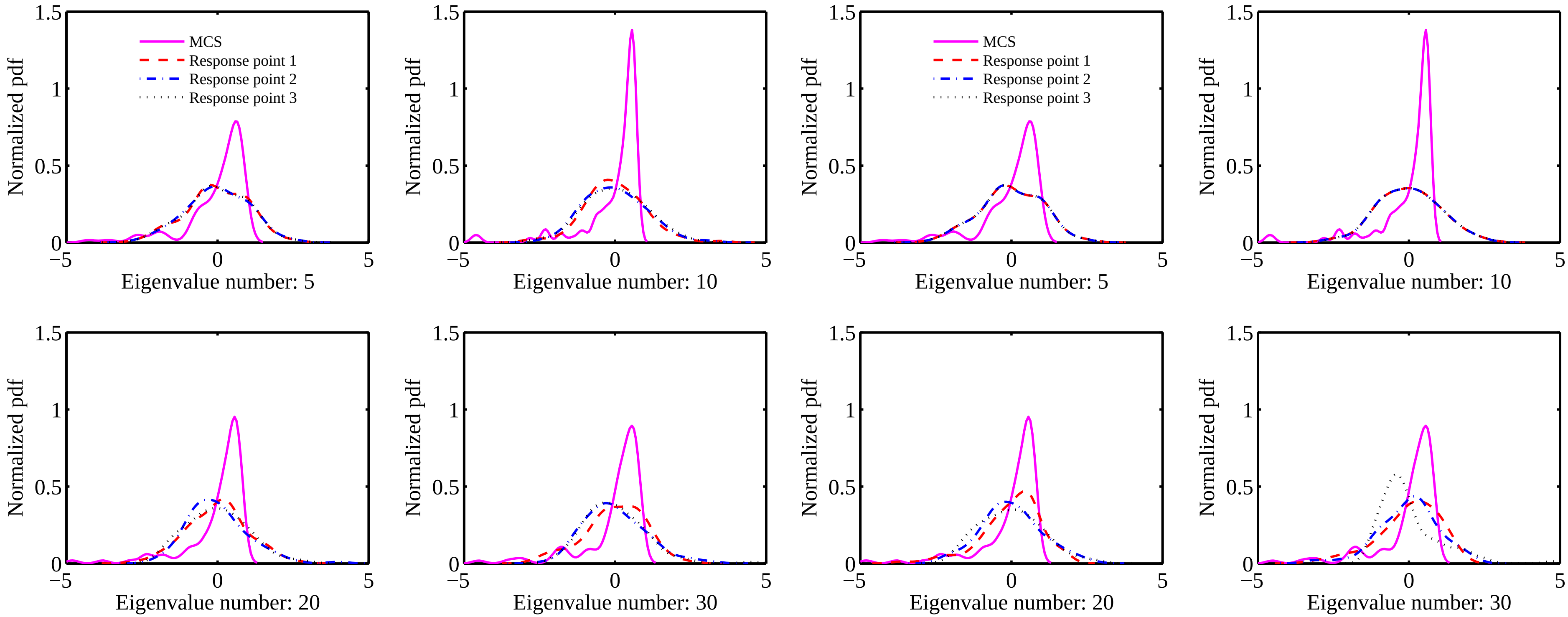
Left: RFP; Right: Nonlinear Leastsquares

Standard Deviation



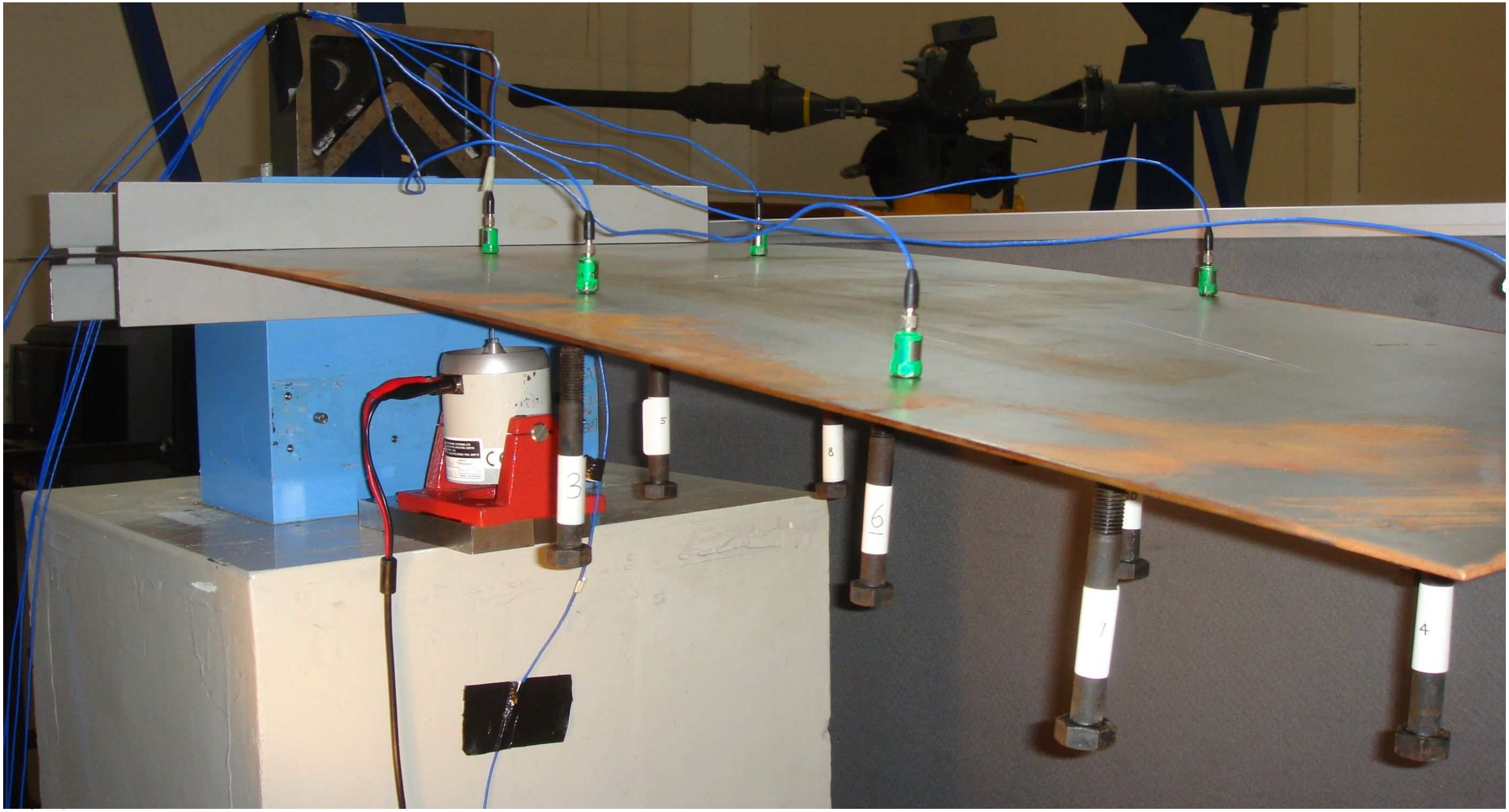
Left: RFP; Right: Nonlinear Least-squares

PDFs



Left: RFP; Right: Nonlinear Least-squares

Cantilever plate



The test rig for the cantilever plate: Front View

Cantilever plate

Cantilever plate: Properties

Plate Properties	Numerical values
Length (L)	998 mm
Width (b)	530 mm
Thickness (t_h)	3.0 mm
Mass density (ρ)	7800 Kg/m^3
Young's modulus (E)	2.0×10^5 MPa
Total weight	12.38 Kg

Table 1: Material and geometric properties of the cantilever plate considered for the experiment

Attached Oscillators

Attached oscillators at random locations. The spring stiffness varies

Properties of Attached Oscillators

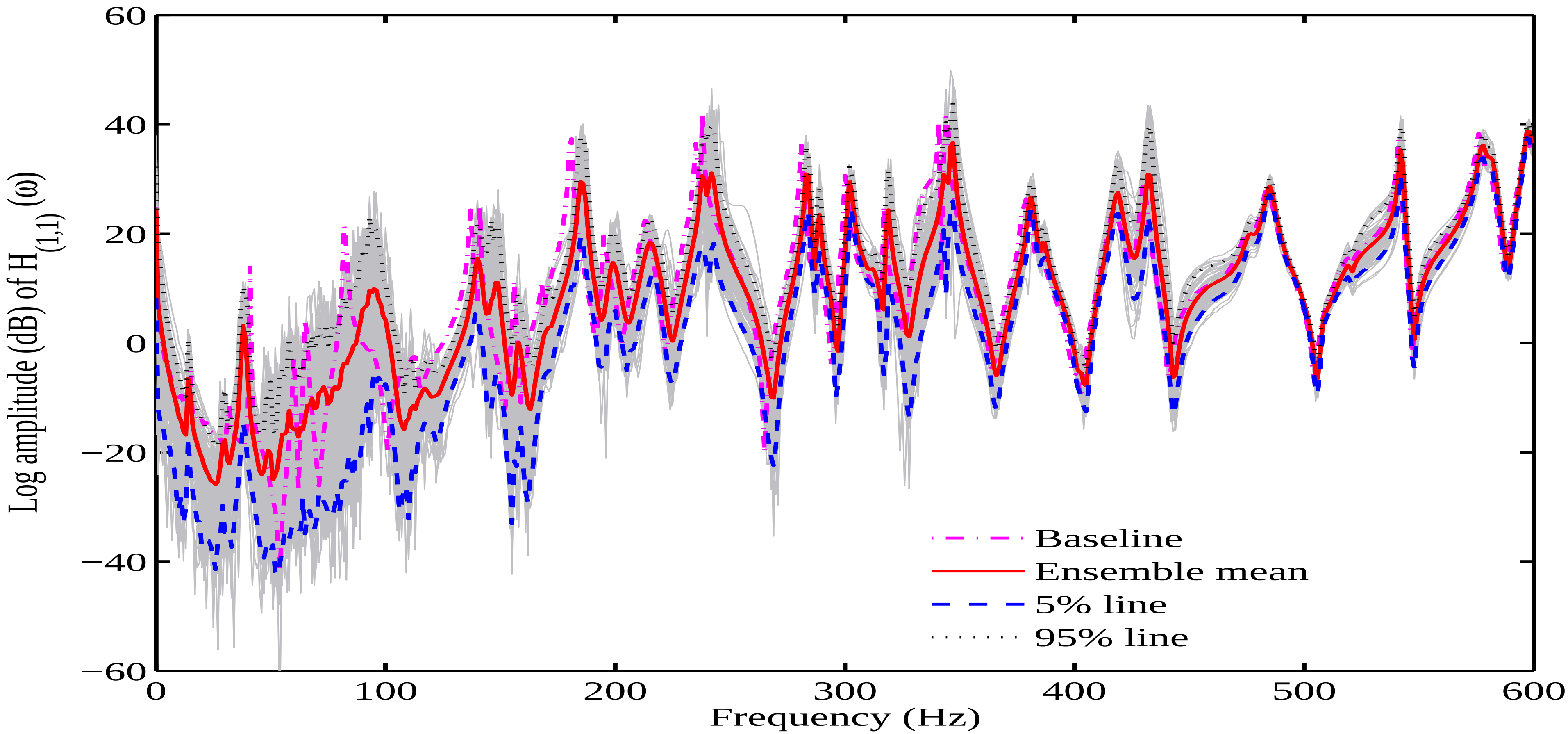
Oscillator Number	Spring stiffness ($\times 10^4$ N/m)	Natural Frequency (Hz)
1	1.6800	59.2060
2	0.9100	43.5744
3	1.7030	59.6099
4	2.4000	70.7647
5	1.5670	57.1801
6	2.2880	69.0938
7	1.7030	59.6099
8	2.2880	69.0938
9	2.1360	66.7592
10	1.9800	64.2752

Table 2: Stiffness of the springs and natural frequency of the oscillators used to simulate unmodelled dynamics (the mass of the each oscillator is 121.4g).

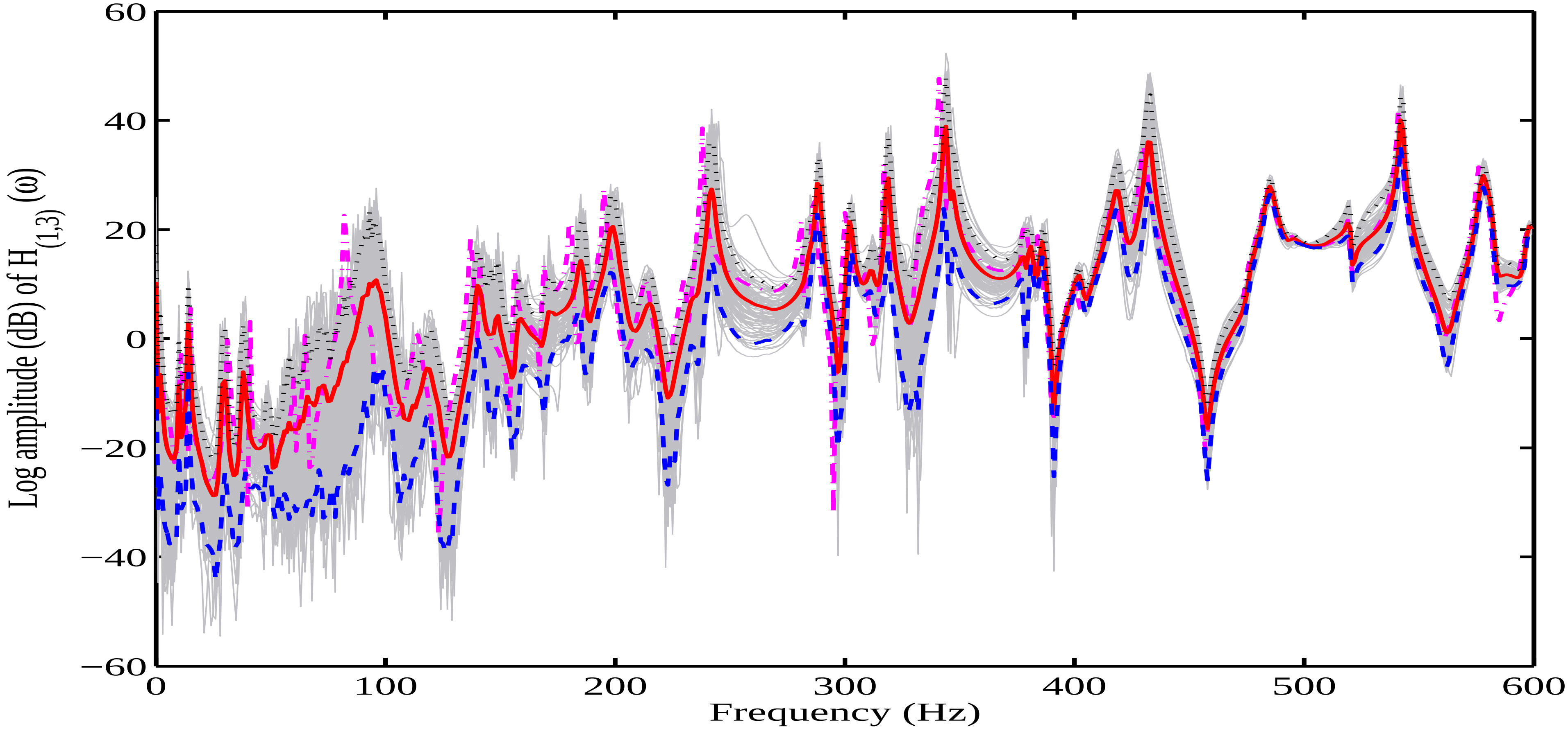
Experiments: Protocol

- Attach the oscillators at random locations (computer generated)
- Measure impulse response at: Point 1: (4,6), Point 2: (6,11), Point 3: (11,3), Point 4: (14,14), Point 5: (18,2), Point 6: (21,10)
- Transform to frequency domain to estimate frequency response function (FRF).
- Curvefit the FRF to estimate the natural frequencies ω_n and damping factors Q_n
  - Rational Fraction Polynomial (RFP) method
  - Nonlinear Leastsquares method
- Calculate the statistics of natural frequencies

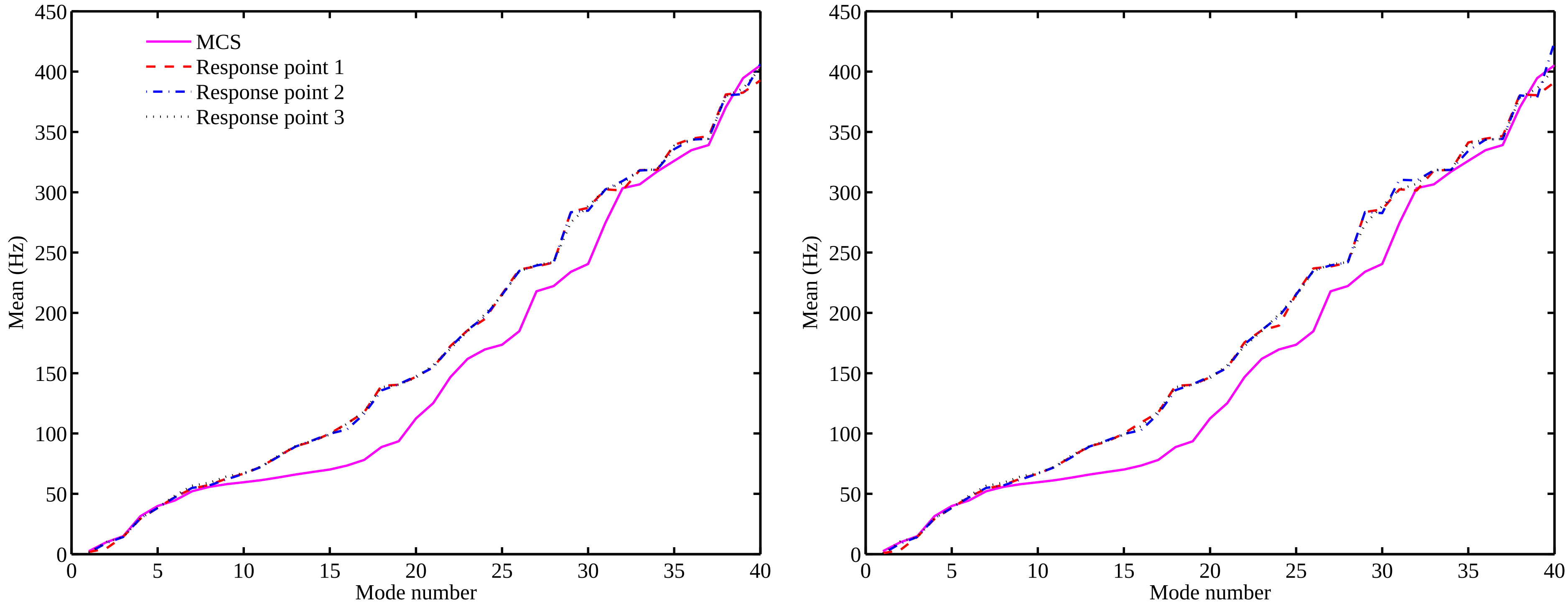
Experiments: FRF at Point 1



Experiments: FRF at point 3

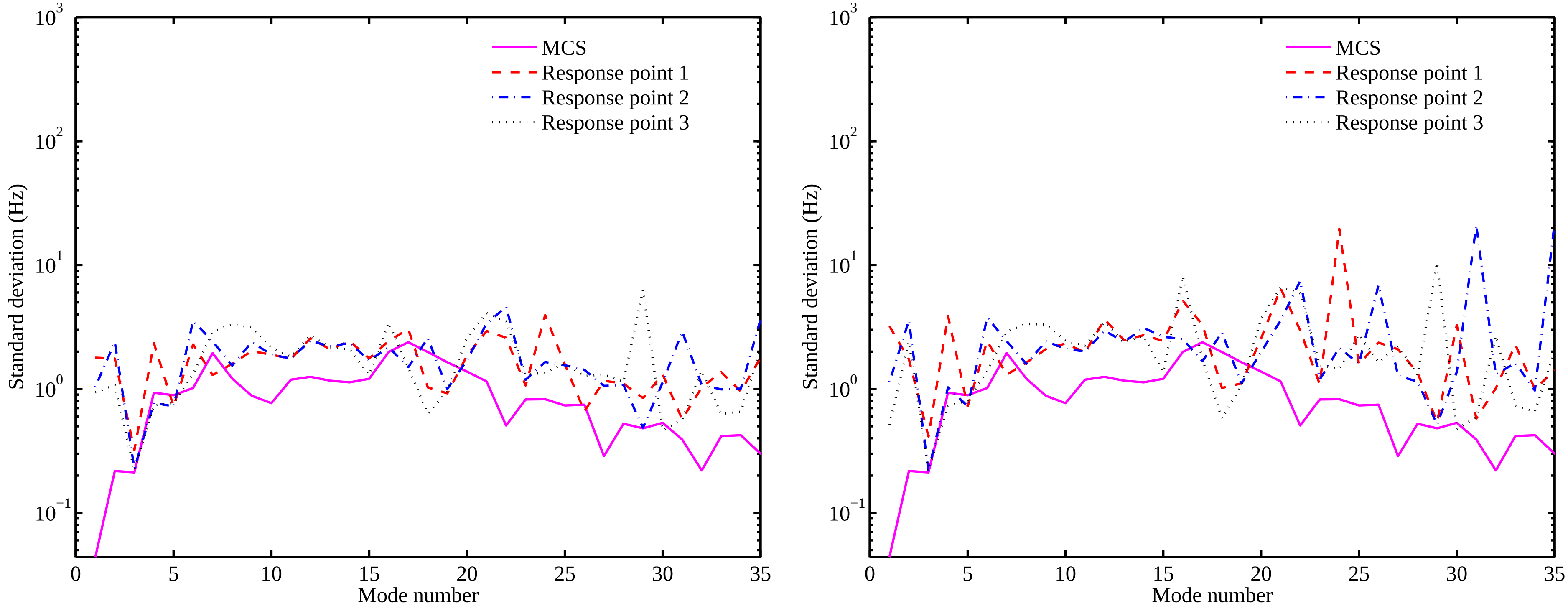


Ensemble Mean



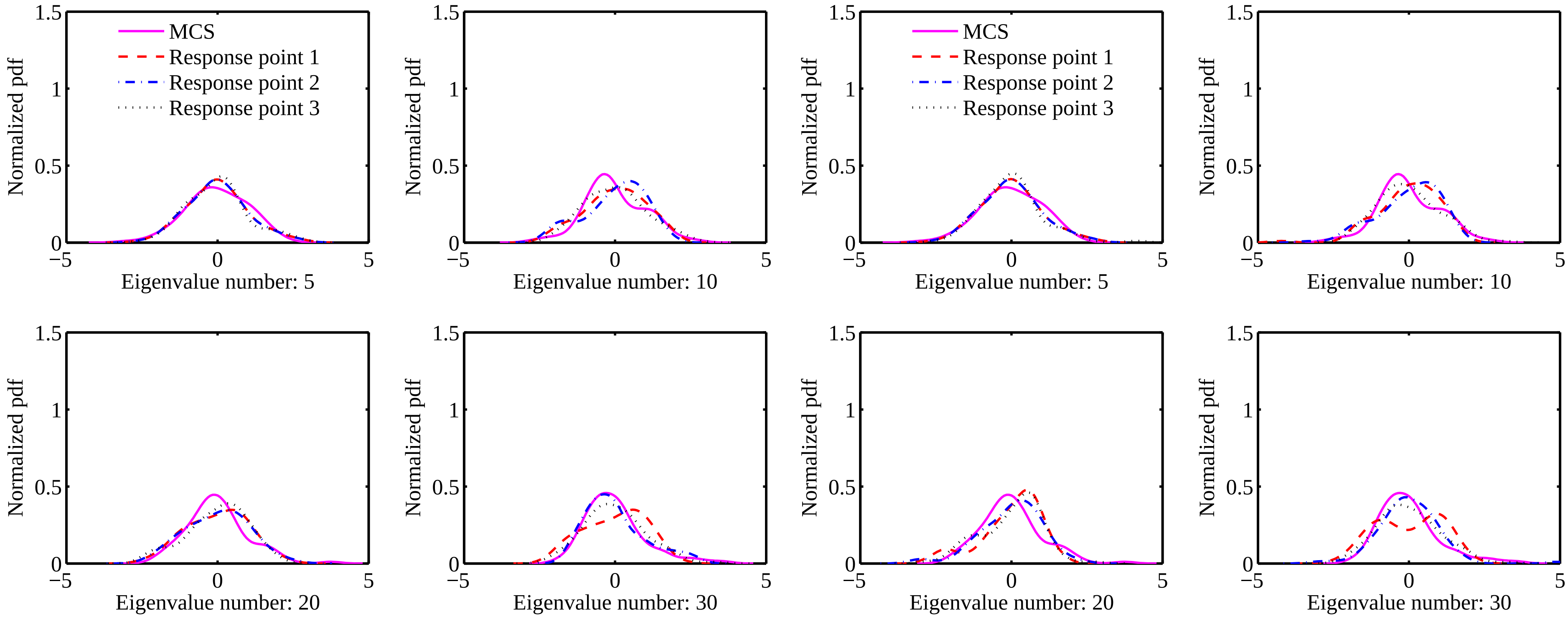
Left: RFP; Right: Nonlinear Leastsquares

Standard Deviation



Left: RFP; Right: Nonlinear Least-squares

PDFs



Left: RFP; Right: Nonlinear Least-squares

Conclusions

- The ensemble statistics such as mean and standard deviation for natural frequencies vary with the spatial location of the measured FRFs and the type of the system identification technique chosen to estimate the natural frequencies.
- Whilst a reasonable predictions for the mean and the standard deviations may be obtained using the Monte Carlo Simulation, higher moments, and hence the pdfs can be significantly different.
- In some cases, the differences in pdfs arising from different points and different identification methods can be more than those obtained from the Monte Carlo Simulation.