Shaped Modal Sensors for Uncertain Dynamical Systems

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Outline of the presentation

This paper aimed at designing shaped polyvinylidene fluoride (PVDF) film modal sensor for Euler-Bernoulli beams with uncertain properties.

- Uncertainty Quantification (UQ) in structural dynamics
- Brief review of existing approaches
  - Stochastic finite element method
- Design of modal sensors - deterministic systems
- Design of modal sensors - stochastic systems
- Numerical results
- Conclusions & future directions

Sources of uncertainty in computational modeling

(a) parametric uncertainty - e.g., uncertainty in geometric parameters, friction coefficient, strength of the materials involved;
(b) model inadequacy - arising from the lack of scientific knowledge about the model which is a-priori unknown;
(c) experimental error - uncertain and unknown error percolate into the model when they are calibrated against experimental results;
(d) computational uncertainty - e.g, machine precession, error tolerance and the so called ‘h’ and ‘p’ refinements in finite element analysis, and
(e) model uncertainty - genuine randomness in the model such as uncertainty in the position and velocity in quantum mechanics, deterministic chaos.

Current UQ approaches - 1

Two different approaches are currently available

- Parametric approaches : Such as the Stochastic Finite Element Method (SFEM):
  - aim to characterize parametric uncertainty (type 'a')
  - assumes that stochastic fields describing parametric uncertainties are known in details
  - suitable for low-frequency dynamic applications (building under earthquake load, steering column vibration in cars)

Current UQ approaches - 2

- Nonparametric approaches : Such as the Statistical Energy Analysis (SEA):
  - aim to characterize nonparametric uncertainty (types ‘b’ - ‘e’)
  - does not consider parametric uncertainties in details
  - suitable for high/mid-frequency dynamic applications (eg, noise propagation in vehicles)

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Stochastic Finite Element Method-1

Problems of structural dynamics in which the uncertainty in specifying mass and stiffness of the structure is modeled within the framework of random fields can be treated using the Stochastic Finite Element Method (SFEM). The application of SFEM in linear structural dynamics typically consists of the following key steps:

1. Selection of appropriate probabilistic models for parameter uncertainties and boundary conditions
2. Replacement of the element property random fields by an equivalent set of a finite number of random variables. This step, known as the 'discretisation of random fields' is a major step in the analysis.

Stochastic Finite Element Method-1

1. Formulation of the equation of motion of the form $\mathbf{D}(\omega)\mathbf{u} = \mathbf{f}$ where $\mathbf{D}(\omega)$ is the random dynamic stiffness matrix, $\mathbf{u}$ is the vector of random nodal displacement and $\mathbf{f}$ is the applied forces. In general $\mathbf{D}(\omega)$ is a random symmetric complex matrix.
2. Calculation of the response statistics by either (a) solving the random eigenvalue problem, or (b) solving the set of complex random algebraic equations.

Distributed Stochastic Dynamical Systems

The equation of motion:

$$\rho(\mathbf{r},\theta)\frac{\partial^2 U(\mathbf{r},t)}{\partial t^2}+L_1\frac{\partial U(\mathbf{r},t)}{\partial t}+L_2U(\mathbf{r},t)=p(\mathbf{r},t);\quad \mathbf{r}\in\mathcal{D},t\in[0,T] \tag{1}$$

$U(\mathbf{r},t)$ is the displacement variable, $\mathbf{r}$ is the spatial position vector and t is time.

- $\rho(\mathbf{r},\theta)$ is the random mass distribution of the system, $p(\mathbf{r},t)$ is the distributed time-varying forcing function, L_1 is the random spatial self-adjoint damping operator, L_2 is the random spatial self-adjoint stiffness operator.
- Eq (1) is a Stochastic Partial Differential Equation (SPDE) [ie, the coefficients are random processes].

Spectral Decomposition of random fields-1

- Just like the displacement fields (or any other continuous state variables) in the deterministic FEM, in SFEM we need to discretise the random fields appearing in the governing SPDE.
- Various approaches (mid-point method, collocation method, weighted integral approach etc) have been proposed in literature.
- Here we use the spectral decomposition of random fields due to its useful mathematical properties (eg, orthogonal eigenfunctions, mean-square convergence etc).

Spectral Decomposition of random fields-2

- Suppose $H(\mathbf{r}, \theta)$ is a random field with a covariance function $C_H(\mathbf{r}_1, \mathbf{r}_2)$ defined in a space Ω. Since the covariance function is finite, symmetric and positive definite it can be represented by a spectral decomposition.
- Using this spectral decomposition, the random process $H(\mathbf{r}, \theta)$ can be expressed in a generalized fourier type of series as

$$H(\mathbf{r}, \theta) = H_0(\mathbf{r}) + \sum_{i=1}^{\infty} \sqrt{\lambda_i} \xi_i(\theta) \varphi_i(\mathbf{r}) \tag{2}$$

where $\xi_i(\theta)$ are uncorrelated random variables.

Spectral Decomposition of random fields-3

- λ_i and $\varphi_i(\mathbf{r})$ are eigenvalues and eigenfunctions satisfying the integral equation

$$\int_{\Omega} C_H(\mathbf{r}_1, \mathbf{r}_2)\varphi_i(\mathbf{r}_1)\mathrm{d}\mathbf{r}_1 = \lambda_i \varphi_i(\mathbf{r}_2), \quad \forall\ i = 1, 2, \cdots \tag{3}$$

- The spectral decomposition in equation (2) is known as the Karhunen-Loève expansion. The series in (2) can be ordered in a decreasing series so that it can be truncated after a finite number of terms with a desired accuracy.

Exponential autocorrelation function

The autocorrelation function:

$$C(x_1, x_2) = e^{-|x_1 - x_2|/b} \tag{4}$$

The underlying random process $H(x, \theta)$ can be expanded using the Karhunen-Loève expansion in the interval $-a \leq x \leq a$ as

$$H(x, \theta) = \sum_{n=1}^{\infty} \left[\xi_n \sqrt{\lambda_n} \varphi_n(x) + \xi_n^* \sqrt{\lambda_n^*} \varphi_n^*(x) \right]. \tag{5}$$

The corresponding eigenvalues and eigenfunctions:

$$\lambda_n = \frac{2c}{\omega_n^2 + c^2}; \quad \varphi_n(x) = \frac{\cos(\omega_n x)}{\sqrt{a + \frac{\sin(2\omega_n a)}{2\omega_n}}} \quad \text{and} \quad \tan(\omega a) = \frac{c}{\omega}; \text{ for even } n \tag{6}$$

$$\lambda_n^* = \frac{2c}{{\omega_n^*}^2 + c^2}; \quad \varphi_n^*(x) = \frac{\sin(\omega_n^* x)}{\sqrt{a - \frac{\sin(2\omega_n^* a)}{2\omega_n^*}}} \quad \text{and} \quad \tan(\omega^* a) = \frac{\omega^*}{-c}; \text{ for odd } n \tag{7}$$

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Equation of motion-1

- Utilizing the series expansion of the random fields describing the uncertain parameter of the system and dicretisation of the displacement fields, the stochastic finite element model of the structure can be represented in the form

$$\mathbf{M}(\theta)\ddot{\mathbf{q}} + \mathbf{D}(\theta)\dot{\mathbf{q}} + \mathbf{K}(\theta)\mathbf{q} = \mathbf{B}\mathbf{u} \tag{8}$$

$$\mathbf{y} = \mathbf{C}\mathbf{q} \tag{9}$$

- Here $\mathbf{M}(\theta)$, $\mathbf{D}(\theta)$ and $\mathbf{K}(\theta)$ are the random mass, damping and stiffness matrices based on the degrees of freedom, $\mathbf{q}$. The inputs to the structure, $\mathbf{u}$, are applied via a matrix $\mathbf{B}$ which determines the location and gain of the actuators (or the actuator shape for distributed actuators).

Equation of motion-2

- The outputs, $\mathbf{y}$, are obtained via the output matrix $\mathbf{C}$ which is determined by the sensor shape. The notation θ is used to denote random natures of the system matrices.
- Due to the presence of uncertainty $\mathbf{M}(\theta)$, $\mathbf{D}(\theta)$ and $\mathbf{K}(\theta)$ become random matrices. These random matrices can be expressed as

$$\mathbf{K}(\theta) = \mathbf{K}_0 + \mathbf{\Delta K}(\theta), \quad \mathbf{M}(\theta) = \mathbf{M}_0 + \mathbf{\Delta M}(\theta) \quad \text{and} \quad \mathbf{D}(\theta) = \mathbf{D}_0 + \mathbf{\Delta D}(\theta) \tag{10}$$

Equation of motion-3

- Here the 'small' random terms are

$$\mathbf{\Delta K}(\theta) = \sum_{j=1}^{N_\mathrm{K}} \xi_{\mathrm{K_j}}(\theta)\sqrt{\lambda_{\mathrm{K_j}}}\mathbf{K}_j, \quad \mathbf{\Delta M}(\theta) = \sum_{j=1}^{N_\mathrm{M}} \xi_{\mathrm{M_j}}(\theta)\sqrt{\lambda_{\mathrm{M_j}}}\mathbf{K}_j$$

$$\mathbf{\Delta D}(\theta) = \sum_{j=1}^{N_\mathrm{D}} \xi_{\mathrm{D_j}}(\theta)\sqrt{\lambda_{\mathrm{D_j}}}\mathbf{K}_j$$

- In the above expression $\xi_{\mathrm{K_j}}(\theta)$, $\xi_{\mathrm{M_j}}(\theta)$ and $\xi_{\mathrm{D_j}}(\theta)$ are set of uncorrelated random variables. The deterministic matrices $\mathbf{K}_j$, $\mathbf{M}_j$ and $\mathbf{D}_j$ are symmetric and non-negative definite. These matrices depend on the eigenvectors corresponding to the eigenvalue $\sqrt{\lambda_{\mathrm{K_j}}}$, $\sqrt{\lambda_{\mathrm{M_j}}}$ and $\sqrt{\lambda_{\mathrm{D_j}}}$ respectively.

Background of distributed transducer-1

- The idea of using modal sensors and actuators for beam- and plate-type structures has been a subject of intense interest for many years.
- Using modal sensors in active control reduces problems of spillover, where high-frequency unmodelled modes affect the stability of the closed-loop system. For example, a modal sensor for a beam-type structure may be obtained by varying the sensor width along the length of the beam.
- An alternative to a large number of discrete transducers is to employ distributed actuators and sensors, often implemented using piezoelectric materials. Most papers concerned with distributed transducers are concerned with beams where the partial differential equations of motion may be solved to derive the continuous mode shapes.

Background of distributed transducer-2

- Here a different approach is taken and the shape functions of the underlying deterministic finite element model are used to approximate the width of the piezoelectric material. In this way, modal transducers may be designed for arbitrary beam-type structures.
- Also, by using additional constraints that not all degrees of freedom are forced or sensed, modal transducers that only cover part of a structure may be designed. Most of the development will concern sensors, although actuators may be dealt with in a similar way.

Defining Shaped Sensors for Beam Structures-1

- The shape of a transducer is a continuous function. However this function needs to be parameterised to enable the optimisation of the sensor shape.
- The main idea: ‘recycle’ FE shape functions - Using the shape functions of the underlying finite element model is a convenient approach to approximate the width of the piezoelectric material. In this way modal transducers may be designed for arbitrary beam type structures. Furthermore modal transducers that only cover part of a structure may be designed.
- Suppose a single polyvinylidene fluoride (PVDF) film sensor is placed on the beam with a shape defined by a variable width $f(\xi)$, where ξ denotes the length along the beam element.

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Defining Shaped Sensors for Beam Structures-2

- Incorporated into $f(\xi)$ is both the physical width of the sensor, and also the polarisation profile of the material.
- For an Euler-Bernoulli beam these shape functions, for element number e, are

$$N_{e1}(\xi) = \left(1 - 3\frac{\xi^2}{\ell_e^2} + 2\frac{\xi^3}{\ell_e^3}\right), \qquad N_{e2}(\xi) = \ell_e\left(\frac{\xi}{\ell_e} - 2\frac{\xi^2}{\ell_e^2} + \frac{\xi^3}{\ell_e^3}\right),$$
$$N_{e3}(\xi) = \left(3\frac{\xi^2}{\ell_e^2} - 2\frac{\xi^3}{\ell_e^3}\right), \qquad N_{e4}(\xi) = \ell_e\left(-\frac{\xi^2}{\ell_e^2} + \frac{\xi^3}{\ell_e^3}\right), \tag{11}$$

where ℓ_e is the length of the element.

Defining Shaped Sensors for Beam Structures-3

- The sensor width within element number e is approximated as

$$f_e(\xi) = \begin{bmatrix} N_{e1}(\xi) & N_{e2}(\xi) & N_{e3}(\xi) & N_{e4}(\xi) \end{bmatrix} \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix} \qquad (12)$$

  where the constants f_{ei} must be determined.

- This approximation has the advantage that the width and slope of the sensor are continuous at the nodes of the finite element model.

Defining Shaped Sensors for Beam Structures-4

- The output (voltage or charge) from the part of the sensor with element number e is

$$y_e(t) = K_s \int_0^{\ell_e} f_e(\xi) \frac{\partial^2 w_e(\xi, t)}{\partial^2 \xi} d\xi \tag{13}$$

where the constant K_s is determined by the properties of the piezoelectric material and w_e is the translational displacement of the beam.

Defining Shaped Sensors for Beam Structures-5

- This displacement is also approximated by the shape functions as

$$w_e(\xi) = \begin{bmatrix} N_{e1}(\xi) & N_{e2}(\xi) & N_{e3}(\xi) & N_{e4}(\xi) \end{bmatrix} \begin{Bmatrix} w_{e1} \\ w_{e2} \\ w_{e3} \\ w_{e4} \end{Bmatrix}. \tag{14}$$

- Combining the preceding 3 equations gives the sensor output for the element as

$$y_e = \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix}^\top \mathbf{C}_e \begin{Bmatrix} w_{e1} \\ w_{e2} \\ w_{e3} \\ w_{e4} \end{Bmatrix} \tag{15}$$

Defining Shaped Sensors for Beam Structures-6

- Here the (i, j)th element of the matrix $\mathbf{C}_e$ is

$$\mathbf{C}_{eij} = K_s \int_0^{\ell_e} N_{ei}(\xi)\, N''_{ej}(\xi)\, d\xi \tag{16}$$

or $$\mathbf{C}_e = -\frac{K_s}{30\ell_e} \begin{bmatrix} 36 & 33\ell_e & -36 & 3\ell_e \\ 3\ell_e & 4\ell_e^2 & -3\ell_e & -\ell_e^2 \\ -36 & -3\ell_e & 36 & -33\ell_e \\ 3\ell_e & -\ell_e^2 & -3\ell_e & 4\ell_e^2 \end{bmatrix}. \tag{17}$$

- The sensor output, y, is the sum of the contributions of the elements given by,

$$y = \sum y_e = \mathbf{f}^\top \mathbf{C}_s \mathbf{q}. \tag{18}$$

Defining Shaped Sensors for Beam Structures-7

- Here the element matrices have been assembled into the global matrix $\mathbf{C}_s$, in the usual way. The element nodal displacements, w_{ei}, have been incorporated into the global displacement vector $\mathbf{q}$, and the sensor nodal *widths* f_{ei} have been assembled into a global vector $\mathbf{f}$. However, the sensor nodal widths at the clamped or pinned boundary conditions are not set to zero, whereas the corresponding displacements are set to zero. Thus in general $\mathbf{C}_s$ is a rectangular matrix.
- Comparing Equations (9) and (18), it is clear that

$$\mathbf{C} = \mathbf{f}^\top \mathbf{C}_s. \tag{19}$$

- *Question*: how to find the sensor nodal width vector $\mathbf{f}$?

Modal Sensors for the Baseline System-1

- Proportional damping will be assumed so that the mode shapes are real, and equal to the mode shapes of the undamped system. For light damping this approximation will introduce small errors.
- The mode shapes, $\mathbf{\Phi}$, are assumed to be normalized arbitrarily so that the modal mass is

$$\mathbf{\Phi}^\top \mathbf{M} \mathbf{\Phi} = \mathbf{M}_m. \tag{20}$$

- Applying the transformation to modal co-ordinates, $\mathbf{q} = \mathbf{\Phi} \mathbf{p}$:

$$\ddot{\mathbf{p}} + 2\mathbf{Z}\Omega\dot{\mathbf{p}} + \Omega^2 \mathbf{p} = \mathbf{M}_m^{-1} \mathbf{\Phi}^\top \mathbf{B} \mathbf{u} \tag{21}$$

$$\mathbf{y} = \mathbf{C}\mathbf{\Phi}\mathbf{p} = \mathbf{C}_{\mathbf{p}}\mathbf{p} \tag{22}$$

Modal Sensors for the Baseline System-2

- Here $\Omega = \text{diag}\left[\omega_1, \omega_2, \ldots, \omega_n\right]$ is a diagonal matrix of the natural frequencies, and $\mathbf{Z} = \text{diag}\left[\zeta_1, \zeta_2, \ldots, \zeta_n\right]$ are the modal damping ratios.
- From Equation (19),

$$\mathbf{C_p} = \mathbf{C}\mathbf{\Phi} = \mathbf{f}^\top \mathbf{C}_s \mathbf{\Phi}. \tag{23}$$

- The modal sensor design problem is then to determine the sensor shape, defined by $\mathbf{f}$, to give the required modal output gain matrix, $\mathbf{C_p}$. Usually the number of elements describing the sensor shape is large and so this equation will be underdetermined. In this case the pseudo inverse solution will produce the minimum norm solution.

Modal Sensors for the Baseline System-3

- An alternative is to minimize transducer curvature, while ensuring zero sensitivity to unwanted modes. The minimum curvature ensures that the transducer may be manufactured as easily as possible.
- Thus, we wish to minimize

$$J_c(\mathbf{f}) = \sum_e \int_0^{\ell_e} f_e''(\xi)^2 \, \mathrm{d}\xi = \sum_e \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix}^\top \mathbf{H}_e \begin{Bmatrix} f_{e1} \\ f_{e2} \\ f_{e3} \\ f_{e4} \end{Bmatrix} \tag{24}$$

where

$$\mathbf{H}_{eij} = \int_0^{\ell_e} N_{ei}''(\xi) \, N_{ej}''(\xi) \, \mathrm{d}\xi. \tag{25}$$

$\mathbf{H}_e$ looks like the element stiffness matrix with a unit flexural rigidity.

Modal Sensors for the Baseline System-4

- Assembling the contributions from all of the elements gives

$$J_c(\mathbf{f}) = \mathbf{f}^\top \mathbf{H} \mathbf{f} \tag{26}$$

  where $\mathbf{H}$ contains the element matrices $\mathbf{H}_e$, and is symmetric.

- The sensor design problem then requires that J_c is minimized, subject to the constraints given by Equation (23).
- This problem may be solved using Lagrange multipliers as the solution of

$$\begin{bmatrix} 2\mathbf{H} & \mathbf{C}_s \boldsymbol{\Phi} \\ \boldsymbol{\Phi}^\top \mathbf{C}_s^\top & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \mathbf{f} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{C}_\mathbf{p}^\top \end{Bmatrix} \tag{27}$$

  where $\boldsymbol{\lambda}$ is the vector of Lagrange multipliers.

Modal Sensors for Uncertain Systems-1

- For uncertain systems, Equation (23) would be

$$\mathbf{C_p} = \mathbf{f}^\top \mathbf{C}_s \mathbf{\Phi}(\theta). \tag{28}$$

  Note that the affect of system uncertainty is reflected by the random nature of the modal matrix $\mathbf{\Phi}(\theta)$.

- $\mathbf{C}_s$ is determined from the element shape functions and is therefore fixed for a given mesh and will not change with the system parameters.
- Clearly the mode shapes of interest, $\mathbf{\Phi}$, will vary with the uncertain parameters.
- The vector $\mathbf{f}$ determines the shape of the sensor, and is obtained from the system optimization.

Modal Sensors for Uncertain Systems-2

- Here we will assume this vector is deterministic, although of course implementing a required sensor shape in practice will be subject to manufacturing errors not considered in this paper.
- Thus for a given sensor the system uncertainty will produce a stochastic modal output vector $\mathrm{C}_{\mathbf{p}}$.
- Suppose the desired modal output vector is denoted $\mathbf{C}_{\mathbf{p}d}$. Then to ensure the correct modal response we will enforce the constraint

$$\mathrm{E}\left[\mathbf{C}_{\mathbf{p}}\right] = \mathbf{C}_{\mathbf{p}d} = \mathbf{f}^{\top}\mathbf{C}_{s}\mathrm{E}\left[\mathbf{\Phi}(\theta)\right] \tag{29}$$

where $\mathrm{E}\left[\;\right]$ denotes the expected value.

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Modal Sensors for Uncertain Systems-3

- Assuming that the mode shapes have mean $\mathbf{\Phi}_0$, we have

$$\mathbf{C}_{\mathbf{p}d} = \mathbf{f}^\top \mathbf{C}_s \mathbf{\Phi}_0 \tag{30}$$

- To ensure robustness we will minimize the sum of the variances of the modal outputs given by

$$J_s = \mathrm{E}\left[\left(\mathbf{C}_{\mathbf{p}} - \mathbf{C}_{\mathbf{p}d}\right) \left(\mathbf{C}_{\mathbf{p}} - \mathbf{C}_{\mathbf{p}d}\right)^\top \right]. \tag{31}$$

- Recalling that the mode shapes have mean $\mathbf{\Phi}_0$, the required optimization is to minimize

$$J_s = \mathbf{f}^\top \mathbf{C}_s \mathrm{E}\left[\left(\mathbf{\Phi} - \mathbf{\Phi}_0\right) \left(\mathbf{\Phi} - \mathbf{\Phi}_0\right)^\top \right] \mathbf{C}_s^\top \mathbf{f} \tag{32}$$

Modal Sensors for Uncertain Systems-4

- with the constraint

$$\mathbf{C}_{\mathbf{p}d} = \mathbf{f}^\top \mathbf{C}_s \mathbf{\Phi}_0. \tag{33}$$

- This optimization is equivalent to minimizing the sensor curvature discussed before, with

$$\mathbf{H} = \mathbf{C}_s \mathrm{E}\left[\left(\mathbf{\Phi} - \mathbf{\Phi}_0 \right) \left(\mathbf{\Phi} - \mathbf{\Phi}_0 \right)^\top \right] \mathbf{C}_s^\top. \tag{34}$$

- The calculation of the above quantity requires the calculation of second-order statistical properties of the mode shapes.
- Here Monte Carlo simulation is used. But analytical results for random eigenvalue problems can be used to obtain the modal statistics.

Illustrative Example

- A clamped-clamped beam example is used to demonstrate the design of modal sensors. The steel beam is 1.5 m long with cross-section 20×5 mm, and bending in the more flexible plane is modeled by using 15 finite elements.
- Only the first nine modes are considered important and damping is assumed to be 1% in all modes.
- The material properties of the baseline beam are assumed to be $\overline{EI} = 43.750$ Nm^2 and $\overline{\rho A} = 0.785$ kg/m.
- The first 12 natural frequencies for the beam are 11.815, 32.569, 63.858, 105.60, 157.84, 220.70, 294.33, 378.97, 474.95, 582.69, 702.61, 834.86 Hz.

Stochastic properties

- It is assumed that the bending stiffness $EI(x)$ and mass per unit length $\rho A(x)$ are random fields of the form

$$EI(x) = \overline{EI}\left(1 + \epsilon_{EI} f_1(x)\right) \tag{35}$$

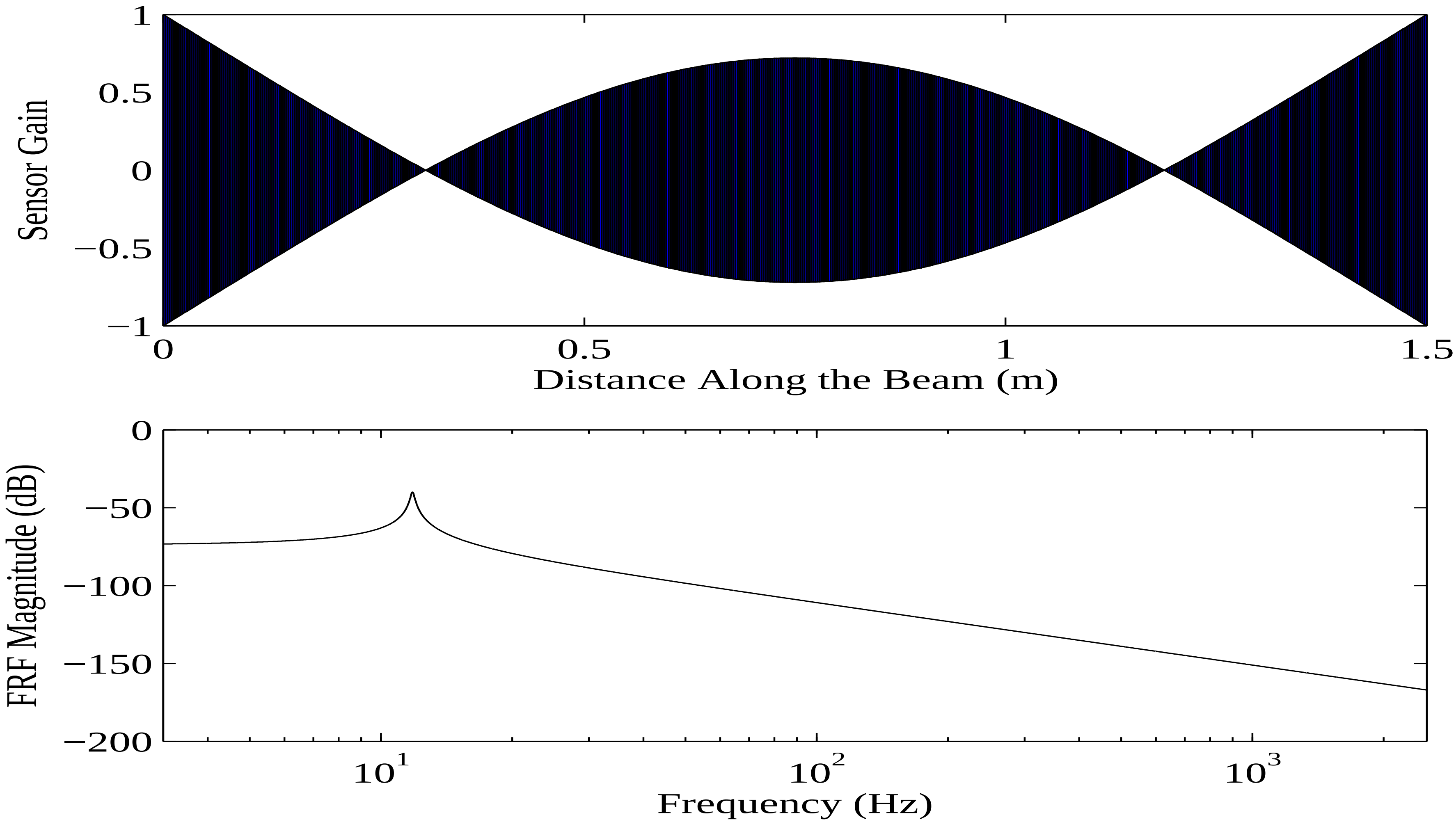
$$\text{and} \quad \rho A(x) = \overline{\rho A}\left(1 + \epsilon_{\rho A} f_2(x)\right) \tag{36}$$

- The strength parameters are assumed to be $\epsilon_{EI} = 0.05$, and $\epsilon_{\rho A} = 0.1$.
- The random fields $f_i(\mathbf{x}), i = 1, \cdots, 4$ are assumed to be delta-correlated homogenous Gaussian random fields. A 1000-sample Monte Carlo simulation is performed to obtain the FRFs and modal statistics.

Sensor design parameters

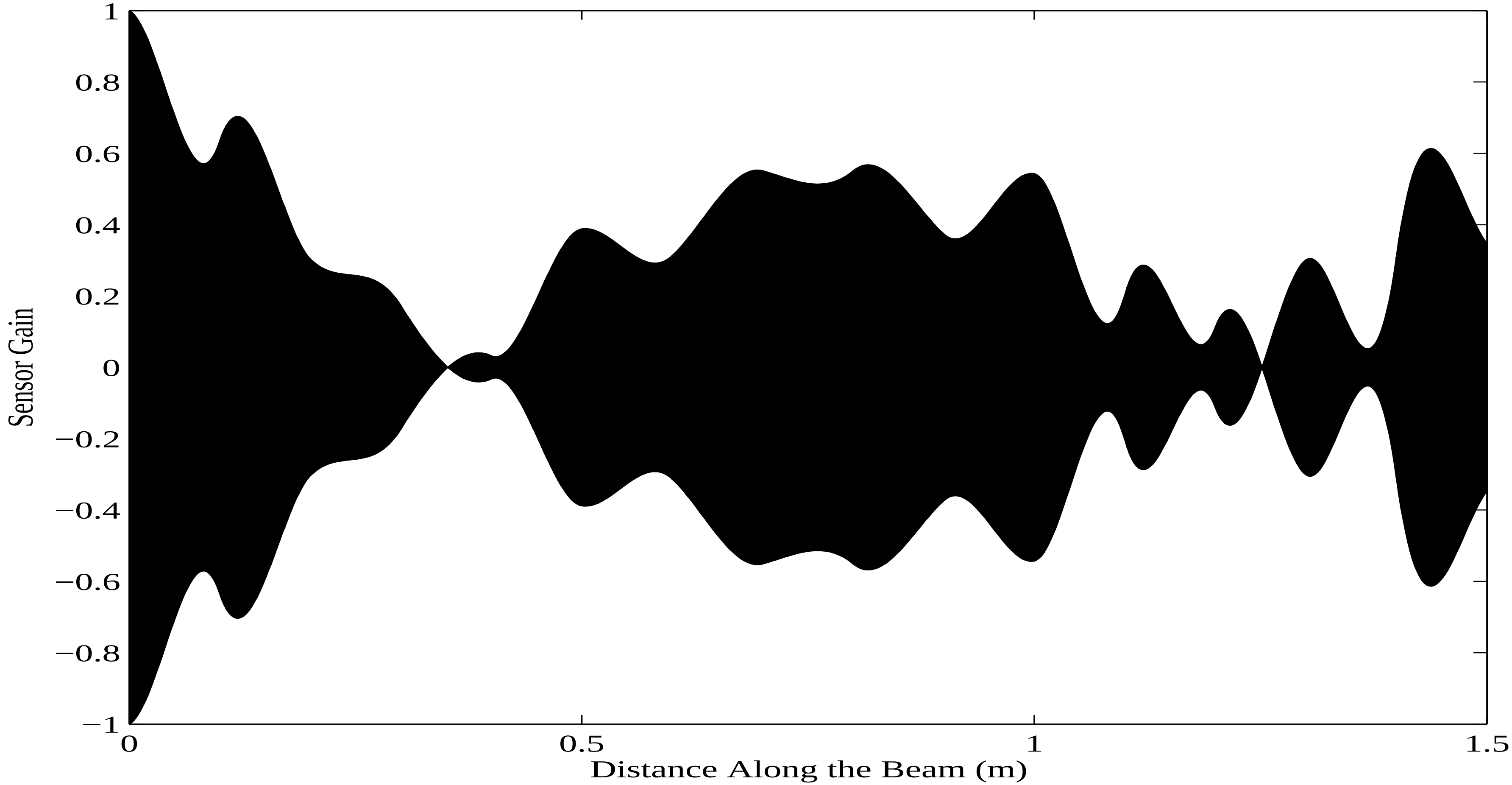
- The force input is applied at node 7 for the sensor design.
- The sensors are designed by considering only the first nine modes of the beam.
- The sensor gain constant is assumed to be unity, $K_s = 1$, since it is most important to compute the sensor shape, rather than the calibration constant.
- Following two cases are considered:
  - Case 1: sensor to be designed to excite only the first mode, with a peak in the receptance of 0.01 m/N.
  - Case 2: sensor to be designed to excite only the third mode, with a peak in the receptance of 0.01 m/N.

Design for the baseline system: Case 1



The distributed sensor shape and receptance designed to excite the first mode.

Design for the stochastic system: Case 1



The distributed sensor shape and receptance designed to excite the first mode with system uncertainty.

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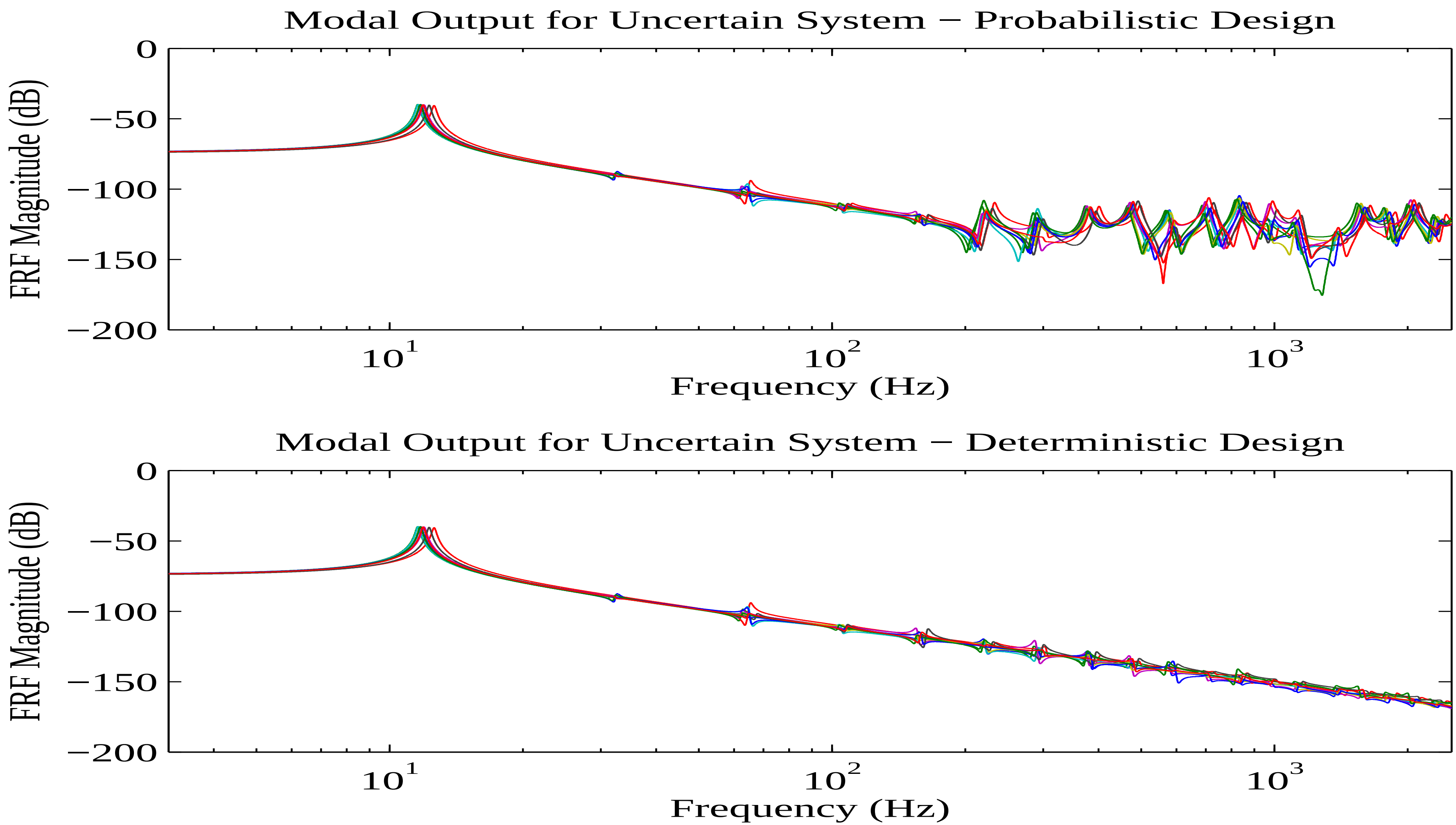
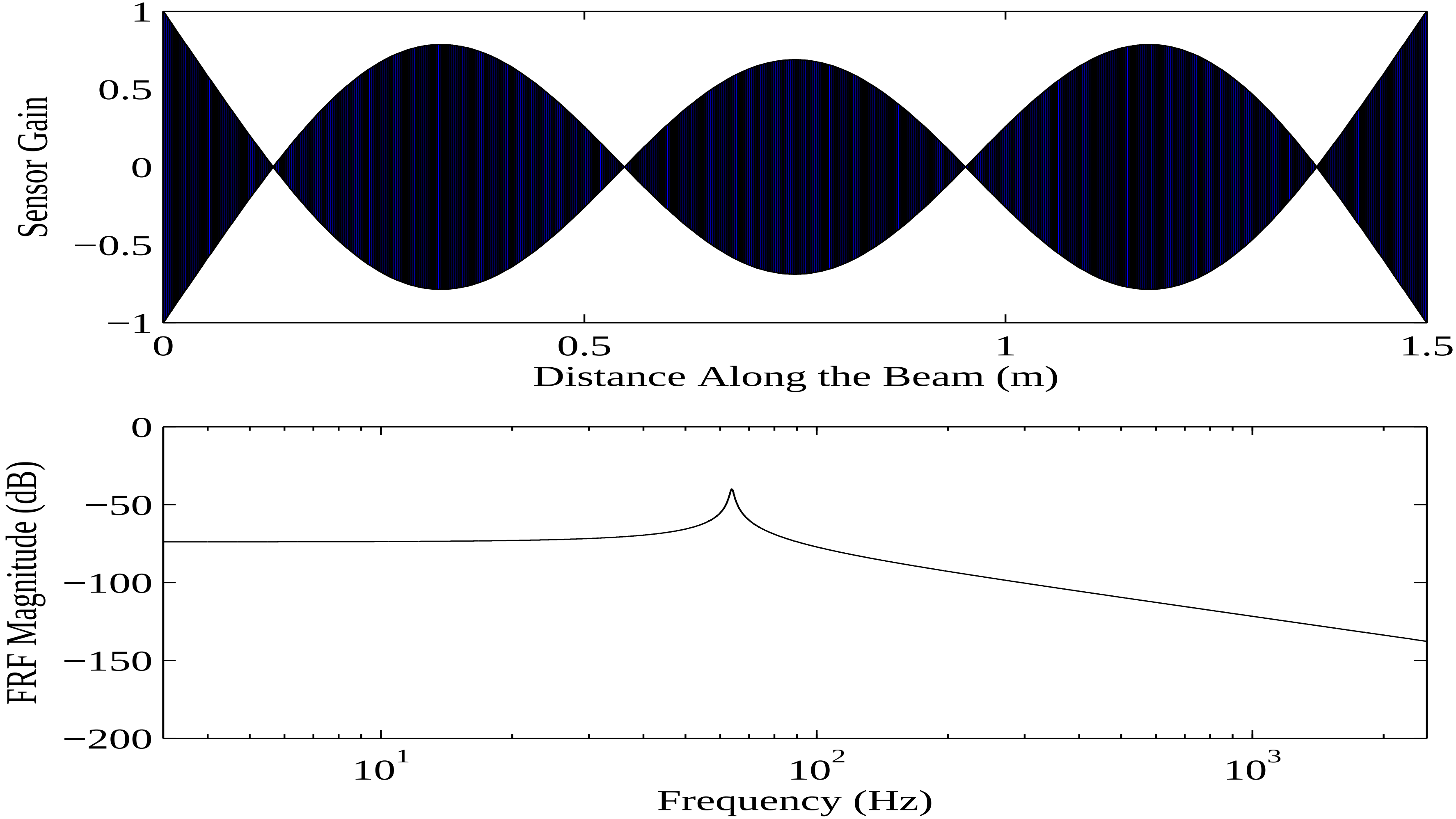
Comparisons of FRFs: Case 1



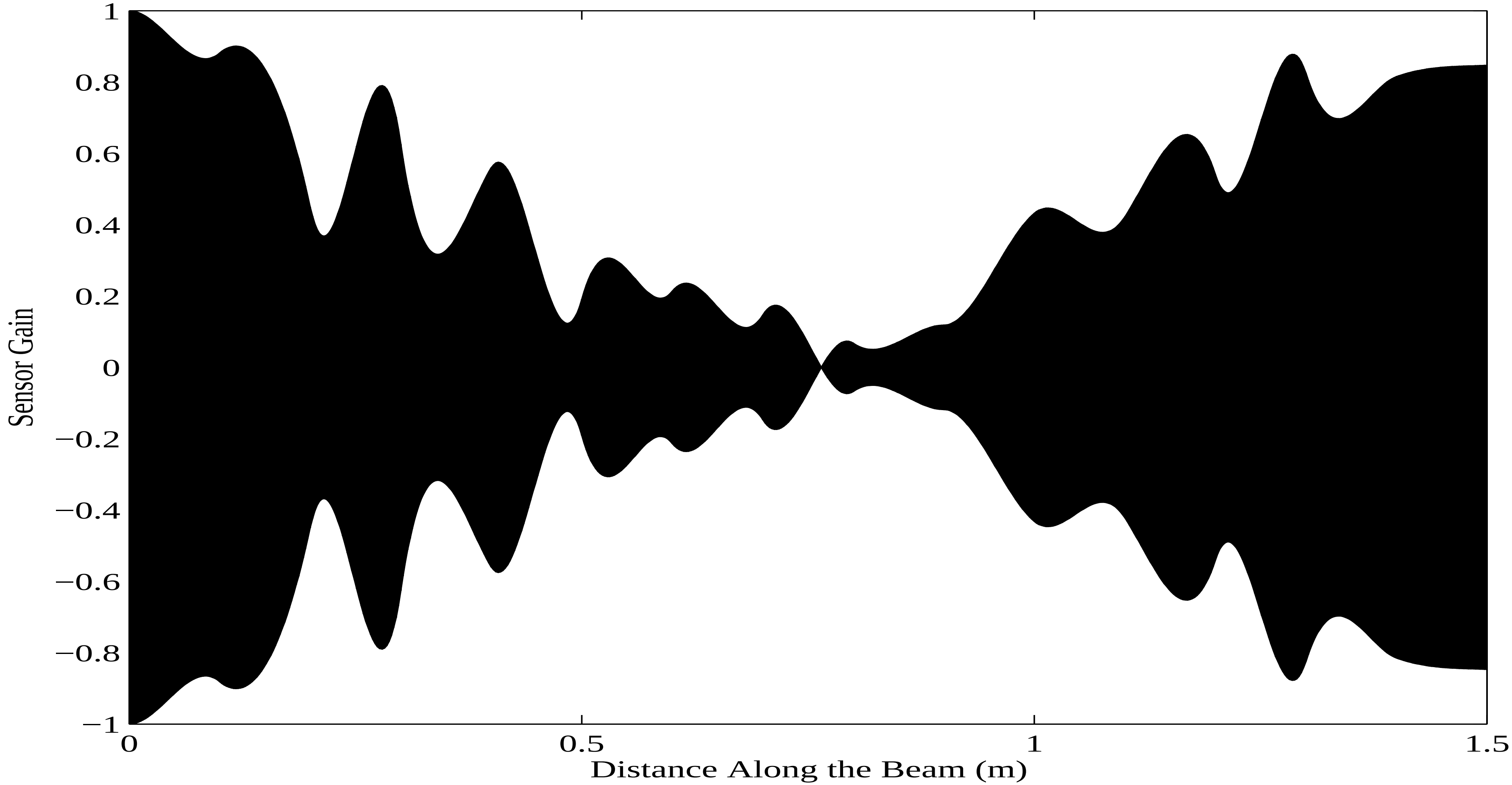
Comparisons of the ensemble of FRFs resulting from two designs.

Design for the baseline system: Case 2



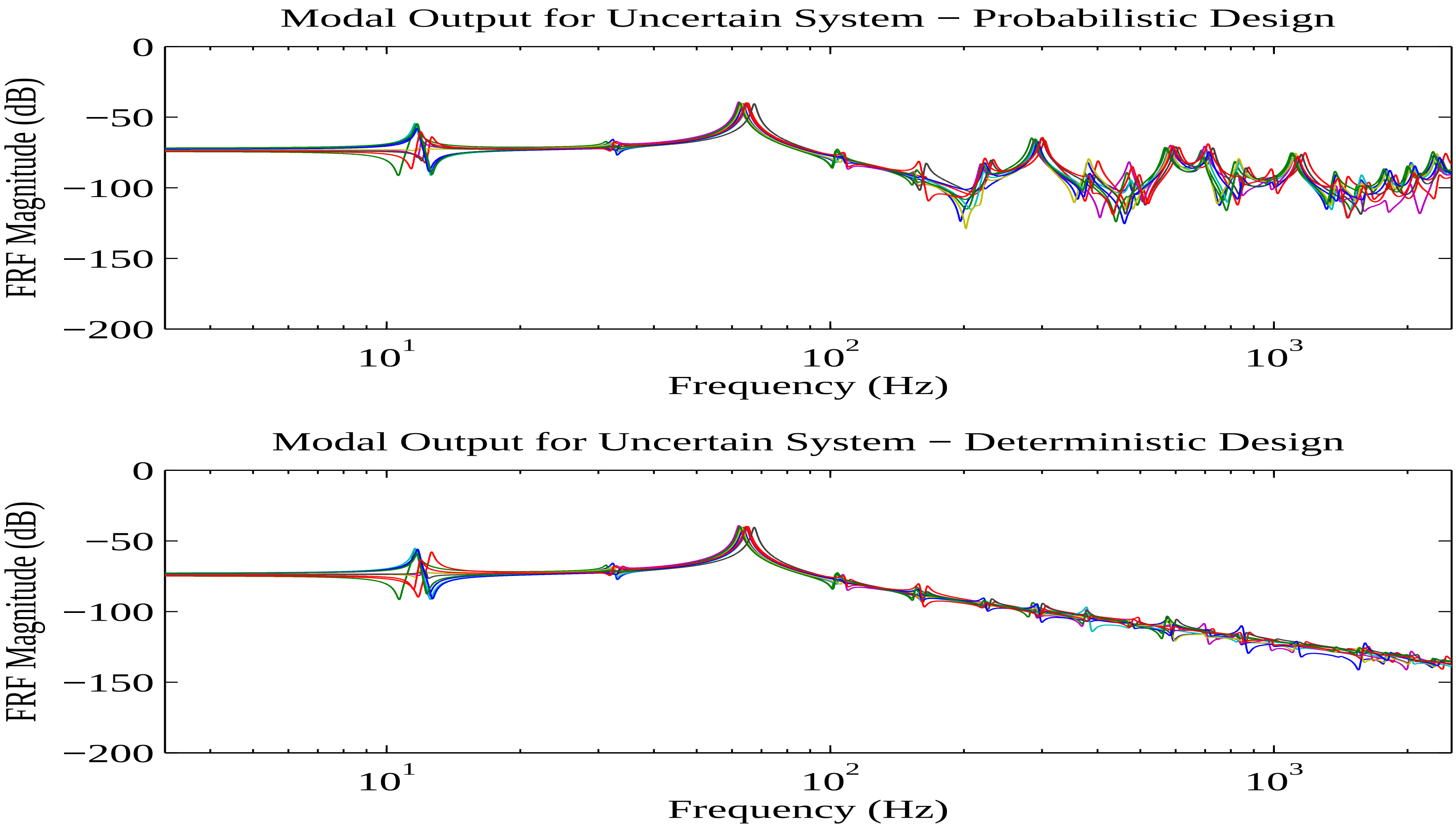
The distributed sensor shape and receptance designed to excite the third mode.

Design for the stochastic system: Case 2



The distributed sensor shape and receptance designed to excite the third mode with system uncertainty.

Comparisons of FRFs: Case 2



Comparisons of the ensemble of FRFs resulting from two designs.

Conclusions - 1

- Uncertainties in the system need to be taken into account for robust design of sensors and actuators for engineering dynamical systems
- This talk has considered the problem of designing modal actuators and sensors using a discrete approximation to the equations of motion for linear stochastic systems.
- Transducer shapes are represented by ‘recycling’ the underlying finite element shape functions. This allows the actuators and sensors to be designed by using the discrete approximation and the shape recovered by using the shape functions.

Conclusions - 2

- Optimal shape design has been coupled with the stochastic finite element method to consider parametric uncertainty.
- It was shown that eigenvector statistic are needed to obtain the optimal shape.
- The shape of the sensors of the deterministic system differs significantly from the random system.

Future directions

- Extension of the proposed approach to more complex 2D and 3D uncertain dynamical systems:
- Alternative design criteria: for example, based on complete covariance tensor of the modal matrix combined with the minimum curvature of the transducers
- Efficient computational methods based on analytical approaches involving random eigenvalue problems
- Health monitoring of uncertain systems using distributed transducers: