# Structural Health Monitoring using Shaped Sensors

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# Vibration based SHM

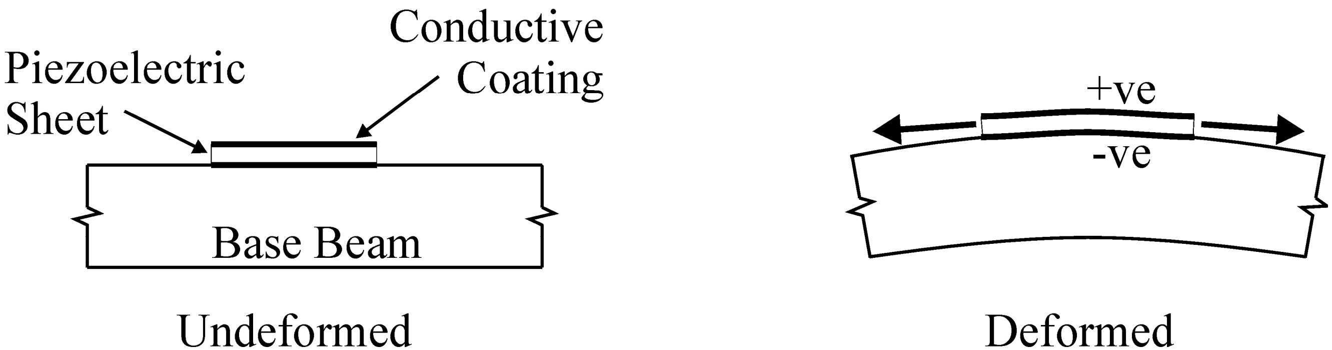
- The identification of the location and severity of cracks, loose bolts and other types of damage in structures using vibration data has received considerable attention.
- Most of the approaches use the modal data of a structure before damage occurs as baseline data, and all subsequent tests are compared to it.
- Any deviation in the modal properties from this baseline data is used to estimate model parameters related to the damage severity and location.
- If changes in the structure are not due to damage (e.g., due to environmental effects), it will be difficult to distinguish them from changes due to damage.

# Vibration based SHM

- The approach adopted in this paper is to use shaped sensors to reduce the sensitivity of the sensor output to the unmodelled parameter changes and environmental effects.
- For structural health monitoring this means that the response can be made sensitive to particular regions of interest, so that, for example, the sensor may be used to monitor the health of a single joint.
- The method is an extension of the selective sensitivity technique which was developed to design excitations that produce strong sensitivities to a subset of the parameters whilst causing the sensitivities to other parameters to vanish.

# Piezoelectric Materials as Sensor / Actuator

- Piezoelectric material may be used as a sensor, an actuator or both
  - electrical charge ⇔ strain
- Mechanical strains are small
  - Use stacks or thin sheets off the neutral axis
- May be poled through thickness, or using inter-digitated electrodes

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# Shaped Piezoelectric Sensors

- Shaped sensors by
  - Changing sensor width on beam
  - Change sensor shape for plate
  - Use constant thickness sensor
- Piezoelectric sensors
  - Ceramic, PVDF film, paint, etc
  - Assume charge proportional to curvature of beam/plate

# Shaped Sensors for Beam Structures

- The shape of a sensor is a continuous function. However this function needs to be parameterised to enable the optimisation of the sensor shape
- *The key idea: 'recycle' FE shape functions-* Using the shape functions of the underlying finite element model is a convenient approach to approximate the width of the piezoelectric material. In this way sensors may be designed for arbitrary beam type structures. Furthermore sensors that only cover part of a structure may be designed.

# Shaped Sensors for Beam Structures

- Suppose a single polyvinylidene fluoride (PVDF) film sensor is placed on the beam with a shape defined by a variable width $f(\xi)$
- Using finite element shape functions for element $e$:

$$f_e(\xi) = [N_{e1}(\xi) \quad N_{e2}(\xi) \quad N_{e3}(\xi) \quad N_{e4}(\xi)] \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix}$$

- $N_{ei}$ are shape functions, $f_{ei}$ to be determined
- Need some criteria to obtain 'nodal parameters' $f_{ei}$

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# Shaped Sensors for Beam Structures

- This displacement is also approximated by the shape functions as

$$w_e(\xi) = \begin{bmatrix} N_{e1}(\xi) & N_{e2}(\xi) & N_{e3}(\xi) & N_{e4}(\xi) \end{bmatrix} \begin{Bmatrix} w_{e1} \\ w_{e2} \\ w_{e3} \\ w_{e4} \end{Bmatrix}.$$

- For an Euler-Bernoulli beam, these shape functions are

$$N_{e1}(\xi) = \left(1 - 3\frac{\xi^2}{\ell_e^2} + 2\frac{\xi^3}{\ell_e^3}\right), \qquad N_{e2}(\xi) = \ell_e\left(\frac{\xi}{\ell_e} - 2\frac{\xi^2}{\ell_e^2} + \frac{\xi^3}{\ell_e^3}\right),$$
$$N_{e3}(\xi) = \left(3\frac{\xi^2}{\ell_e^2} - 2\frac{\xi^3}{\ell_e^3}\right), \qquad N_{e4}(\xi) = \ell_e\left(-\frac{\xi^2}{\ell_e^2} + \frac{\xi^3}{\ell_e^3}\right), \tag{11}$$

# Sensor Output for Element

- The sensor output for element $e$, is

$$y_e(t) = K_s \int_0^{\ell_e} f_e(\xi) \frac{\partial^2 w_e(\xi, t)}{\partial^2 \xi} d\xi$$

- Thus

$$y_e = \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix}^\top \mathbf{C}_e \begin{Bmatrix} w_{e1} \\ w_{e2} \\ w_{e3} \\ w_{e4} \end{Bmatrix}$$

- where

$$\mathbf{C}_{eij} = K_s \int_0^{\ell_e} N_{ei}(\xi) N''_{ej}(\xi) d\xi$$

# Sensor Output

- Total sensor output: $$y = \sum_e y_e = \mathbf{f}^\top \mathbf{C}_s \mathbf{q}$$

- Equations of motion: $$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{B}\mathbf{u}$$ $$\mathbf{y} = \mathbf{C}\mathbf{q}$$

- where $\mathbf{C} = \mathbf{f}^\top \mathbf{C}_s$

# Transform to Modal Coordinates

- Real mode shape matrix, $\mathbf{\Phi}$
- Assume proportional damping

- Modal output matrix is $\mathbf{C_p} = \mathbf{C}\mathbf{\Phi} = \mathbf{f}^\top \mathbf{C}_s \mathbf{\Phi}$

- Problem: specify $\mathbf{C}_{\mathrm{p}}$, find $\mathbf{f}$
  - Usually underdetermined
  - Pseudo inverse, gives minimum norm solution
  - Or, minimise curvature (with modal output constraint)

# Minimise Curvature

- We wish to minimize

$$J_c(\mathbf{f}) = \sum_e \int_0^{\ell_e} f_e''(\xi)^2 \, \mathrm{d}\xi = \sum_e \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix}^\top \mathbf{H}_e \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix}$$

where

$$\mathbf{H}_{eij} = \int_0^{\ell_e} N_{ei}''(\xi)\, N_{ej}''(\xi) \, \mathrm{d}\xi.$$

***H**e* looks like the element stiffness matrix with a unit flexural rigidity

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# Minimise Curvature

- Minimise $J_c(\mathbf{f}) = \mathbf{f}^\top \mathbf{H}\mathbf{f}$
  - $\mathbf{H}$ is free-free beam stiffness with unit stiffness
- Constraint $\mathbf{C_p} = \mathbf{C}\boldsymbol{\Phi} = \mathbf{f}^\top \mathbf{C}_s \boldsymbol{\Phi}$
  - $\mathbf{C}_\mathrm{p}$ given, $\mathbf{C}_s$ and $\boldsymbol{\Phi}$ known
- Sensor shape from solution to

$$\begin{bmatrix} 2\mathbf{H} & \mathbf{C}_s \boldsymbol{\Phi} \\ \boldsymbol{\Phi}^\top \mathbf{C}_s^\top & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{f} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{C}_\mathbf{p}^\top \end{Bmatrix}$$

  - Lagrange multipliers $\lambda$

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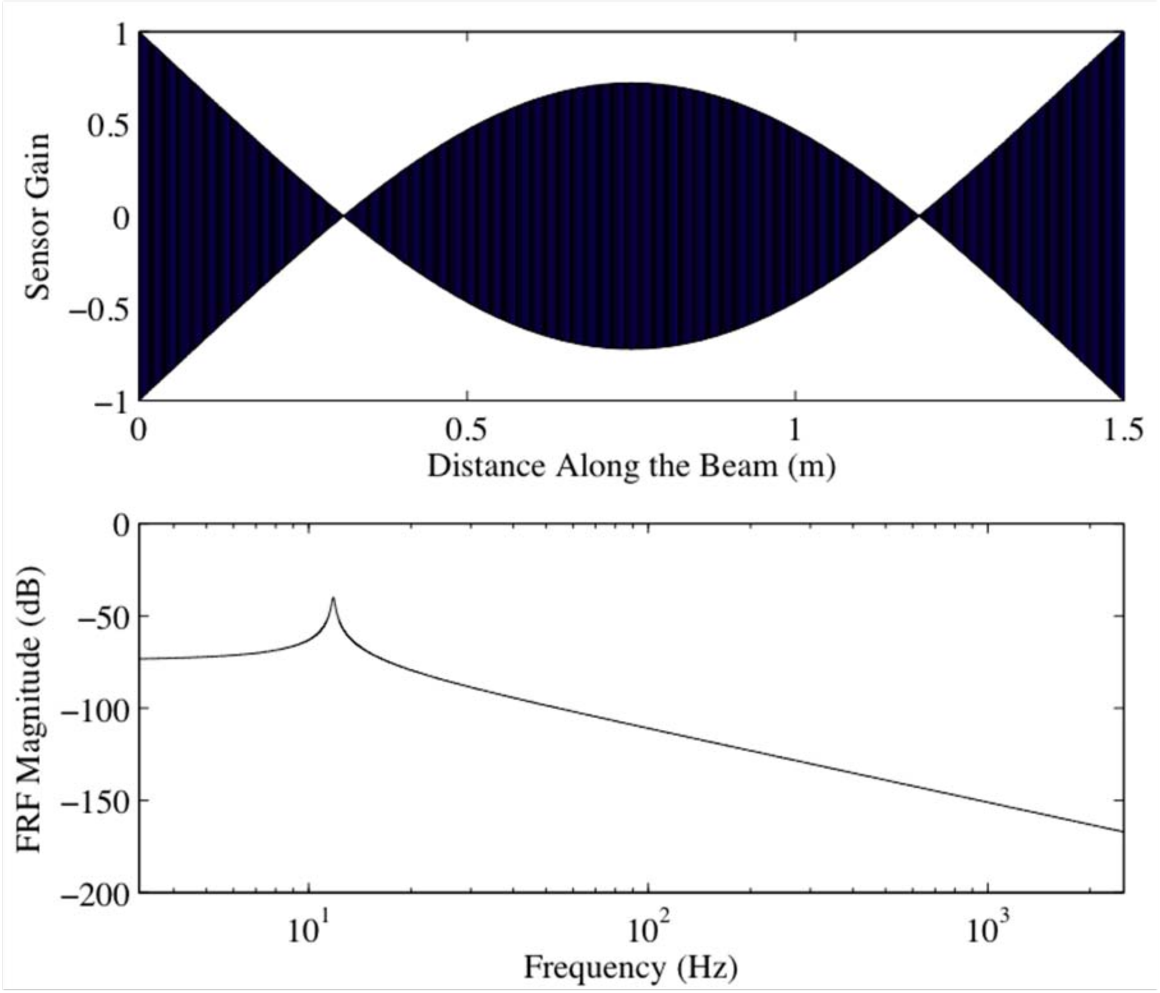
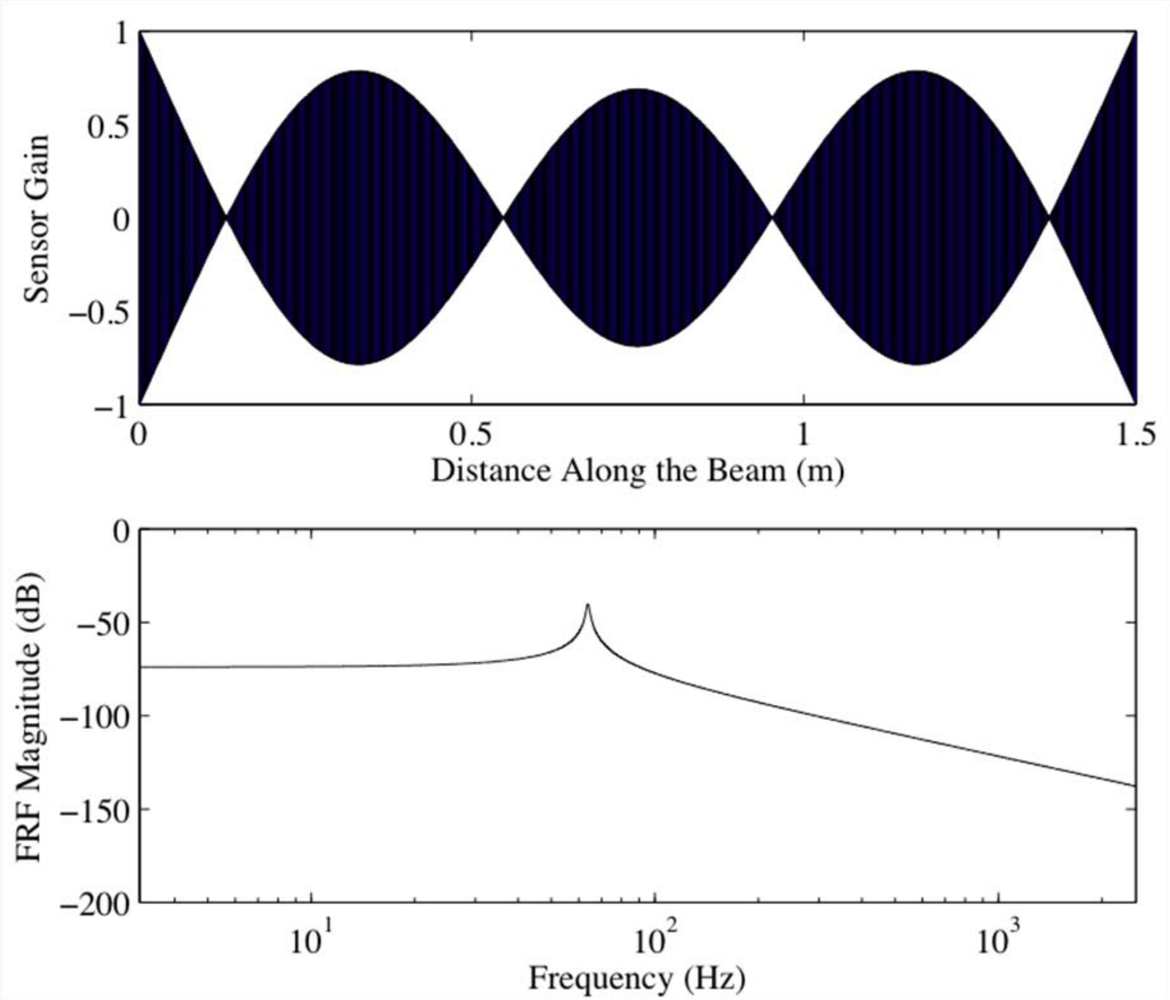
# Beam Example – Modal Sensor

- Clamped-clamped beam
- Length 1.5m, cross-section 20x5mm
- 15 elements, first 9 modes considered
- Beam forced at node 7
- Natural frequencies at 11.8Hz, 32.6Hz, 63.9Hz, 105.6Hz, …

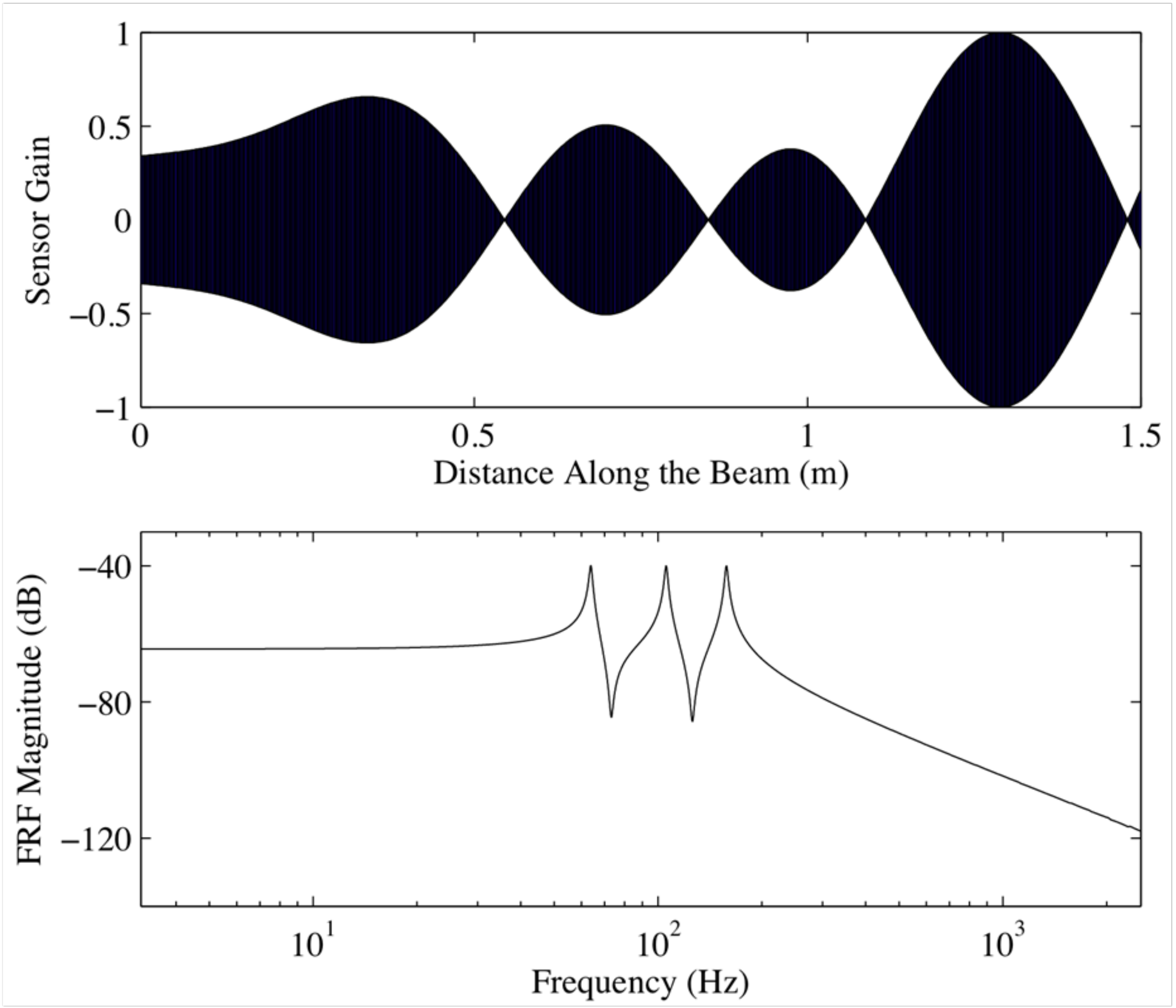
# Sensor for Mode 1 only



# Sensor for Mode 3 only

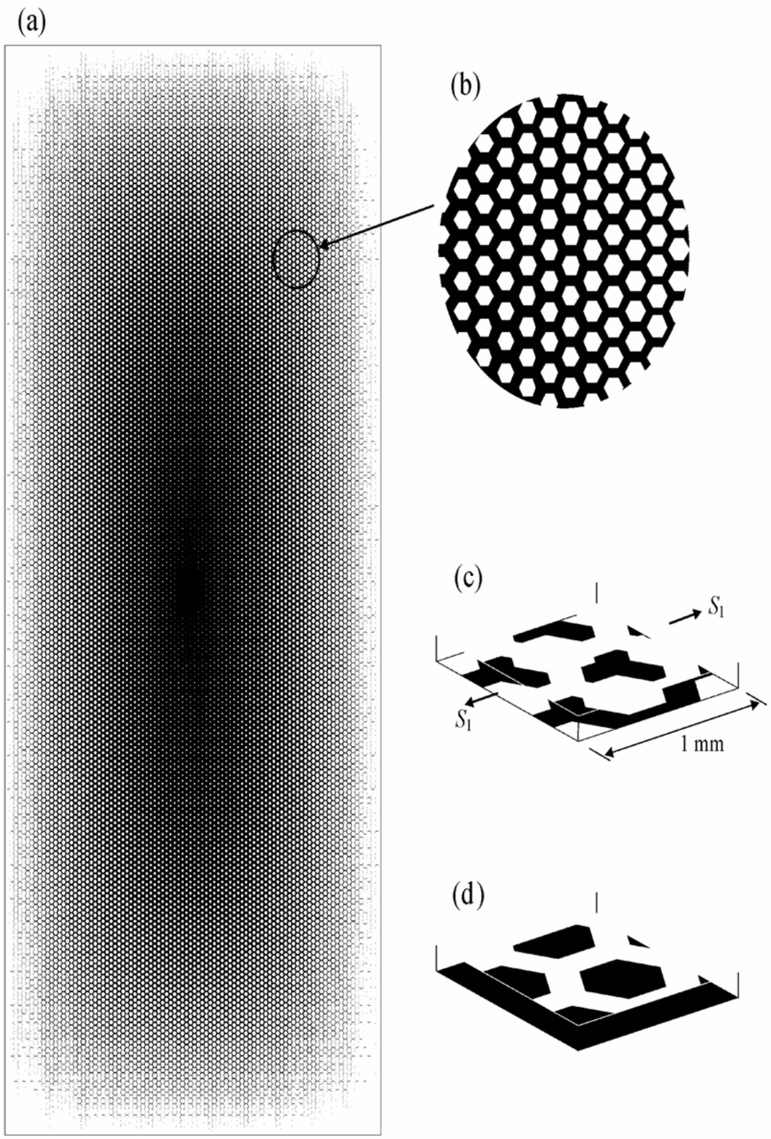


# Sensor for Modes 3 to 5 only



# Modal Sensors for Plate Structures

- Can vary effectiveness
  - Vary thickness
  - Porous electrode (Preumont *et al.*)
  - Method the same as beams
- Or, constant thickness
  - Parameterise boundary using splines
  - Optimise shape
  - Use finite elements

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# Modal Sensors for Plate Structures

- For a plate element, the output (voltage or charge) from the part of the sensor covering element number *e* is

$$y_e(t) = K_s \iint f_e(\xi, \eta) \left( \frac{\partial^2 w_e(\xi, \eta, t)}{\partial \xi^2} + \frac{\partial^2 w_e(\xi, \eta, t)}{\partial \eta^2} \right) \mathrm{d}\xi \mathrm{d}\eta$$

where $f_e(\xi, \eta)$ defines the effectiveness of the sensor at location $(\xi, \eta)$

- The total sensor output is obtained by summing the output from all of the elements, thus

$$y = \sum_e y_e = \mathbf{f}_s^\top \mathbf{C}_s \mathbf{q}$$

- Where $\mathbf{C}_s = \sum \mathbf{C}_{se}$

$$\mathbf{C}_{se} = K_s \iint \mathbf{N}(\xi, \eta)^\top \left( \frac{\partial^2 \mathbf{N}(\xi, \eta)}{\partial \xi^2} + \frac{\partial^2 \mathbf{N}(\xi, \eta)}{\partial \eta^2} \right) \mathrm{d}\xi \mathrm{d}\eta.$$

# Modal Sensors for Plate Structures

- As an example, for a square element with side *2a*:

$$
\frac{K_s}{720}\begin{bmatrix}
-600 & -120a & -120a & 168 & 48a & 0 & 264 & -72a & -72a & 168 & 0 & 48a \\
-552a & -143a^2 & -159a^2 & -240a & -79a^2 & 111a^2 & 24a & 31a^2 & 15a^2 & 48a & 47a^2 & -15a^2 \\
-552a & -159a^2 & -143a^2 & 48a & -15a^2 & 47a^2 & 24a & 15a^2 & 31a^2 & -240a & 111a^2 & -79a^2 \\
168 & 48a & 0 & -600 & -120a & 120a & 168 & 0 & -48a & 264 & -72a & 72a \\
-240a & -79a^2 & -111a^2 & -552a & -143a^2 & 159a^2 & 48a & 47a^2 & 15a^2 & 24a & 31a^2 & -15a^2 \\
-48a & 15a^2 & 47a^2 & 552a & 159a^2 & -143a^2 & 240a & -111a^2 & -79a^2 & -24a & -15a^2 & 31a^2 \\
264 & 72a & 72a & 168 & 0 & -48a & -600 & 120a & 120a & 168 & -48a & 0 \\
-24a & 31a^2 & 15a^2 & -48a & 47a^2 & -15a^2 & 552a & -143a^2 & -159a^2 & 240a & -79a^2 & 111a^2 \\
-24a & 15a^2 & 31a^2 & 240a & 111a^2 & -79a^2 & 552a & -159a^2 & -143a^2 & -48a & -15a^2 & 47a^2 \\
168 & 0 & 48a & 264 & 72a & -72a & 168 & -48a & 0 & -600 & 120a & -120a \\
-48a & 47a^2 & 15a^2 & -24a & 31a^2 & -15a^2 & 240a & -79a^2 & -111a^2 & 552a & -143a^2 & 159a^2 \\
-240a & -111a^2 & -79a^2 & 24a & -15a^2 & 31a^2 & 48a & 15a^2 & 47a^2 & -552a & 159a^2 & -143a^2
\end{bmatrix}
$$

# Structural Health Monitoring

- Make sensor sensitive to some parameters
  - e.g.   Joint stiffnesses
- But insensitive to most other parameters
- Reduce processing
  - Monitor single joints

- Method of selective sensitivity
  - Originally for forces (Ben-Haim *et al.*)
  - Frequency domain method
  - Possibly use for high frequencies?

# Parameter Sensitivities

- The response is

$$\mathbf{y}(\omega,\boldsymbol{\theta}) = \mathbf{C}\left[-\omega^2\mathbf{M}(\boldsymbol{\theta}) + \mathrm{j}\omega\mathbf{D}(\boldsymbol{\theta}) + \mathbf{K}(\boldsymbol{\theta})\right]^{-1}\mathbf{B}\mathbf{u}(\omega)$$

$$= \mathbf{f}^\top\mathbf{C}_s\mathbf{H}(\omega,\boldsymbol{\theta})\mathbf{B}\mathbf{u}(\omega)$$

where $\mathbf{H} = \left[-\omega^2\mathbf{M} + \mathrm{j}\omega\mathbf{D} + \mathbf{K}\right]^{-1}$

- The sensitivity is $S_j(\mathbf{f},\omega) = \left\| \frac{\partial y}{\partial \theta_j} \right\|^2$, or

$$S_j(\mathbf{f},\omega) = \mathbf{f}^\top\mathbf{C}_s\frac{\partial \mathbf{H}}{\partial \theta_j}\mathbf{B}\mathbf{u}\mathbf{u}^\mathrm{H}\mathbf{B}^\top\frac{\partial \mathbf{H}^\mathrm{H}}{\partial \theta_j}\mathbf{C}_s^\top\mathbf{f} = \mathbf{f}^\top\mathbf{G}_j(\omega)\mathbf{f}$$

# Optimisation of Selective Sensitivity

$$S_j(\mathbf{f},\omega) = \mathbf{f}^\top \mathbf{C}_s \frac{\partial \mathbf{H}}{\partial \theta_j} \mathbf{B}\mathbf{u}\mathbf{u}^\mathrm{H} \mathbf{B}^\top \frac{\partial \mathbf{H}^\mathrm{H}}{\partial \theta_j} \mathbf{C}_s^\top \mathbf{f} = \mathbf{f}^\top \mathbf{G}_j(\omega) \mathbf{f}$$

- Suppose simple threshold is required
  - Single output required
  - Depends on source of excitation
- Frequency weighting

$$\hat{S}_j(\mathbf{f}) = \int_{\omega_1}^{\omega_2} W(\omega) S_j(\mathbf{f}) \, \mathrm{d}\omega$$

$$\hat{S}_j(\mathbf{f}) = \mathbf{f}^\top \left[ \int_{\omega_1}^{\omega_2} W(\omega) \mathbf{G}_j(\omega) \, \mathrm{d}\omega \right] \mathbf{f} = \mathbf{f}^\top \hat{\mathbf{G}}_j(\omega) \mathbf{f}$$

# Optimisation at a Single Frequency

- Make response sensitive to parameter $s$

$$S_j(\mathbf{f}) = \begin{cases} \neq 0 & \text{if } j = s \\ = 0 & \text{otherwise} \end{cases}$$

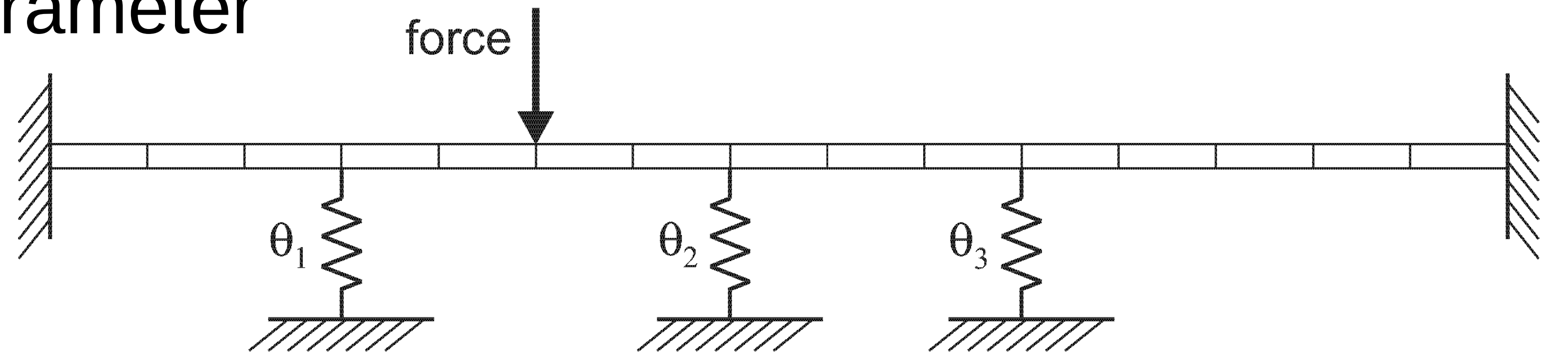
- Transform to ensure zero sensitivity to unwanted parameters
- Then minimise curvature (or other objective)

- Alternative – direct minimisation

$$J(\mathbf{f}) = \left\{ \sum_{j \neq s} S_j(\mathbf{f}) \right\} / S_s(\mathbf{f})$$

# SHM Beam Example

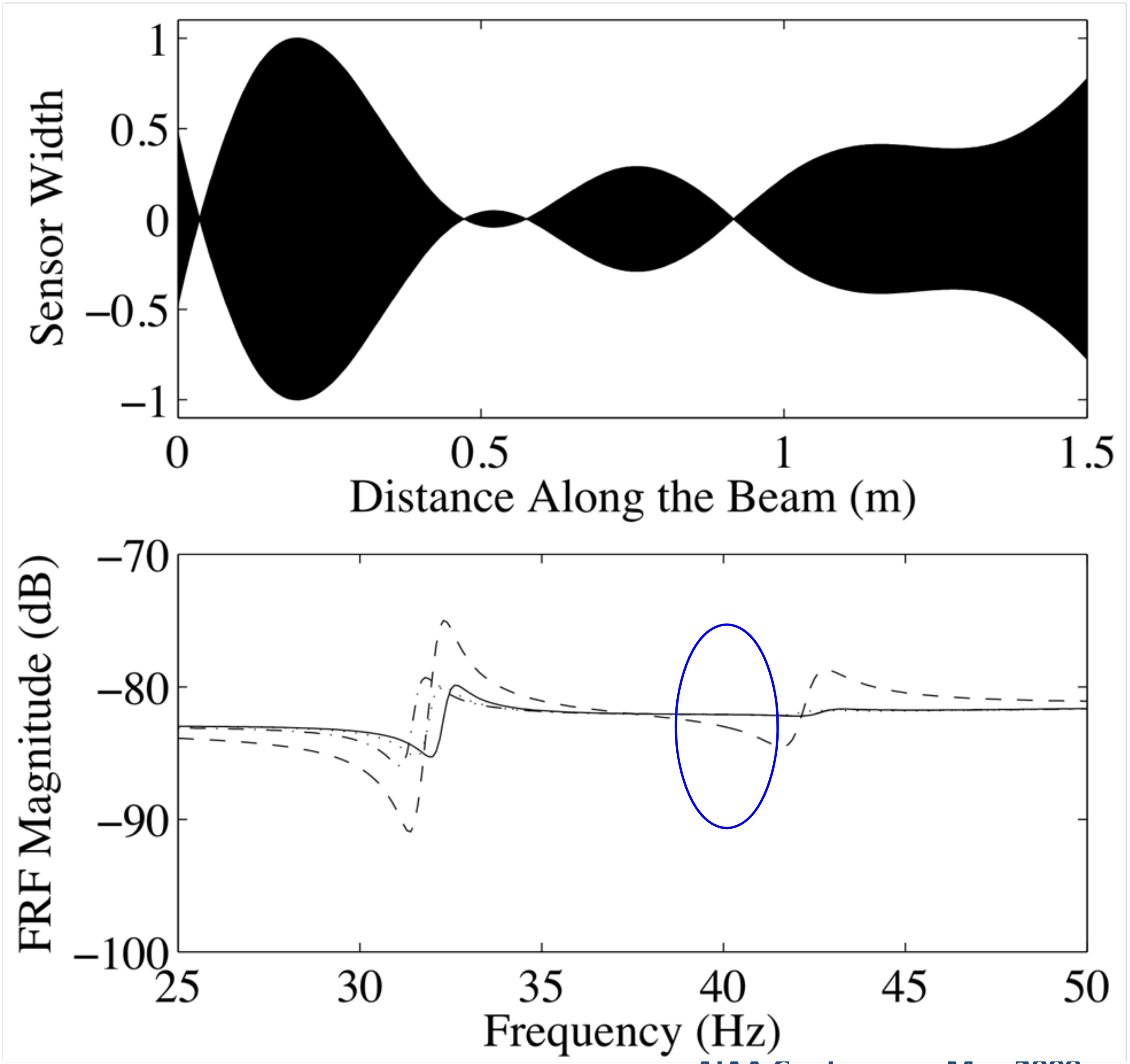
- Beam with 3 supports
- Sensor covers whole beam
- Single force at 40Hz – perhaps a rotating machine
- Sensitive to one parameter
- Checked using a 10% change in each parameter



# Sensitive to Support 1

- Minimise curvature
- Dashed line is 10% change to support 1

- $S_1/S_2 = 1.2\text{x}10^{15}$
- $S_1/S_3 = 8.7\text{x}10^{12}$

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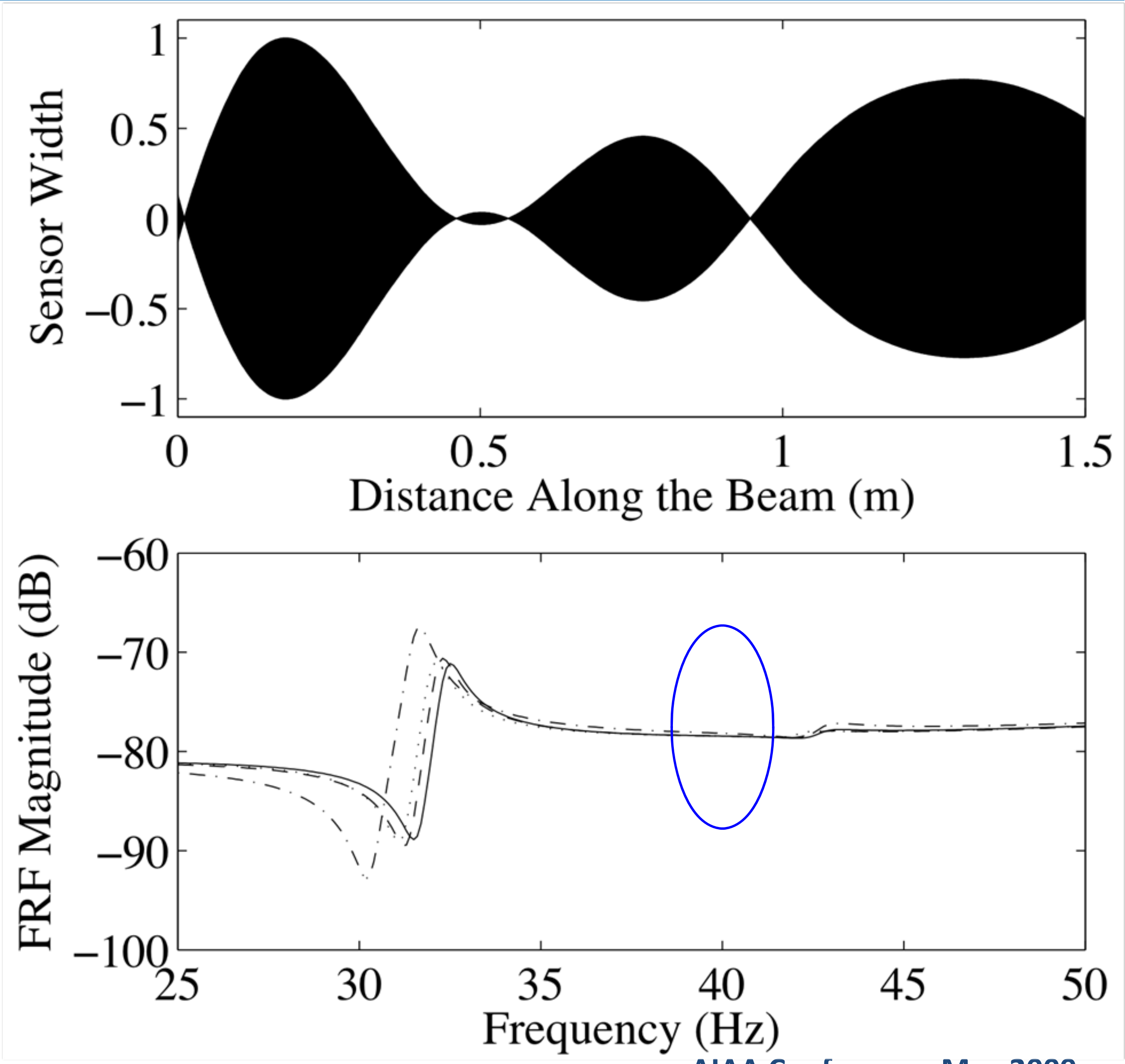
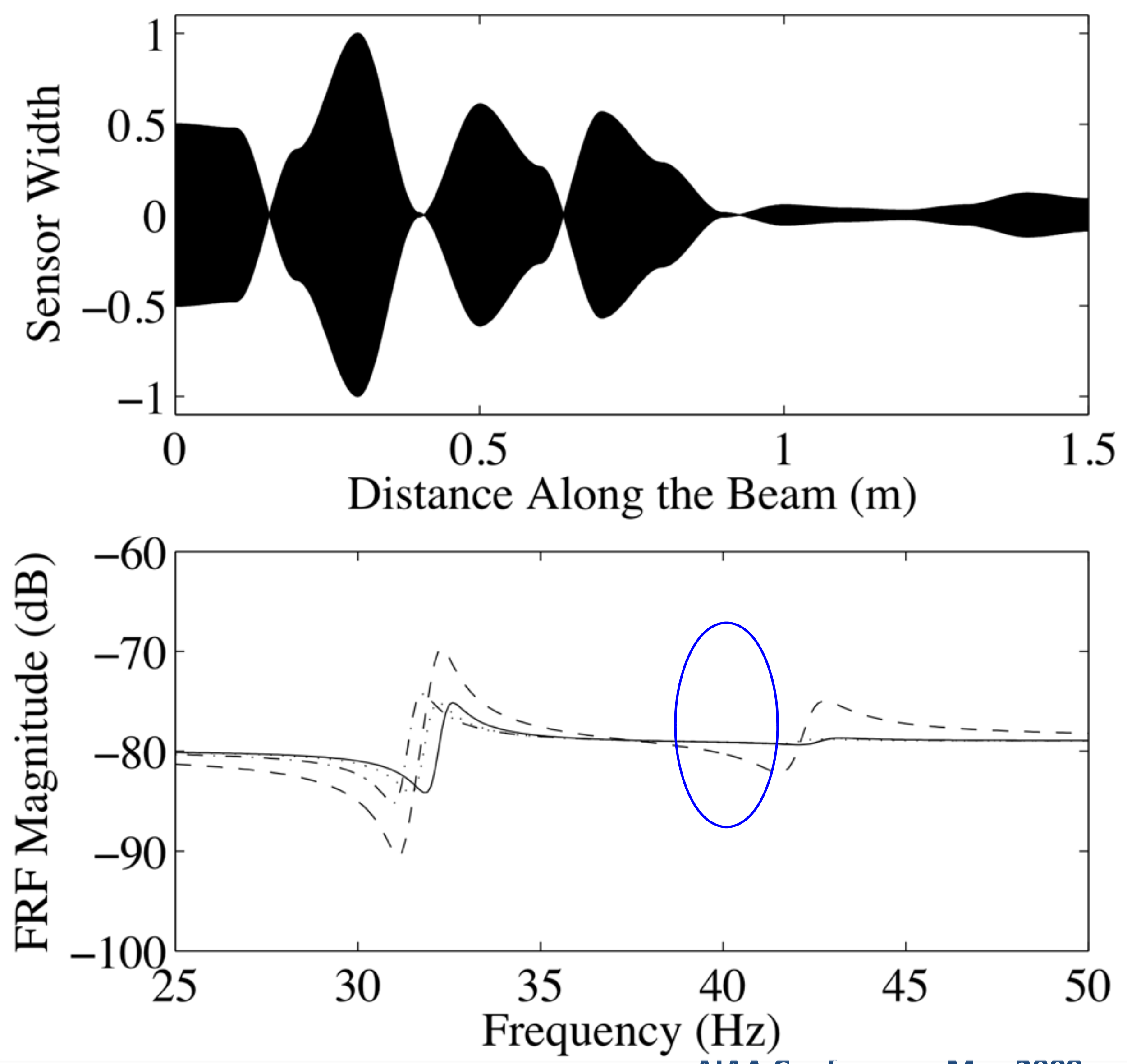
# Sensitive to Support 2

- Minimise curvature
- Dot-dashed line is 10% change to support 2



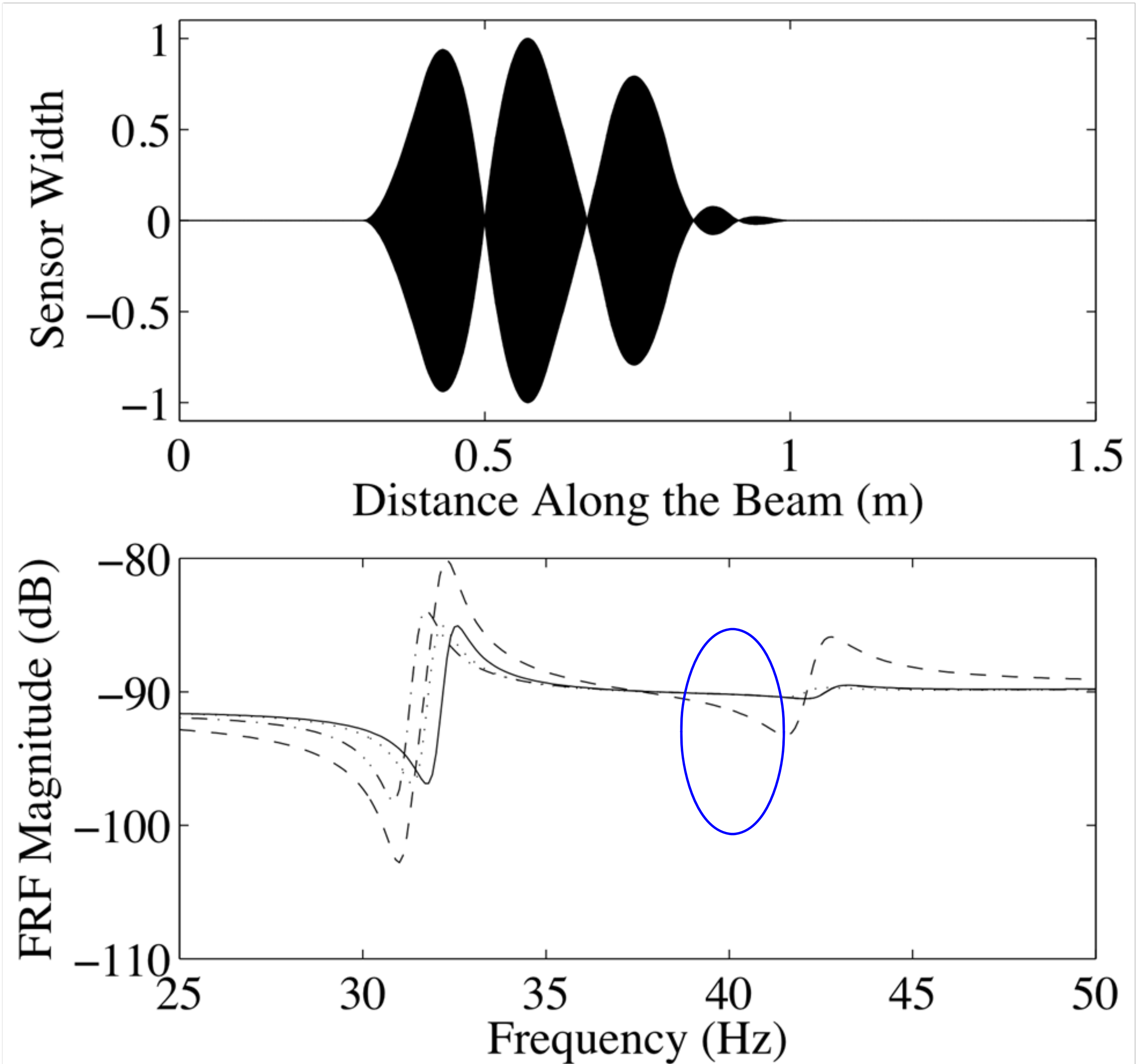
# Maximise Sensitivity to Support 1

- More sensitive to parameter
- Shape more complex



# Restrict Sensor Region

- Sensitive to support 1

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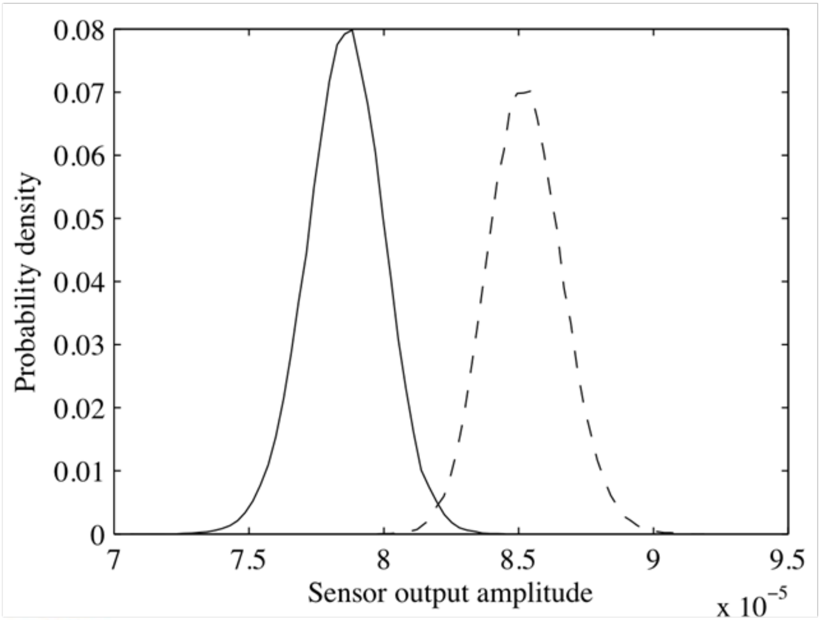
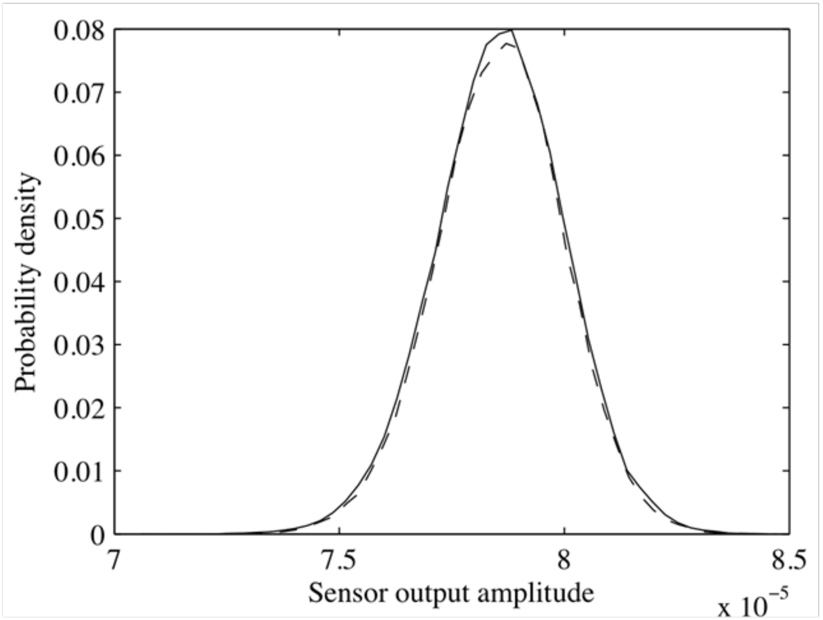
# Response to Modelling Errors

- Sensor designed to be sensitive to parameter 1
- 2% random variation in parameters
- Dashed line is 10% variation in parameters: 1 (left) and 2 (right)

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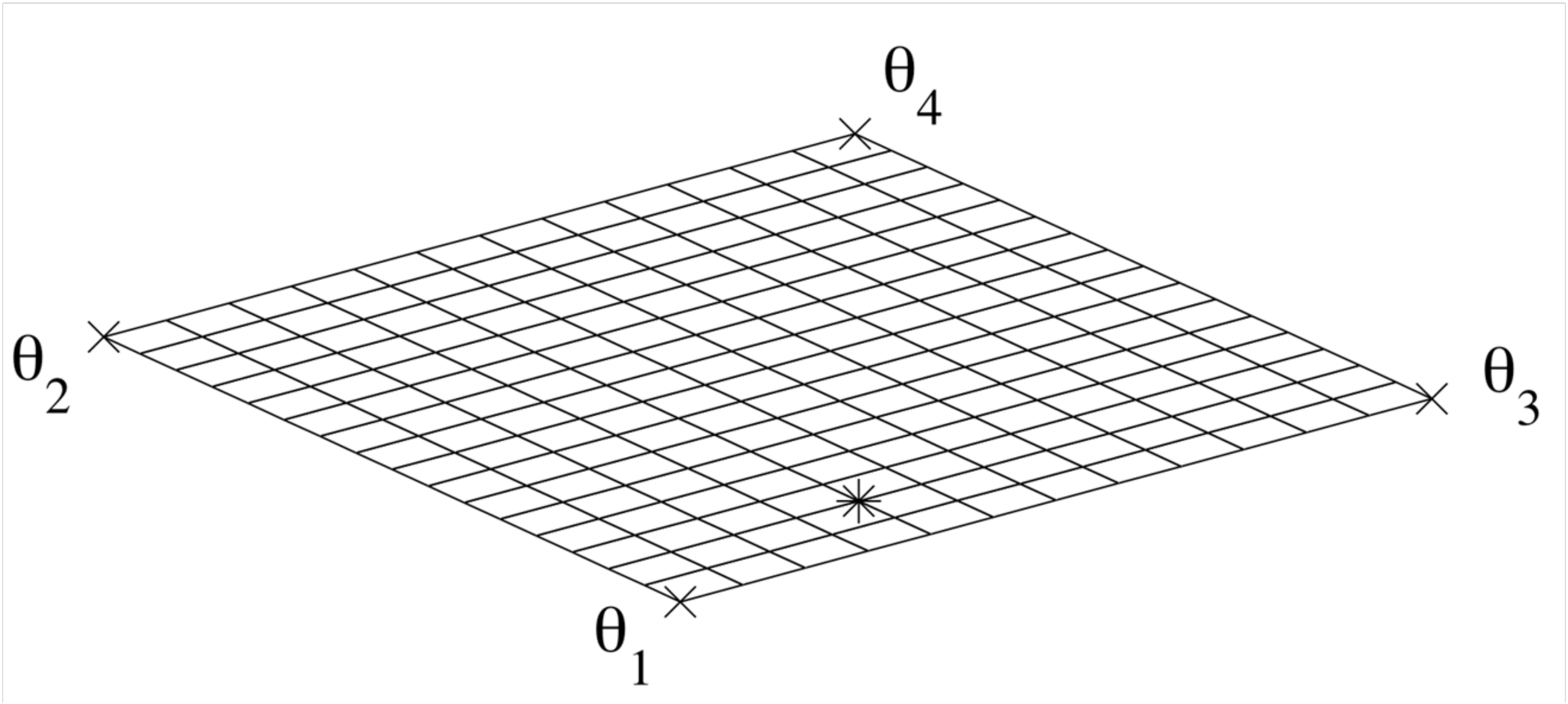
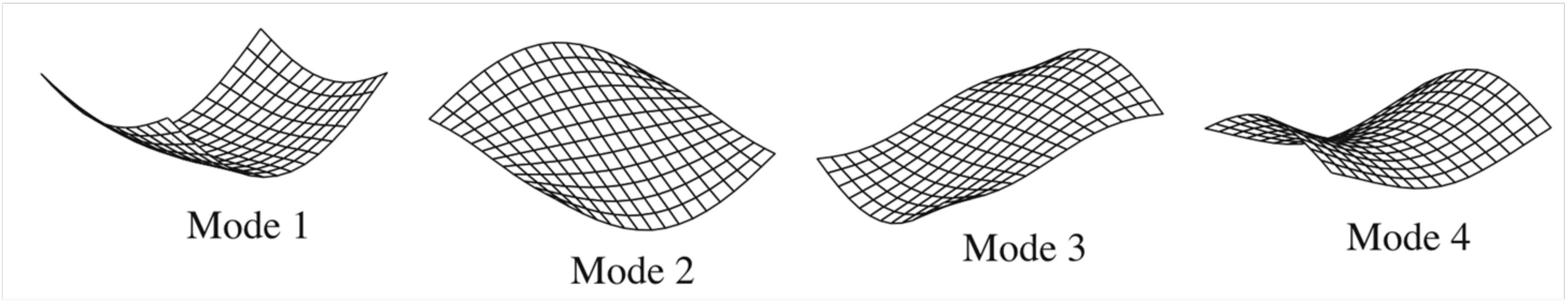
# Plate Example

- Plate 400x300mm, 7mm thick
- Supported at corners, spring constant 500kN/m
- Force at 40Hz at starred node



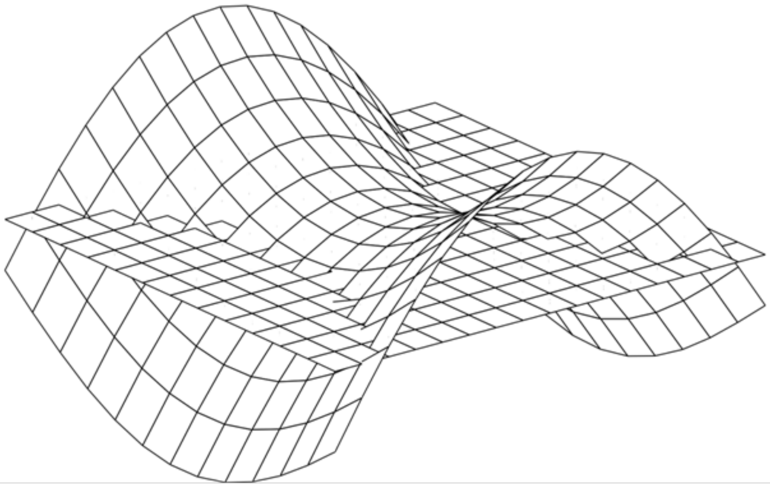
# Plate Modes

- Natural frequencies:  21.0, 44.7, 48.8, 72.7 Hz

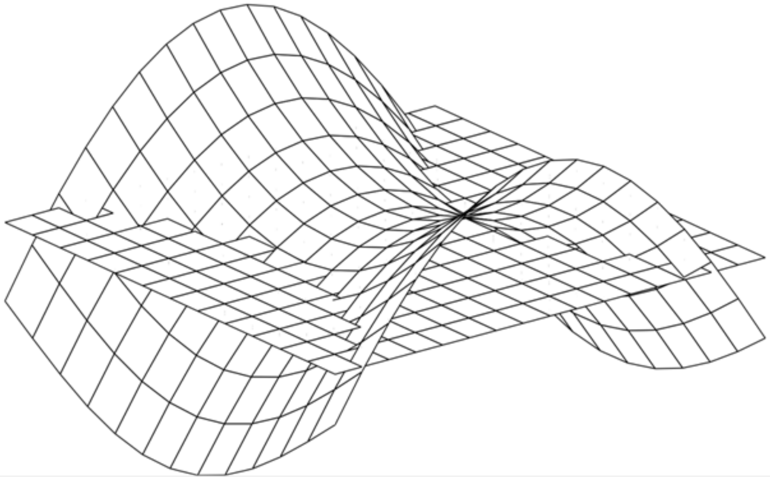


# Sensor Designs for Supports 1 and 2

- The meshes represent sensor thicknesses
- Note the shapes are very similar



Sensitive to Support 1



Sensitive to Support 2

# Conclusions

- Shaped sensors designed by parameterising shape using FE analysis and optimisation – FE shape functions are reused
- Structural health monitoring
  - Make sensors sensitive to a single parameter (selective sensitivity)
  - More work required on frequency weighting
  - Could provide a cheap method to monitor joints
- Need to test the methods experimentally
- Need to extend to general systems and develop computational methods.