Uncertainty quantification in structural dynamics using random matrix theory

S Adhikari

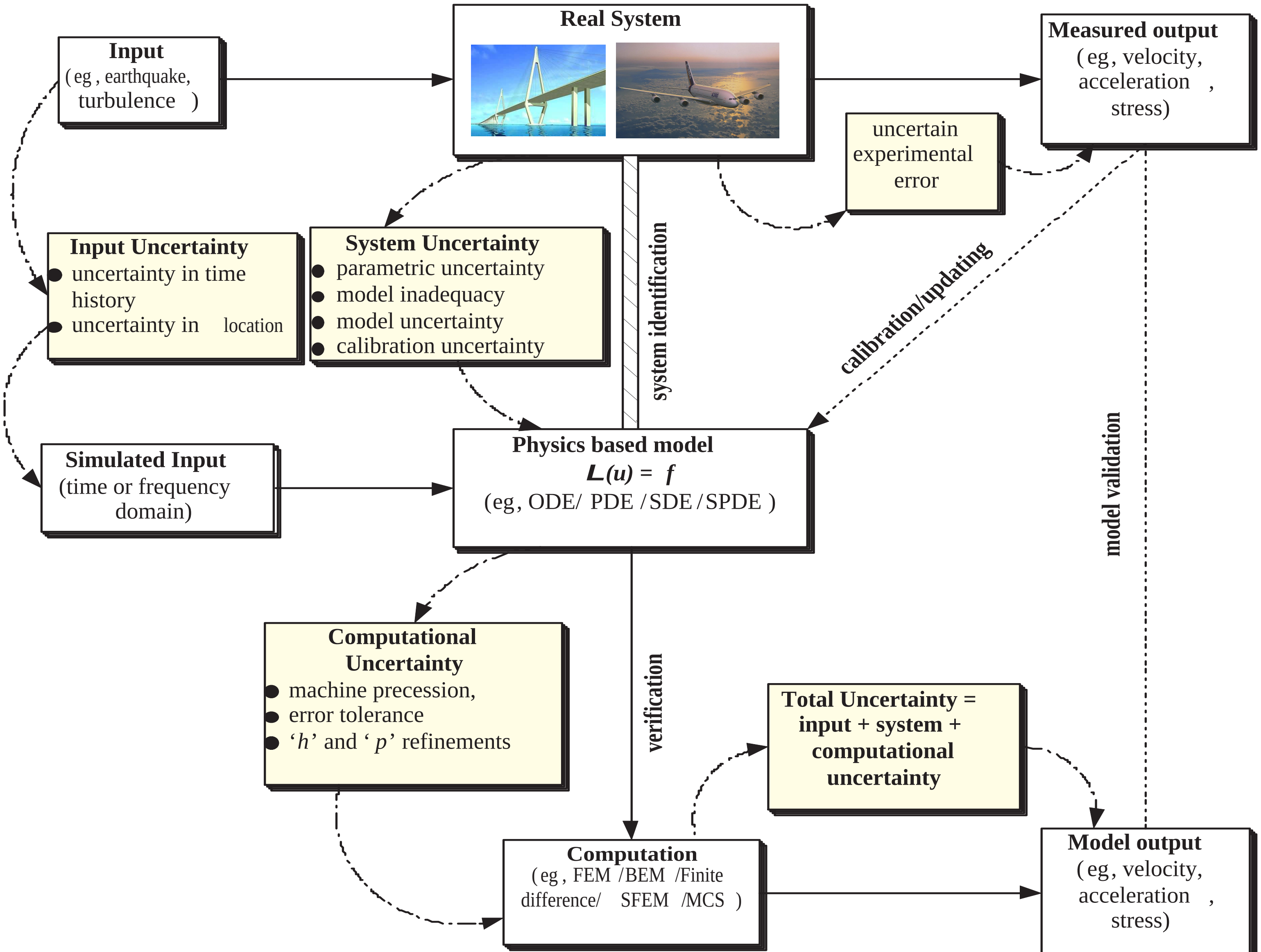
School of Engineering, Swansea University, Swansea, UK

Email: S.Adhikari@swansea.ac.uk

URL: http://engweb.swan.ac.uk/∼adhikaris

Swansea University
Prifysgol Abertawe

A general overview of computational structural dynamics



Swansea University
Prifysgol Abertawe

Ensembles of structural dynamical systems

Many structural dynamic systems are manufactured in a production line (nominally identical systems)

A complex structural dynamical system

Complex aerospace system can have millions of degrees of freedom and significant 'errors' and/or 'lack of knowledge' in its numerical (Finite Element) model

Sources of uncertainty

(a) parametric uncertainty - e.g., uncertainty in geometric parameters, friction coefficient, strength of the materials involved;
(b) model inadequacy - arising from the lack of scientific knowledge about the model which is a-priori unknown;
(c) experimental error - uncertain and unknown error percolate into the model when they are calibrated against experimental results;
(d) computational uncertainty - e.g, machine precession, error tolerance and the so called ‘h’ and ‘p’ refinements in finite element analysis, and
(e) model uncertainty - genuine randomness in the model such as uncertainty in the position and velocity in quantum mechanics, deterministic chaos.

Outline of the presentation

- Uncertainty Propagation (UP) in structural dynamics
- Brief review of parametric approach
  - Stochastic finite element method
- Non-parametric approach: Wishart random matrices
  - Analytical derivation
  - Parameter selection
  - Computational results
- Experimental results
- Conclusions & future directions

Swansea University
Prifysgol Abertawe

UP approaches: key challenges

The main difficulties are:

- the computational time can be prohibitively high compared to a deterministic analysis for real problems,
- the volume of input data can be unrealistic to obtain for a credible probabilistic analysis,
- the predictive accuracy can be poor if considerable resources are not spend on the previous two items, and
- the need for general purpose software tools: as the state-of-the art methodology stands now (such as the Stochastic Finite Element Method), only very few highly trained professionals (such as those with PhDs) can even attempt to apply the complex concepts (e.g., random fields) and methodologies to real-life problems.

Main objectives

Our work is aimed at developing methodologies [the 10-10-10 challenge] with the ambition that they should:

- not take more than 10 times the computational time required for the corresponding deterministic approach;
- result a predictive accuracy within 10% of direct Monte Carlo Simulation (MCS);
- use no more than 10 times of input data needed for the corresponding deterministic approach; and
- enable engineering graduates to perform probabilistic structural dynamic analyses with a reasonable amount of training.

Current UP approaches - 1

Two different approaches are currently available

- Parametric approaches : Such as the Stochastic Finite Element Method (SFEM):
  - aim to characterize parametric uncertainty (type 'a')
  - assumes that stochastic fields describing parametric uncertainties are known in details
  - suitable for low-frequency dynamic applications (building under earthquake load, steering column vibration in cars)

Swansea University
Prifysgol Abertawe

Current UP approaches - 2

- Nonparametric approaches : Such as the Statistical Energy Analysis (SEA):
  - aim to characterize nonparametric uncertainty (types ‘b’ - ‘e’)
  - does not consider parametric uncertainties in details
  - suitable for high/mid-frequency dynamic applications (eg, noise propagation in vehicles)

Random continuous dynamical systems

The equation of motion:

$$\rho(\mathbf{r},\theta)\frac{\partial^2 U(\mathbf{r},t)}{\partial t^2} + L_1 \frac{\partial U(\mathbf{r},t)}{\partial t} + L_2 U(\mathbf{r},t) = p(\mathbf{r},t); \quad \mathbf{r} \in \mathcal{D}, t \in [0,T] \tag{1}$$

$U(\mathbf{r},t)$ is the displacement variable, $\mathbf{r}$ is the spatial position vector and t is time.

- $\rho(\mathbf{r},\theta)$ is the random mass distribution of the system, $p(\mathbf{r},t)$ is the distributed time-varying forcing function, L_1 is the random spatial self-adjoint damping operator, L_2 is the random spatial self-adjoint stiffness operator.
- Eq (1) is a Stochastic Partial Differential Equation (SPDE) [ie, the coefficients are random processes].

Stochastic Finite Element Method

Problems of structural dynamics in which the uncertainty in specifying mass and stiffness of the structure is modeled within the framework of random fields can be treated using the Stochastic Finite Element Method (SFEM). The application of SFEM in linear structural dynamics typically consists of the following key steps:

1. Selection of appropriate probabilistic models for parameter uncertainties and boundary conditions
2. Replacement of the element property random fields by an equivalent set of a finite number of random variables. This step, known as the 'discretisation of random fields' is a major step in the analysis.
3. Formulation of the equation of motion of the form $\mathbf{D}(\omega)\mathbf{u} = \mathbf{f}$ where $\mathbf{D}(\omega)$ is the random dynamic stiffness matrix, $\mathbf{u}$ is the vector of random nodal displacement and $\mathbf{f}$ is the applied forces. In general $\mathbf{D}(\omega)$ is a random symmetric complex matrix.
4. Calculation of the response statistics by either (a) solving the random eigenvalue problem, or (b) solving the set of complex random algebraic equations.

Dynamics of a general linear system

The equation of motion:

$$\mathbf{M}\ddot{\mathbf{q}}(t) + \mathbf{C}\dot{\mathbf{q}}(t) + \mathbf{K}\mathbf{q}(t) = \mathbf{f}(t) \tag{2}$$

- Due to the presence of (parametric/nonparametric or both) uncertainty $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ become random matrices.
- The main objectives in the 'forward problem' are:
  - to quantify uncertainties in the system matrices
  - to predict the variability in the response vector $\mathbf{q}$
- Probabilistic solution of this problem is expected to have more credibility compared to a deterministic solution

Random Matrix Method (RMM)

- The methodology:
  - Derive the matrix variate probability density functions of $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ [a] using available information.
  - Propagate the uncertainty (using Monte Carlo simulation or analytical methods) to obtain the response statistics (or pdf)

[a] AIAA Journal, 45[7] (2007), pp. 1748-1762

Swansea University
Prifysgol Abertawe

Matrix variate distributions

- The probability density function of a random matrix can be defined in a manner similar to that of a random variable.
- If $\mathbf{A}$ is an $n \times m$ real random matrix, the matrix variate probability density function of $\mathbf{A} \in \mathbb{R}_{n,m}$, denoted as $p_{\mathbf{A}}(\mathbf{A})$, is a mapping from the space of $n \times m$ real matrices to the real line, i.e., $p_{\mathbf{A}}(\mathbf{A}) : \mathbb{R}_{n,m} \to \mathbb{R}$.

Swansea University
Prifysgol Abertawe

Gaussian random matrix

The random matrix $\mathbf{X} \in \mathbb{R}_{n,p}$ is said to have a matrix variate Gaussian distribution with mean matrix $\mathbf{M} \in \mathbb{R}_{n,p}$ and covariance matrix $\mathbf{\Sigma} \otimes \mathbf{\Psi}$, where $\mathbf{\Sigma} \in \mathbb{R}_n^+$ and $\mathbf{\Psi} \in \mathbb{R}_p^+$ provided the pdf of $\mathbf{X}$ is given by

$$p_{\mathbf{X}}(\mathbf{X}) = (2\pi)^{-np/2} \det\left\{\mathbf{\Sigma}\right\}^{-p/2} \det\left\{\mathbf{\Psi}\right\}^{-n/2} \operatorname{etr}\left\{-\frac{1}{2}\mathbf{\Sigma}^{-1}(\mathbf{X}-\mathbf{M})\mathbf{\Psi}^{-1}(\mathbf{X}-\mathbf{M})^T\right\} \quad (3)$$

This distribution is usually denoted as $\mathbf{X} \sim N_{n,p}(\mathbf{M}, \mathbf{\Sigma} \otimes \mathbf{\Psi})$.

Swansea University
Prifysgol Abertawe

Wishart matrix

A $n \times n$ symmetric positive definite random matrix $\mathbf{S}$ is said to have a Wishart distribution with parameters $p \geq n$ and $\mathbf{\Sigma} \in \mathbb{R}_n^+$, if its pdf is given by

$$p_{\mathbf{S}}(\mathbf{S}) = \left\{ 2^{\frac{1}{2}np} \, \Gamma_n\left(\frac{1}{2}p\right) \det\left\{\mathbf{\Sigma}\right\}^{\frac{1}{2}p} \right\}^{-1} |\mathbf{S}|^{\frac{1}{2}(p-n-1)} \mathrm{etr}\left\{ -\frac{1}{2}\mathbf{\Sigma}^{-1}\mathbf{S} \right\} \tag{4}$$

This distribution is usually denoted as $\mathbf{S} \sim W_n(p, \mathbf{\Sigma})$.

Note: If $p = n + 1$, then the matrix is non-negative definite.

Swansea University
Prifysgol Abertawe

Distribution of the system matrices

The distribution of the random system matrices $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ should be such that they are

- symmetric
- positive-definite, and
- the moments (at least first two) of the inverse of the dynamic stiffness matrix $\mathbf{D}(\omega) = -\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}$ should exist $\forall\,\omega$. This ensures that the moments of the response exist for all frequency values.

Swansea University
Prifysgol Abertawe

Maximum Entropy Distribution

Suppose that the mean values of $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are given by $\overline{\mathbf{M}}$, $\overline{\mathbf{C}}$ and $\overline{\mathbf{K}}$ respectively. Using the notation $\mathbf{G}$ (which stands for any one the system matrices) the matrix variate density function of $\mathbf{G} \in \mathbb{R}_n^+$ is given by $p_{\mathbf{G}}(\mathbf{G}) : \mathbb{R}_n^+ \to \mathbb{R}$. We have the following constrains to obtain $p_{\mathbf{G}}(\mathbf{G})$:

$$\int_{\mathbf{G}>0} p_{\mathbf{G}}(\mathbf{G})\, d\mathbf{G} = 1 \quad \text{(normalization)} \tag{5}$$

and
$$\int_{\mathbf{G}>0} \mathbf{G}\, p_{\mathbf{G}}(\mathbf{G})\, d\mathbf{G} = \overline{\mathbf{G}} \quad \text{(the mean matrix)} \tag{6}$$

Swansea University
Prifysgol Abertawe

Further constraints

- Suppose that the inverse moments up to order ν of the system matrix exist. This implies that $\mathrm{E}\left[\|\mathbf{G}^{-1}\|_{\mathrm{F}}^{\nu}\right]$ should be finite. Here the Frobenius norm of matrix $\mathbf{A}$ is given by $\|\mathbf{A}\|_{\mathrm{F}} = \left(\mathrm{Trace}\left(\mathbf{A}\mathbf{A}^T\right)\right)^{1/2}$.
- Taking the logarithm for convenience, the condition for the existence of the inverse moments can be expresses by

$$\mathrm{E}\left[\ln\det\left\{\mathbf{G}\right\}^{-\nu}\right] < \infty$$

Swansea University
Prifysgol Abertawe

MEnt distribution - 1

The Lagrangian becomes:

$$\mathcal{L}\left(p_{\mathbf{G}}\right) = -\int_{\mathbf{G}>0} p_{\mathbf{G}}\left(\mathbf{G}\right) \ln\left\{p_{\mathbf{G}}\left(\mathbf{G}\right)\right\} \, d\mathbf{G} + \left(\lambda_0 - 1\right)\left(\int_{\mathbf{G}>0} p_{\mathbf{G}}\left(\mathbf{G}\right) \, d\mathbf{G} - 1\right) - \nu \int_{\mathbf{G}>0} \ln\det\left\{\mathbf{G}\right\} \, p_{\mathbf{G}} \, d\mathbf{G} + \operatorname{Trace}\left(\mathbf{\Lambda}_1\left[\int_{\mathbf{G}>0} \mathbf{G}\, p_{\mathbf{G}}\left(\mathbf{G}\right) \, d\mathbf{G} - \overline{\mathbf{G}}\right]\right) \tag{7}$$

Note: ν cannot be obtained uniquely!

MEnt distribution - 2

Using the calculus of variation

$$\frac{\partial \mathcal{L}\left(p_{\mathbf{G}}\right)}{\partial p_{\mathbf{G}}} = 0$$

or $-\ln\left\{p_{\mathbf{G}}\left(\mathbf{G}\right)\right\} = \lambda_0 + \mathrm{Trace}\left(\mathbf{\Lambda}_1 \mathbf{G}\right) - \ln\det\left\{\mathbf{G}\right\}^{\nu}$

or $p_{\mathbf{G}}\left(\mathbf{G}\right) = \exp\left\{-\lambda_0\right\}\det\left\{\mathbf{G}\right\}^{\nu}\mathrm{etr}\left\{-\mathbf{\Lambda}_1 \mathbf{G}\right\}$

MEnt distribution - 3

Using the matrix variate Laplace transform
($\mathbf{T} \in \mathbb{R}_{n,n}, \mathbf{S} \in \mathbb{C}_{n,n}, a > (n+1)/2$)

$$\int_{\mathbf{T}>0} \operatorname{etr}\left\{-\mathbf{S}\mathbf{T}\right\} \det\left\{\mathbf{T}\right\}^{a-(n+1)/2} d\mathbf{T} = \Gamma_n(a) \det\left\{\mathbf{S}\right\}^{-a}$$

and substituting $p_{\mathbf{G}}\left(\mathbf{G}\right)$ into the constraint equations it can be shown that

$$p_{\mathbf{G}}\left(\mathbf{G}\right) = r^{-nr}\left\{\Gamma_n(r)\right\}^{-1} \det\left\{\overline{\mathbf{G}}\right\}^{-r} \det\left\{\mathbf{G}\right\}^{\nu} \operatorname{etr}\left\{-r\overline{\mathbf{G}}^{-1}\mathbf{G}\right\} \quad (8)$$

where $r = \nu + (n+1)/2$.

MEnt Distribution - 4

Comparing it with the Wishart distribution we have: If ν-th order inverse-moment of a system matrix $\mathbf{G} \equiv \{\mathbf{M}, \mathbf{C}, \mathbf{K}\}$ exists and only the mean of $\mathbf{G}$ is available, say $\overline{\mathbf{G}}$, then the maximum-entropy pdf of $\mathbf{G}$ follows the Wishart distribution with parameters $p = (2\nu + n + 1)$ and $\mathbf{\Sigma} = \overline{\mathbf{G}}/(2\nu + n + 1)$, that is $\mathbf{G} \sim W_n\left(2\nu + n + 1, \overline{\mathbf{G}}/(2\nu + n + 1)\right)$.

Properties of the distribution

- Covariance tensor of $\mathbf{G}$:

$$\mathrm{cov}\left(G_{ij}, G_{kl}\right) = \frac{1}{2\nu + n + 1}\left(\overline{G}_{ik}\overline{G}_{jl} + \overline{G}_{il}\overline{G}_{jk}\right)$$

- Normalized standard deviation matrix

$$\sigma_G^2 = \frac{\mathrm{E}\left[\|\mathbf{G} - \mathrm{E}\left[\mathbf{G}\right]\|_{\mathrm{F}}^2\right]}{\|\mathrm{E}\left[\mathbf{G}\right]\|_{\mathrm{F}}^2} = \frac{1}{2\nu + n + 1}\left\{1 + \frac{\{\mathrm{Trace}\left(\overline{\mathbf{G}}\right)\}^2}{\mathrm{Trace}\left(\overline{\mathbf{G}}^2\right)}\right\}$$

- $\sigma_G^2 \leq \dfrac{1+n}{2\nu + n + 1}$ and $\nu \uparrow \Rightarrow \delta_{\mathbf{G}}^2 \downarrow$.

Swansea University
Prifysgol Abertawe

Wishart random matrix approach

- Suppose we 'know' (e.g, by measurements or stochastic finite element modeling) the mean ($\mathbf{G}_0$) and the (normalized) standard deviation (σ_G) of the system matrices:

$$\sigma_G^2 = \frac{\mathrm{E}\left[\|\mathbf{G} - \mathrm{E}\left[\mathbf{G}\right]\|_{\mathrm{F}}^2\right]}{\|\mathrm{E}\left[\mathbf{G}\right]\|_{\mathrm{F}}^2}. \tag{9}$$

- The parameters of the Wishart distribution can be identified using the expressions derived before.

Swansea University
Prifysgol Abertawe

Stochastic dynamic response-1

- The dynamic response of the system can be expressed in the frequency domain as

$$\mathbf{q}(\omega) = \mathbf{D}^{-1}(\omega)\mathbf{f}(\omega) \tag{10}$$

where the dynamic stiffness matrix is defined as

$$\mathbf{D}(\omega) = -\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}. \tag{11}$$

This is a complex symmetric random matrix.

- The calculation of the response statistics requires the calculation of statistical moments of the inverse of this matrix.

Swansea University
Prifysgol Abertawe

Stochastic dynamic response-2

Using the eigenvectors ($\mathbf{\Phi}$) and eigenvalues ($\mathbf{\Omega}^2$) of $\mathbf{M}$ and $\mathbf{K}$ and assuming $\mathbf{C}$ is simultaneously diagonalisable

$$\mathbf{D}^{-1}(\omega) = \left[-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K}\right]^{-1} \tag{12}$$

$$= \mathbf{\Phi}\left[-\omega^2\mathbf{I}_n + i\zeta\omega\mathbf{\Omega} + \mathbf{\Omega}^2\right]^{-1}\mathbf{\Phi}^T \tag{13}$$

Because the system is random, we assume that $\mathbf{\Omega}^2$ is a random matrix. Note that $\mathbf{\Omega}^2$ is actually a diagonal (therefore, trivially symmetric) and positive definite matrix. We model $\mathbf{\Omega}^2$ by a Wishart random matrix (can be derived using the maximum entropy approach), $\mathbf{\Omega}^2 \sim W_n(p, \mathbf{\Sigma})$

Swansea University
Prifysgol Abertawe

Parameter-selection of Wishart matrices

- Approach 1: M and K are fully correlated Wishart (most complex)
- Approach 2: (Scalar Wishart) $\mathbf{\Sigma} = c_1 \mathbf{I}_n$ (most simple)
- Approach 3: (Diagonal Wishart with different entries) $\mathbf{\Sigma} = c_2 \mathbf{\Omega}_0^2$ (where $\mathbf{\Omega}_0^2$ is the matrix containing the eigenvalues of the baseline system) (something in the middle)

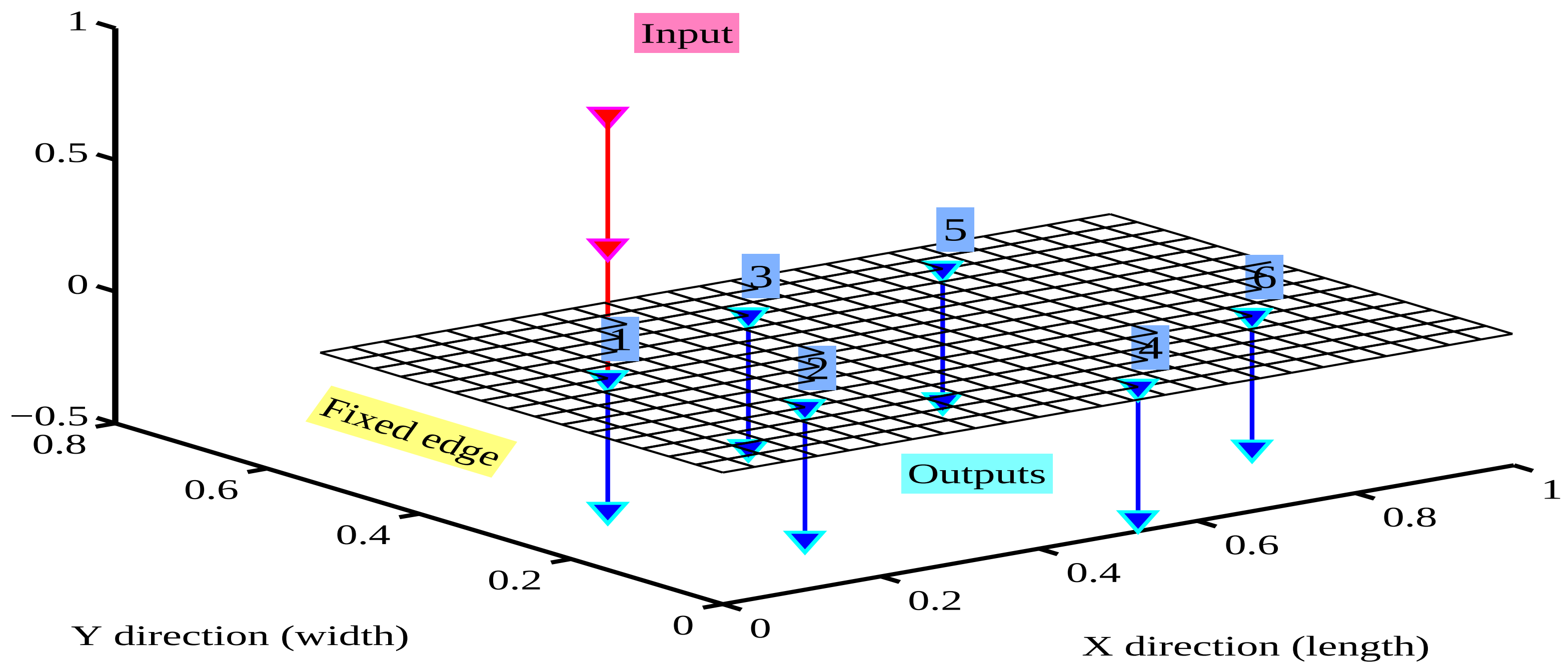
The parameter p can be related to the standard deviation of the system:

$$p = (1+\beta)/\widetilde{\sigma}_{\mathbf{\Omega}^2}, \;\; \beta = \left\{ \text{Trace}\left(\mathbf{\Omega}_0^2\right) \right\}^2 / \text{Trace}\left(\mathbf{\Omega}_0^4\right) \tag{14}$$

Numerical Examples

Swansea University
Prifysgol Abertawe

A vibrating cantilever plate



Baseline Model: Thin plate elements with 0.7% modal damping assumed for all the modes.

Physical properties

Plate Properties	Numerical values
Length (L_x)	998 mm
Width (L_y)	530 mm
Thickness (t_h)	3.0 mm
Mass density (ρ)	7860 kg/m^3
Young's modulus (E)	2.0×10^5 MPa
Poisson's ratio (μ)	0.3
Total weight	12.47 kg

Material and geometric properties of the cantilever plate considered for the experiment. The data presented here are available from http://engweb.swan.ac.uk/∼adhikaris/uq/.

Uncertainty type 1: random fields

The Young's modulus, Poissons ratio, mass density and thickness are random fields of the form

$$E(\mathbf{x}) = \bar{E}\left(1 + \epsilon_E f_1(\mathbf{x})\right) \quad (15)$$

$$\mu(\mathbf{x}) = \bar{\mu}\left(1 + \epsilon_\mu f_2(\mathbf{x})\right) \quad (16)$$

$$\rho(\mathbf{x}) = \bar{\rho}\left(1 + \epsilon_\rho f_3(\mathbf{x})\right) \quad (17)$$

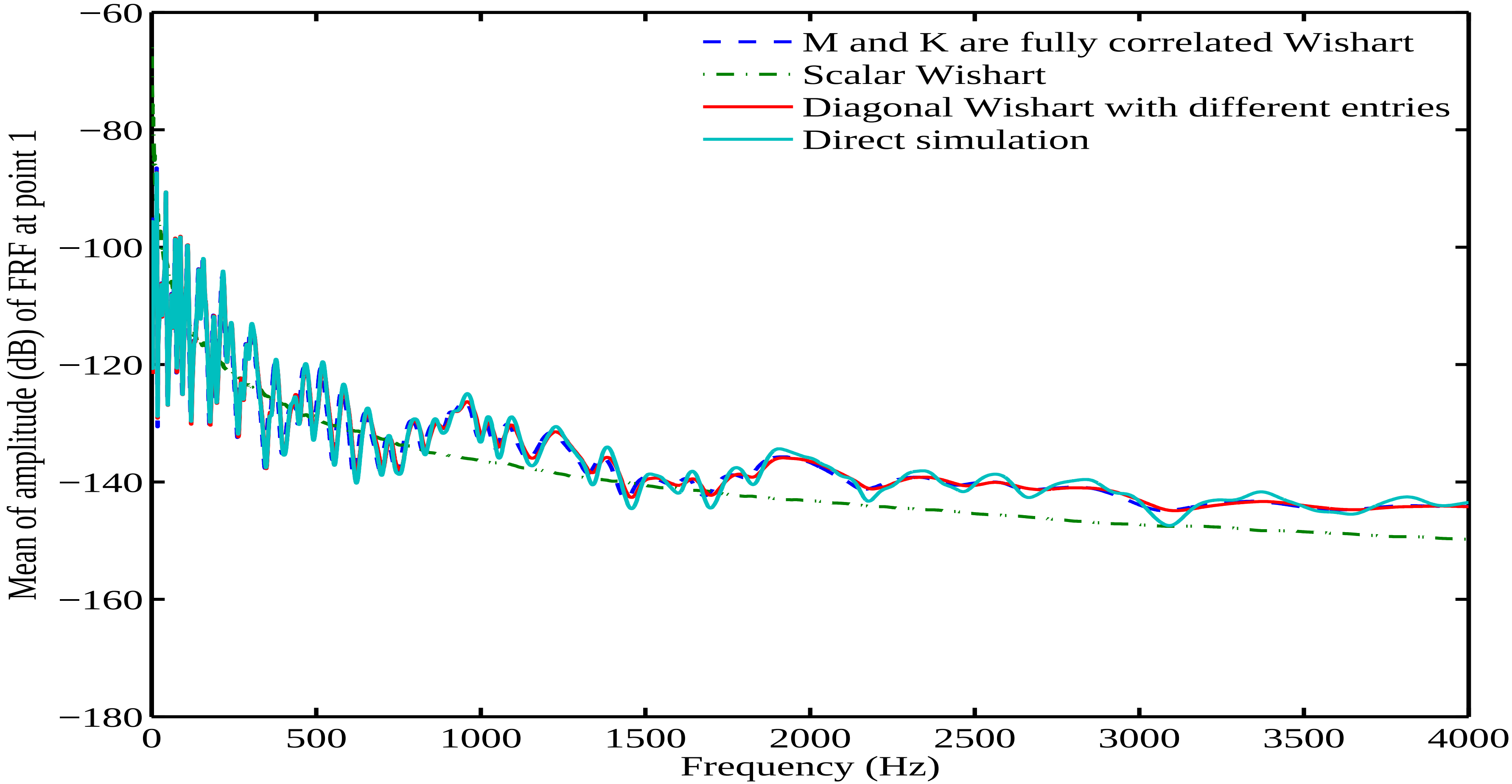
$$\text{and} \quad t(\mathbf{x}) = \bar{t}\left(1 + \epsilon_t f_4(\mathbf{x})\right) \quad (18)$$

- The strength parameters: $\epsilon_E = 0.15$, $\epsilon_\mu = 0.15$, $\epsilon_\rho = 0.10$ and $\epsilon_t = 0.15$.
- The random fields $f_i(\mathbf{x}), i = 1, \cdots, 4$ are delta-correlated homogenous Gaussian random fields.

Uncertainty type 2: random attached oscillators

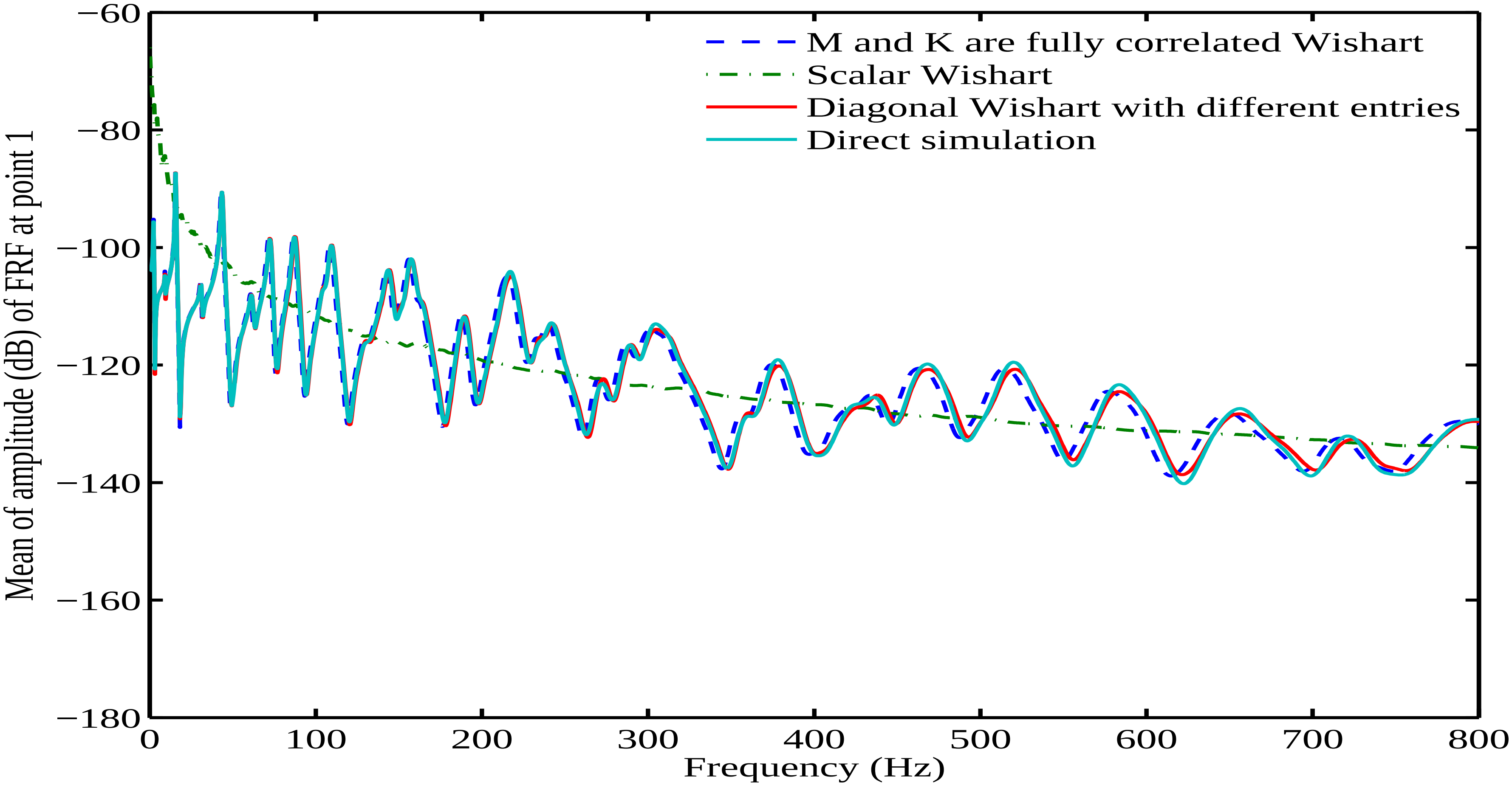
- Here we consider that the baseline plate is ‘perturbed’ by attaching 10 oscillators with random spring stiffnesses at random locations
- This is aimed at modeling non-parametric uncertainty.
- This case will be investigated experimentally later.

Mean of cross-FRF: Utype 1



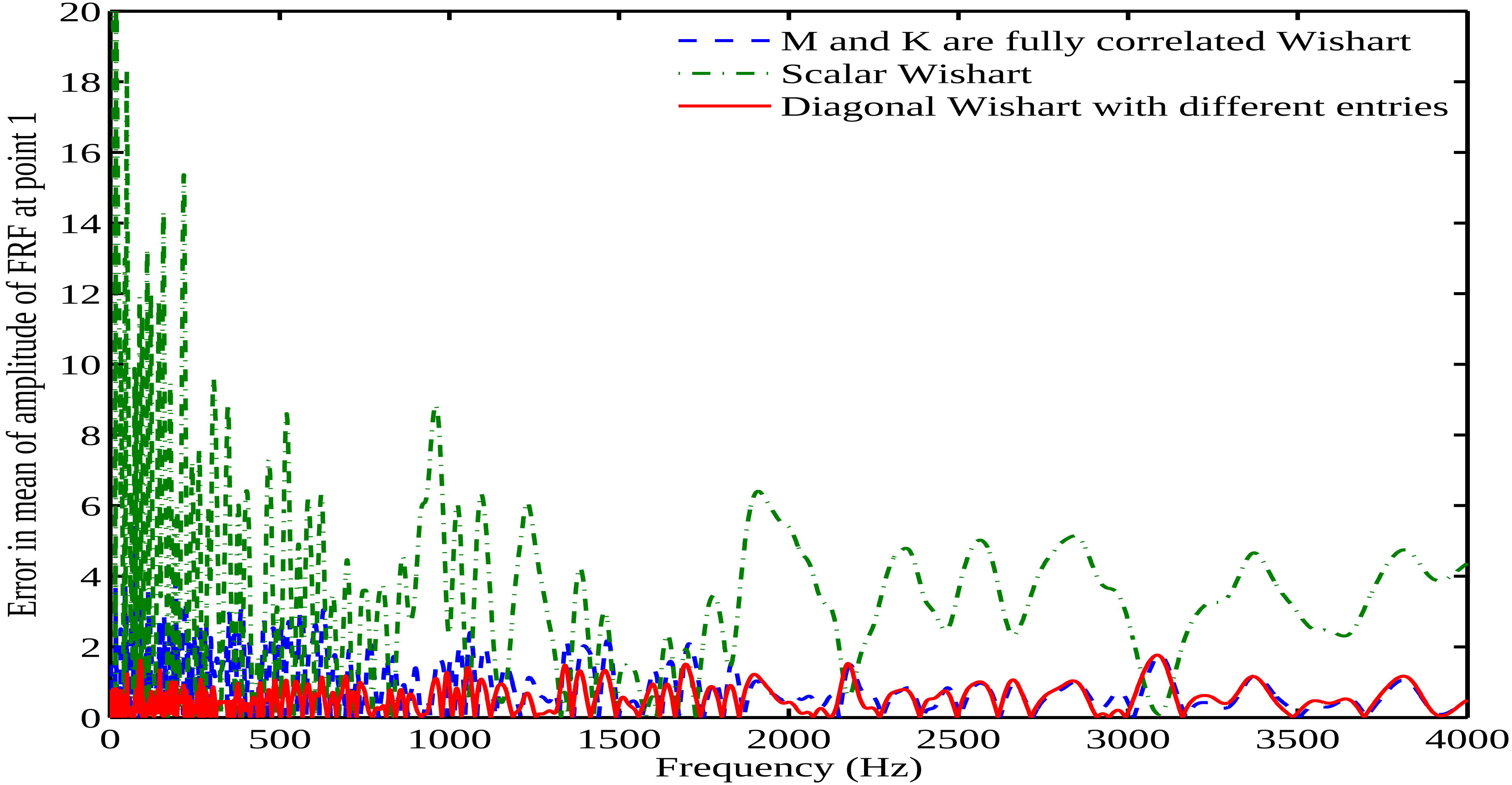
Mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

Mean of cross-FRF in the Low Frequency: Utype 1



Mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

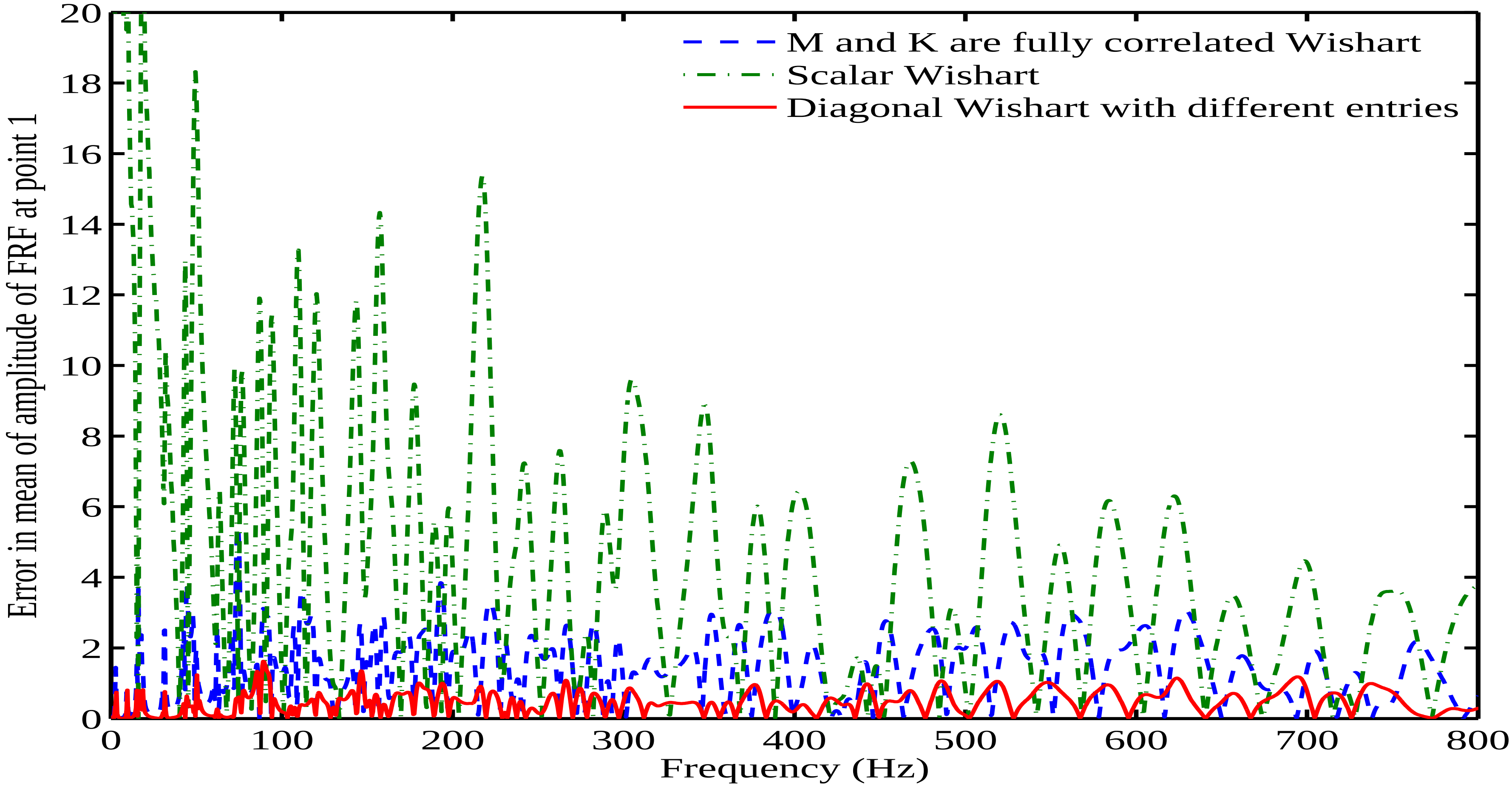
Error in the mean of cross-FRF: Utype 1



Error in the mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

Swansea University
Prifysgol Abertawe

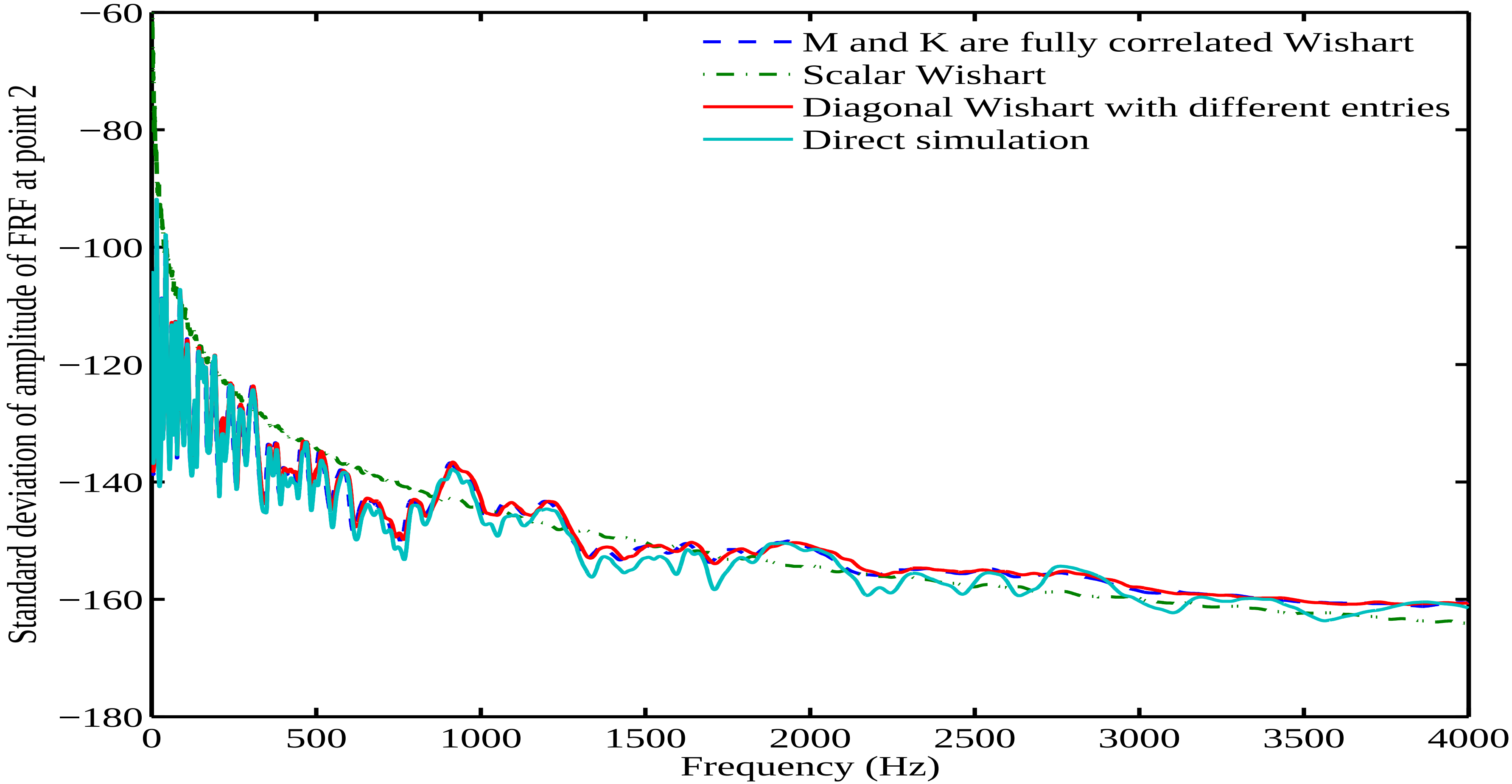
Error in the mean of cross-FRF in the Low Frequency: Utype 1



Error in the mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

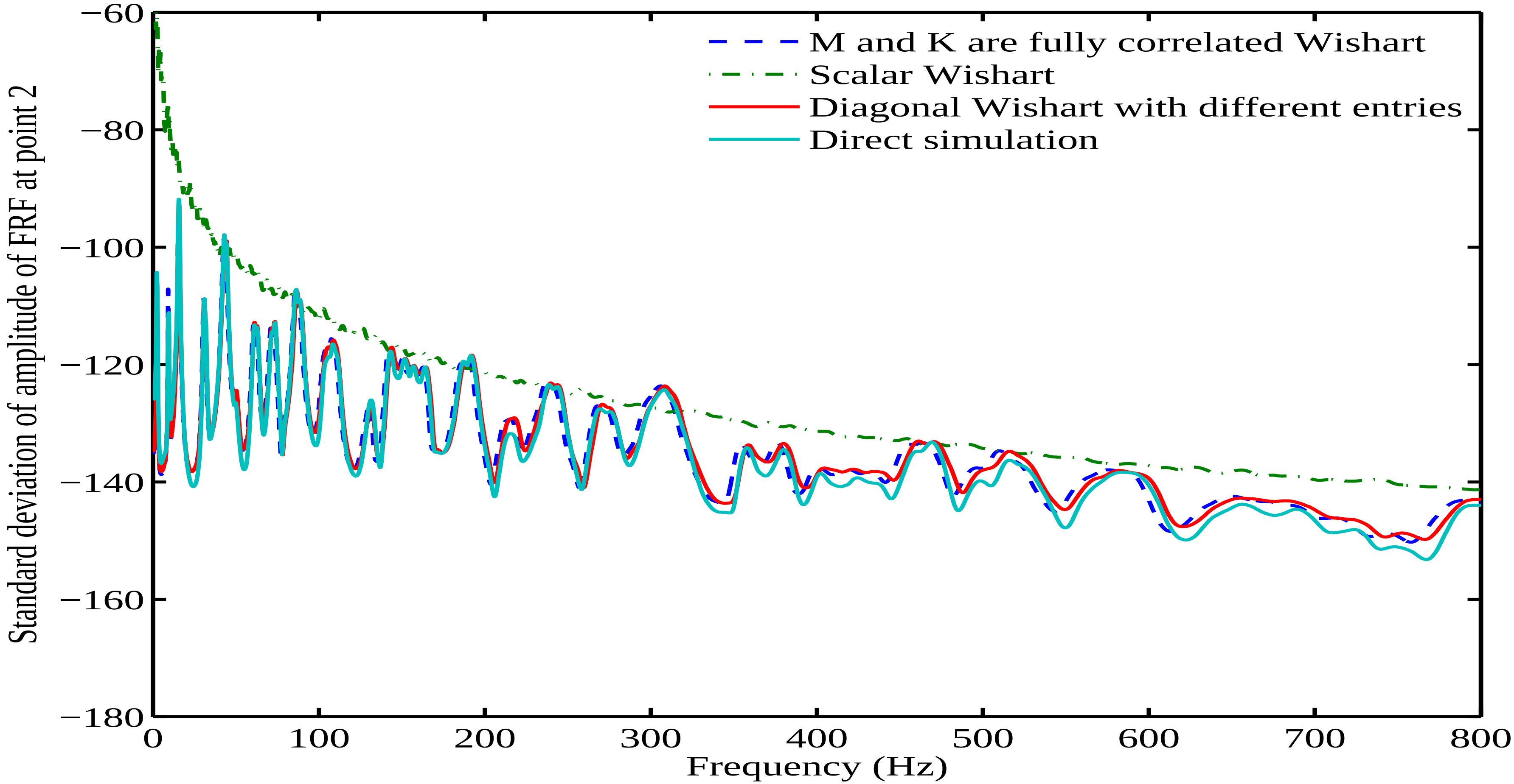
Swansea University
Prifysgol Abertawe

Standard deviation of driving-point-FRF: Utype 1



Standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

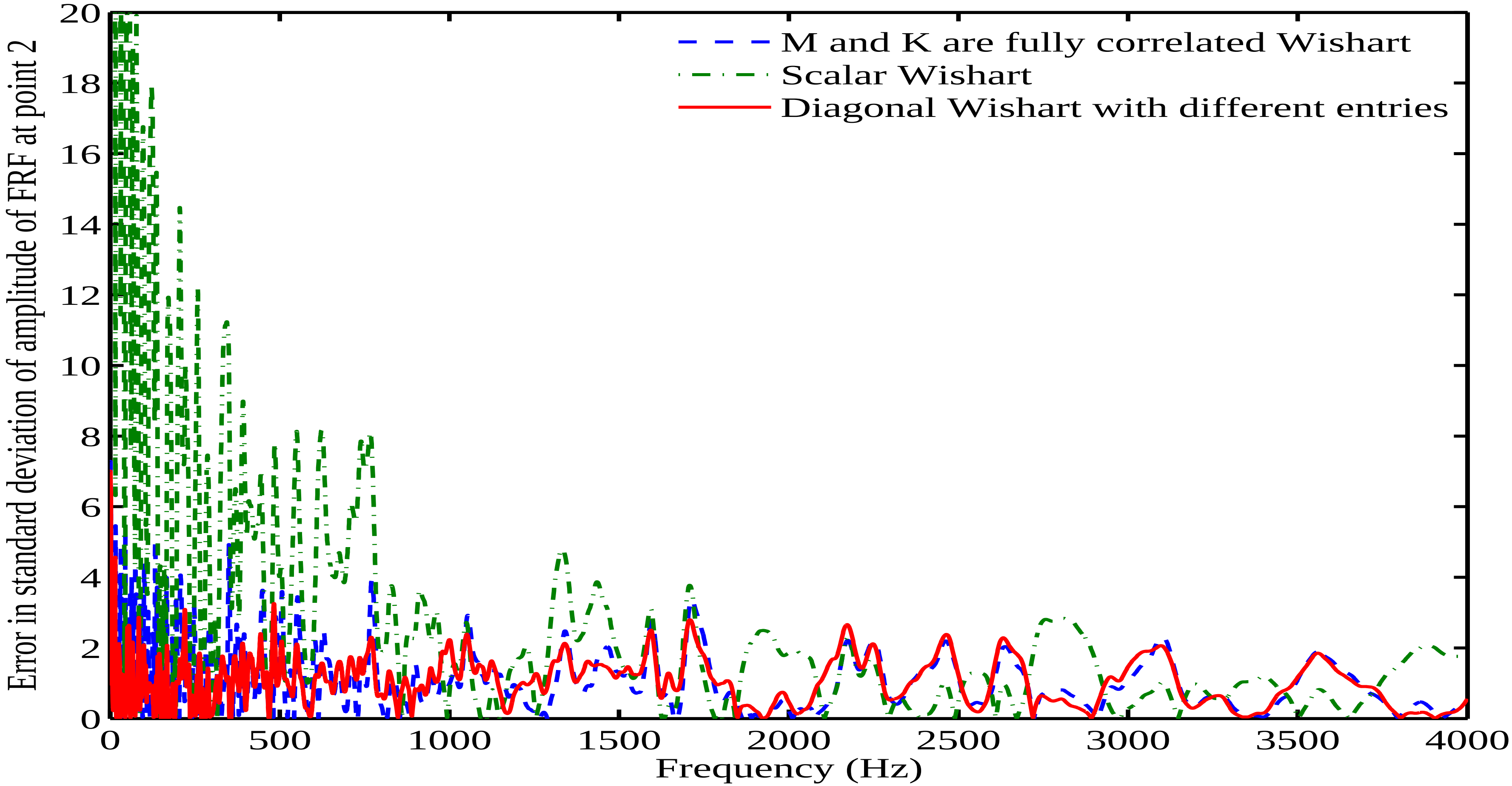
Standard deviation of driving-point-FRF in the Low Frequency: Utype 1



Standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

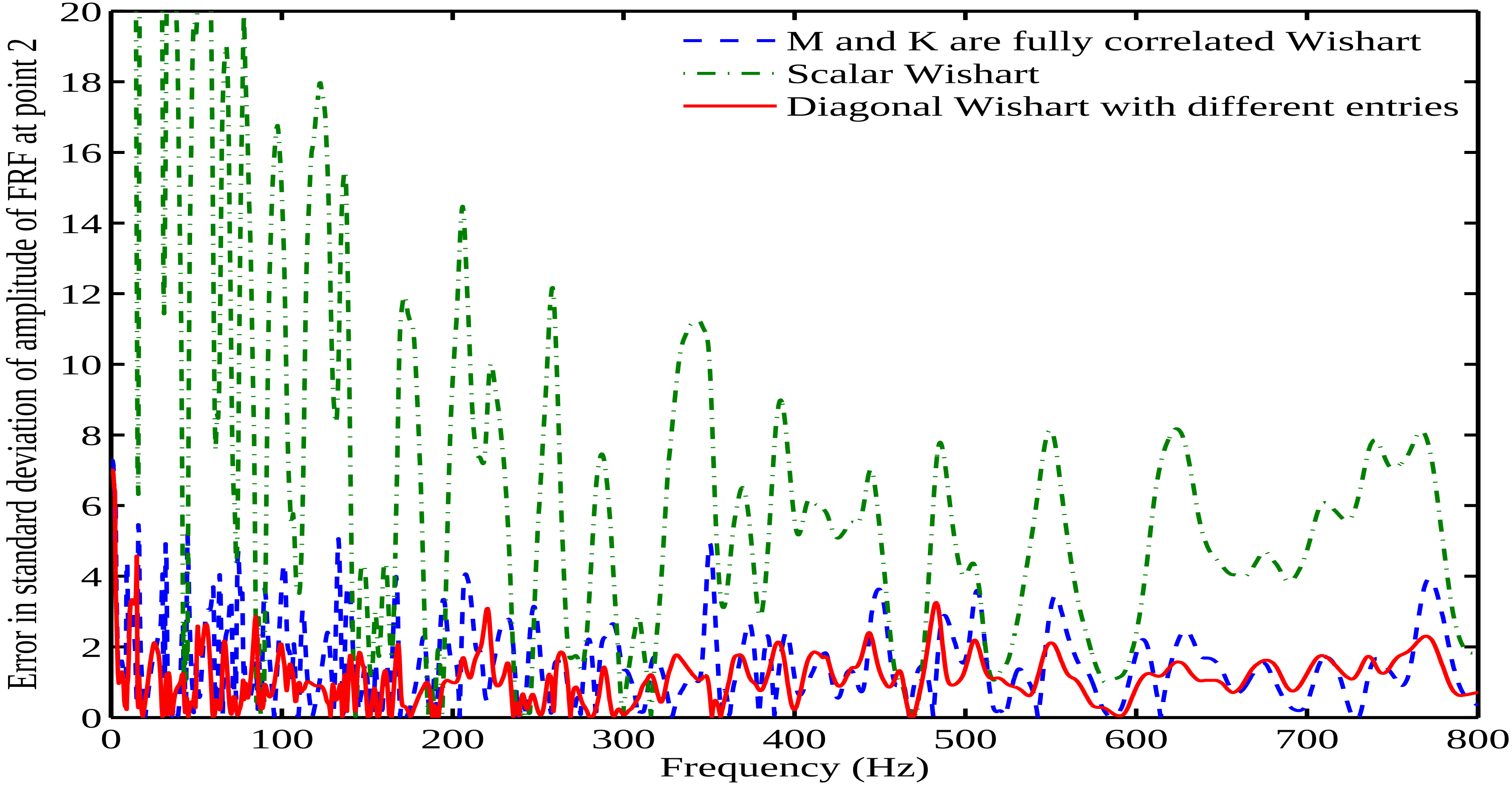
Swansea University
Prifysgol Abertawe

Error in the standard deviation of driving-point-FRF: Utype 1



Error in the standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

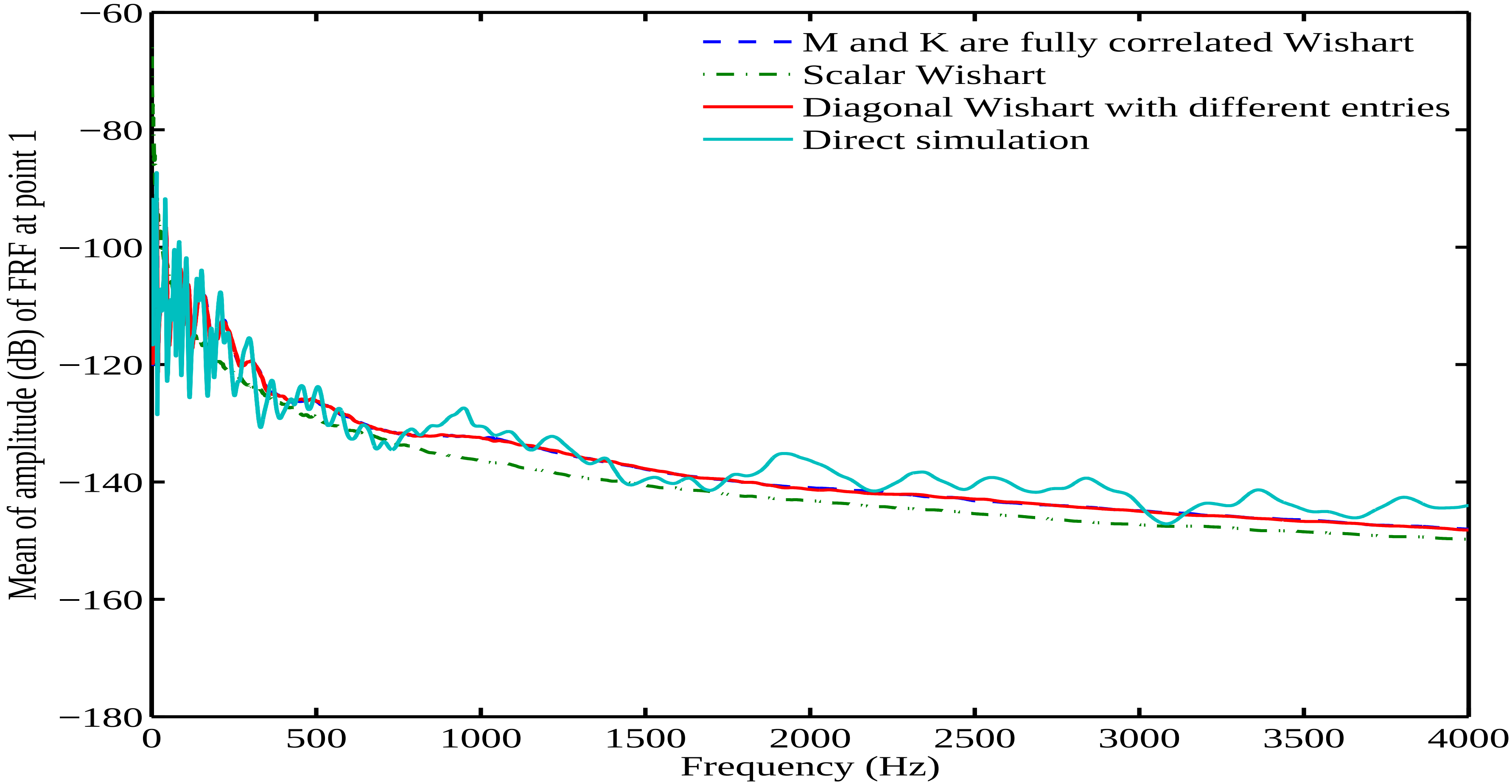
Error in the standard deviation of driving-point-FRF in the Low Frequency: Utype 1



Error in the standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.078$ and $\sigma_K = 0.205$.

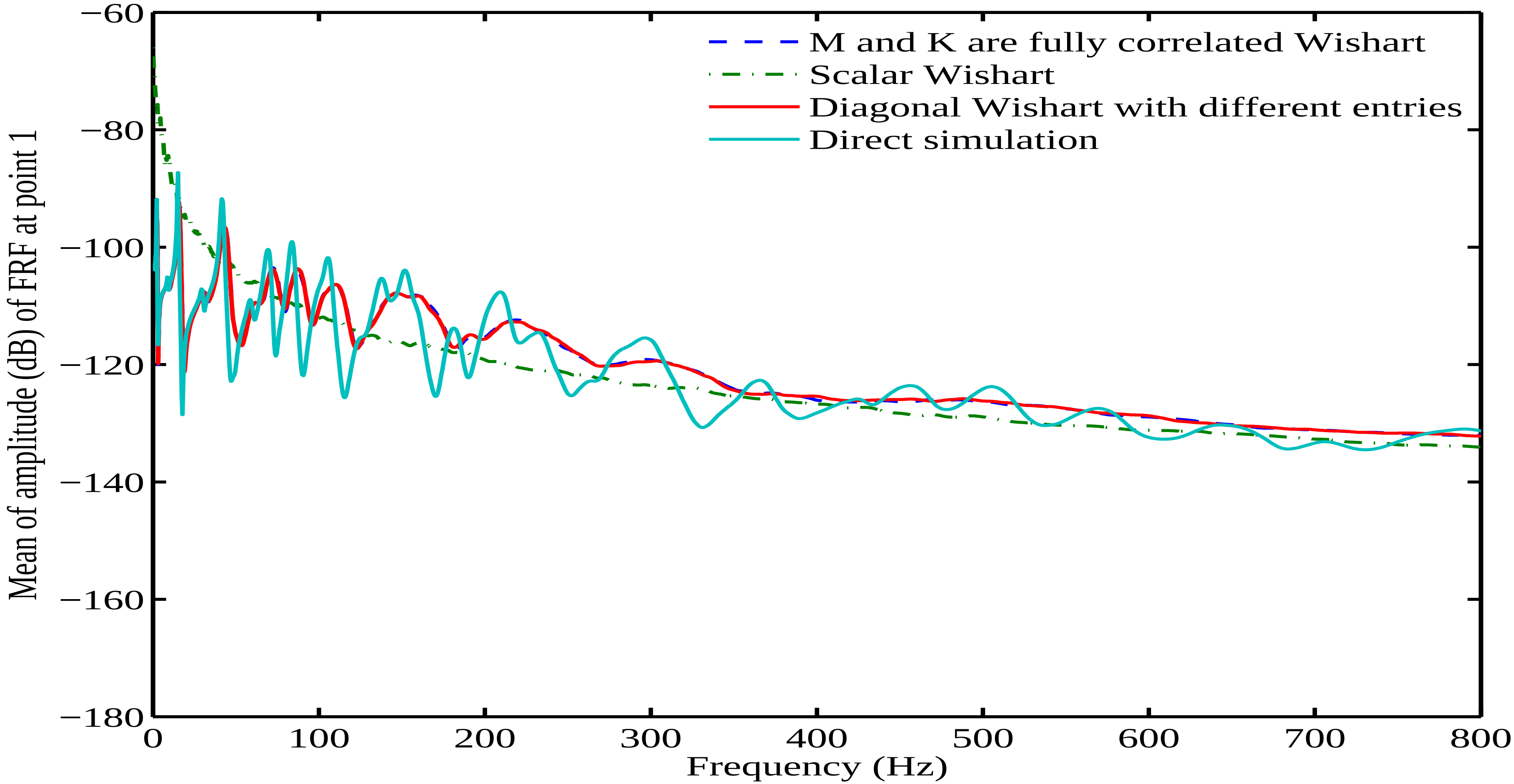
Swansea University
Prifysgol Abertawe

Mean of cross-FRF: Utype 2



Mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

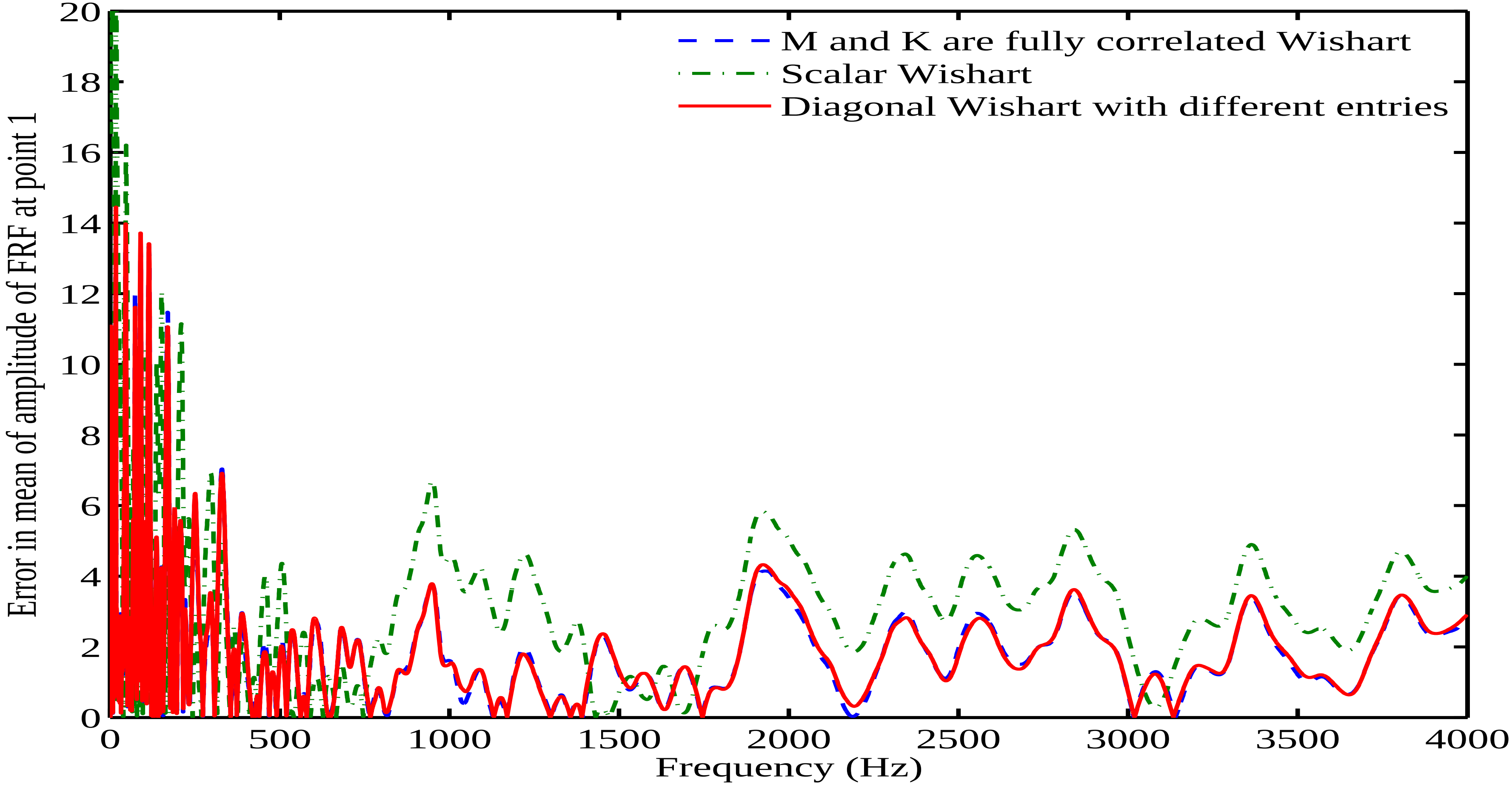
Mean of cross-FRF in the Low Frequency: Utype 2



Mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

Swansea University
Prifysgol Abertawe

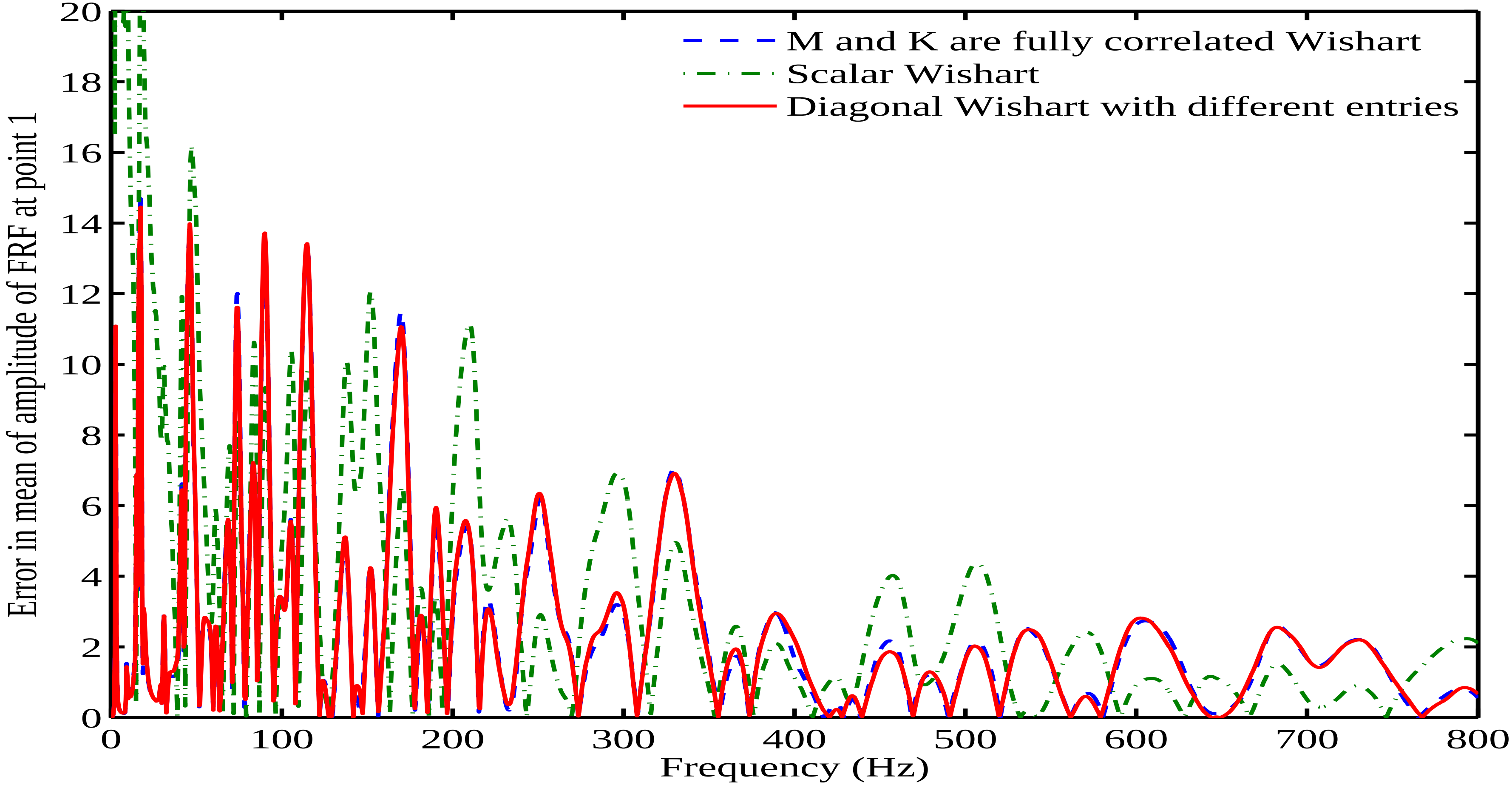
Error in the mean of cross-FRF: Utype 2



Error in the mean of the amplitude of the response of the cross-FRF of the plate,
$n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

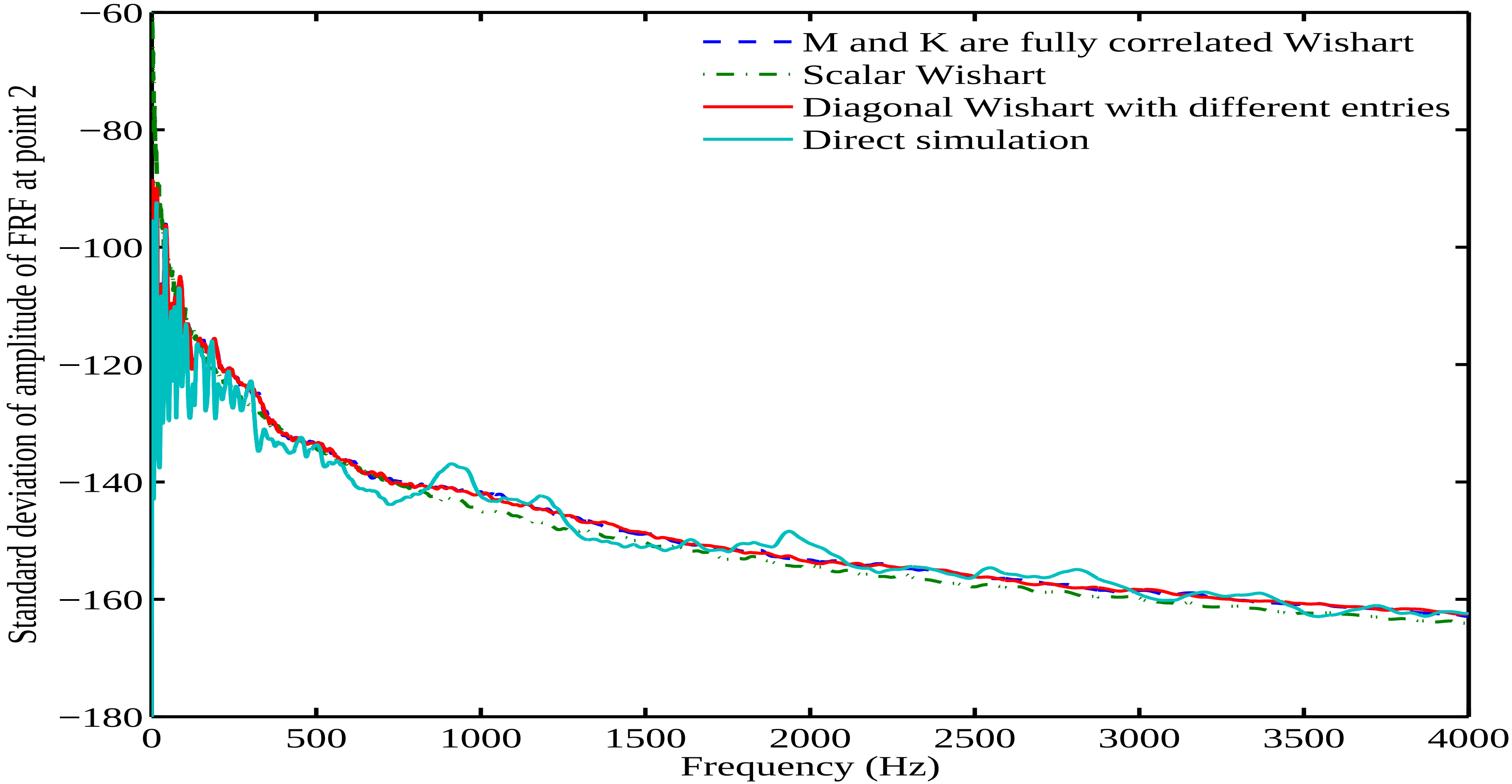
Swansea University
Prifysgol Abertawe

Error in the mean of cross-FRF in the Low Frequency: Utype 2



Error in the mean of the amplitude of the response of the cross-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

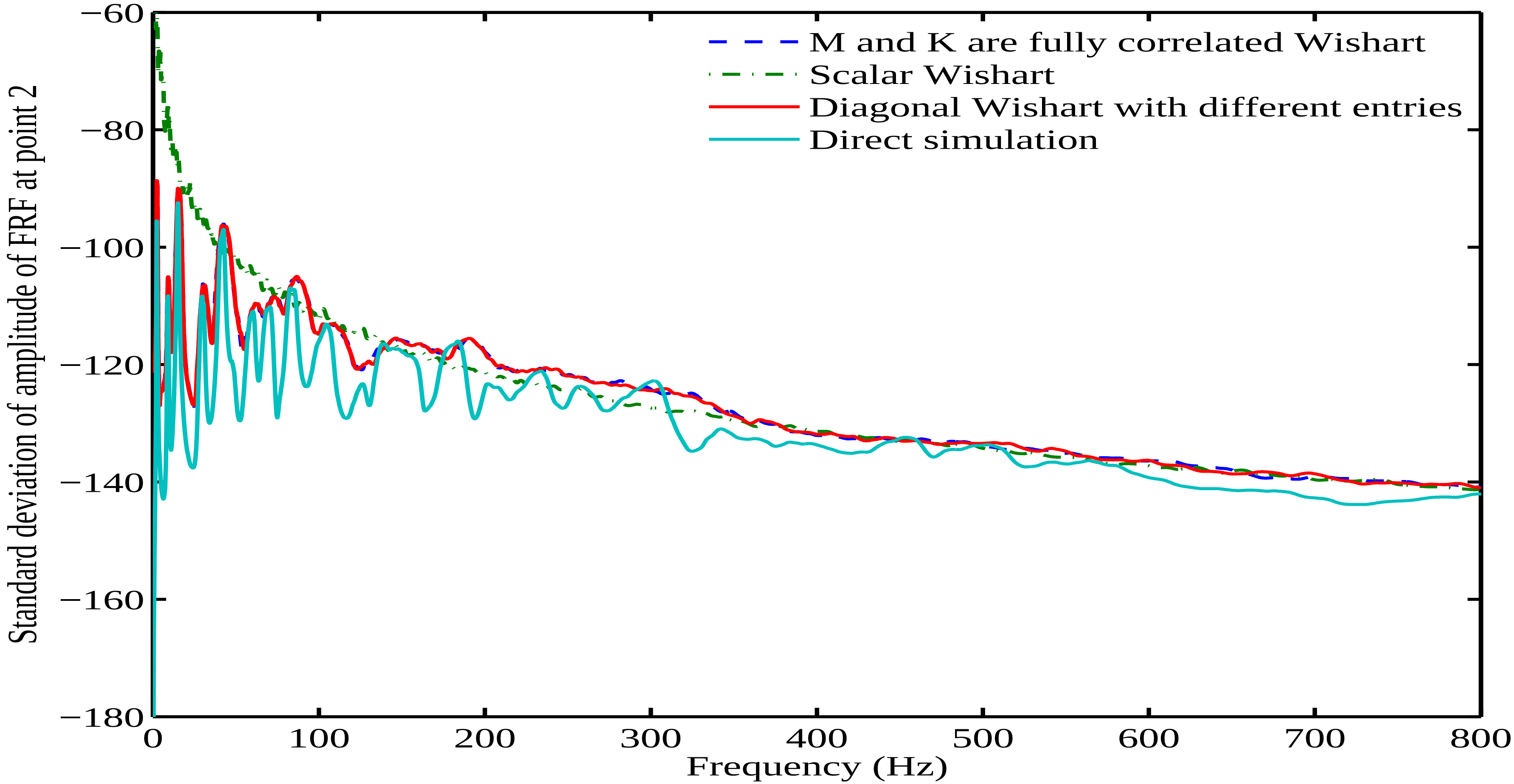
Standard deviation of driving-point-FRF: Utype 2



Standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

Swansea University
Prifysgol Abertawe

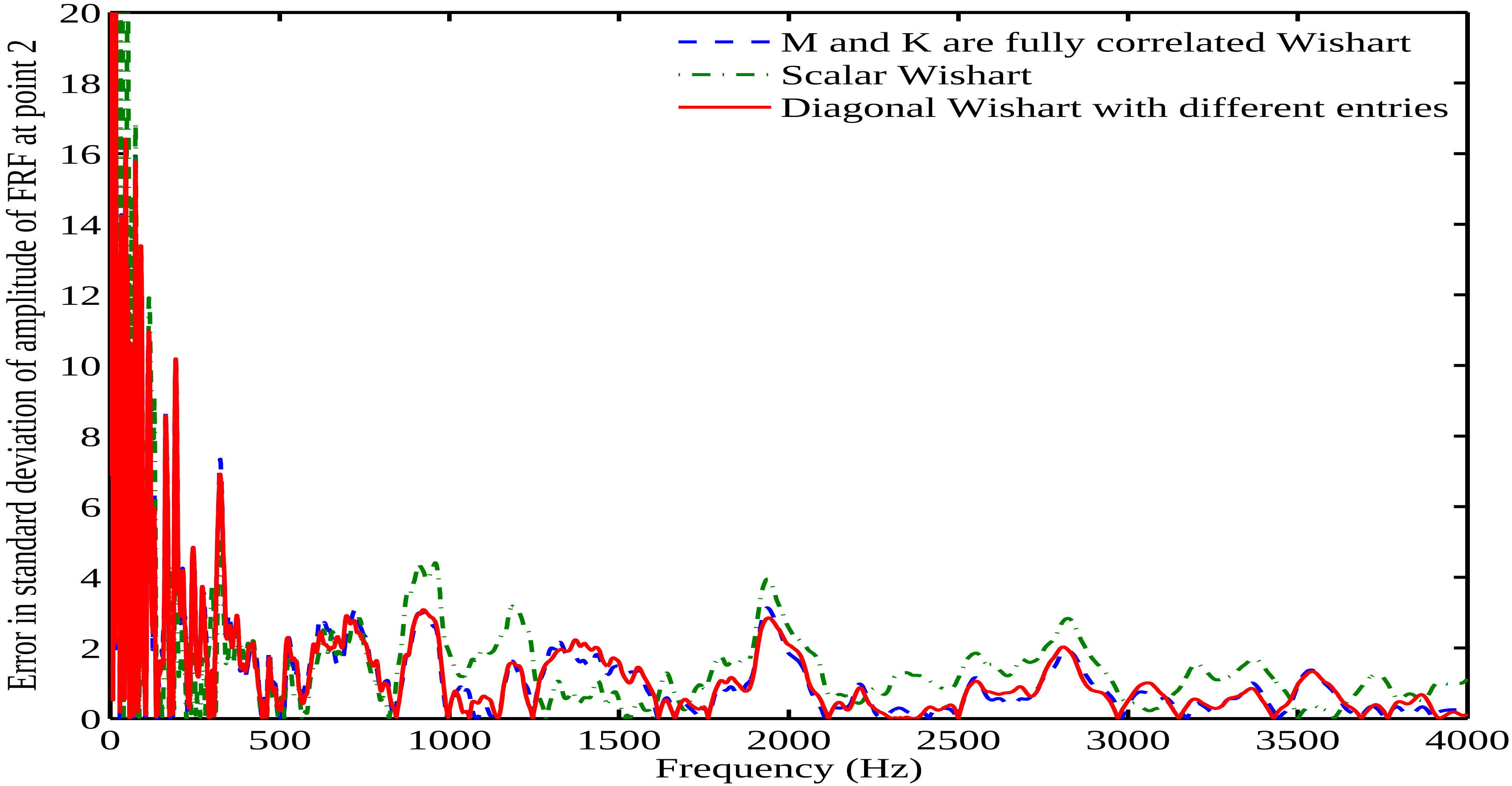
Standard deviation of driving-point-FRF in the Low Frequency: Utype 2



Standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

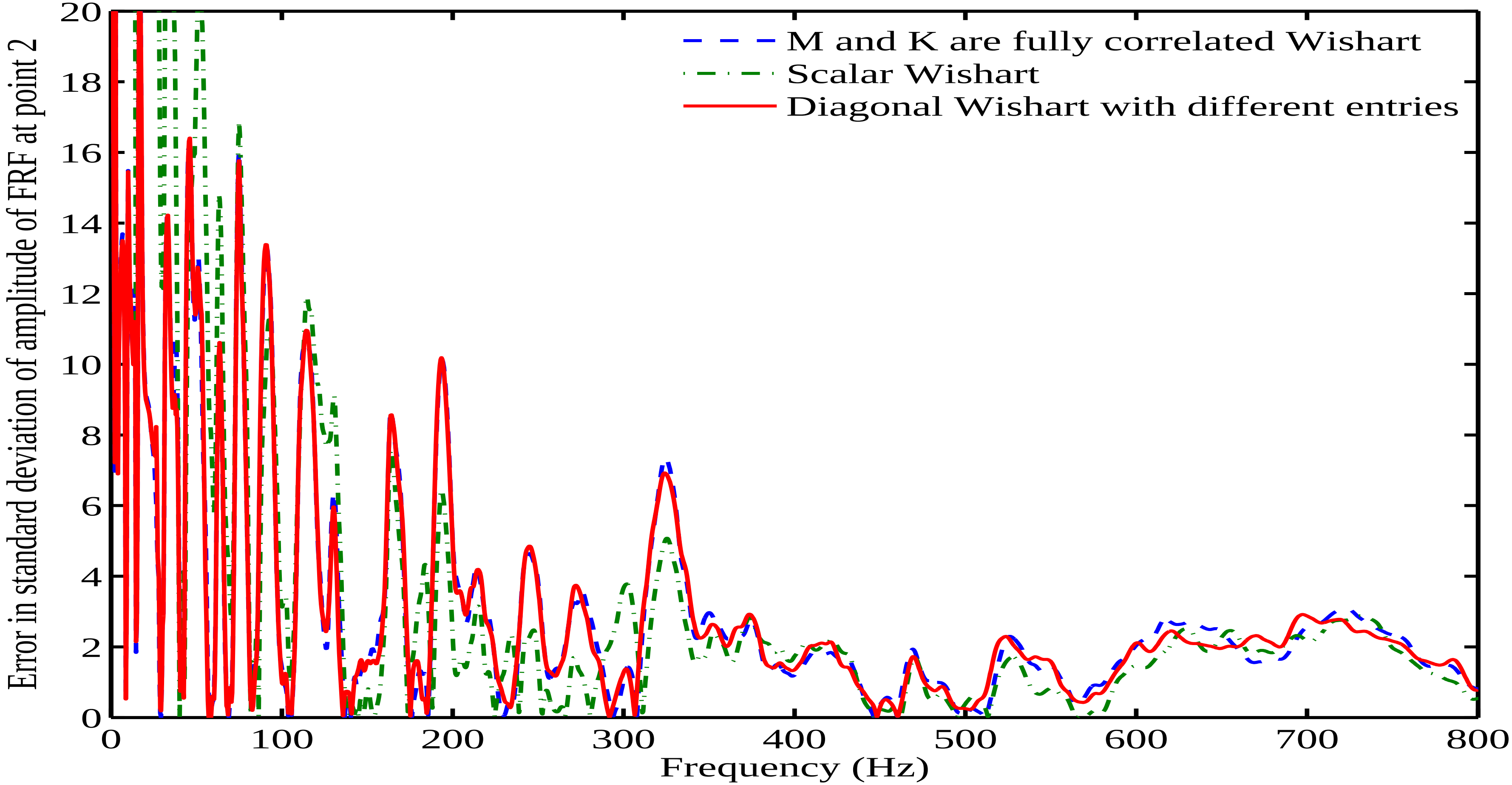
Swansea University
Prifysgol Abertawe

Error in the standard deviation of driving-point-FRF: Utype 2



Error in the standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

Error in the standard deviation of driving-point-FRF in the Low Frequency: Utype 2



Error in the standard deviation of the amplitude of the response of the driving-point-FRF of the plate, $n = 1200$, $\sigma_M = 0.133$ and $\sigma_K = 0.420$.

Swansea University
Prifysgol Abertawe

Main observations

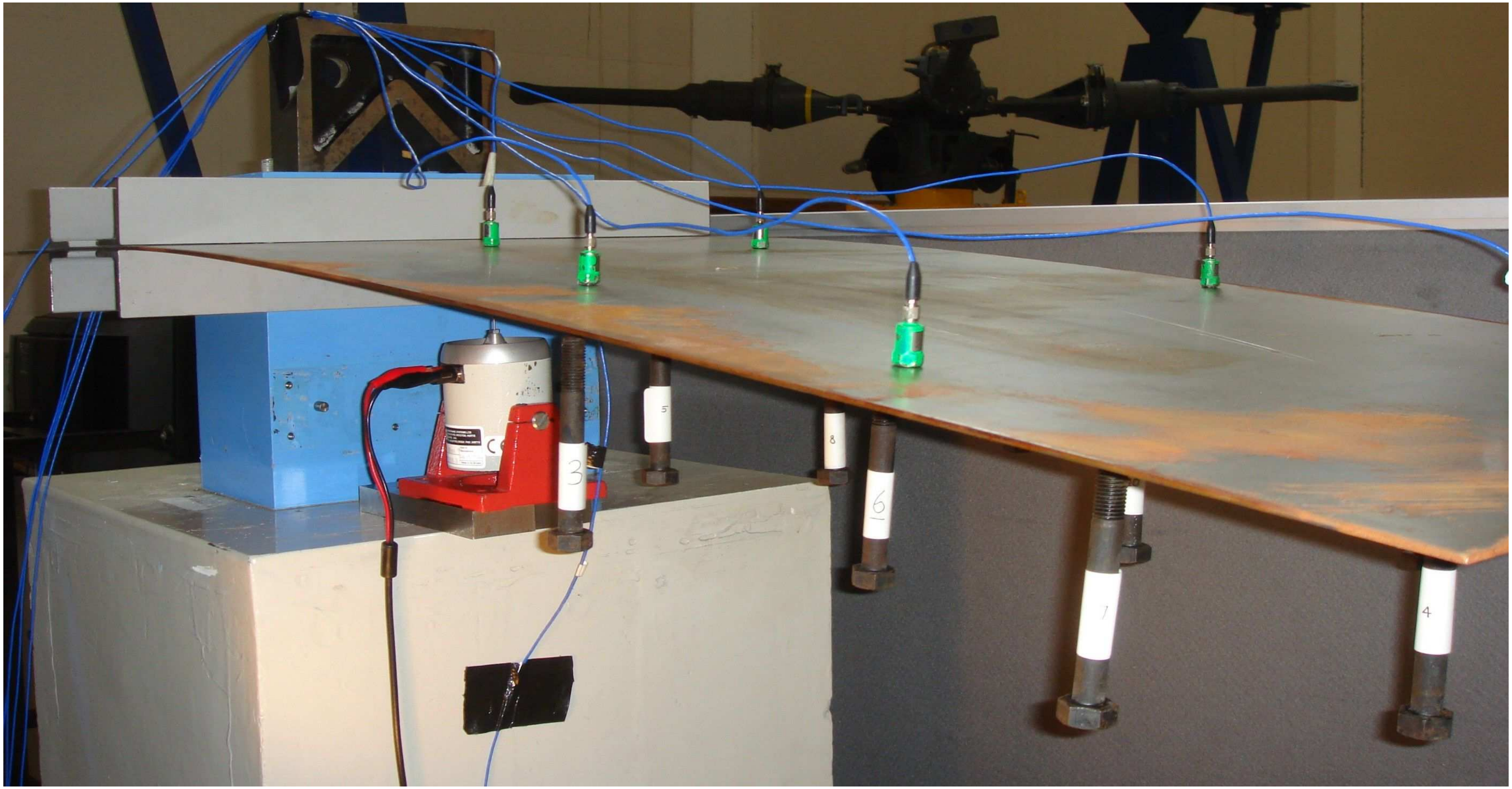
- Error in the low frequency region is higher than that in the higher frequencies [a]
- In the high frequency region all methods are similar
- Overall, parameter selection 3 turns out to be most cost effective.

[a] to appear in ASCE J. of Engineering Mechanics

Swansea University
Prifysgol Abertawe

Experimental investigation for uncertainty type 2 (randomly attached oscillators)

A cantilever plate: front view



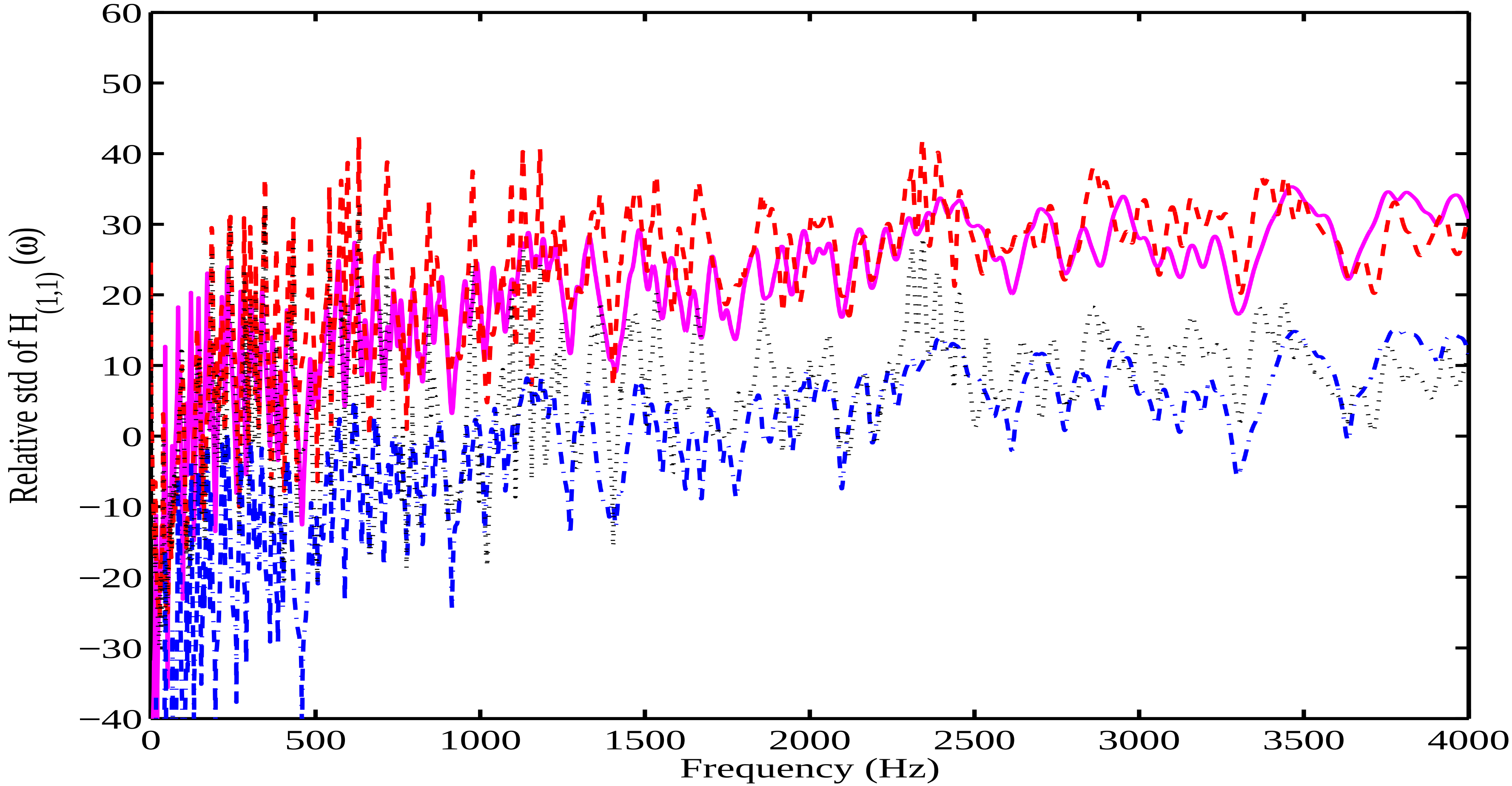
The test rig for the cantilever plate; front view (to appear in Probabilistic Engineering Mechanics).

Swansea University
Prifysgol Abertawe

A cantilever plate: side view

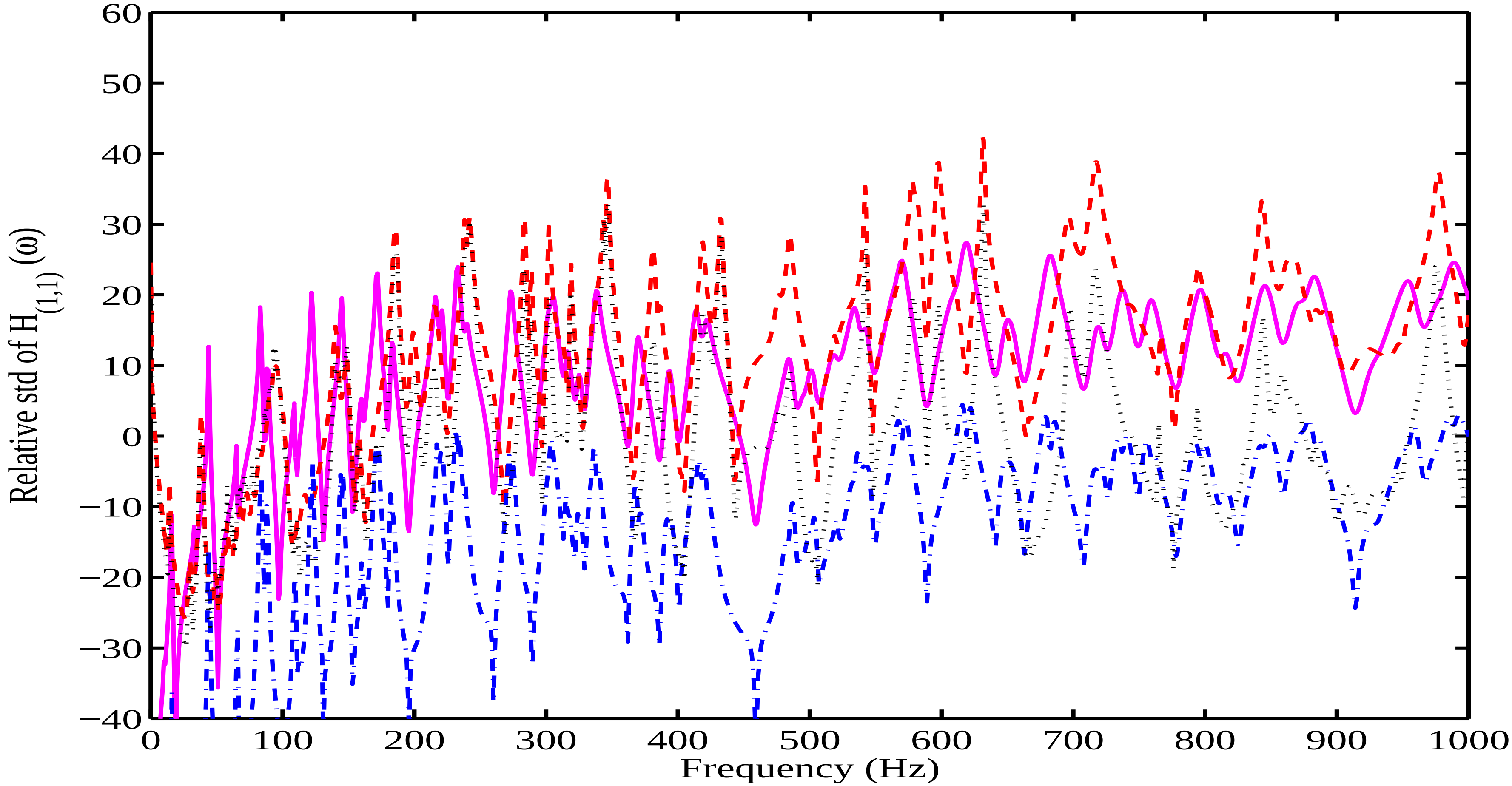
The test rig for the cantilever plate; side view.

Comparison of driving-point-FRF



Comparison of the mean and standard deviation of the amplitude of the driving-point-FRF, $n = 1200$, $\delta_M = 0.1166$ and $\delta_K = 0.2711$. (dash and dot lines are from experiment)

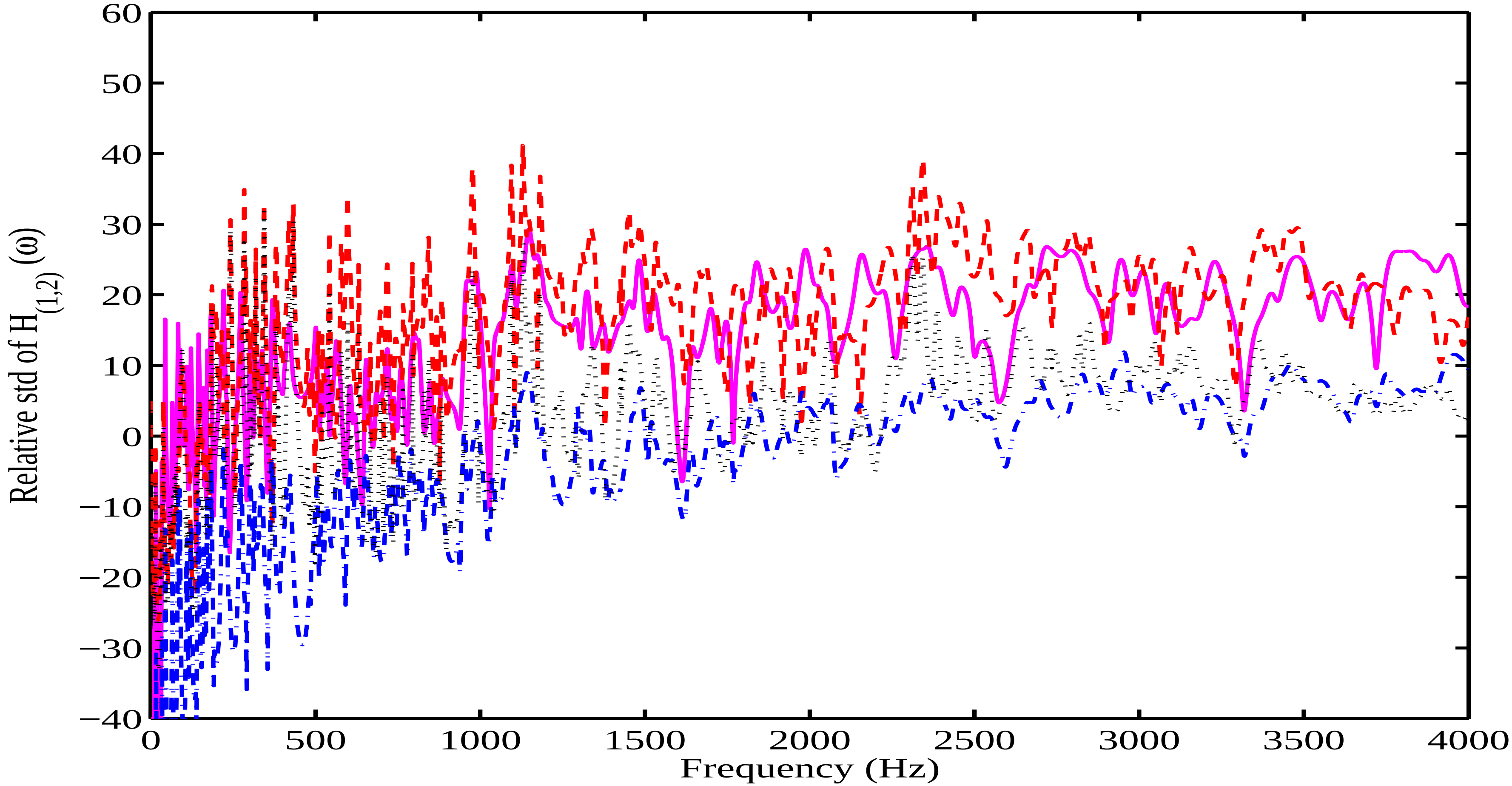
Comparison of driving-point-FRF: Low Freq



Comparison of the mean and standard deviation of the amplitude of the driving-point-FRF, $n = 1200$, $\delta_M = 0.1166$ and $\delta_K = 0.2711$. (dash and dot lines are from experiment)

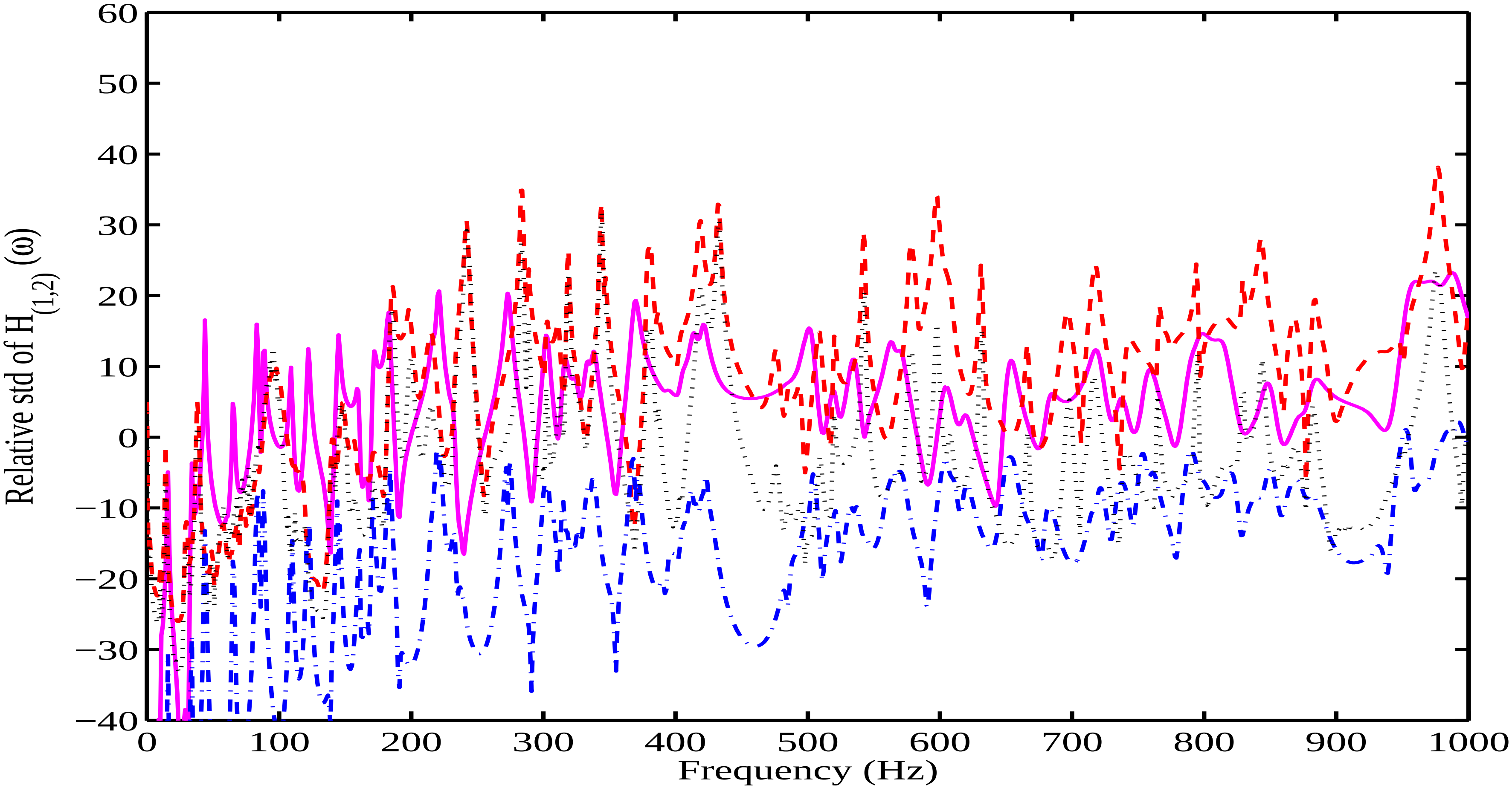
Swansea University
Prifysgol Abertawe

Comparison of cross-FRF



Comparison of the mean and standard deviation of the amplitude of the cross-FRF, $n = 1200$, $\delta_M = 0.1166$ and $\delta_K = 0.2711$. (dash and dot lines are from experiment)

Comparison of cross-FRF: Low Freq



Comparison of the mean and standard deviation of the amplitude of the cross-FRF, $n = 1200$, $\delta_M = 0.1166$ and $\delta_K = 0.2711$. (dash and dot lines are from experiment)

Swansea University
Prifysgol Abertawe

Future works on random matrix theory

- Random matrix inversion based computational method:
  - utilize analytical inverted matrix variate probability density functions for response moment calculation
  - explore different random matrix parameter fitting options
- Random eigenvalue based computational method:
  - utilize eigensolution density function of Wishart matrices in response statistics calculation
  - simple analytical expressions via asymptotic approach applicable for large systems

Conclusions

- Uncertainties need to be taken into account for credible predictions using computational methods.
- This talk concentrated on Uncertainty Propagation (UP) in linear structural dynamic problems.
- A general UP approach based on Wishart random matrix is discussed and the results are compared with experimental results.
- Based on numerical and experimental studies, a suitable simple Wishart random matrix model has been identified.

Future directions

- Efficient computational methods based on analytical approaches involving random eigenvalue problems
- Model calibration/updating: from experimental measurements (with uncertainties) how to identify/update the model (ie, the system matrices) and its associated uncertainty.
- High performance computing software for uncertain systems: how the UP approaches can be integrated with high performance computing and general purpose commercial software? This is becoming a very important issue due the availability of relatively inexpensive ‘clusters’.