Equivalent elastic properties of irregular hexagonal lattices: An analytical approach

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Introduction

Hexagonal lattices/ lattice like structural forms are present in materials and structures across different length scales. Spatial irregularity in such hexagonal honeycombs may occur due to various reasons such as manufacturing/fabrication uncertainty, structural defects, variation in temperature, pre-stressing and micro-structural variability.

Quantifying the effects of such irregularities in a stochastic framework using finite element based approach is computationally very expensive. We have developed an efficient analytical approach leading to closed form formulae for different elastic moduli of irregular honeycombs.

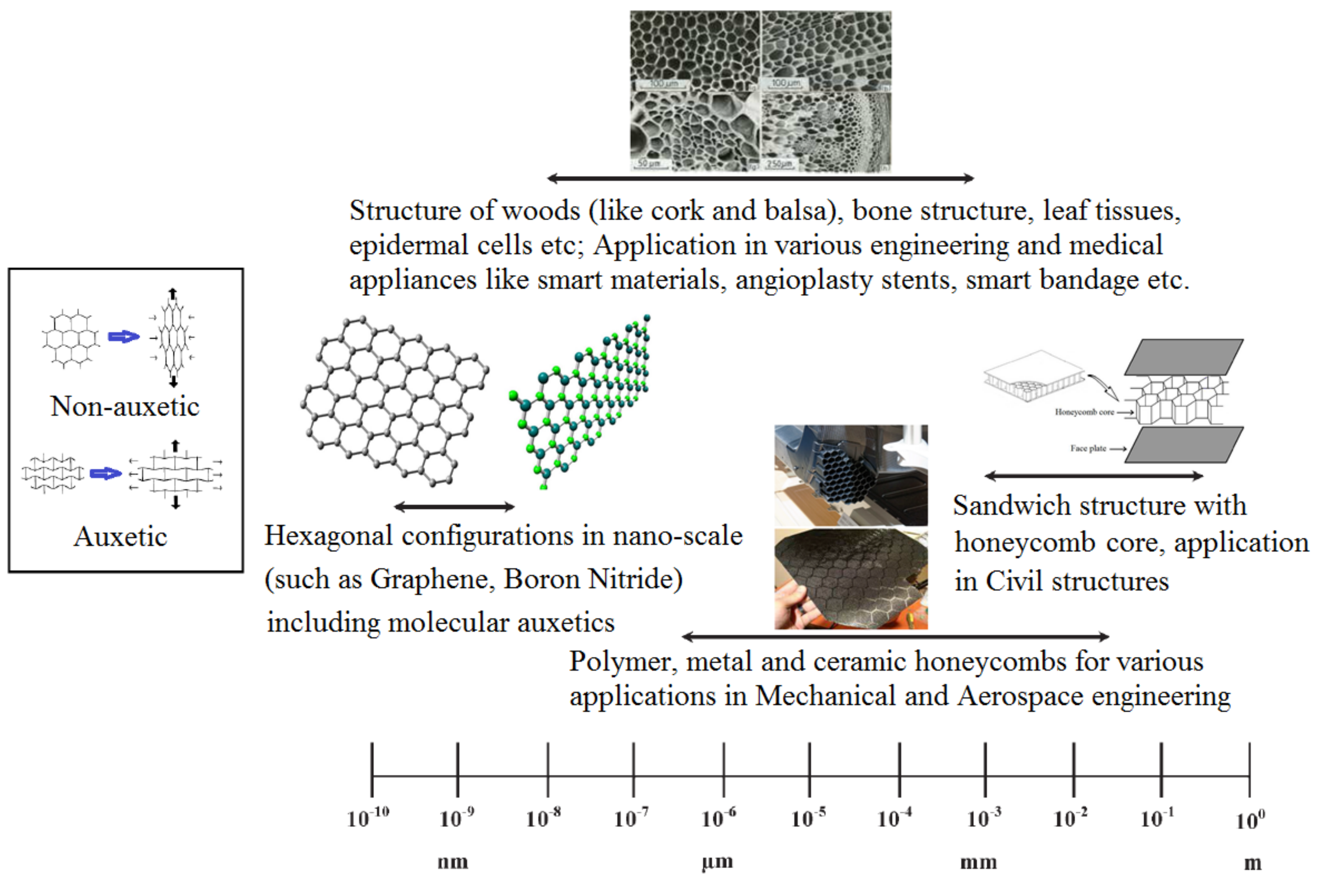

Figure: Occurance and application of hexagonal lattices across different length scales

Key Idea

A bottom up approach has been proposed for deriving expressions of effective elastic moduli for irregular honeycombs, wherein the entire irregular honeycomb structure is considered to be consisted of several representative unit cell elements having different individual elastic moduli depending on its structural geometry and material properties.

In the elementary local level, effect of irregularity is accounted by analysing the representative unit cell elements (RUCEs) first and then this effect of irregularity is propagated towards the global properties of the entire structure in a multi-scale framework through a multi-stage process [1-2].

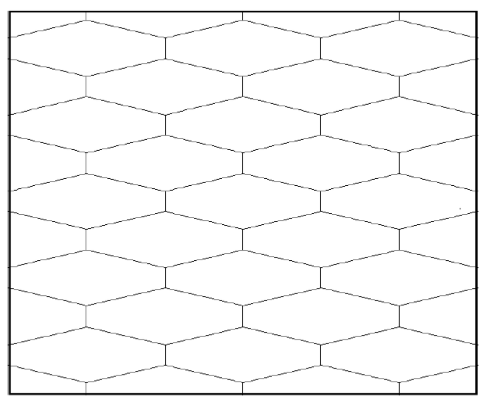

Figure: Regular honeycomb

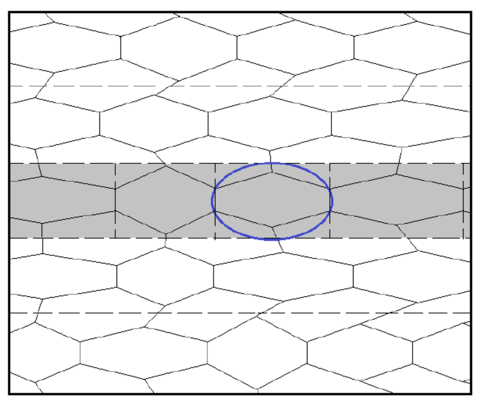

Figure: Irregular honeycomb

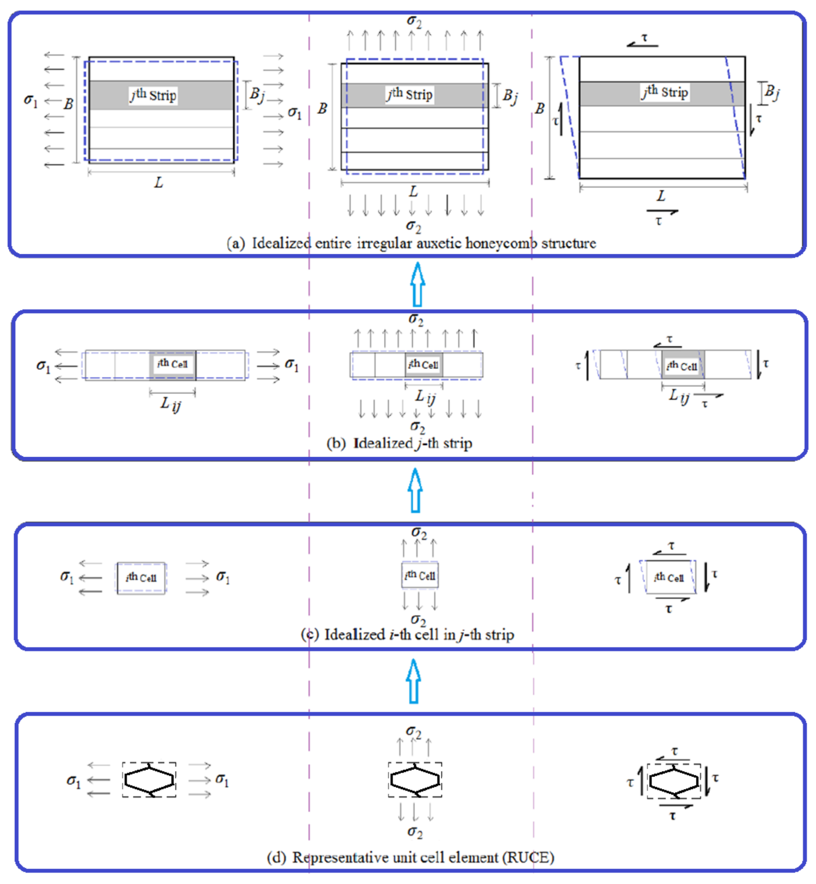

Figure: Bottom up analytical framework for analyzing irregular honeycombs

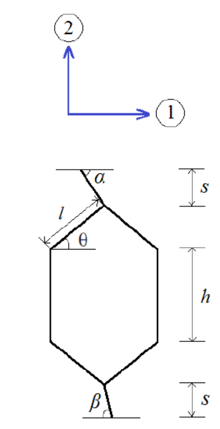

Figure: Representative unit cell element

Typical expression for equivalent elastic modulus E_2:

$$E_{2eq} = \frac{1}{\left(\sum_{j=1}^{n} B_j \frac{\sum_{i=1}^{m} l_{ij}\cos\theta_{ij}}{\sum_{i=1}^{m} E_{2Uij} l_{ij}\cos\theta_{ij}}\right)} \sum_{j=1}^{n} B_j$$

where,

$$E_{2U} = E_s \left(\frac{t}{l}\right)^3 \times \frac{\left(\frac{h}{l}+2\frac{s}{l}+2\sin\theta\right)}{\cos\theta\left(2\cos^2\theta+8\left(\frac{s}{l}\right)^3\left(\frac{\cos^2\alpha}{\sin^3\alpha}+\frac{\cos^2\beta}{\sin^3\beta}\right)+2\left(\frac{s}{l}\right)^2(cot^2\alpha+cot^2\beta)\right)}$$

Results and discussion

Results are presented for random spatial variation of cell angles considering very small t/l ratio (mean cell angle of 45° and degree of irregularity as $\Delta\theta$ = 5°) [1]. The proposed analytical framework has been validated with the results of finite element simulations. Typical validation results for E_2 (as a ratio of E_2 for irregular honeycomb and that of regular honeycomb) are presented below:

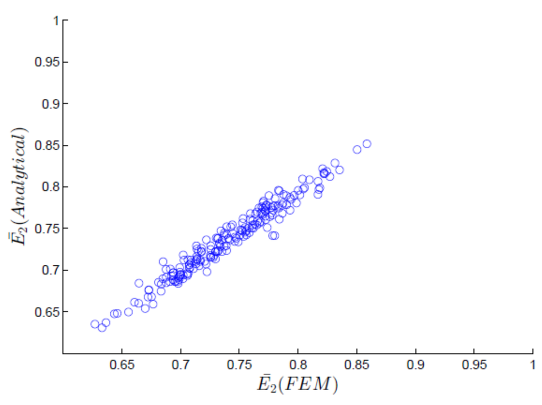

Figure: Scatter plot for E_2

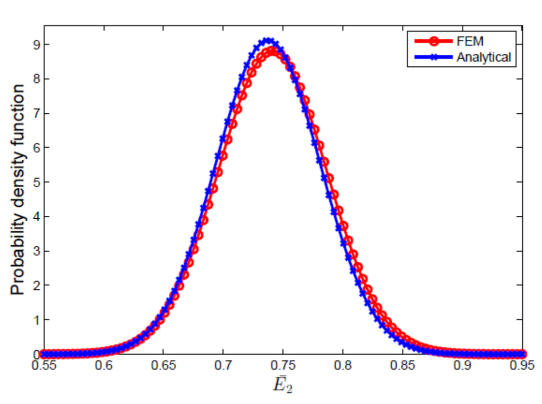

Figure: Probability density function for E_2

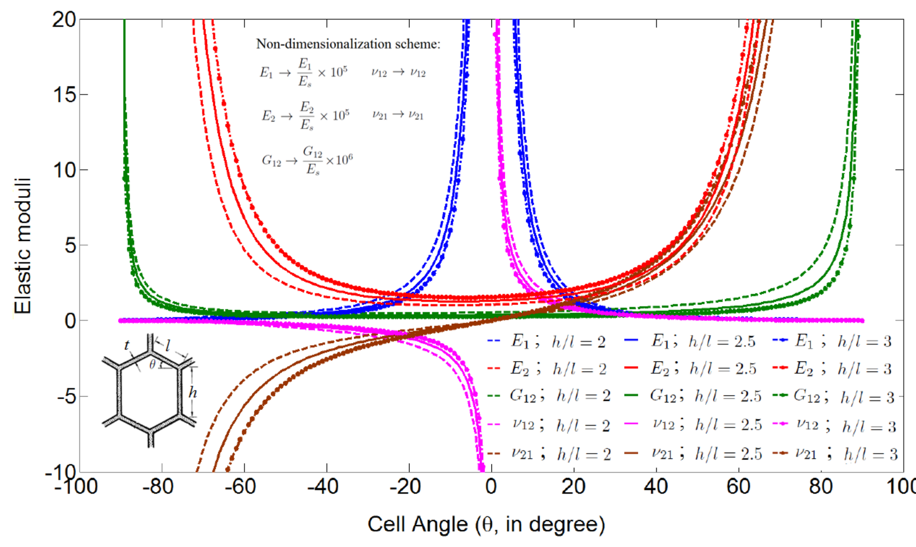

Figure: Variation of non-denominational elastic moduli with cell angle for regular honeycombs

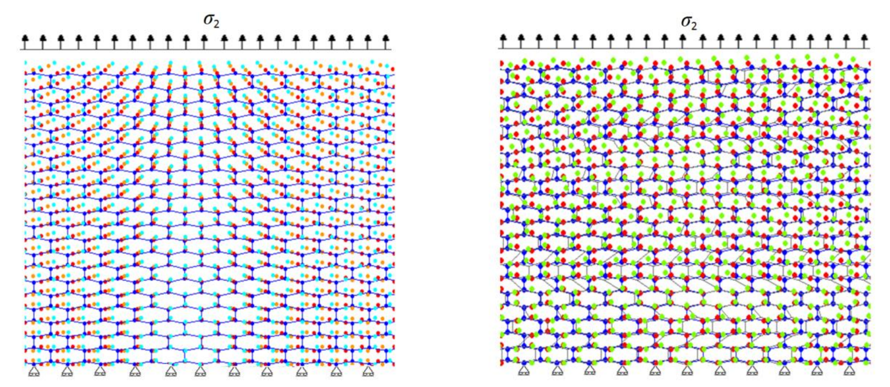

Figure: Deformed honeycomb configuration (Legends: — regular undeformed configuration; — irregular undeformed configuration; • Location of nodes for regular undeformed honeycomb; • Location of nodes for regular deformed honeycomb; • Location of nodes for irregular deformed honeycomb; • • • • Location of nodes for regular deformed honeycomb corresponding to three different stress levels in increasing order respectively)

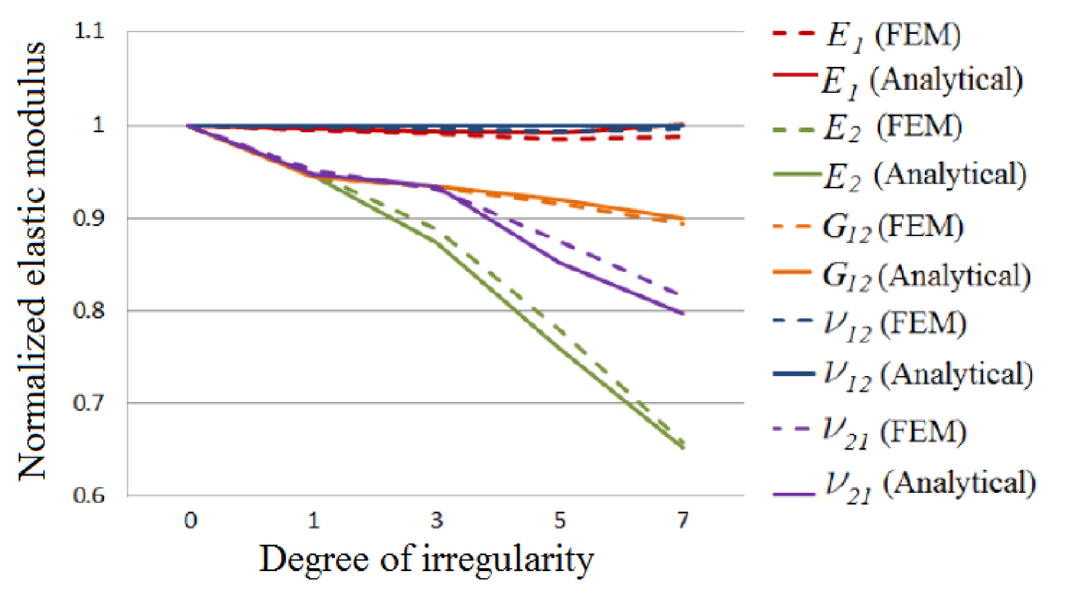

Figure: Variation of normalized elastic moduli with different degree of irregularity

Variation of mean normalized elastic moduli with different degree of irregularities are shown in the above figure, wherein it is evident that E_2, G_{12} and ν_{12} are most sensitive to spatially random structural irregularity.

Conclusion

A novel and computationally efficient analytical framework has been developed for predicting effective elastic moduli of hexagonal lattices with spatial irregularity. The closed form formulae can also be used to predict effective elastic properties of honeycombs having spatially random variation in intrinsic material properties.

The proposed analytical approach can be extended further for other forms of lattices considering appropriate representative unit cell element. Future research will follow dynamic characterization of irregular lattices.

Reference

1. Mukhopadhyay T., Adhikari S. (2016) Equivalent in-plane elastic properties of irregular honeycombs: An analytical approach, *International Journal of Solids and Structures*, (In Press) doi:10.1016/j.ijsolstr.2015.12.006

2. Mukhopadhyay T., Adhikari S. (2016) Effective in-plane elastic properties of auxetic honeycombs with spatial irregularity, *Mechanics of Materials*, (In Press) doi: 10.1016/j.mechmat.2016.01.009

Acknowledgement

TM gratefully acknowledges the financial support from Swansea University through the award of Zienkiewicz Scholarship.