Uncertainty Quantification in the Dynamics of Composite Structures

C. Scarth[1], S. Adhikari[2]

[1]Research Assistant, [2]Chair of Aerospace Engineering
College of Engineering, Swansea University, Bay Campus, Fabian Way, Swansea, SA1 8EN, UK

Aeroelastic Tailoring Workshop, Seoul, Korea, 2016

Swansea University
Prifysgol Abertawe

Outline

- Introduction – Project overview
- Uncertainty quantification of the EMB benchmark wing
  - Uncertainty in the engine mass position
  - Uncertainty in the engine mass
  - Variation in the fuel mass
  - Uncertainty in the pylon stiffness
- Random tow variation
  - Problem formulation
  - Challenges
  - Energy approach
- Conclusions

Project overview

Present and past staff

Sondipon Adhikari

Carl Scarth

Madelein Raunholm Midtoy (left after MSc, in Jan 2016 – got a 1st class degree)

Work Plan

1. Modelling of uncertain variability in the system (*months 0-8*):
2. Stochastic modelling of the gust load (*months 9-12*):
3. Global sensitivity analysis and reduced order modelling (*months 13-18*):
4. Robust response analysis (months 19-22):
5. Optimal design approach for aeroelastic tailoring (*months 23-30*):
6. Application and validation using Embraer benchmark wing (*months 31-36*):

Related external funding

Swansea University
Prifysgol Abertawe

• Swansea University funded an international PhD studentship in stochastic dynamics area (£25,000 per year for three years): £75,000

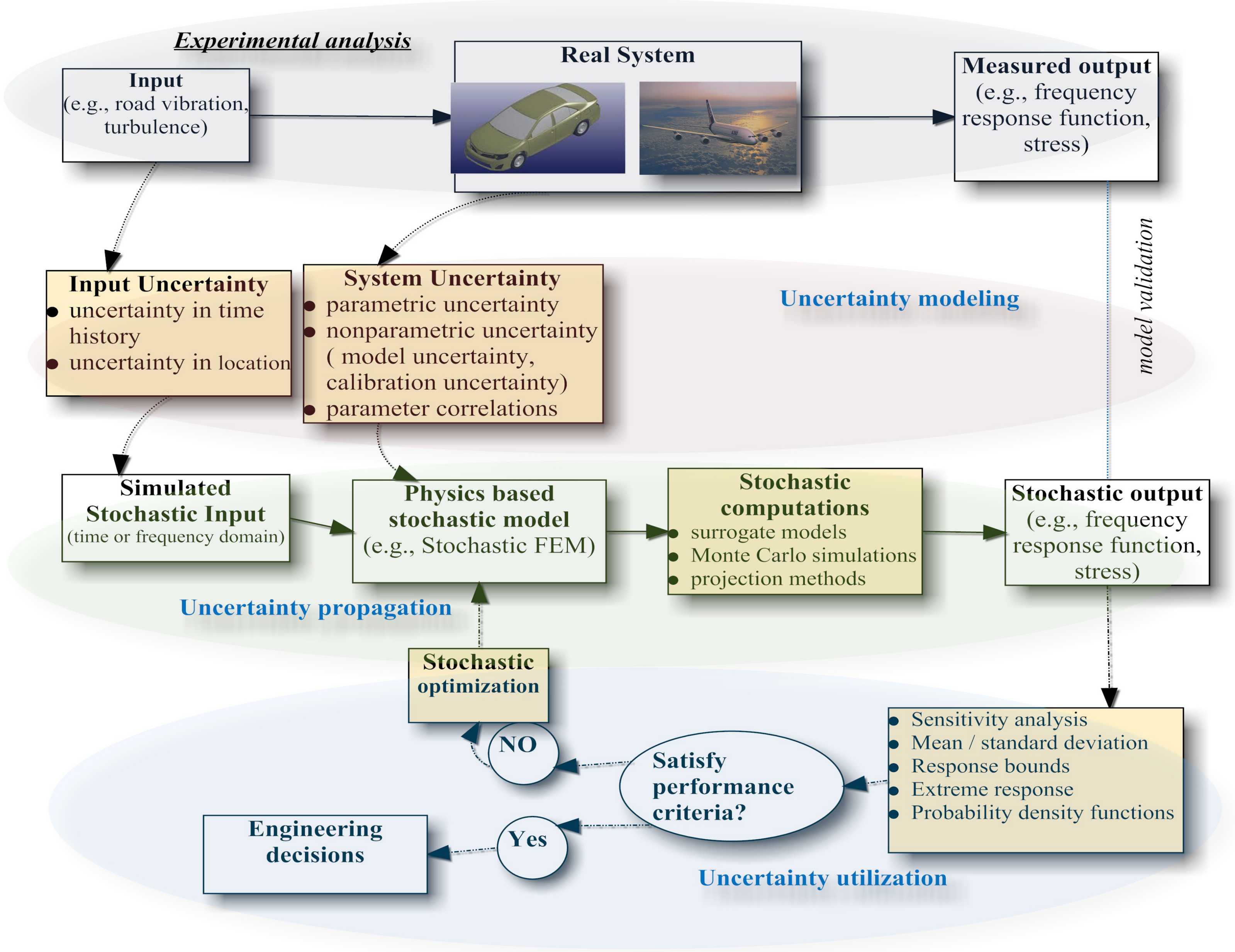
• Wales Research Network (NRN 102) awarded a grant of £83,000 on "A multiscale approach for uncertainty quantification in composite structures" (two-years Research Associate)

•Wales Research Network (NRN 125) awarded a grant of £59,500 on "Reduced order modelling and error estimates for time varying stochastic systems" (three-year EU PhD studentship)

ENGINEERING RESEARCH NETWORK WALES
RHWYDWAITH YMCHWIL PEIRIANNEG CYMRU

Swansea University
Prifysgol Abertawe

UQ in Computational Modeling



Why Uncertainty: The Sources

Experimental error

uncertain and unknown error percolate into the model when they are calibrated against experimental results

Parametric Uncertainty

uncertainty in the geometric parameters, boundary conditions, forces, strength of the materials involved

Computational uncertainty

machine precession, error tolerance and the so called ‘h’ and ‘p’ refinements in finite element analysis

Model Uncertainty

arising from the lack of scientific knowledge about the model which is a-priori unknown (damping, nonlinearity, joints)

Broad approaches to UQ

UQ

Physics based UQ

[1] Kundu, A., Adhikari, S., Friswell, M. I., "Transient response analysis of randomly parametrized finite element systems based on approximate balanced reduction", *Computer Methods in Applied Mechanics and Engineering*, 285[3] (2015), pp. 542-570.

[2] Kundu, A. and Adhikari, S., "Dynamic analysis of stochastic structural systems using frequency adaptive spectral functions", *Probabilistic Engineering Mechanics*, 39[1] (2015), pp. 23-38.

[3] DiazDelaO , F. A., Kundu, A., Adhikari, S. and Friswell, M. I., "A hybrid spectral and metamodeling approach for the stochastic finite element analysis of structural dynamic systems, *Computer Methods in Applied Mechanics and Engineering*, 270[3] (2014), pp. 201-209.

[4] Kundu, A., Adhikari, S., "Transient response of structural dynamic systems with parametric uncertainty", *ASCE Journal of Engineering Mechanics*, 140[2] (2014), pp. 315-331.

[5] Kundu, A., Adhikari, S. and Friswell, M. I., "Stochastic finite elements of discretely parametrized random systems on domains with boundary uncertainty", *International Journal for Numerical Methods in Engineering*, 100[3] (2014), pp. 183-221.

Black-box UQ

[1[Dey, S., Mukhopadhyay, T., Sahu, S. K., Li, G., Rabitz, H. and Adhikari, S., "Thermal uncertainty quantification in frequency responses of laminated composite plates", *Composites Part B: Engineering*, 80[6] (2015), pp. 186-197.

[2] Dey, S., Mukhopadhyay, T., Adhikari, S. Khodaparast, H. H. and Kerfriden, P., "Rotational and ply-level uncertainty in response of composite conical shells", *Composite Structures*, 131[6] (2015), pp. 594-605.

[3] Dey, S., Mukhopadhyay, T., Adhikari, S. and Khodaparast, H. H., "Stochastic natural frequency of composite conical shells", *Acta Mechanica*, 226[8] (2015), pp. 2537-2553.

[4] Dey, S., Mukhopadhyay, T., and Adhikari, S., "Stochastic free vibration analyses of composite doubly curved shells - A Kriging model approach", *Composites Part B: Engineering*, 70[3] (2015), pp. 99-112.

[5] Dey, S., Mukhopadhyay, T., and Adhikari, S., "Stochastic free vibration analysis of angle-ply composite plates - A RS-HDMR approach", *Composite Structures*, 122[4] (2015), pp. 526-536.

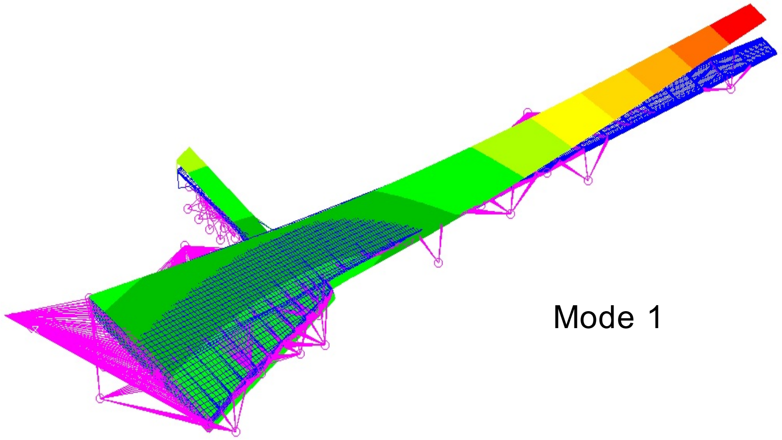
UQ on EMB Benchmark wing

Benchmark wing – v1

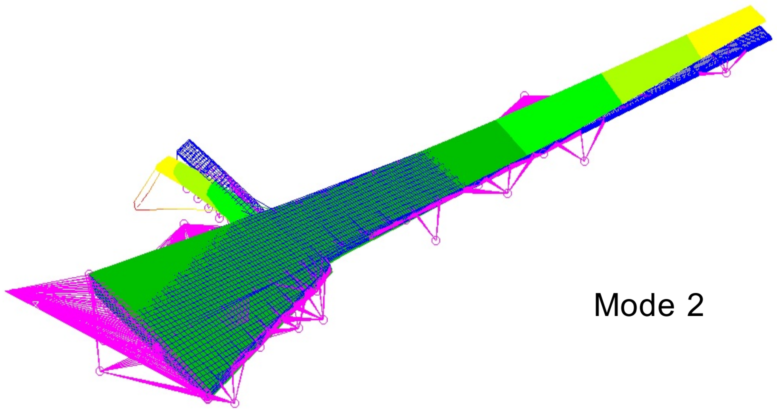
Mode no	Frequency (radian/s)	Frequency (Hz)
1	17.73323	2.822331
2	21.24172	3.380725
3	36.46753	5.803988
4	43.43028	6.912143
5	54.2736	8.637911
6	83.9236	13.35686
7	100.7871	16.04077
8	119.4018	19.00338
9	146.0667	23.24724
10	173.8495	27.669

The frequency of the first ten modes for the initial model

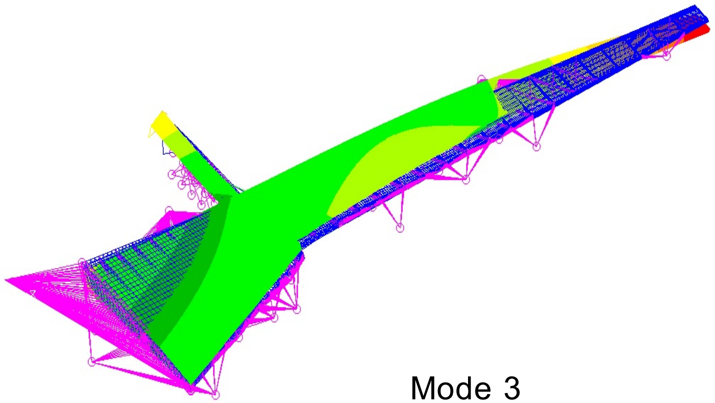
Mode shapes



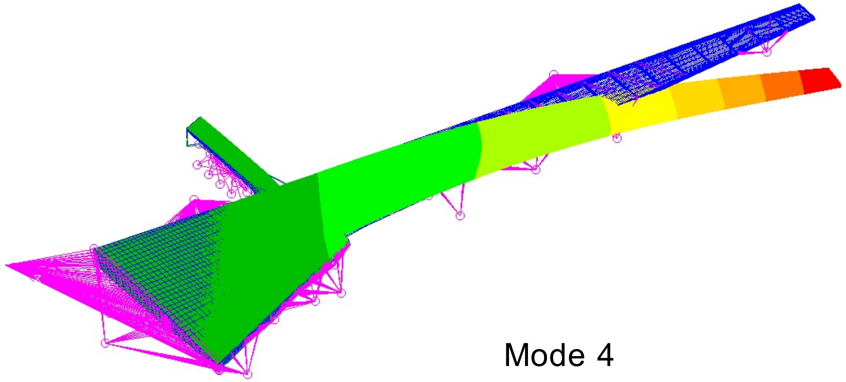
Mode 1



Mode 2

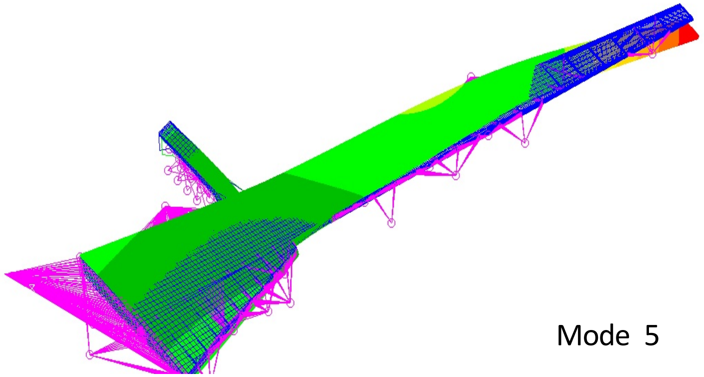


Mode 3

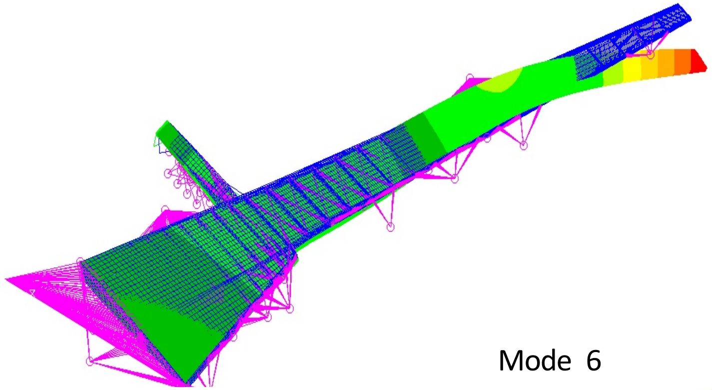


Mode 4

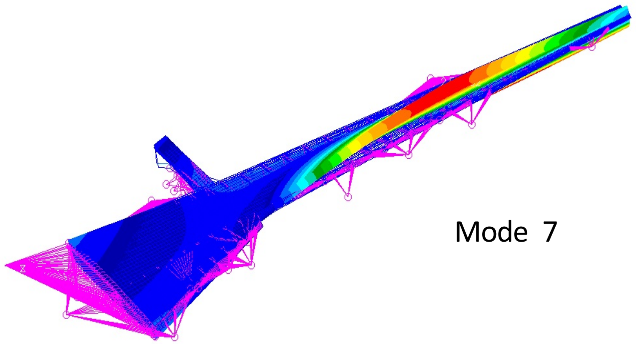
Mode shapes



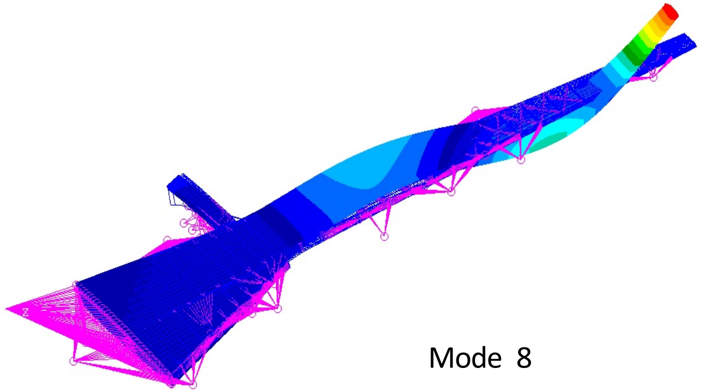
Mode 5



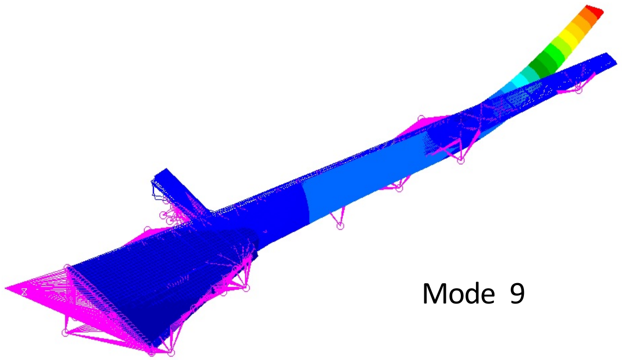
Mode 6



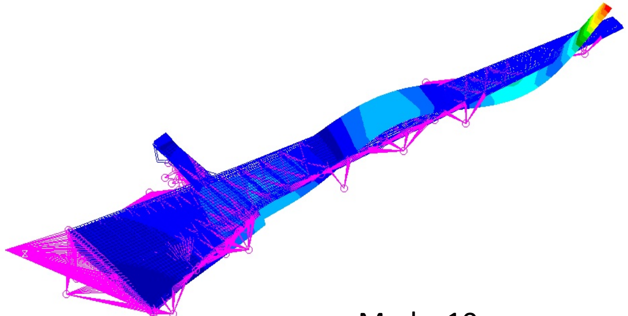
Mode 7



Mode 8

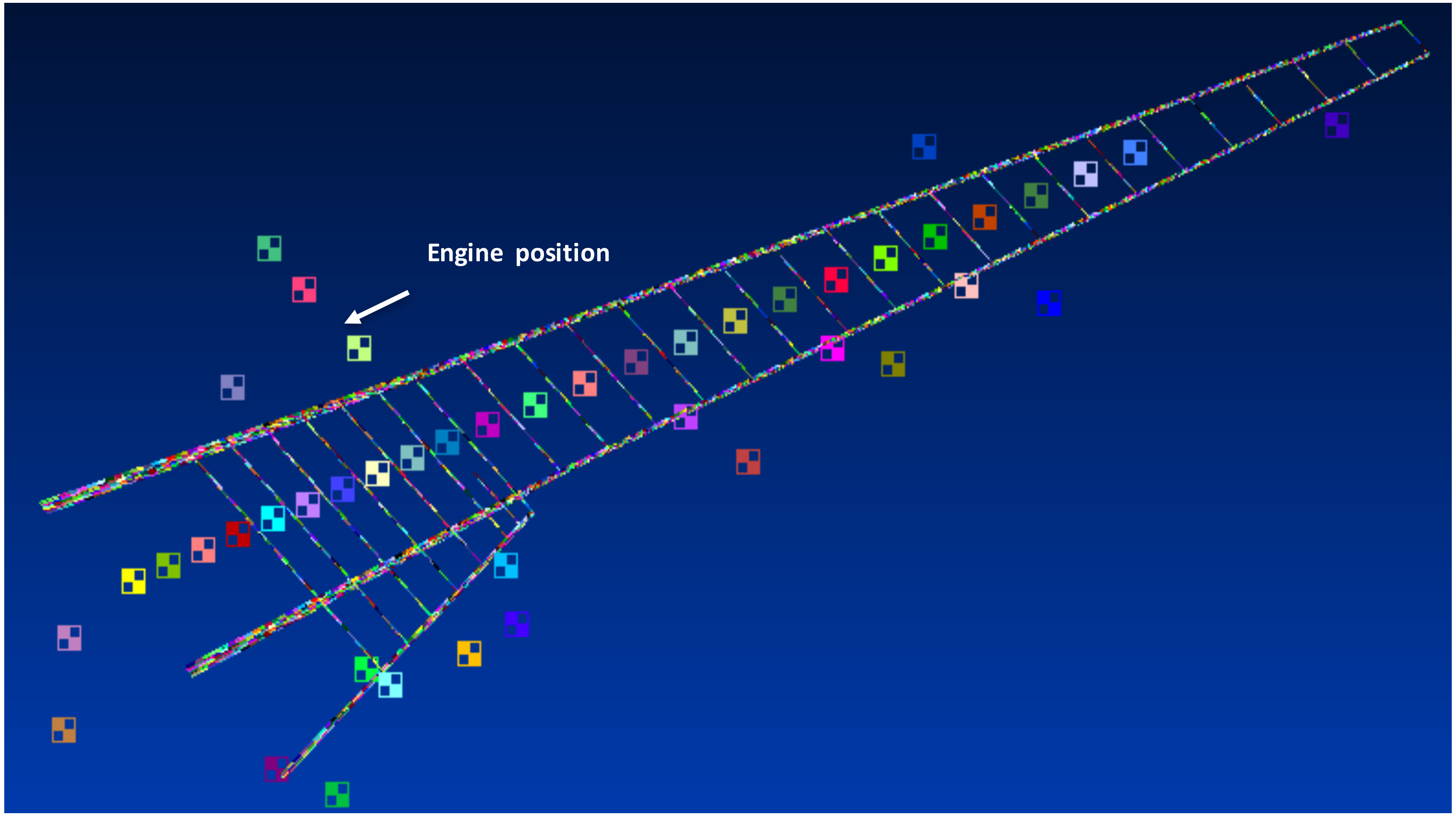


Mode 9



Mode 10

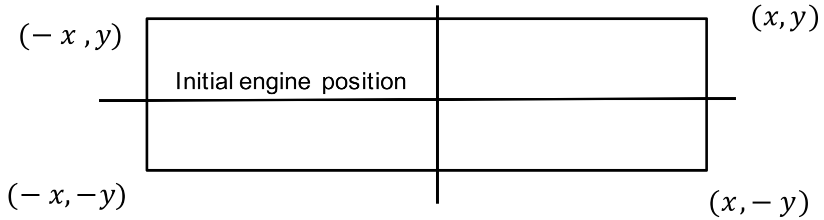
Uncertainty in the engine mass position

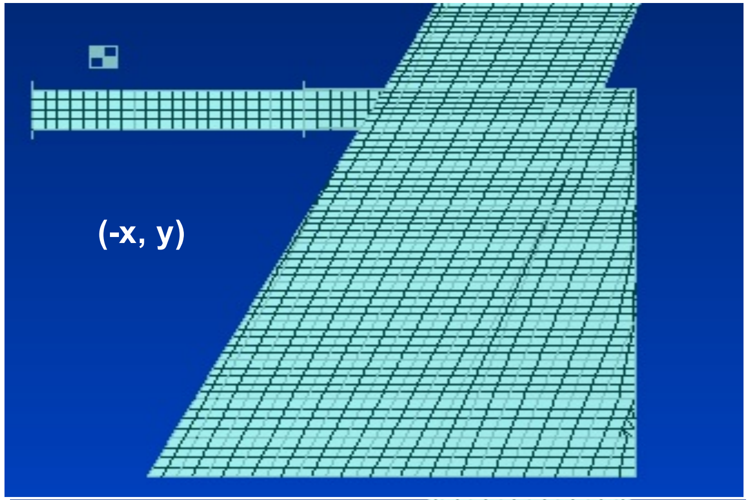


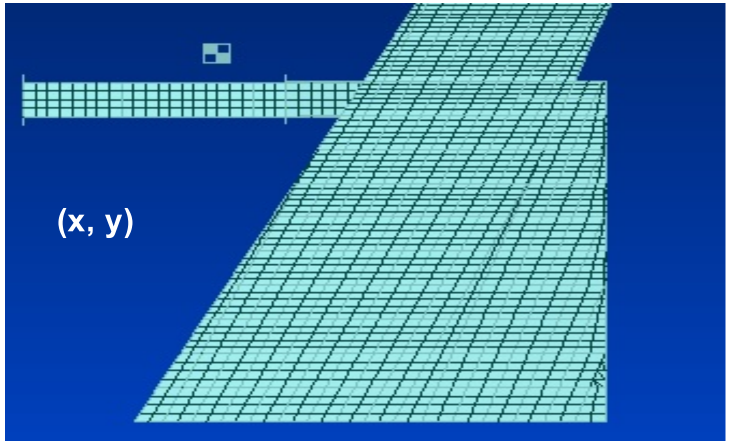
Non-Structural masses on the wing structure

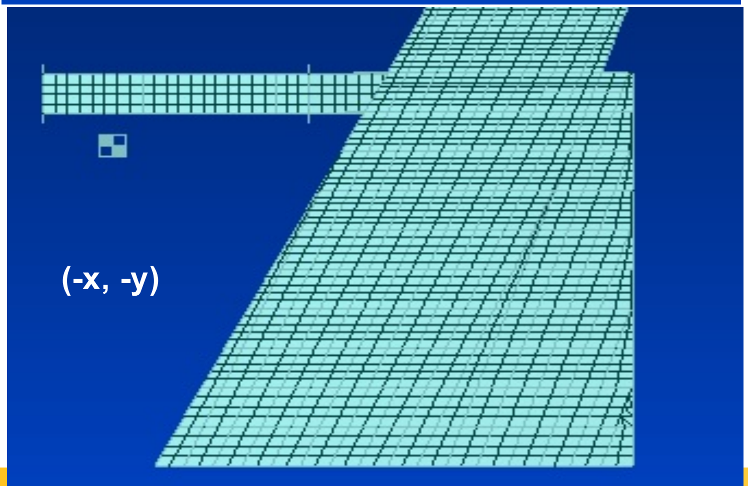
Engine location variability

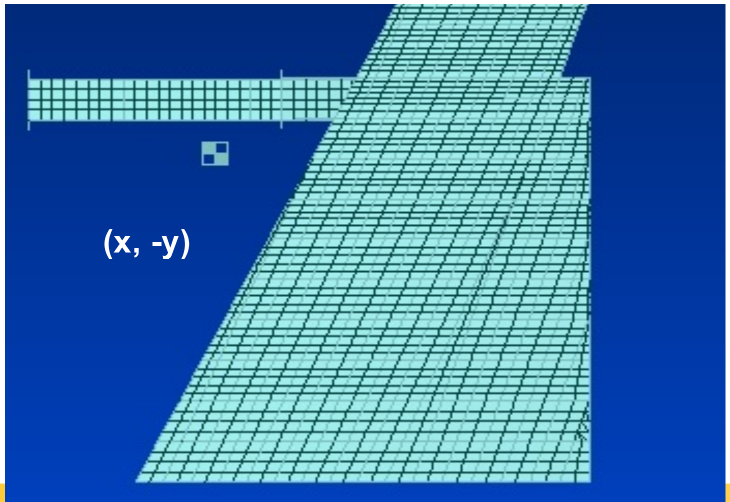
- $\pm$ 0.5m from its original coordinate
- At steps of 0.1m



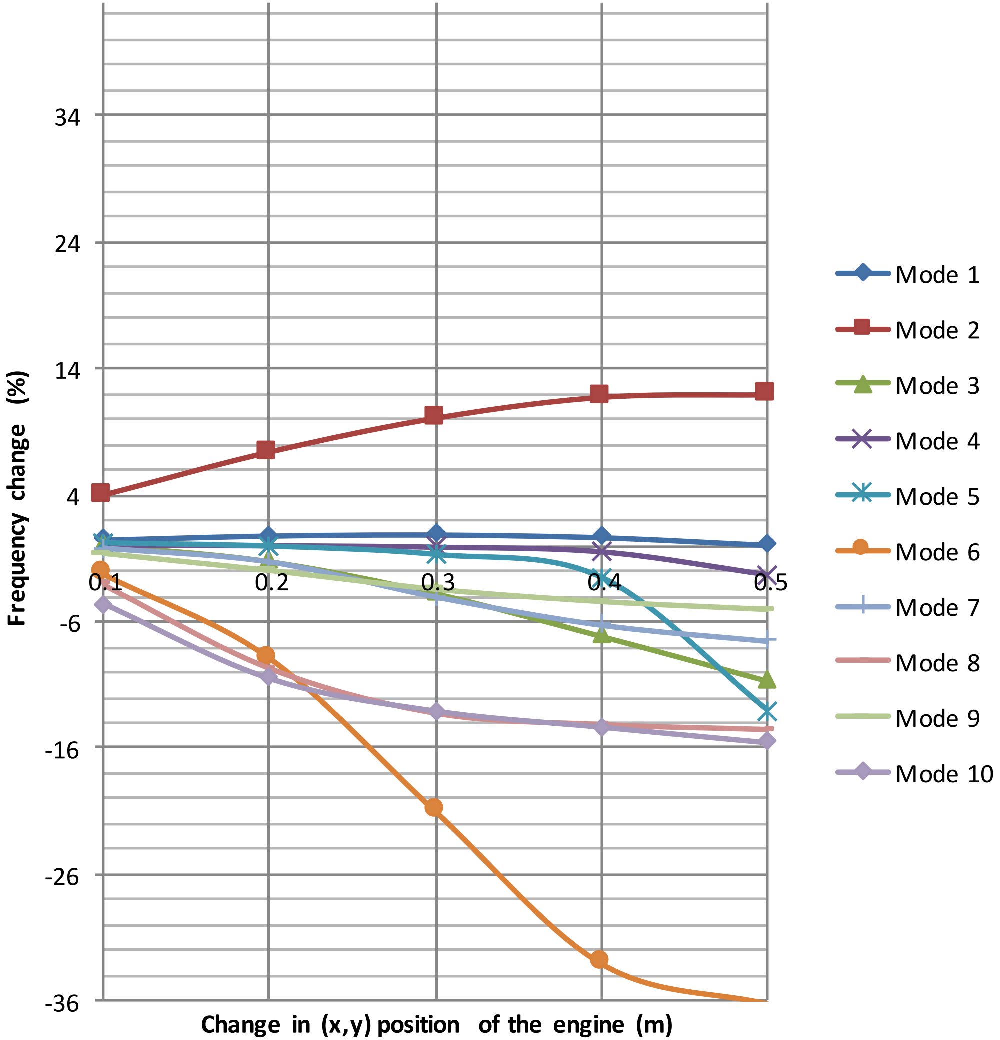


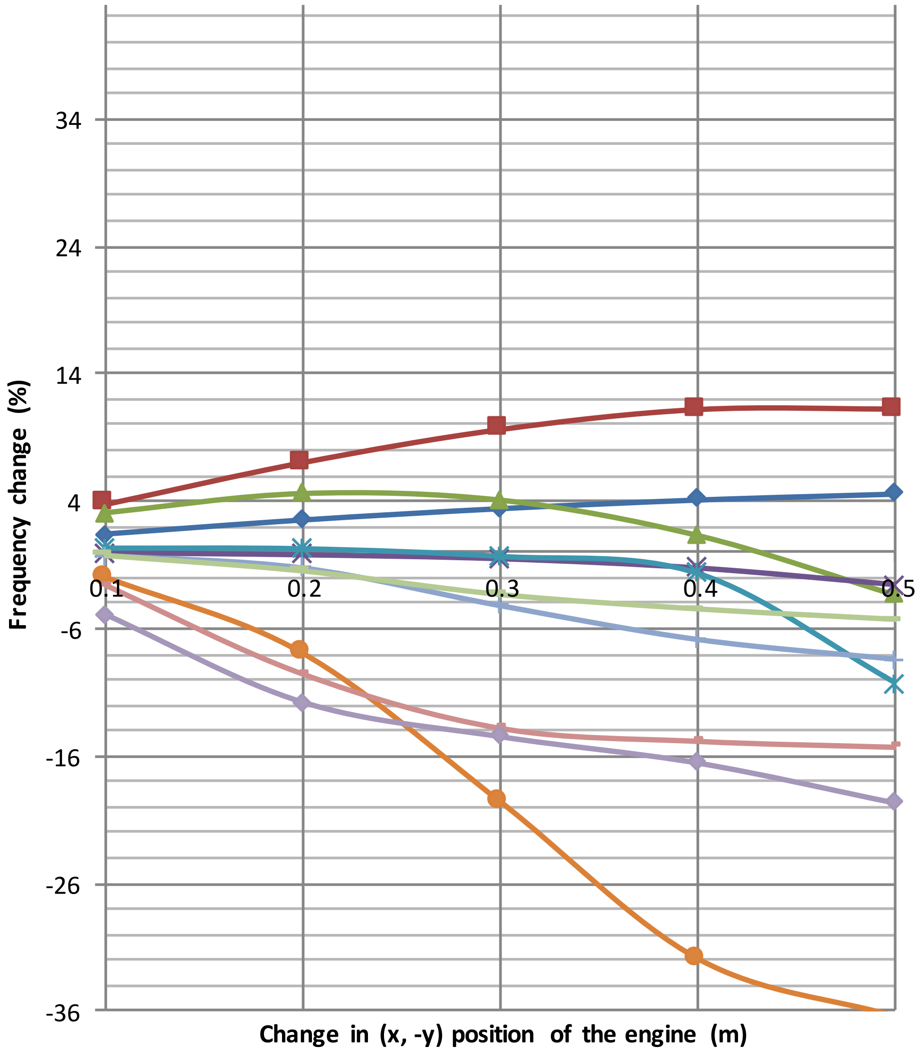




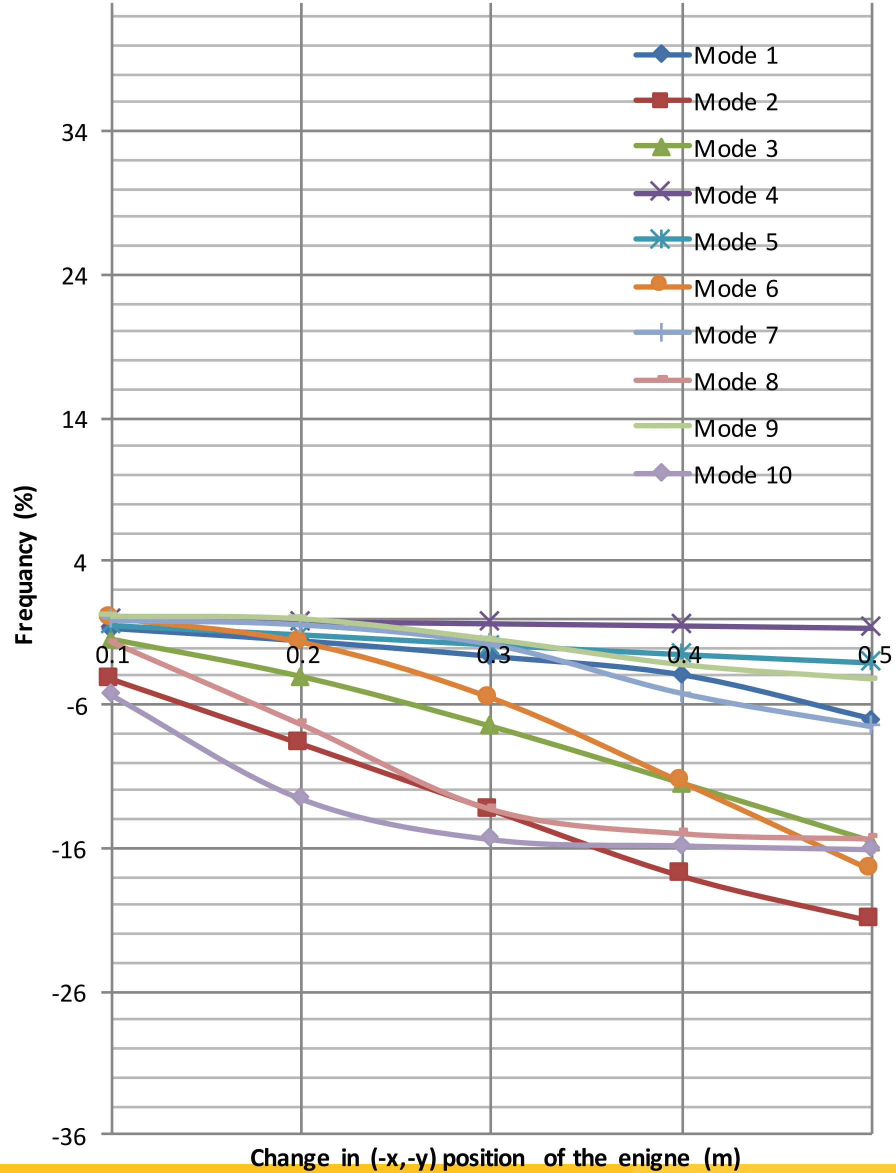


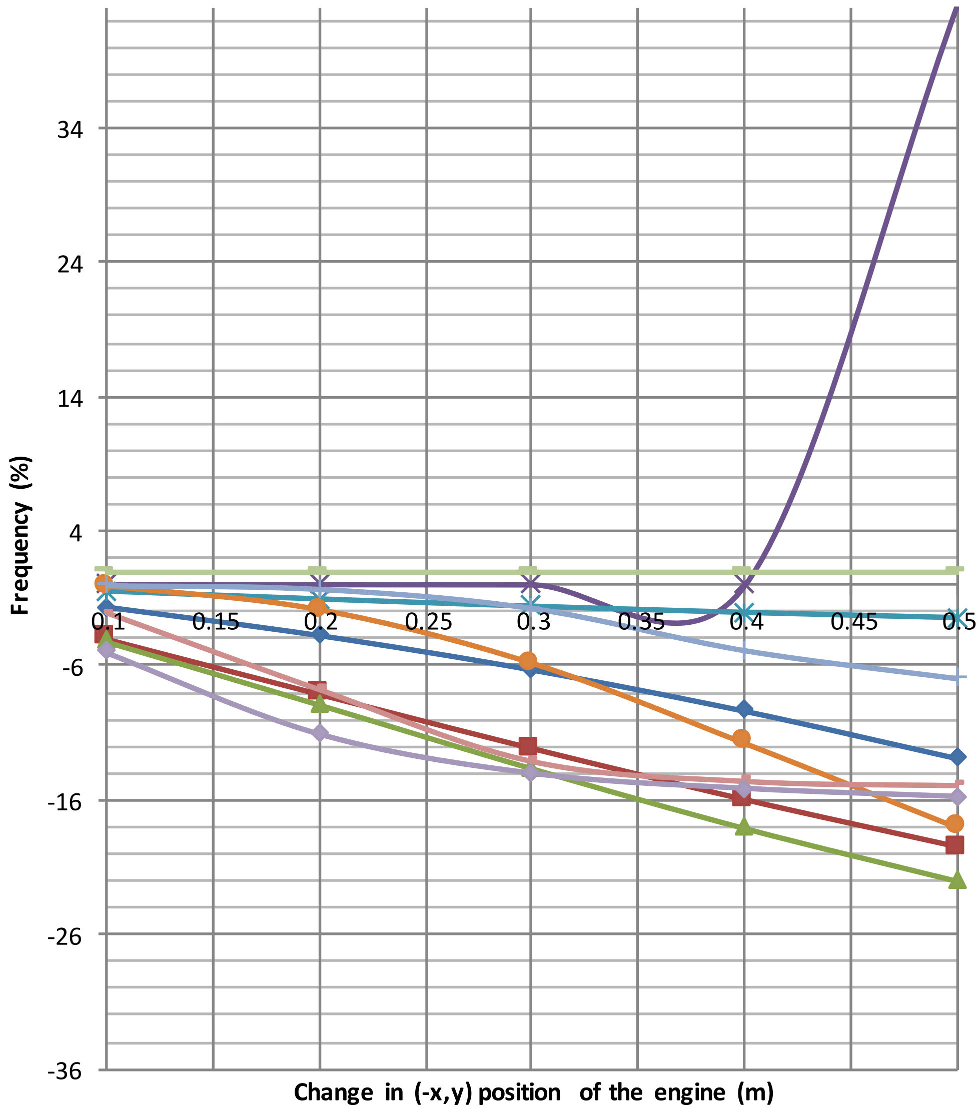
Changes in the natural frequencies due to engine location variability





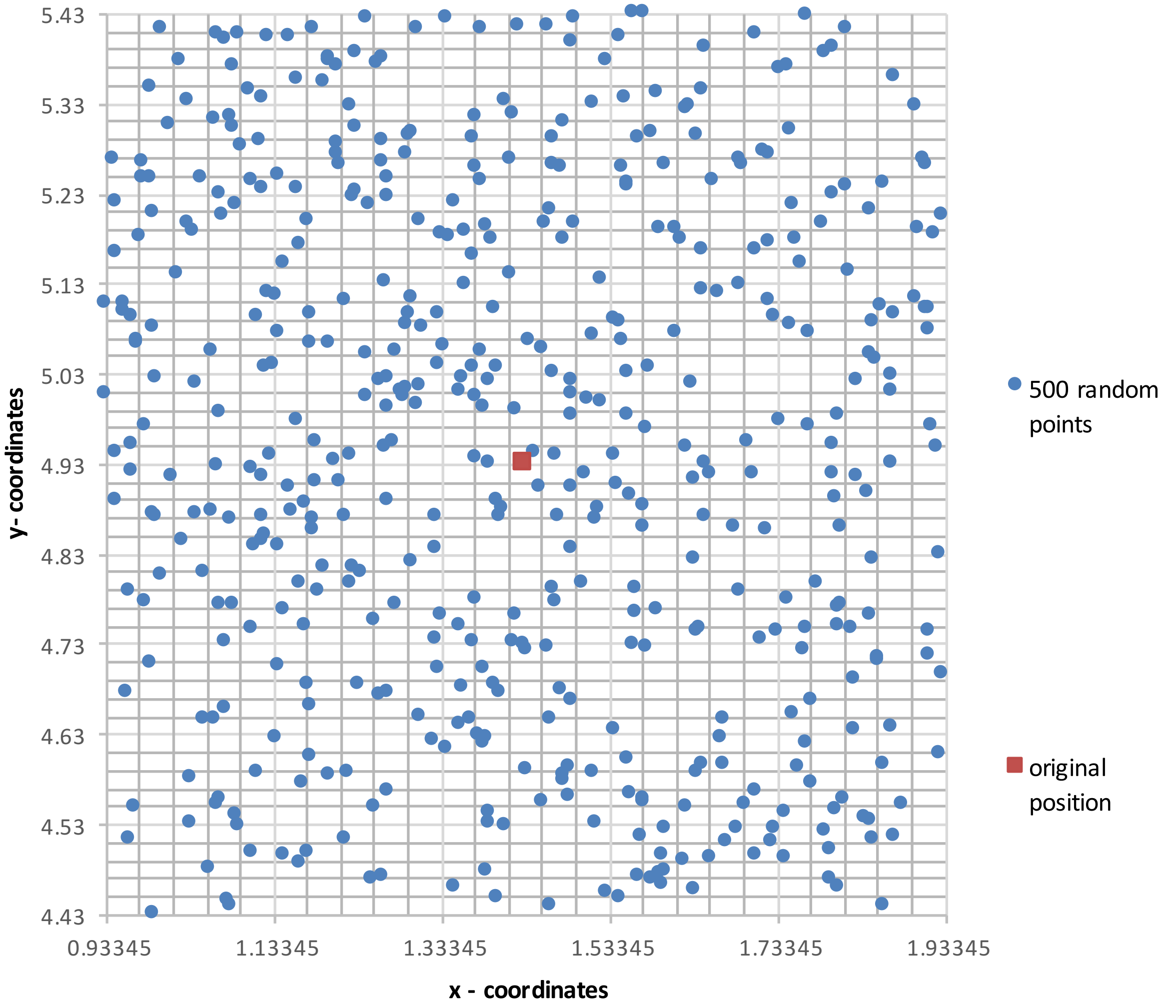
Changes in the natural frequencies due to engine location variability



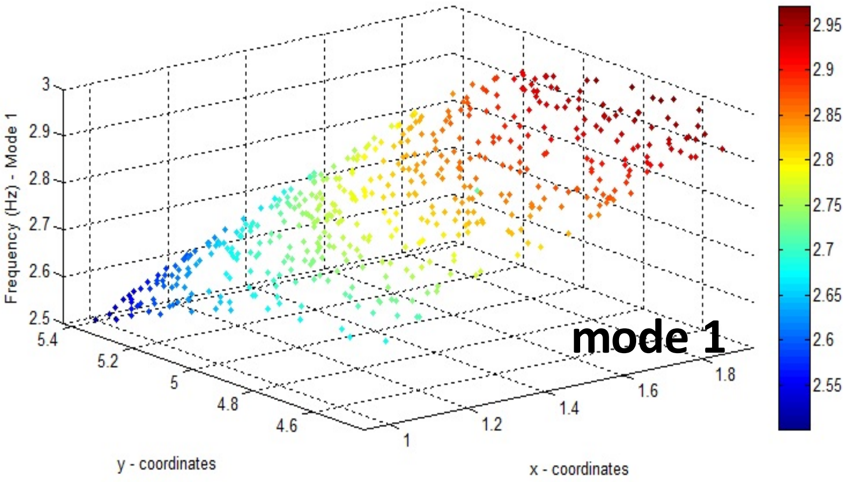


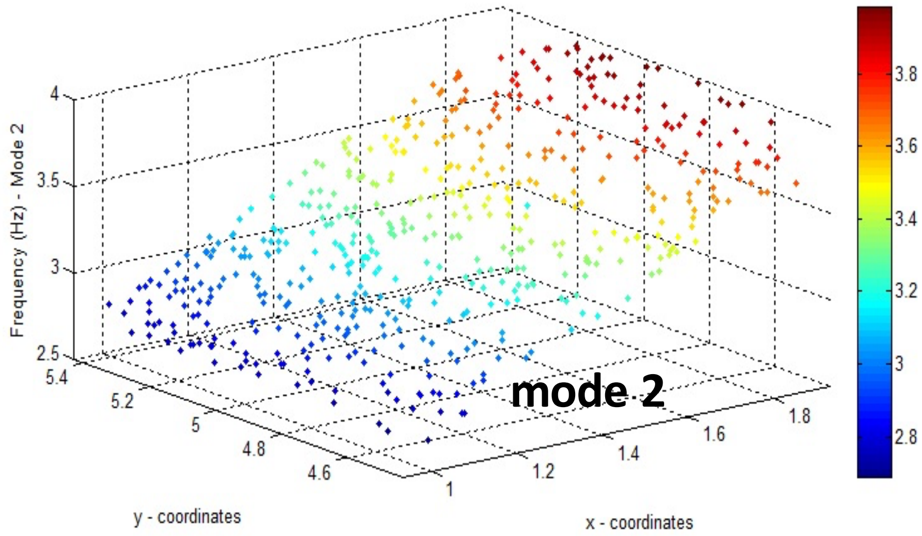
Random engine positions

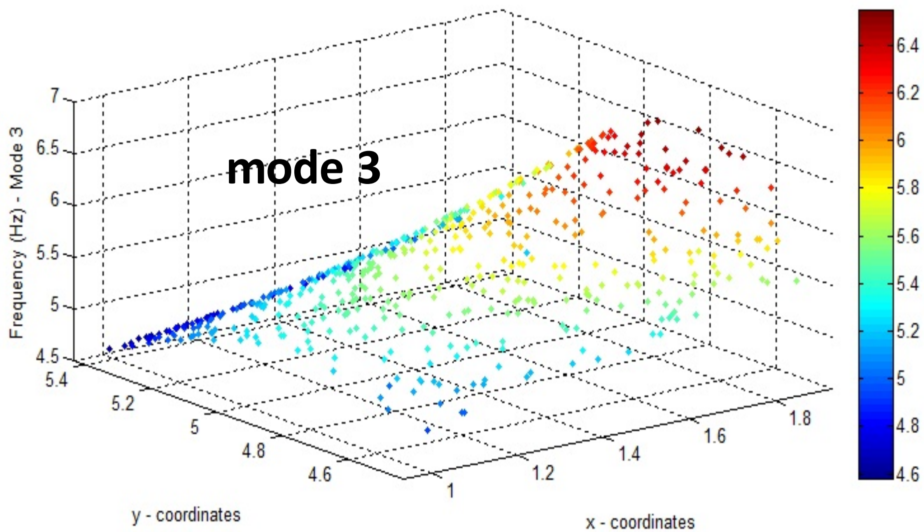
- The study explained in section (4.2.2) was extended to include more coordinates within a given limit.
- The engine position was randomly changed within the limits +-0.5m from the original (x,y) position.
- The coordinates were chosen using the excel RAND function where 500 (x,y) coordinates were randomly selected, within the x and y limits.

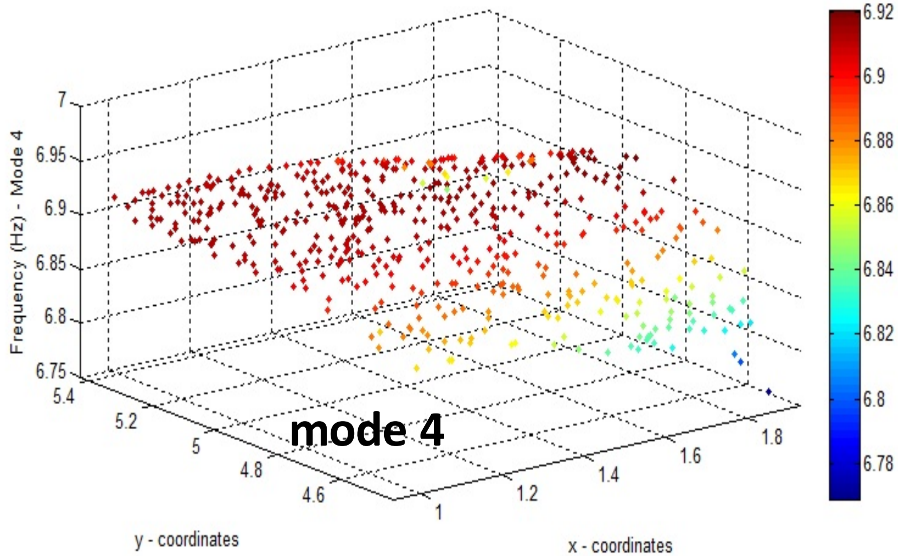


Randomness in the natural frequencies

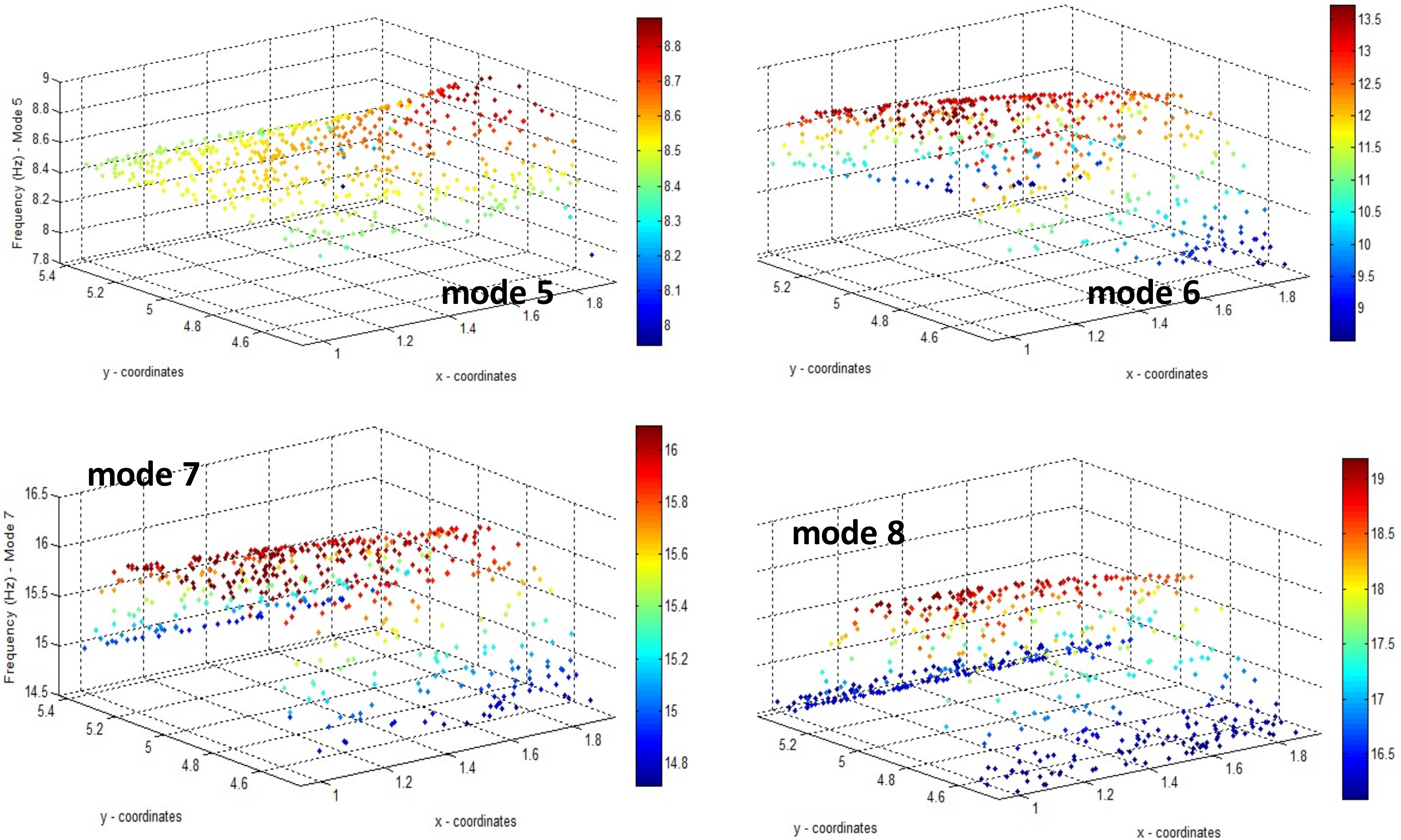








Randomness in the natural frequencies

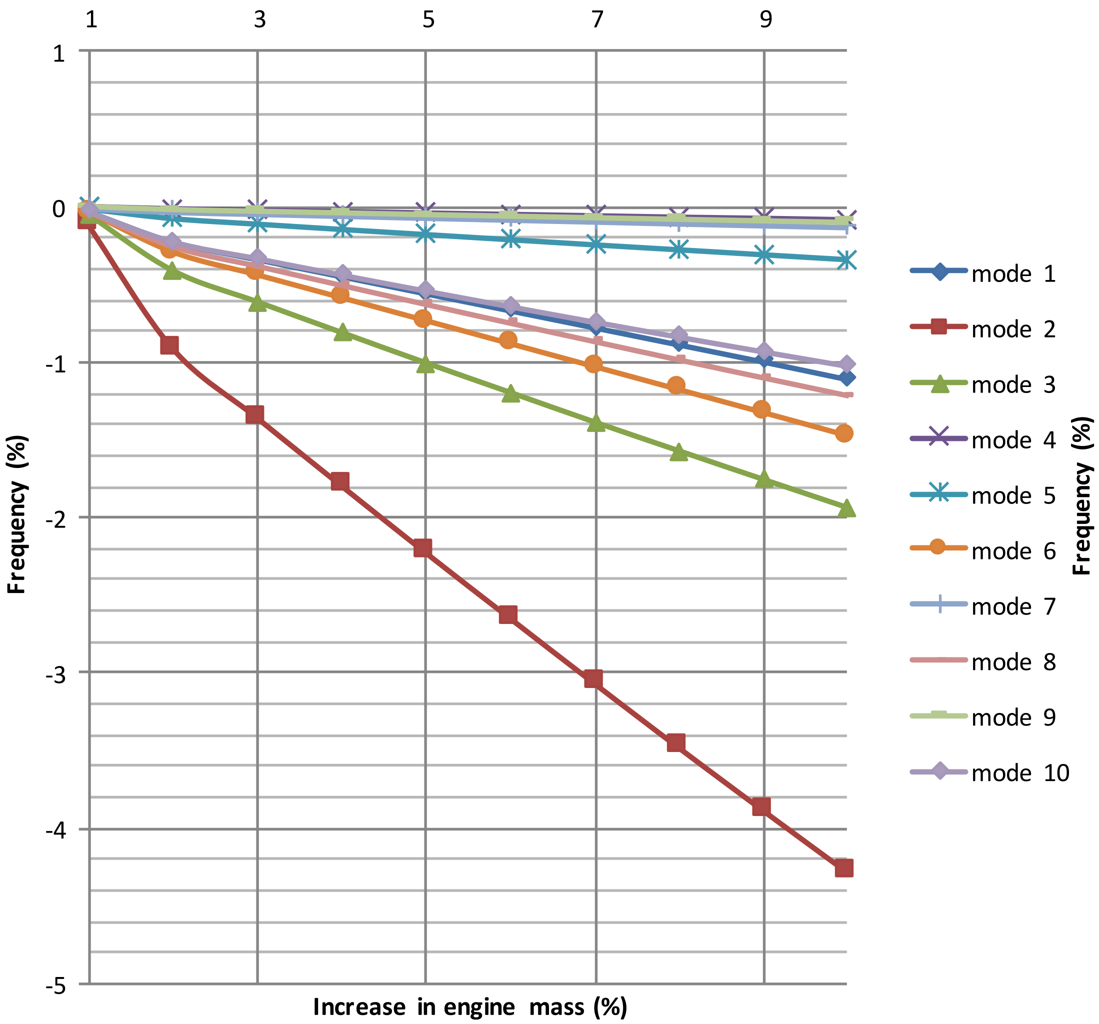


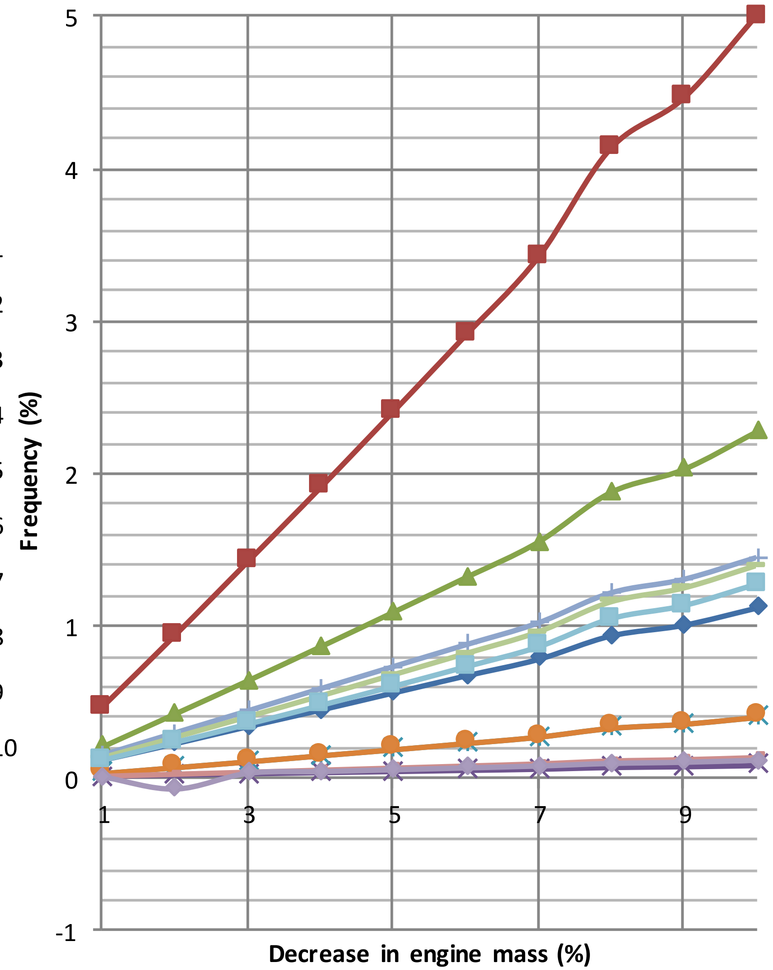
Frequency bounds due to random engine positions

	Frequency bounds (Hz)	
mode	min	max
1	2.500773265	2.971083786
2	2.689028245	3.986899084
3	4.581720923	6.551350079
4	6.768835967	6.920839845
5	7.944959284	8.882580244
6	8.501260652	13.73100129
7	14.71434644	16.08875428
8	16.09331312	19.18565409
9	22.07522894	23.3612546
10	22.55190219	27.75692596

Minimum and maximum frequencies measured in the 500 coordinates

Variability of the engine mass





Change in Frequency of mode 1-10 (in %) when the engine mass was gradually decreased (from -1-10%)

Change in Frequency of mode 1-10 (in % when the engine mass was gradually increased (from 1-10%)

Variability of the engine mass

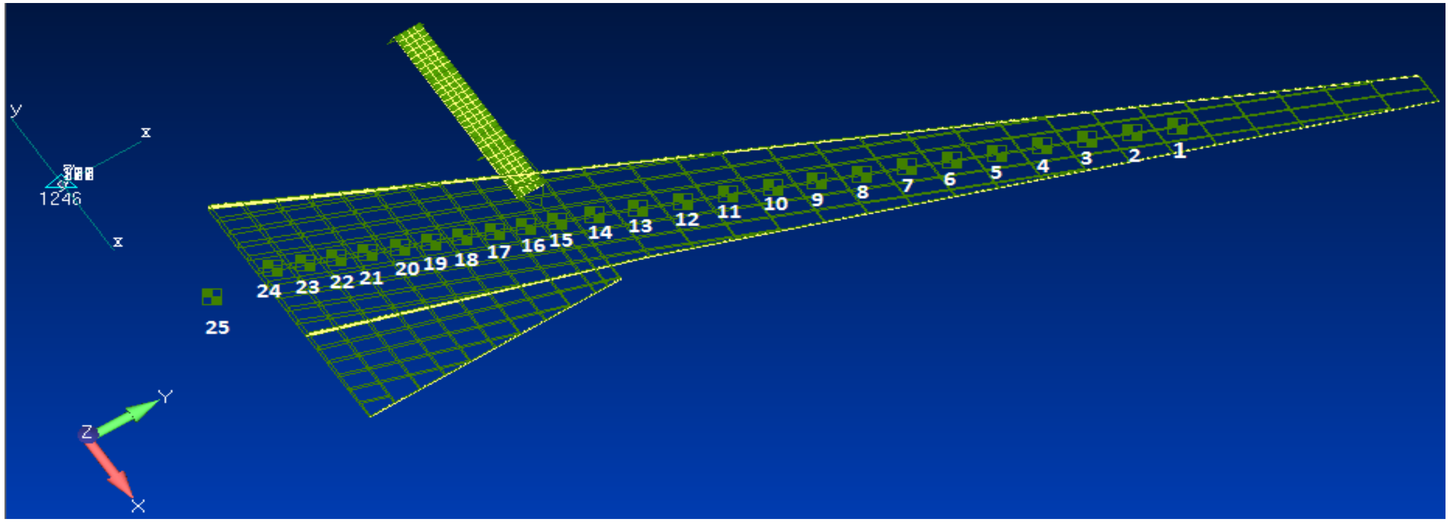
Mode	Original weight	10% decrease in weight	Weight Increase (%)
1	2.8223	2.854	1.123197
2	3.3807	3.5497	4.998965
3	5.804	5.9358	2.270848
4	6.9121	6.9183	0.089698
5	8.6379	8.6727	0.402876
6	13.357	13.5504	1.44793
7	16.041	16.0628	0.135902
8	19.003	19.2674	1.391359
9	23.247	23.2732	0.112703
10	27.669	28.0202	1.269291

Change in frequency (%) when the engine mass is decreased 10% from its original weight

Mode	Original weight	10% increase in weight	Weight Decrease (%)
1	2.8223	2.7911	-1.10548
2	3.3807	3.2358	-4.28609
3	5.804	5.6911	-1.94521
4	6.9121	6.9063	-0.08391
5	8.6379	8.6085	-0.34036
6	13.357	13.1602	-1.47338
7	16.041	16.02	-0.13091
8	19.003	18.771	-1.22086
9	23.247	23.2251	-0.09421
10	27.669	27.387	-1.01919

Change in frequency (%) when the engine mass is increased 10% from its original weight

Variability in the fuel mass distribution

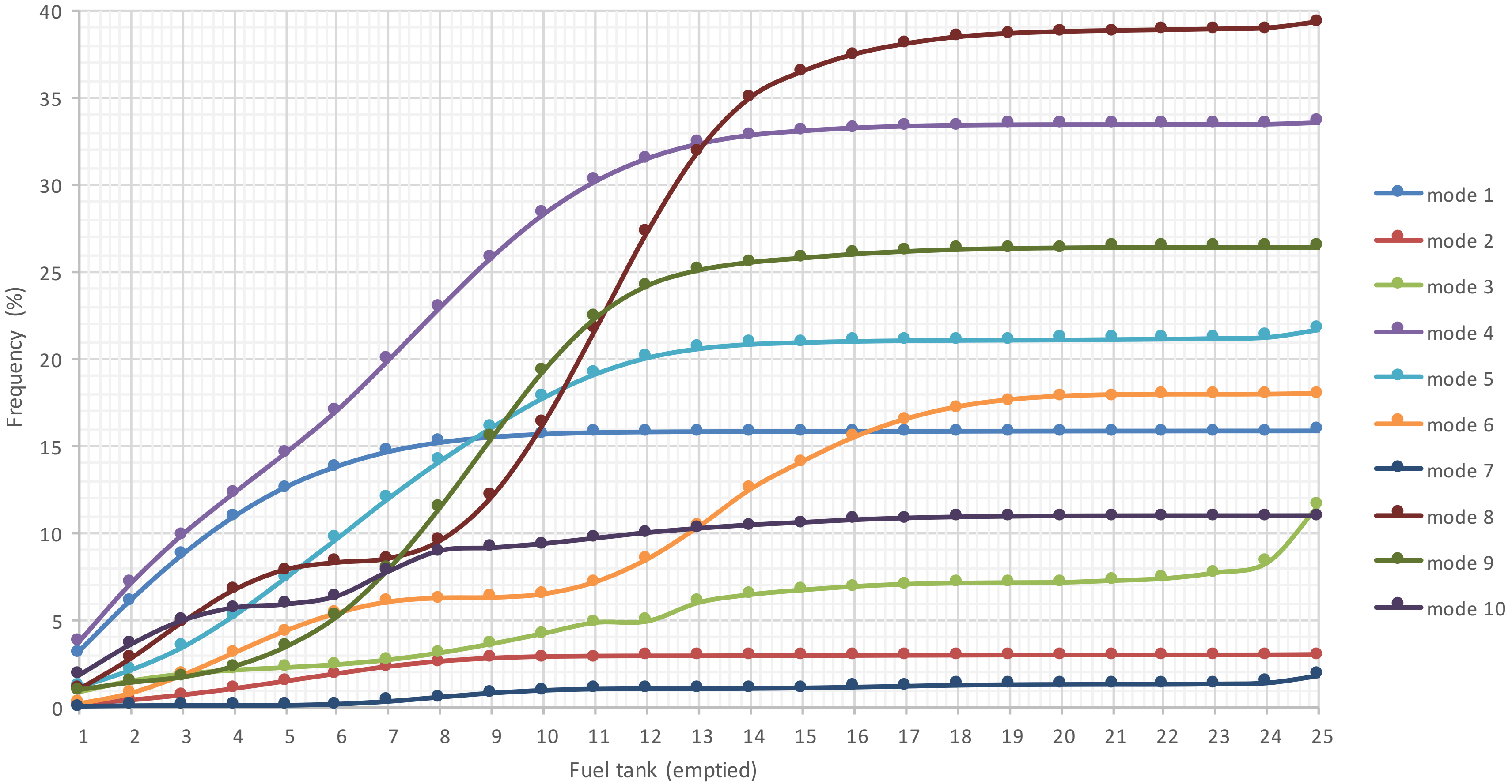


The location of the 25 fuel tanks

Fuel tank	kg	Fuel tank	kg
1	76.842	14	238.682
2	85.298	15	195.059
3	94.205	16	218.283
4	103.556	17	242.381
5	113.348	18	267.252
6	123.58	19	297.123
7	144.329	20	332.041
8	156.26	21	368.069
9	168.694	22	405.327
10	182.291	23	443.698
11	196.424	24	483.203
12	211.092	25	1960
13	226.274		

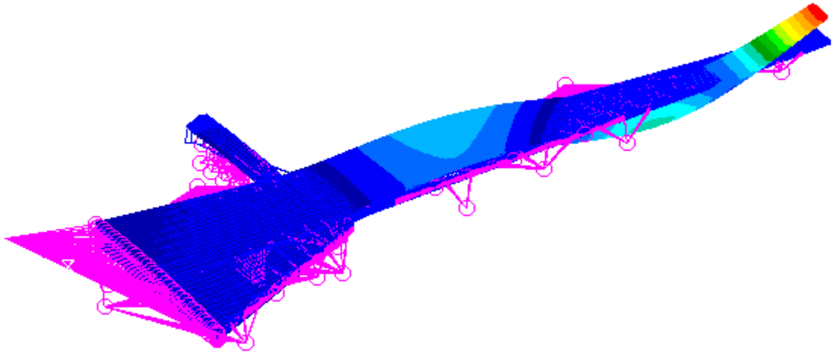
Fuel tank's masses under fully loaded conditions.

Natural frequencies due to variability in the fuel mass distribution

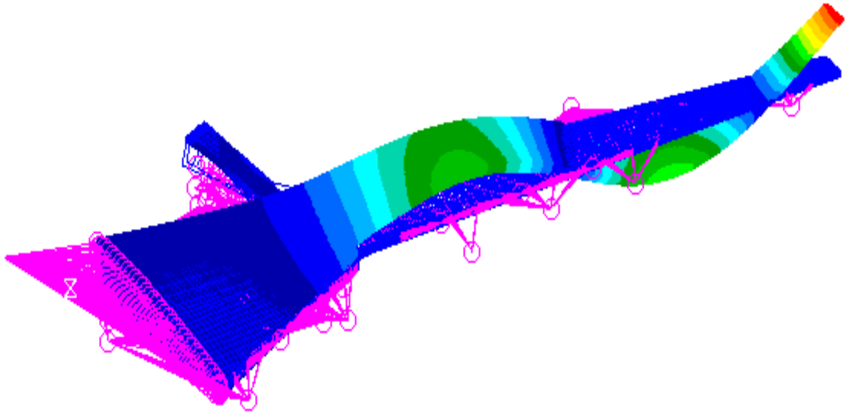


Frequency of mode 1-10 when the fuel tanks are emptied from tip to root

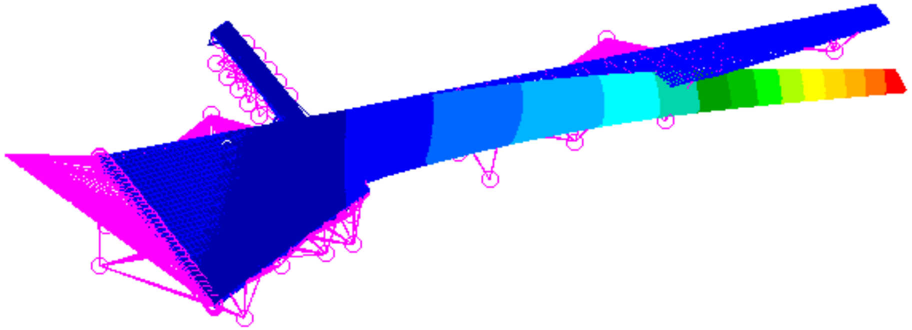
Mode shapes due to variability in the fuel mass distribution



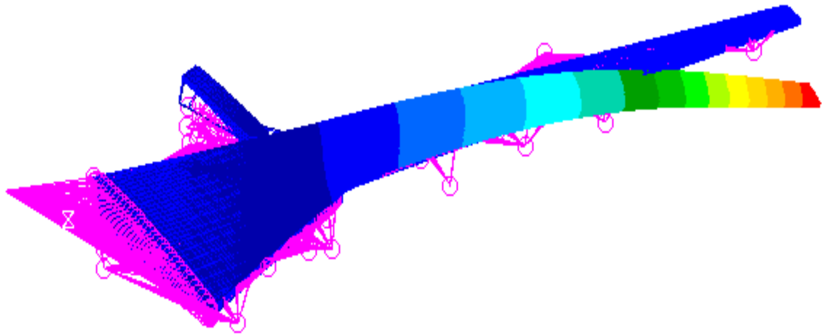
Full fuel conditions (mode 8)



Empty fuel conditions (mode 8)



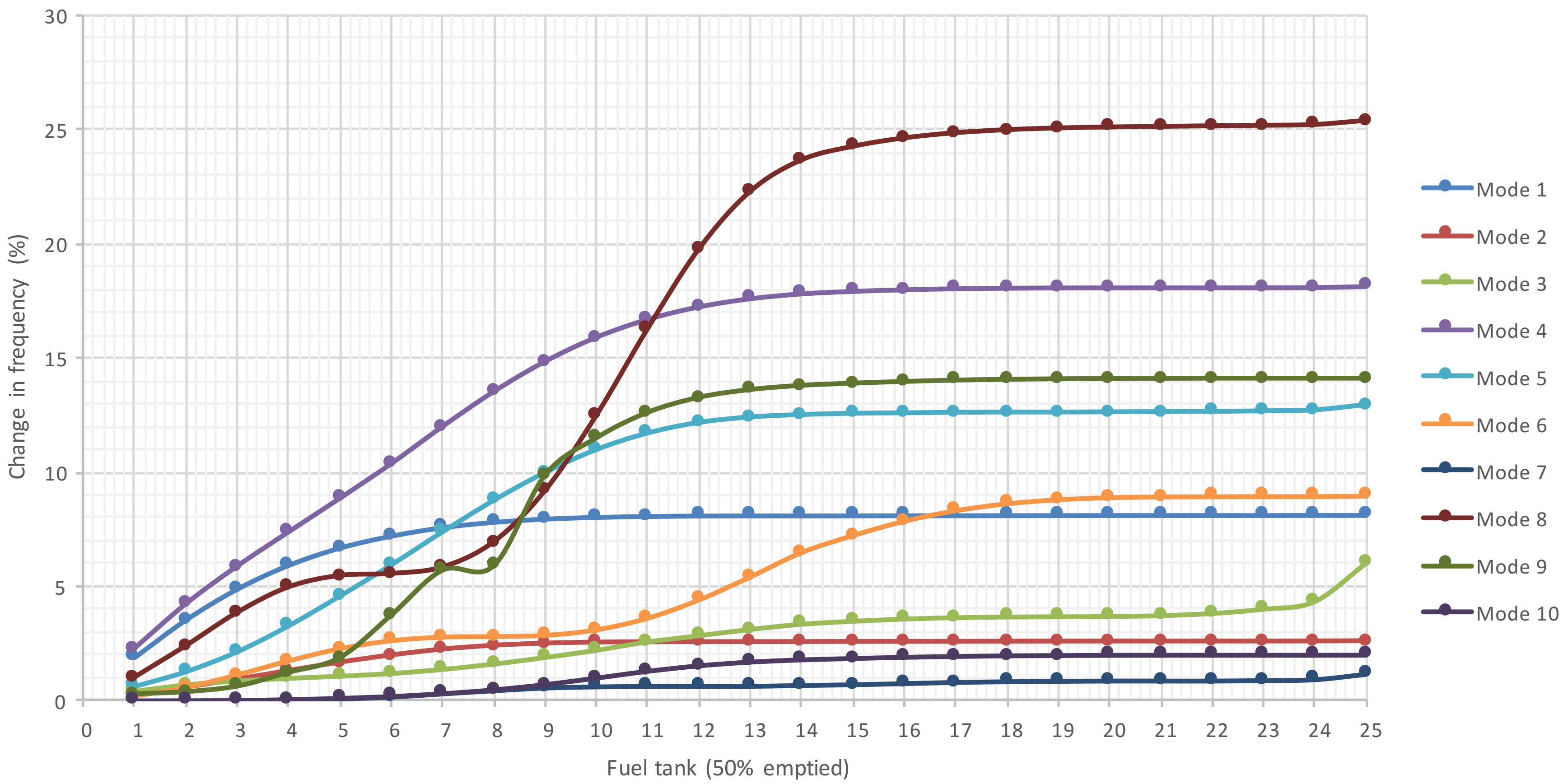
Full fuel conditions (mode 4)



Empty fuel conditions (mode 4)

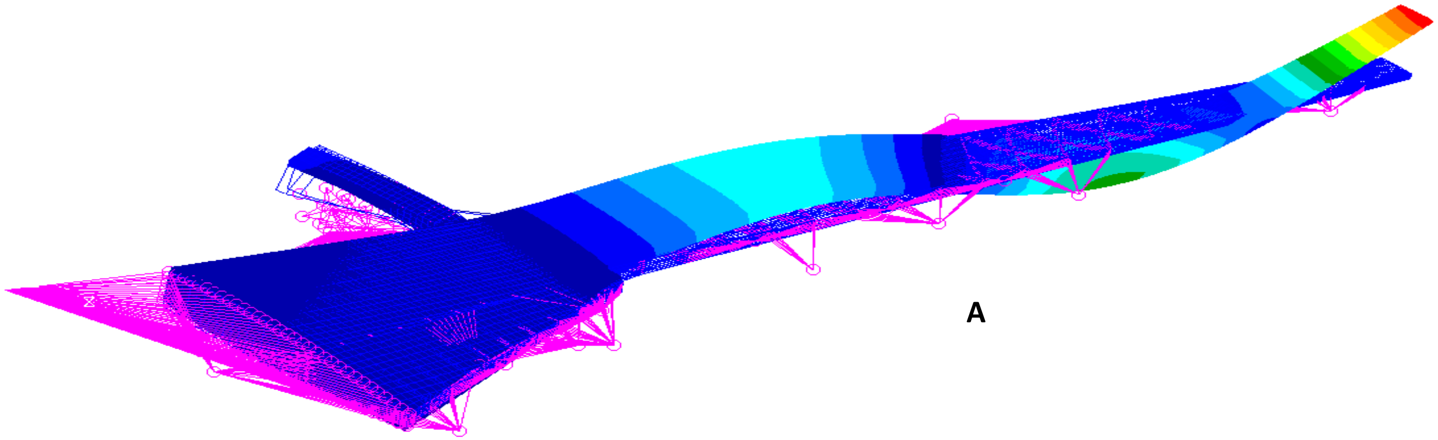
26

Natural frequencies due to variability in the fuel mass distribution

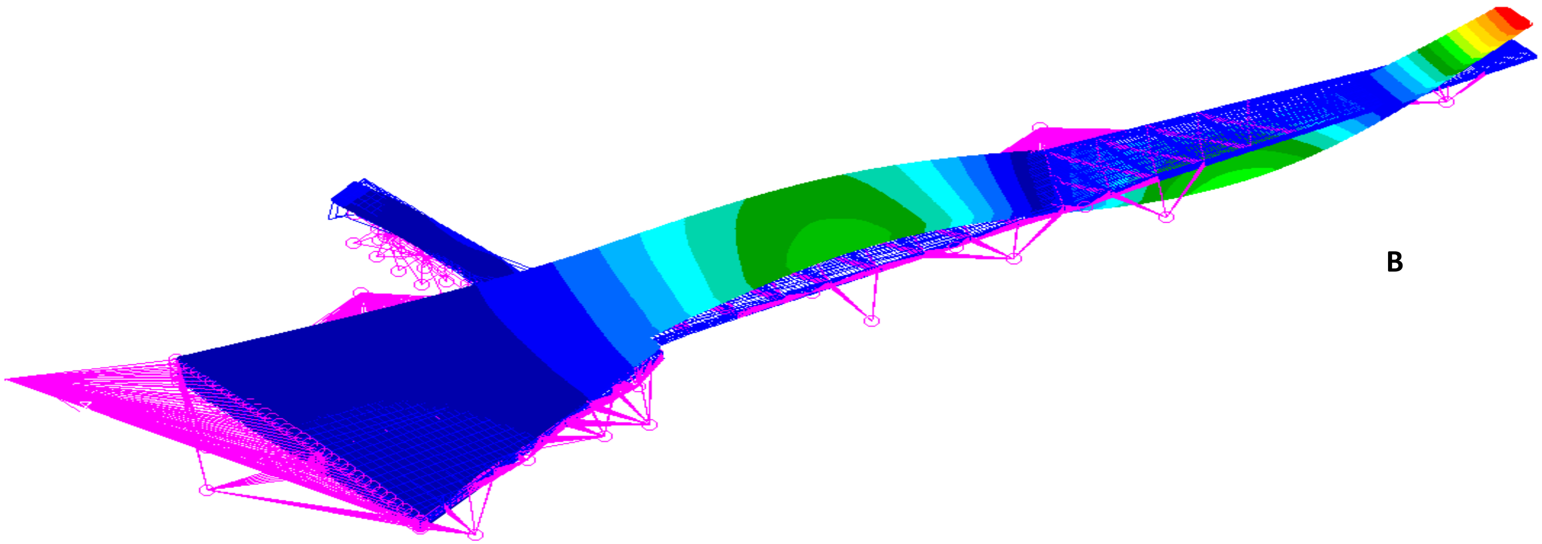


Changes in the frequency from the original frequency when the fuel tank is 50% full.

Mode shapes due to variability in the fuel mass distribution



A

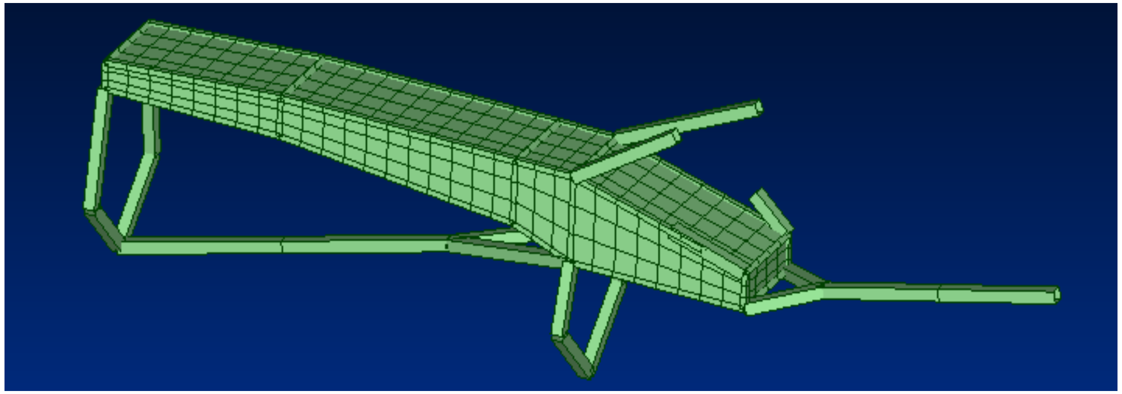


B

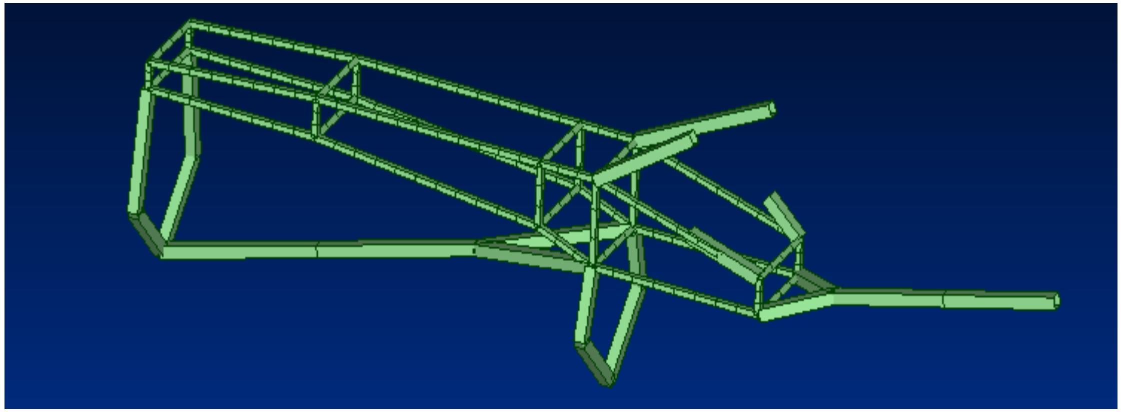
Mode Shapes for mode 8 (A= 50% fuel in all tanks) (B=All tanks have been emptied)

Variability in the pylon stiffness

- The wing structure is equipped with 21 pylon panels and 36 pylon frames
- Initial pylon stiffness was changed gradually from 1 to 20% of the original stiffness by changing the area of the pylon links
- The pylon is attached to the engine casing at one side and to the wing on the other side.

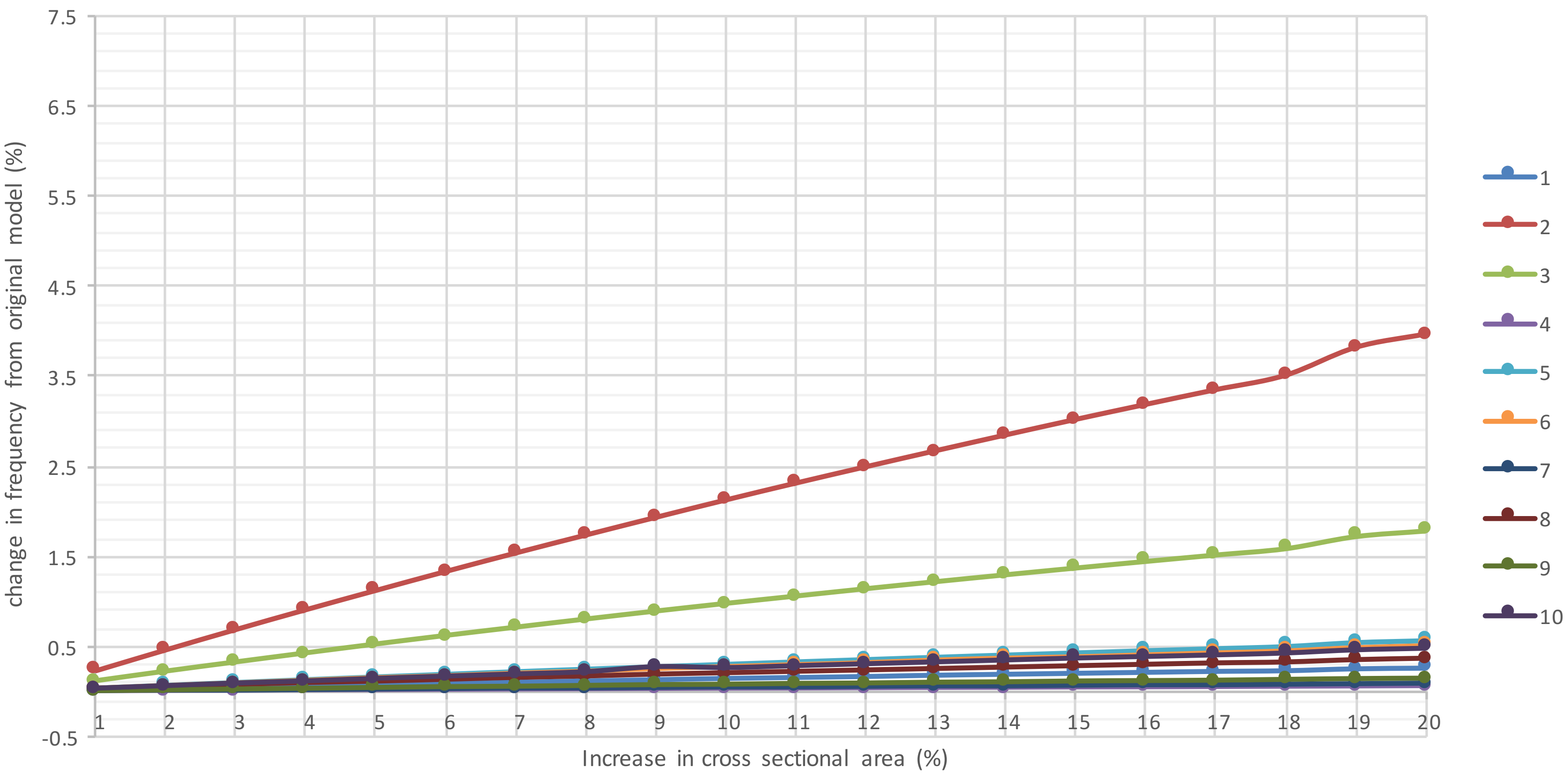


Pylon links and pylon panels (FEM model)



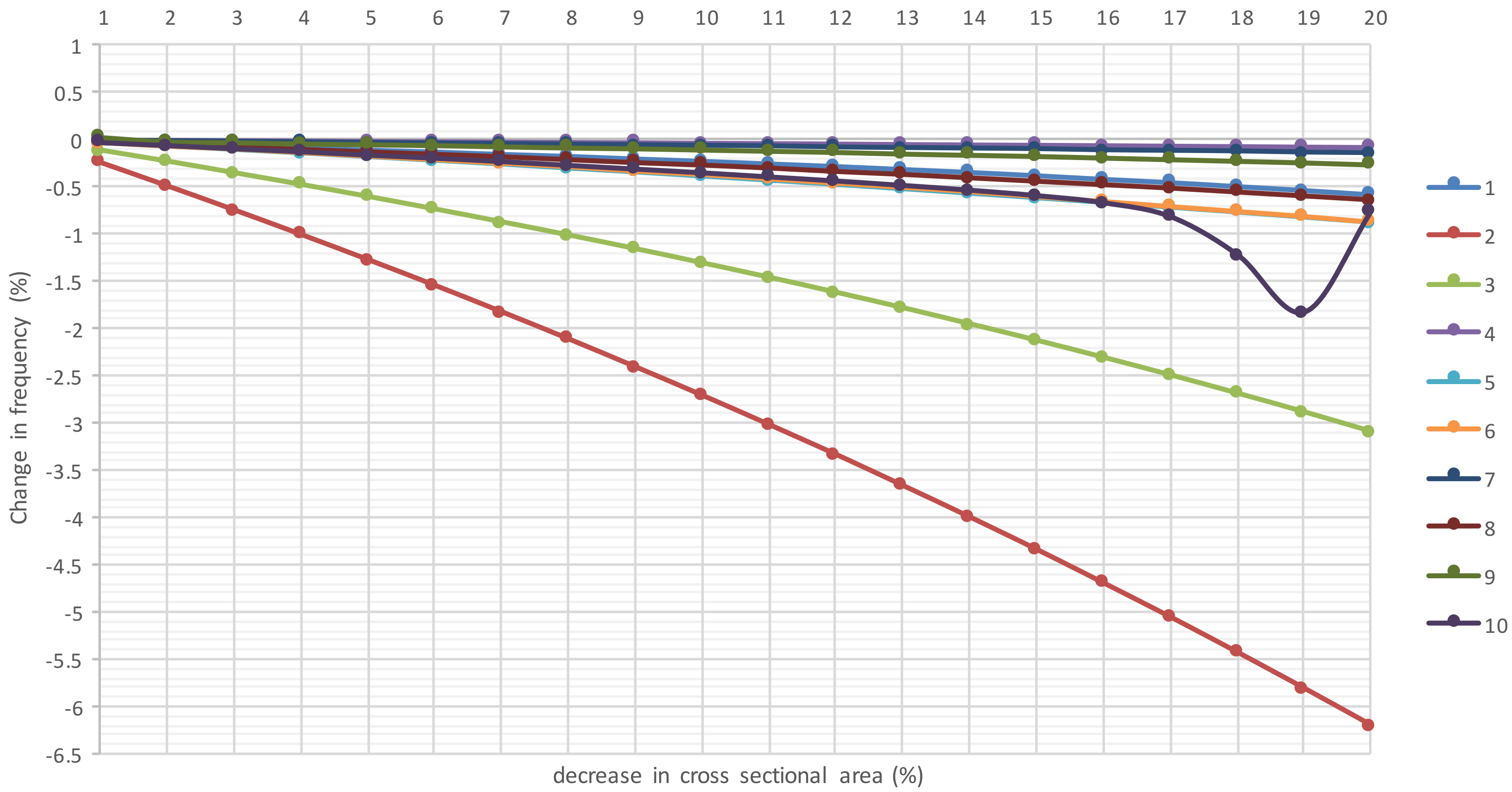
Pylon links and pylon frames (FEM model)

Natural frequencies due to the variability in the stiffness of the pylon



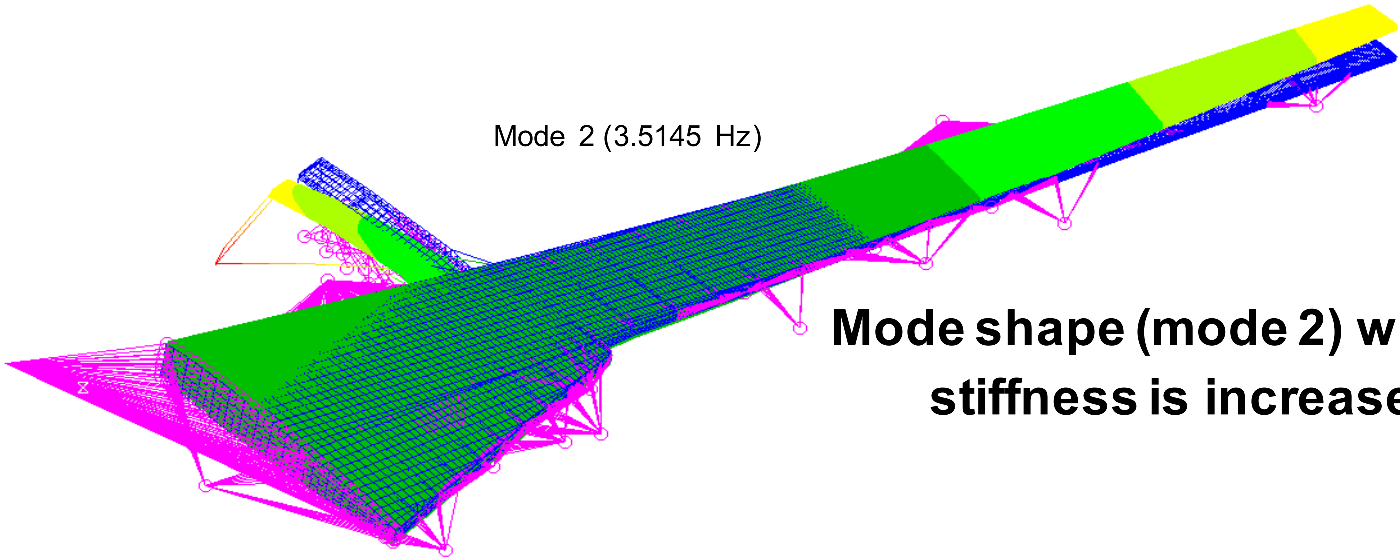
Change in the frequency (from original model- %) when increasing the cross sectional area

Natural frequencies due to the variability in the stiffness of the pylon

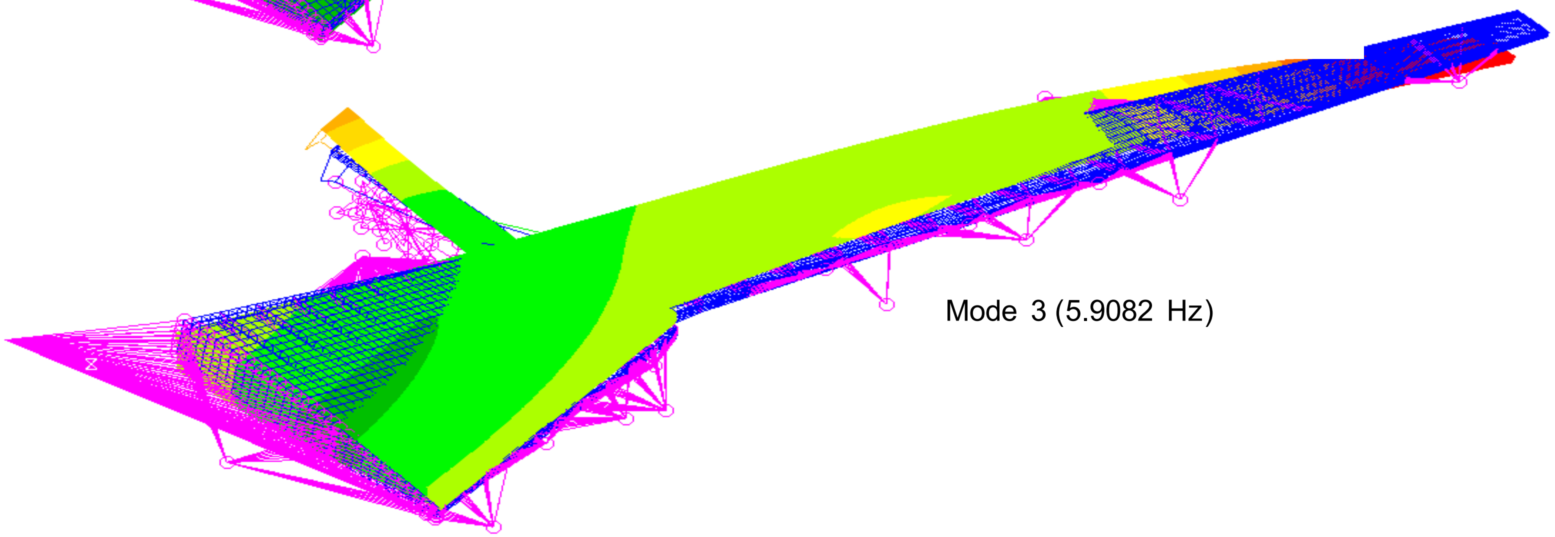


Change in frequency (from original model-%) when decreasing the cross sectional area

Mode shapes due to variability in the fuel mass distribution



Mode shape (mode 2) when the pylon stiffness is increased by 20%



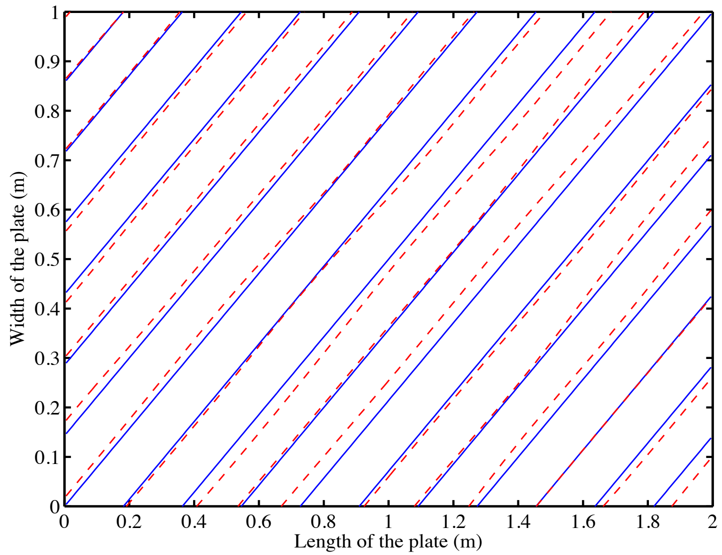
Mode shape (mode 3) when the pylon stiffness is increased by 20%

Main observations

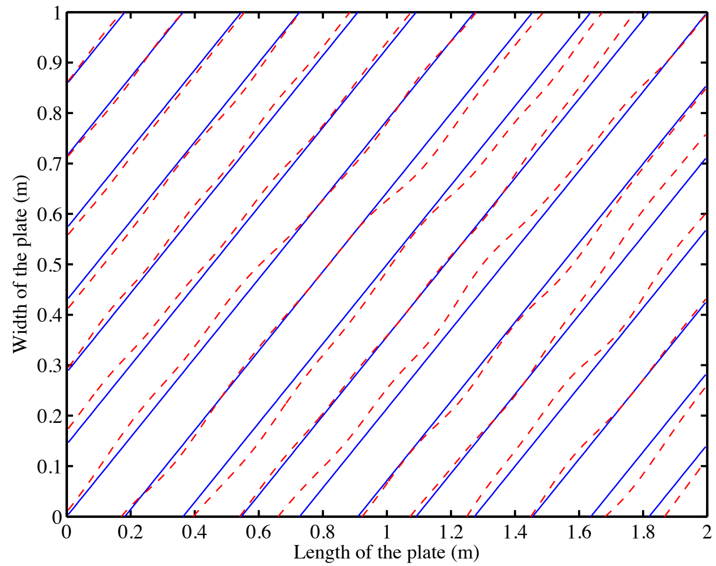
1. Randomness in the position of the engine has a significant effect on the natural frequencies. A variability of $\pm$ 0.5m from its original coordinate can result into approximately $\pm 30\%$ variability in the first 10 natural frequencies.
2. Variability in the engine mass has a moderate effect on the natural frequencies: $\pm$10% variation leads to ± 5% variation in the frequencies
3. Changes in fuel tank mass have a significant effect on the natural frequencies – empting the fuel tanks can change the frequencies up to 35%
4. Uncertainty in the pylon stiffness has little impact on the natural frequencies. A change of $\pm$ 20% in the cross sectional area leads to a maximum of ± 5% change in the natural frequencies (mainly in the second mode).

Random tow variations in composites

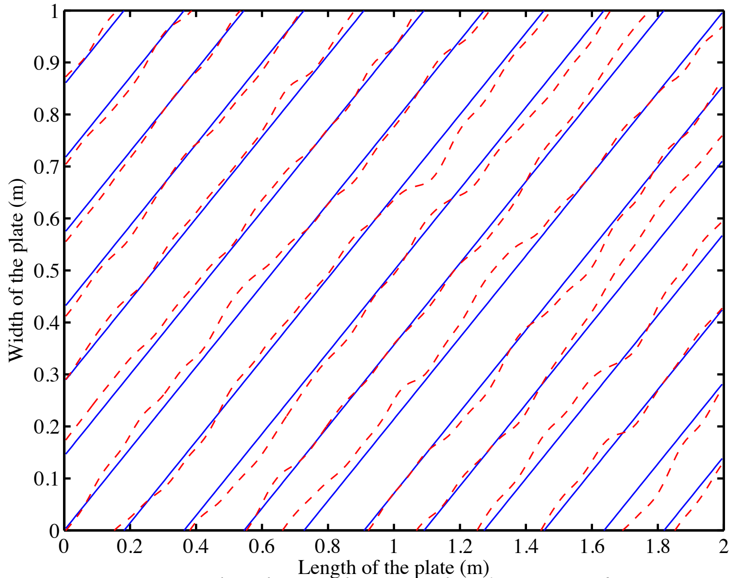
Random tow variations - examples



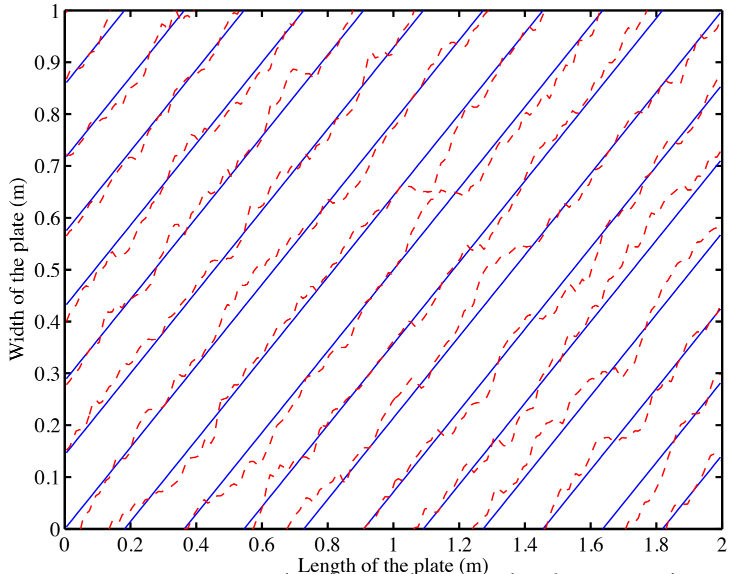
Correlation length $b = 2L$



Correlation length $b = L/2$



Correlation length $b = L/5$



Correlation length $b = L/20$

Random field model

- A random field is a function is space whose values are statistically correlated – defined by an auto correlation function
- The autocorrelation funciton is ofted expressed by a correlation length such as C(x1,x2) =e−|x1−x2|/b.

Assuming the mean is zero, then the underlying random field $F(x,\theta)$ can be expanded using the Karhunen-Loève expansion in the interval $-a \leq x \leq a$ as

$$F(x,\theta) = \sum_{k=1}^{\infty} \xi_k(\theta)\sqrt{\lambda_k}\varphi_k(x). \tag{4}$$

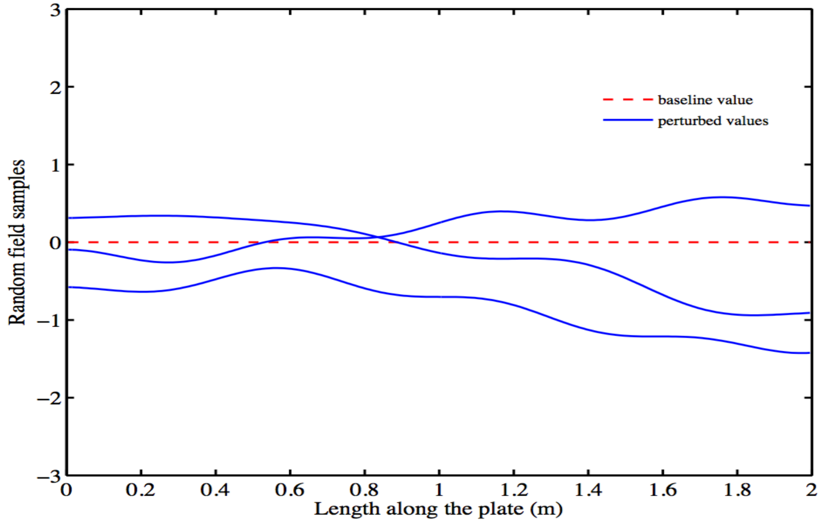
Using the notation $c = 1/b$, the corresponding eigenvalues and eigenfunctions for odd k are given by

$$\lambda_k = \frac{2c}{\beta_k^2 + c^2}, \qquad \varphi_k(x) = \frac{\cos(\beta_k x)}{\sqrt{a + \frac{\sin(2\beta_k a)}{2\beta_k}}}, \qquad \text{where} \quad \tan(\beta_k a) = \frac{c}{\beta_k}, \tag{5}$$

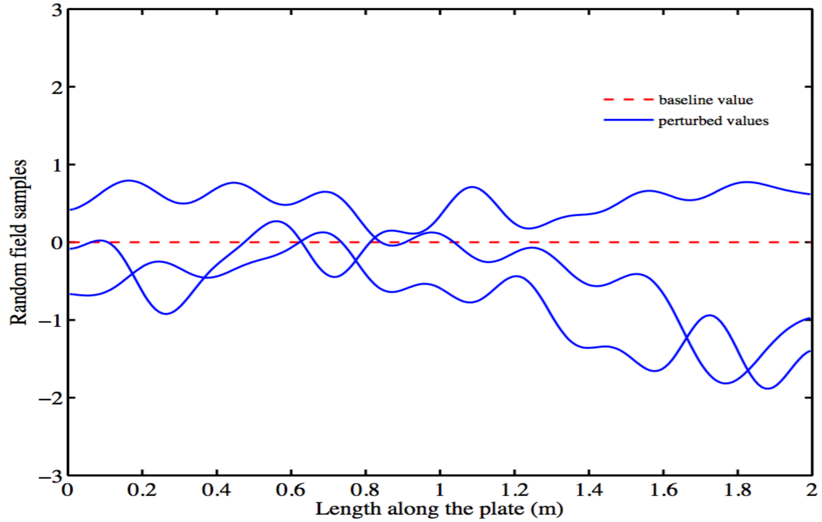
and for even k are given by

$$\lambda_k = \frac{2c}{{\beta_k}^2 + c^2}, \qquad \varphi_k(x) = \frac{\sin(\beta_k x)}{\sqrt{a - \frac{\sin(2\beta_k a)}{2\beta_k}}}, \qquad \text{where} \quad \tan(\beta_k a) = \frac{\beta_k}{-c}. \tag{6}$$

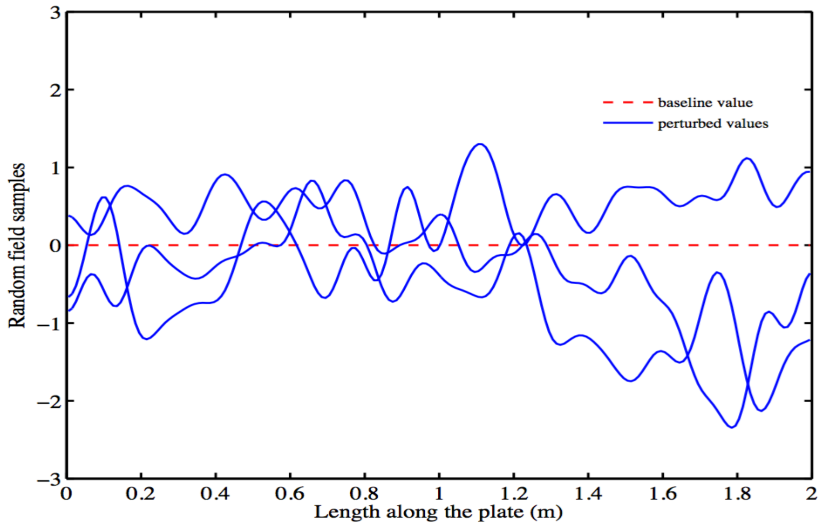
Samples of the random field



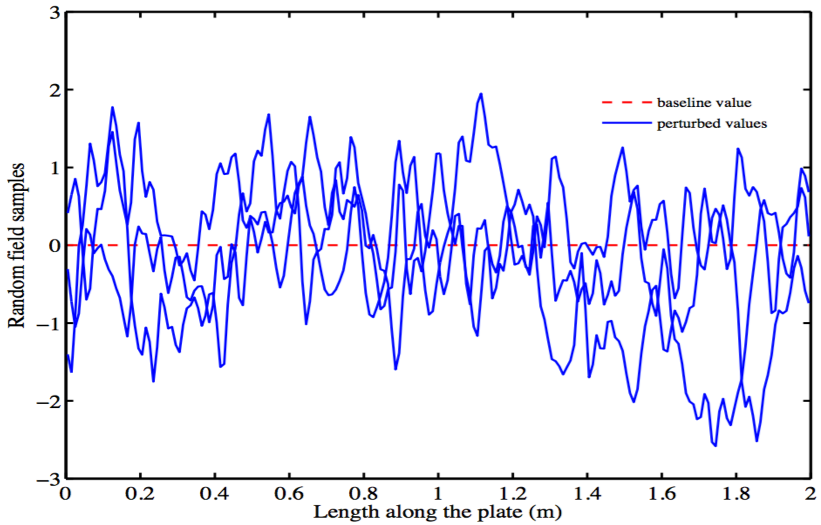
(a) Correlation length $b = 2a$ (8 terms).



(b) Correlation length $b = a/2$ (18 terms).

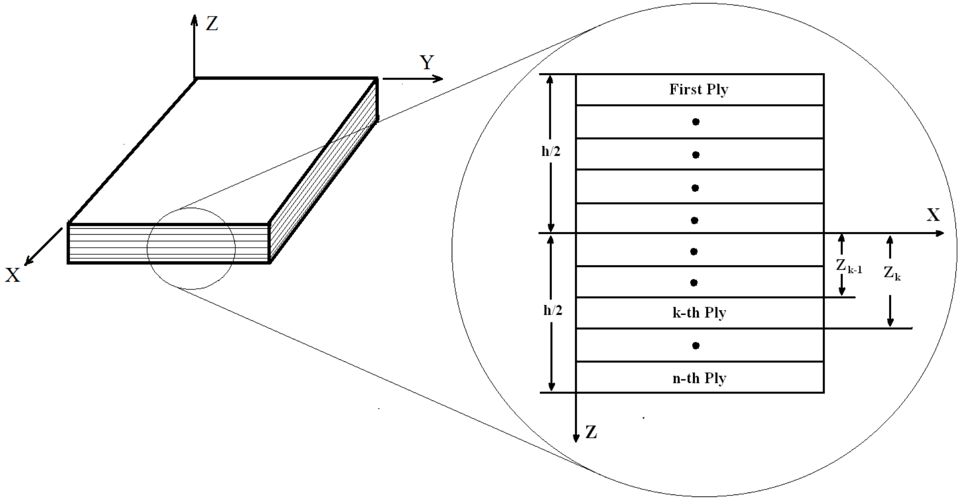


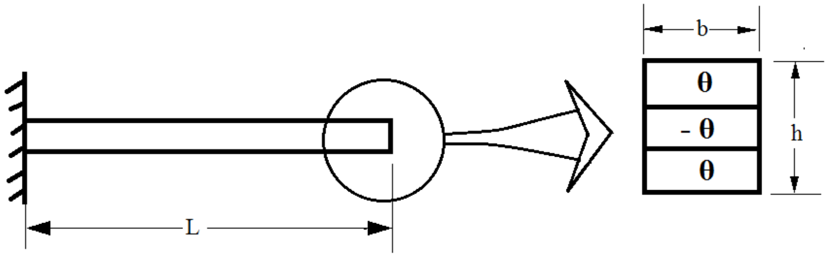
(c) Correlation length $b = a/5$ (36 terms).

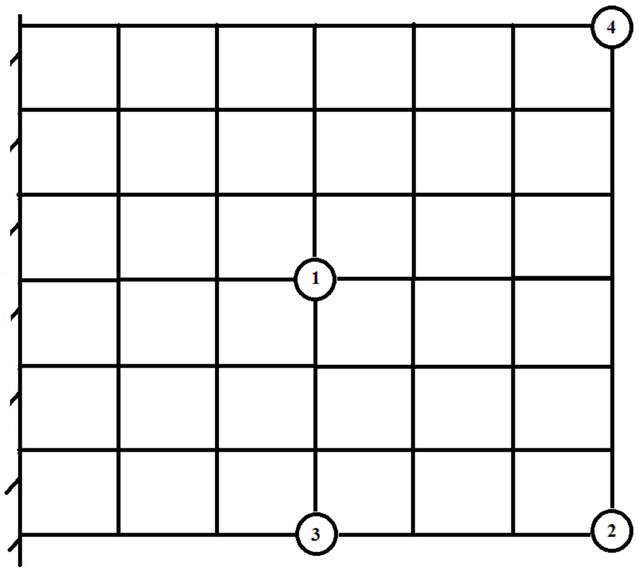


(d) Correlation length $b = a/20$ (130 terms).

Composite Plate Model







Composite mechanics

- Stress and strain relationship: $\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}$

$\bar{Q}_{11} = Q_{11}c^4 + Q_{22}s^4 + 2(Q_{12} + 2Q_{66})s^2c^2$
$\bar{Q}_{12} = (Q_{11} + Q_{22} - 4Q_{66})s^2c^2 + Q_{12}(c^4 + s^4)$
$\bar{Q}_{22} = Q_{11}s^4 + Q_{22}c^4 + 2(Q_{12} + 2Q_{66})\, s^2c^2$
$\bar{Q}_{16} = (Q_{11} + Q_{12} - 2Q_{66})c^3s - (Q_{22} - Q_{12} - 2Q_{66})cs^3$
$\bar{Q}_{26} = (Q_{11} + Q_{12} - 2Q_{66})cs^3 - (Q_{22} - Q_{12} - 2Q_{66})c^3s$
$\bar{Q}_{66} = (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})s^2c^2 + Q_{66}(s^4 + c^4)$

$c = \cos\theta$ and $s = \sin\theta$ and ***θ= θ*(x,y)** is a random field

$$D_{ij} = \int_{-1/2}^{1/2} \left(\bar{Q}_{ij}\right)_k z^2 dz = \frac{1}{3}\sum_{k=1}^{N} \left(\bar{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right)$$

Become also random fields

Solution of stochastic partial differential equations

- Stochastic field material properties leads to stochastic partial differential equations
- Conventional FE packages does not solve this type of equations
- Needs simplified approaches and engineering approximations