Experimental Case Studies on Uncertainty Quantification in Structural Dynamics

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Outline of the presentation

- Introduction
- Probabilistic structural dynamics
- Experimental case study 1: Fixed beam with randomly placed masses
- Experimental case study 2: Cantilever plate with randomly placed oscillators
- Conclusions & discussions

Overview of Predictive Methods in Engineering

There are five key steps:

- Physics (mechanics) model building
- Uncertainty Quantification (UQ)
- Uncertainty Propagation (UP)
- Model Verification & Validation (V & V)
- Prediction

Tools are available for each of these steps. Focus of this talk is mainly on UQ in linear dynamical systems.

Why uncertainty?

Different sources of uncertainties in the modeling and simulation of dynamic systems may be attributed, but not limited, to the following factors:

- Mathematical models: equations (linear, non-linear), geometry, damping model (viscous, non-viscous, fractional derivative), boundary conditions/initial conditions, input forces;
- Model parameters: Young's modulus, mass density, Poisson's ratio, damping model parameters (damping coefficient, relaxation modulus, fractional derivative order)

Why uncertainty?

- Numerical algorithms: weak formulations, discretisation of displacement fields (in finite element method), discretisation of stochastic fields (in stochastic finite element method), approximate solution algorithms, truncation and roundoff errors, tolerances in the optimization and iterative methods, artificial intelligent (AI) method (choice of neural networks)
- Measurements: noise, resolution (number of sensors and actuators), experimental hardware, excitation method (nature of shakers and hammers), excitation and measurement point, data processing (amplification, number of data points, FFT), calibration

Structural dynamics

- The equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{p}(t)$$

- Due to the presence of uncertainty $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ become random matrices.
- The main objectives in the ‘forward problem’ are:
  - to quantify uncertainties in the system matrices
  - to predict the variability in the response vector $\mathbf{x}$

Current Methods

Two different approaches are currently available

- Low frequency : Stochastic Finite Element Method (SFEM) - assumes that stochastic fields describing parametric uncertainties are known in details
- High frequency : Statistical Energy Analysis (SEA) - do not consider parametric uncertainties in details

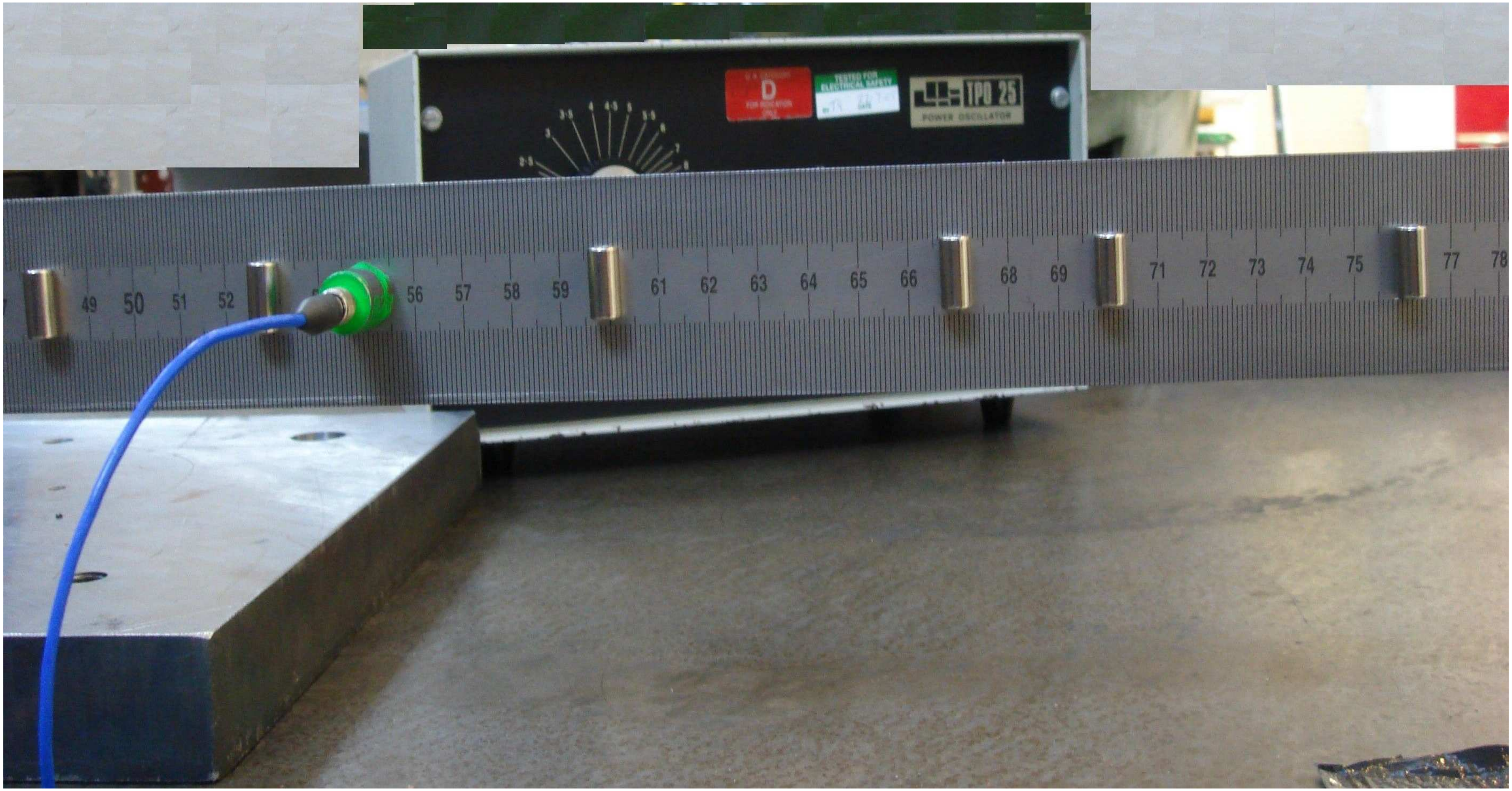
Experimental Study: Fixed beam

A fixed-fixed beam

Beam properties

Beam Properties	Numerical values
Length (L)	1200 mm
Width (b)	40.06 mm
Thickness (t_h)	2.05 mm
Mass density (ρ)	7800 Kg/m^3
Young's modulus (E)	2.0×10^5 MPa
Mass per unit length (ρ_l)	0.641 Kg/m
Total weight	0.7687 Kg

Randomly placed masses



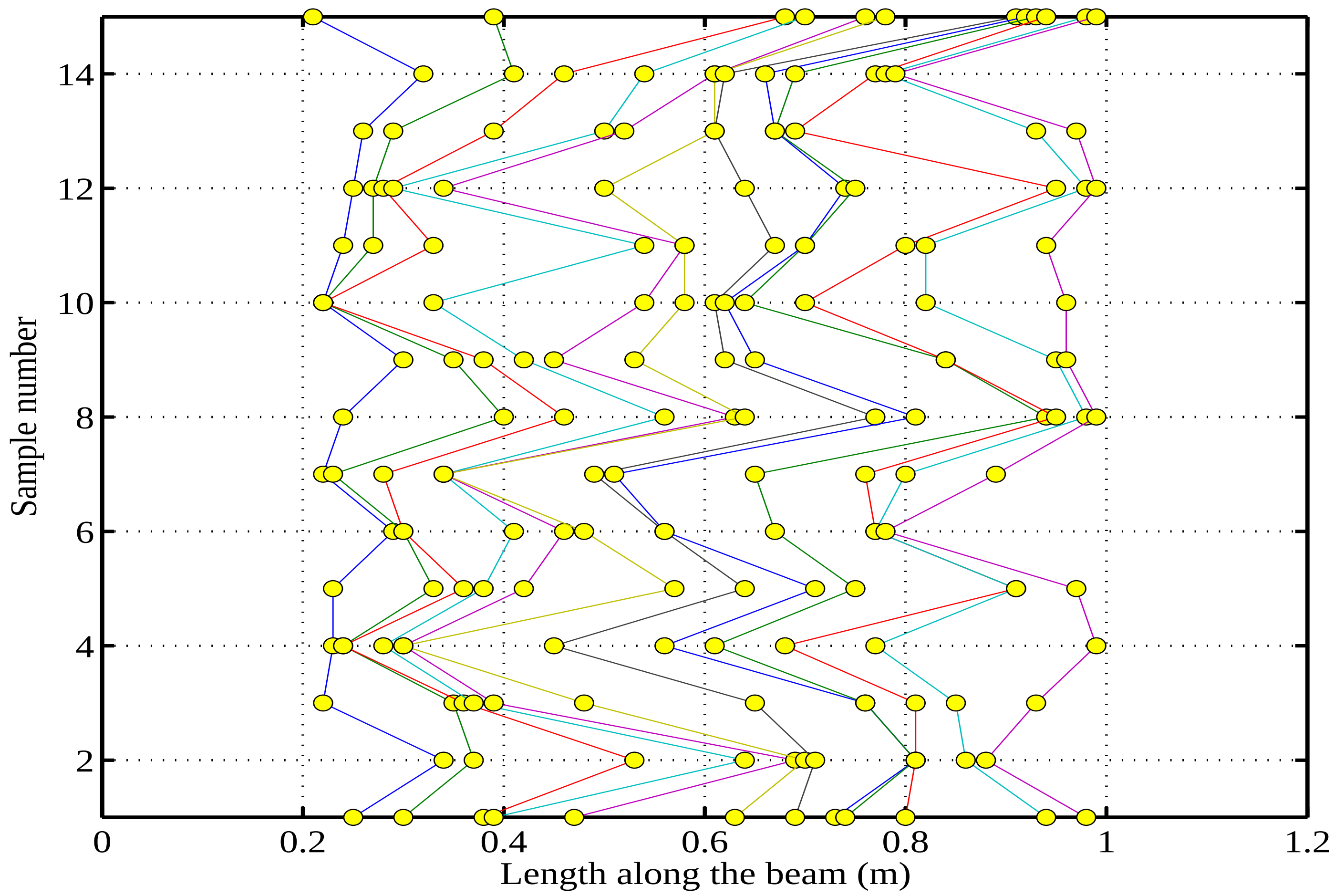
12 randomly placed masses (magnets), each weighting 2 g (total variation: 3.2%): mass locations are generated using uniform distribution

Randomly placed masses

Mean (m)	Standard deviation (m)
0.2709	0.0571
0.3390	0.0906
0.3972	0.1043
0.4590	0.1034
0.5215	0.1073
0.5769	0.1030
0.6398	0.1029
0.6979	0.1021
0.7544	0.0917
0.8140	0.0837
0.8757	0.0699
0.9387	0.0530

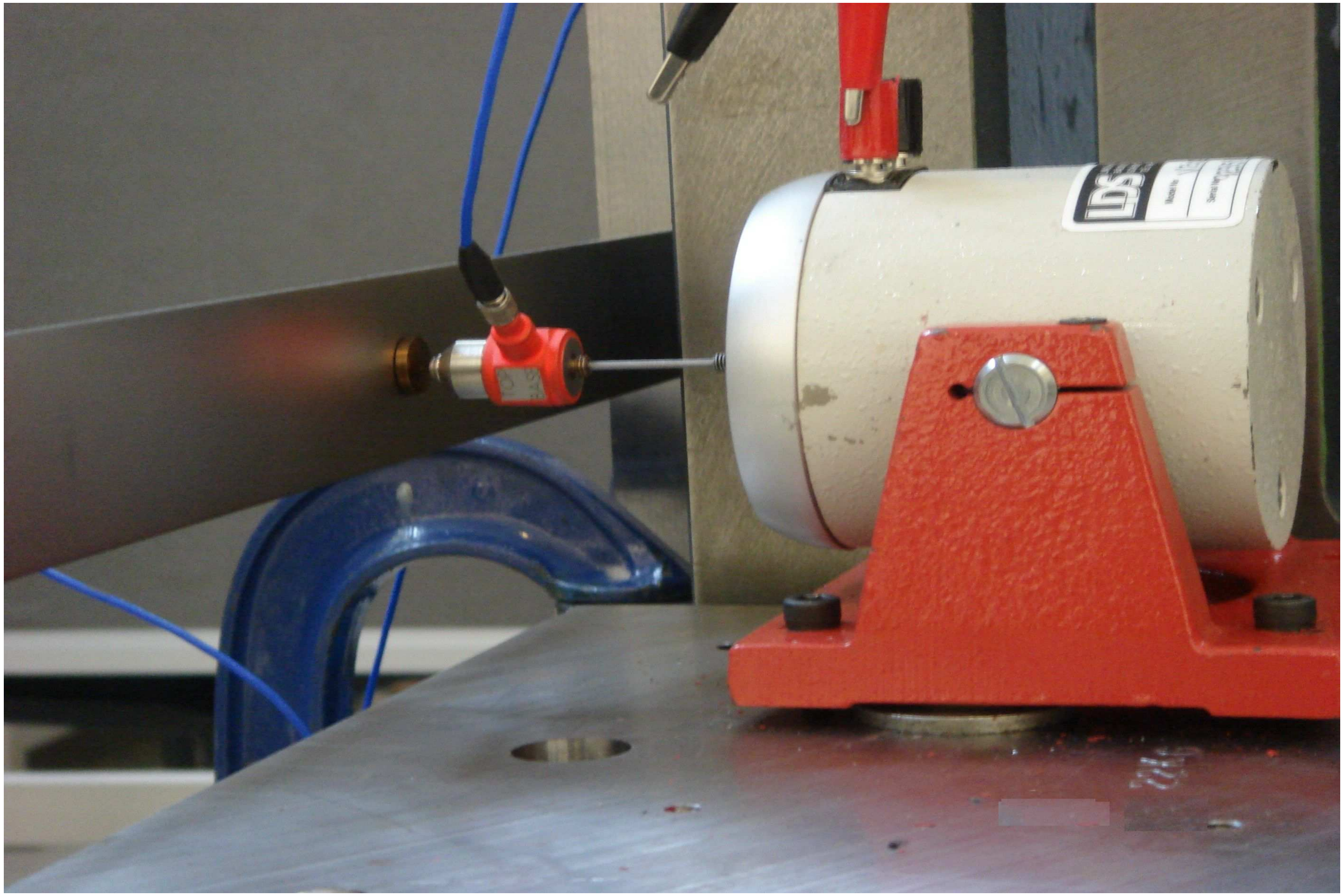
Gaussian distribution of mass locations along the beam

Randomly placed masses



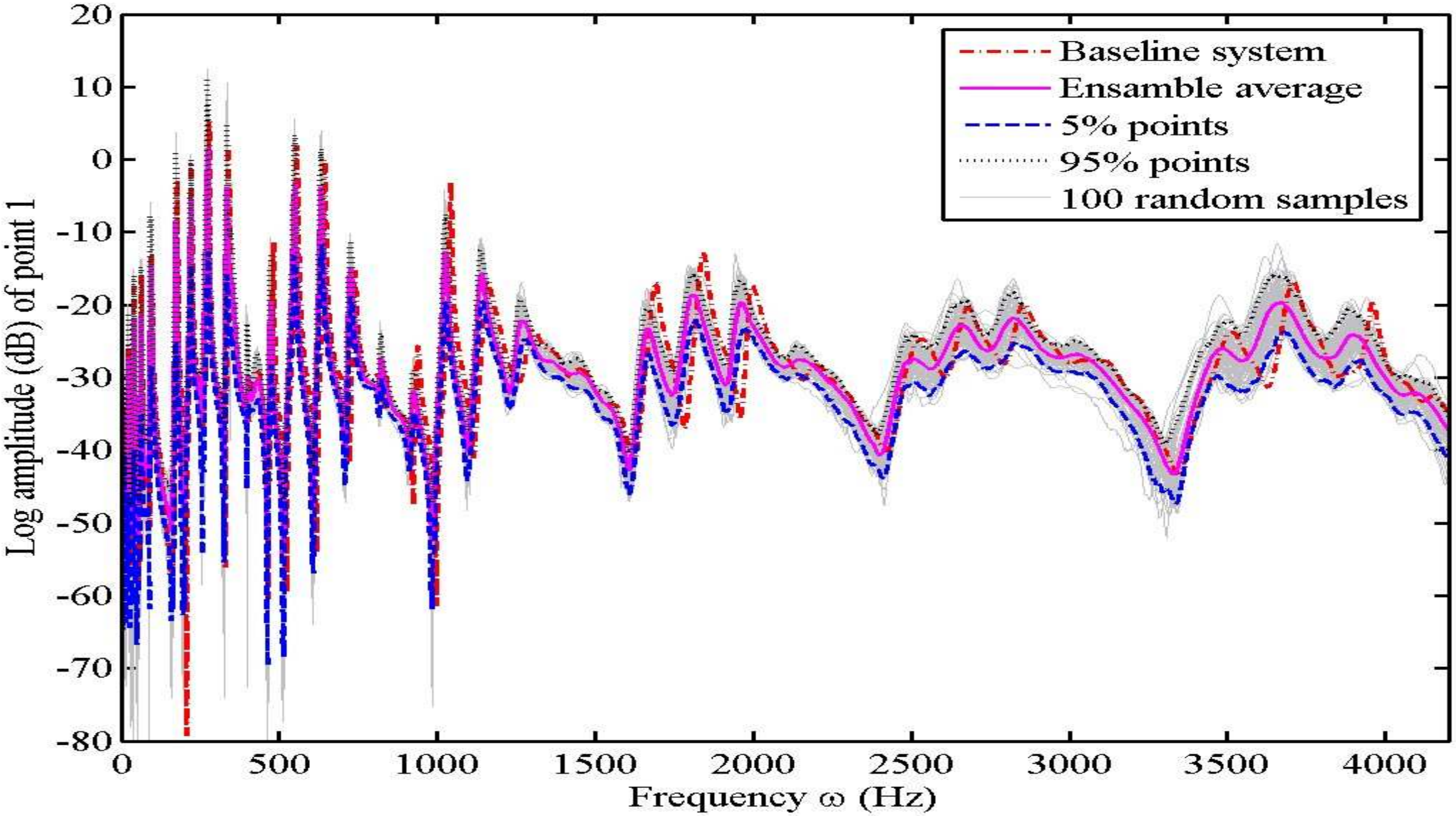
First 15 samples of the locations of 12 masses along the length of the beam.

Impulse excitation using a shaker



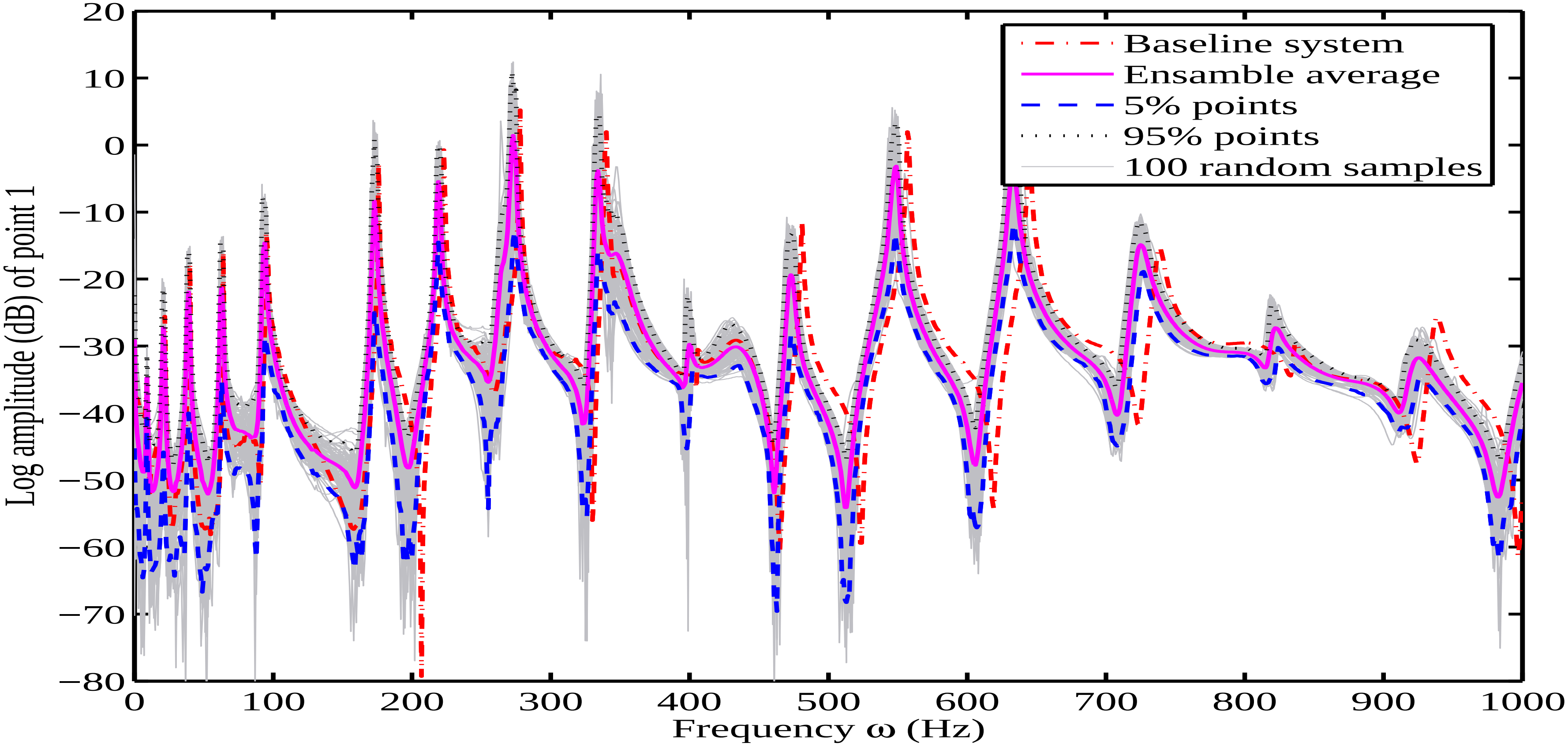
The shaker used as an impulse hammer using Simulink™. A hard steel tip used.

FRF Variability: complete spectrum



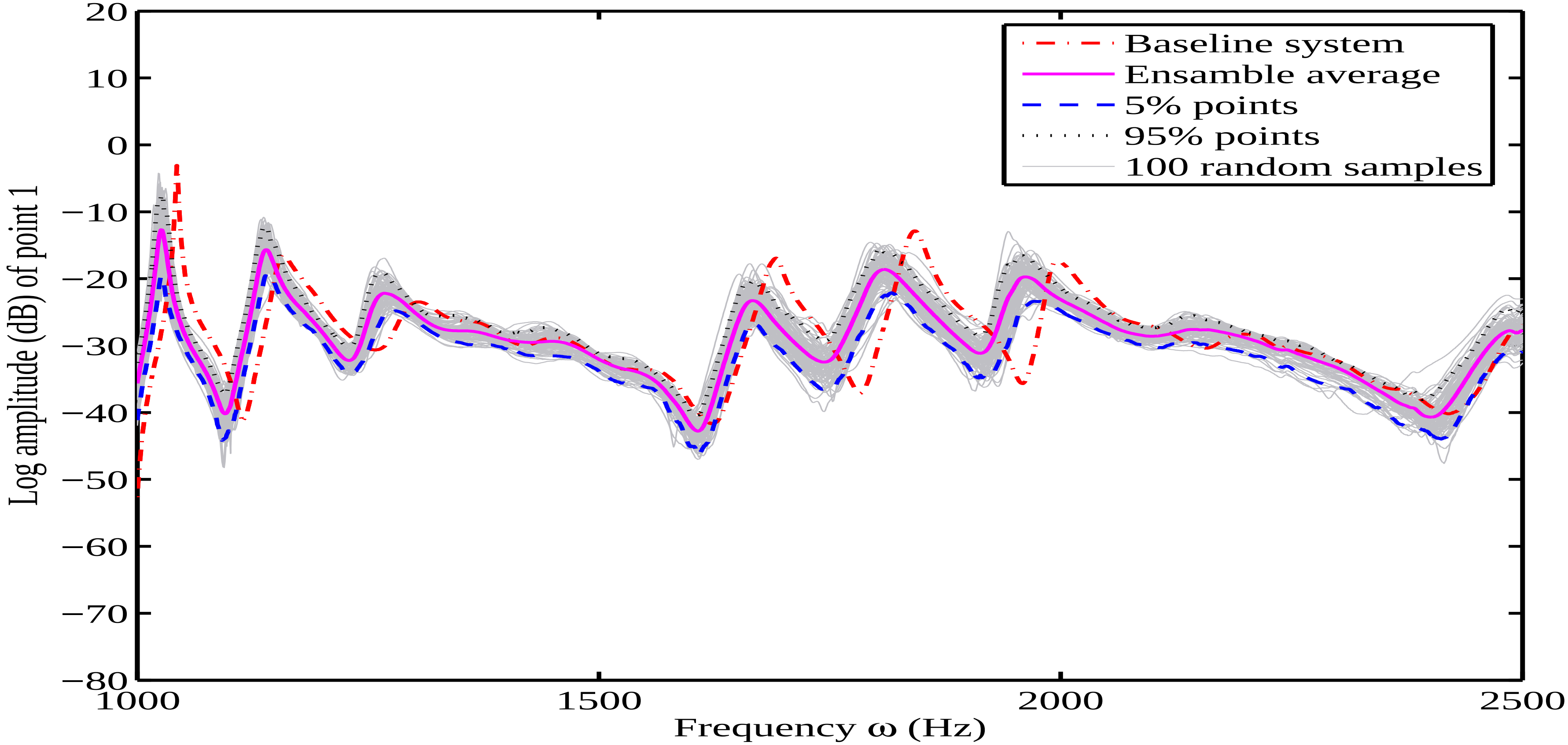
Variability in the amplitude of the driving-point-FRF of the beam.

FRF Variability: Low Freq



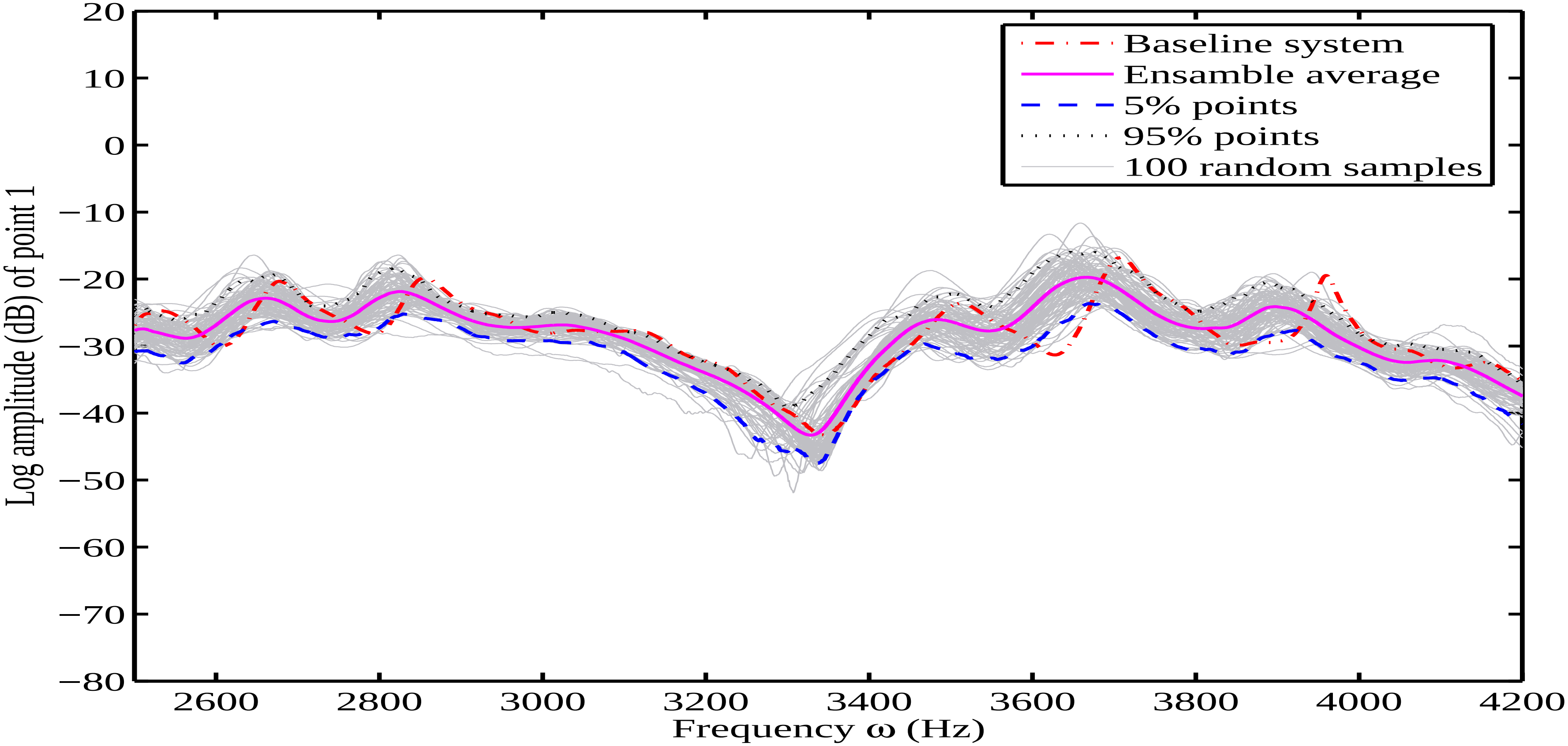
Variability in the amplitude of the driving-point-FRF of the beam.

FRF Variability: Mid Freq



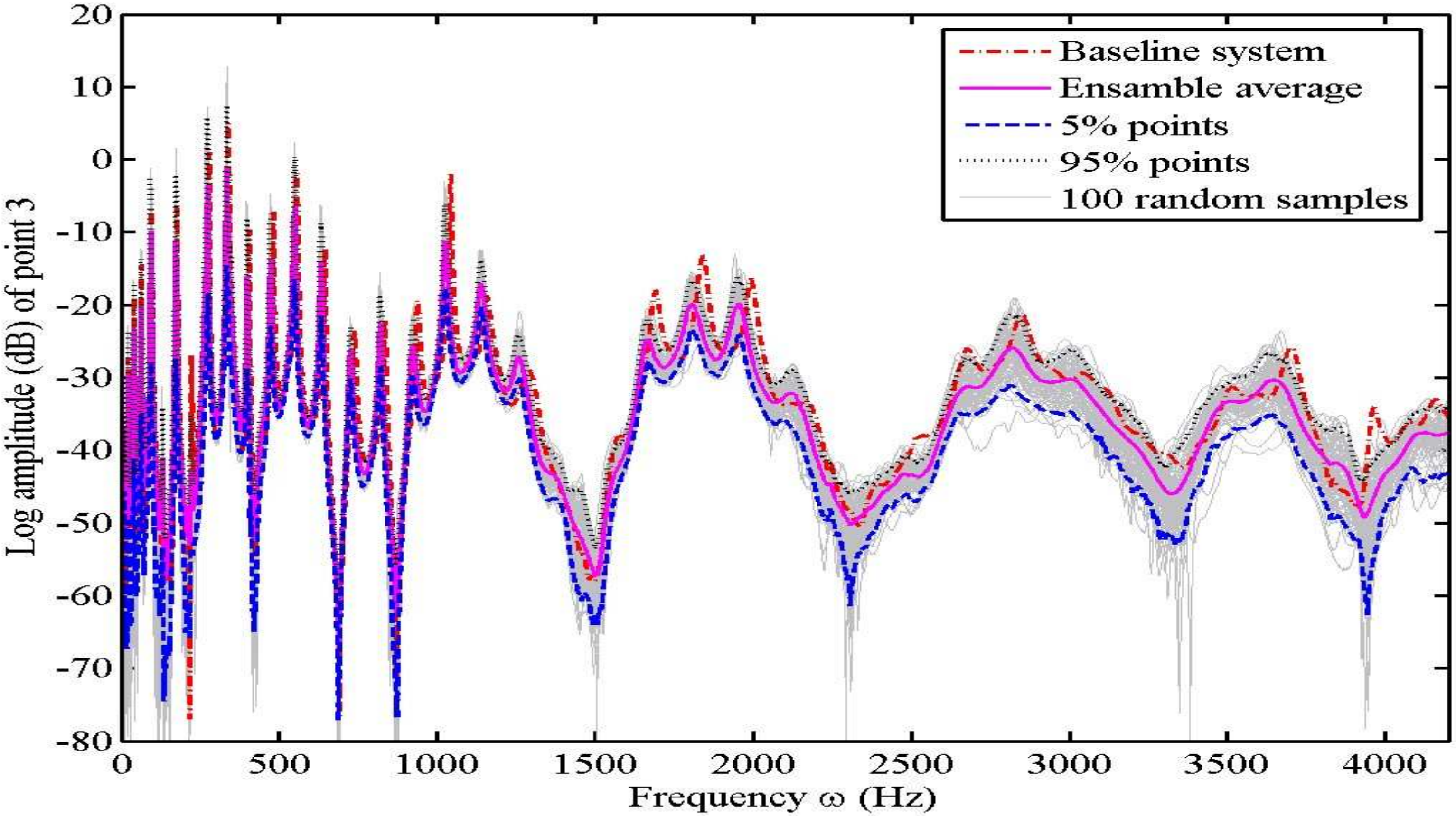
Variability in the amplitude of the driving-point-FRF of the beam.

FRF Variability: High Freq



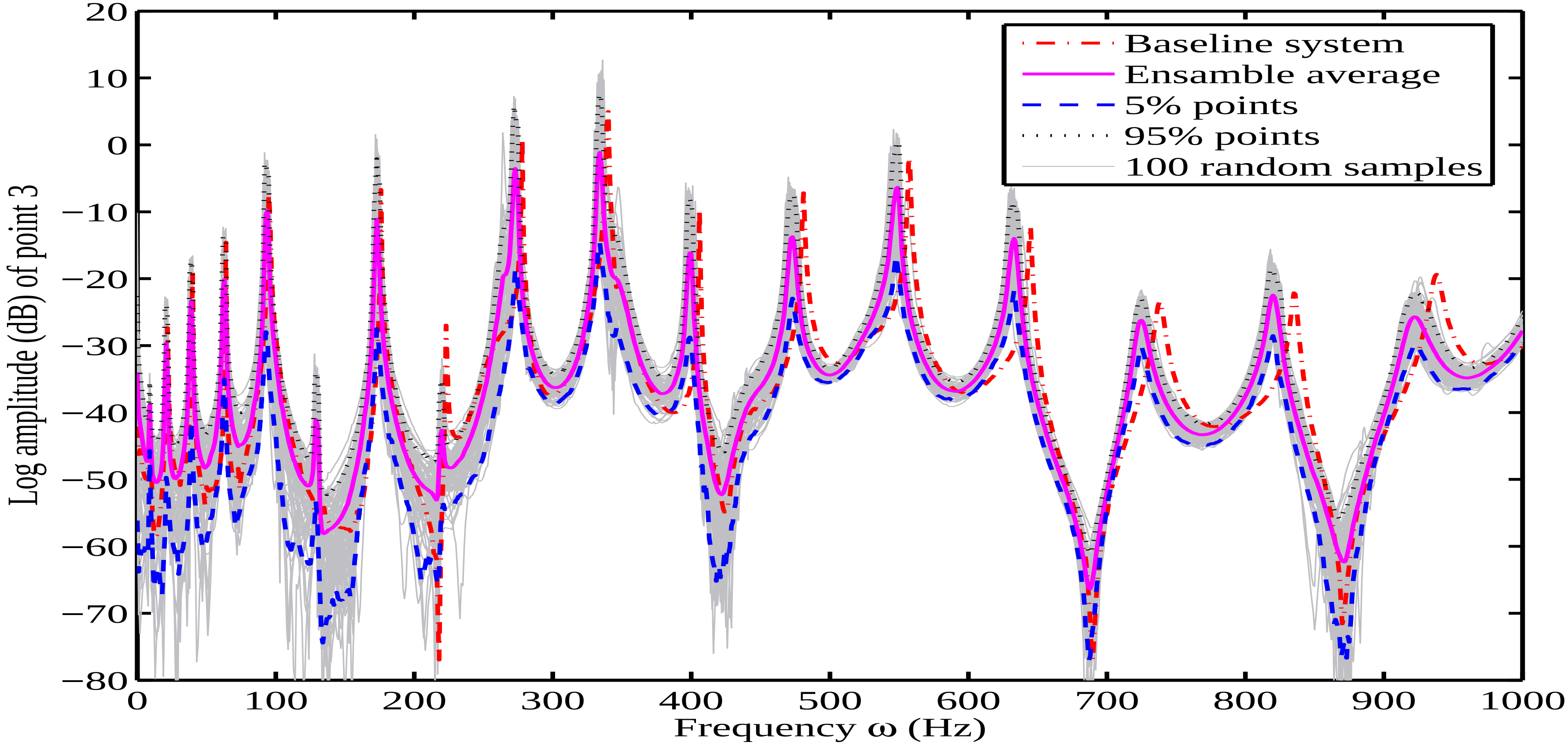
Variability in the amplitude of the driving-point-FRF of the beam.

FRF Variability: complete spectrum



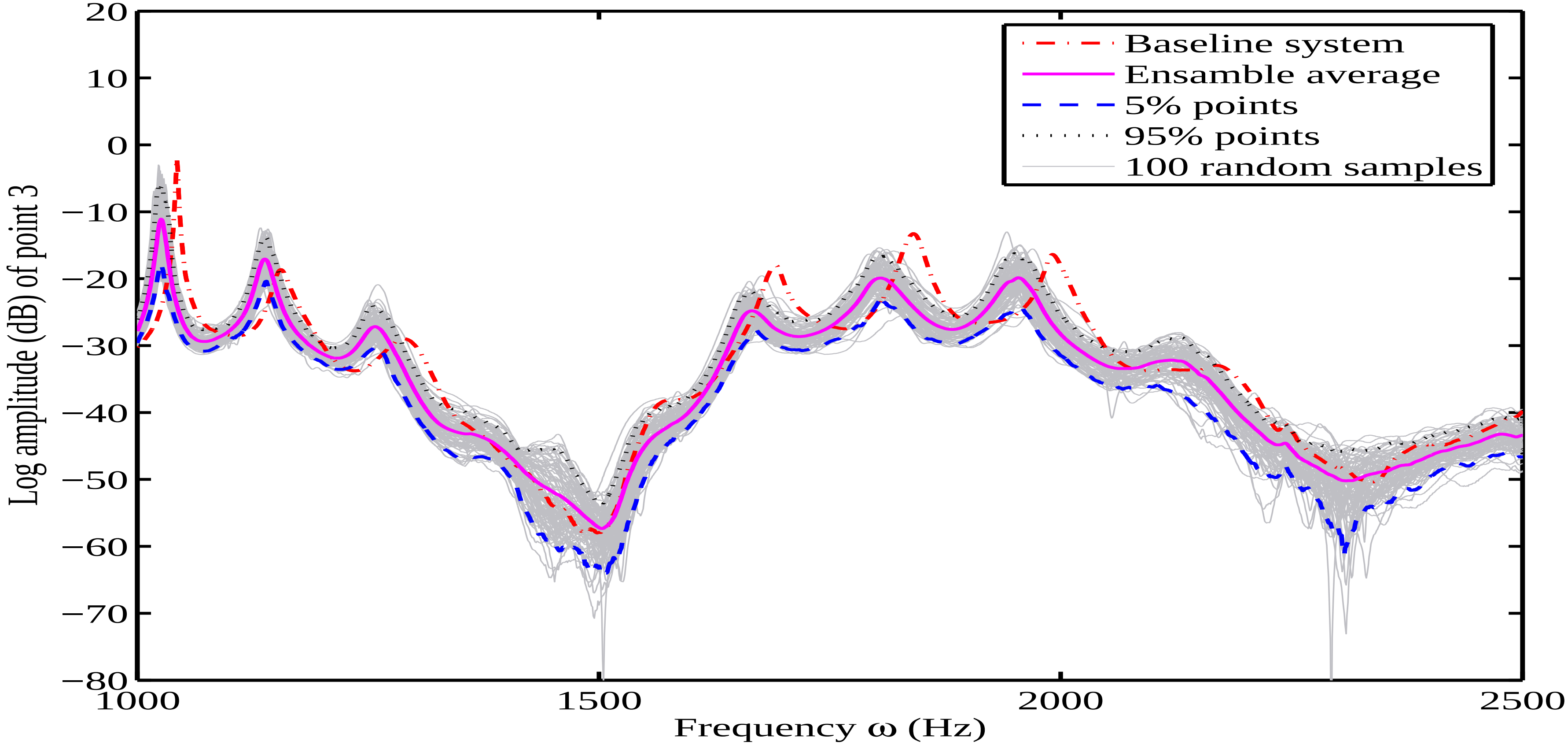
Variability in the amplitude of a cross-FRF of the beam.

FRF Variability: Low Freq



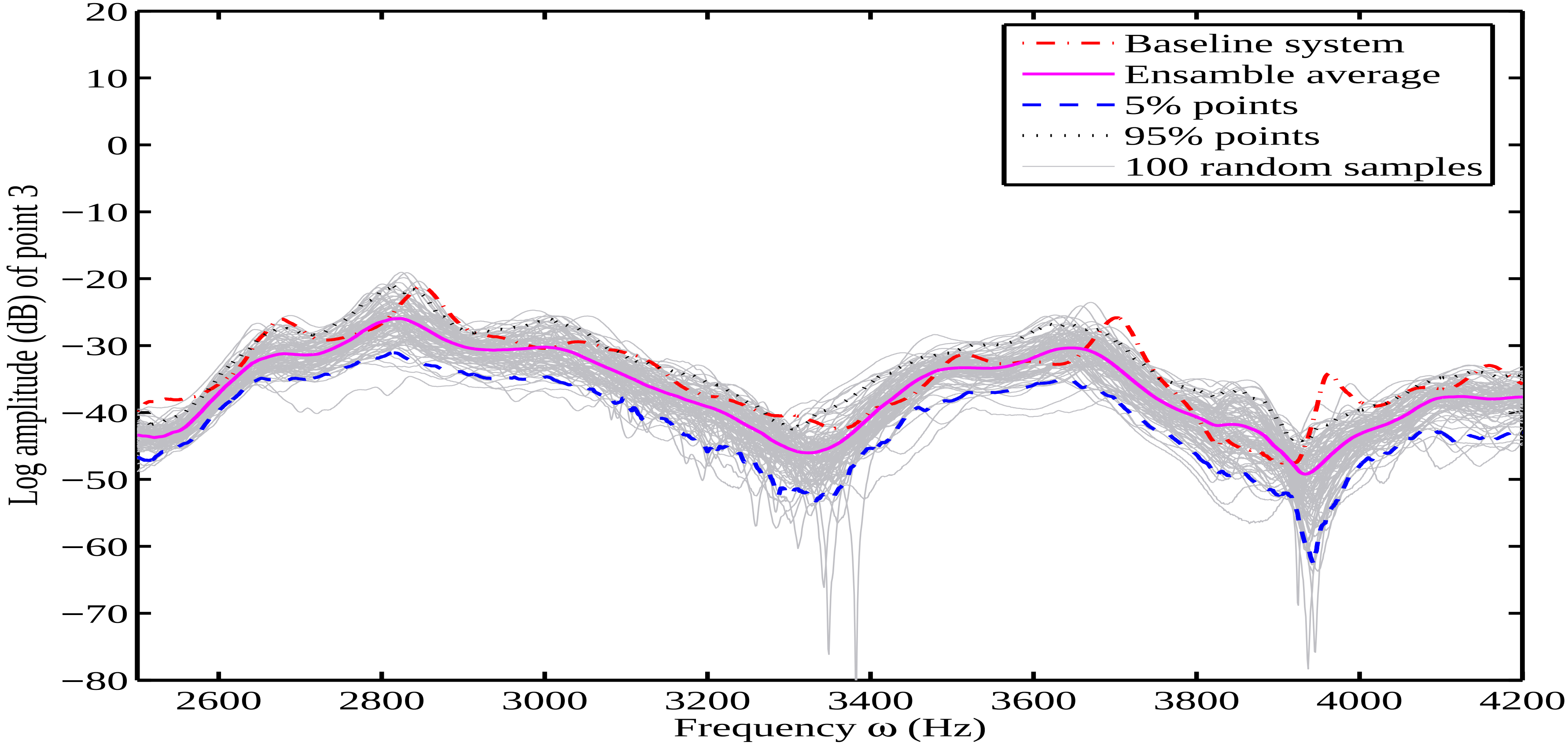
Variability in the amplitude of a cross-FRF of the beam.

FRF Variability: Mid Freq



Variability in the amplitude of a cross-FRF of the beam.

FRF Variability: High Freq



Variability in the amplitude of a cross-FRF of the beam.

Experimental Study: cantilever plate

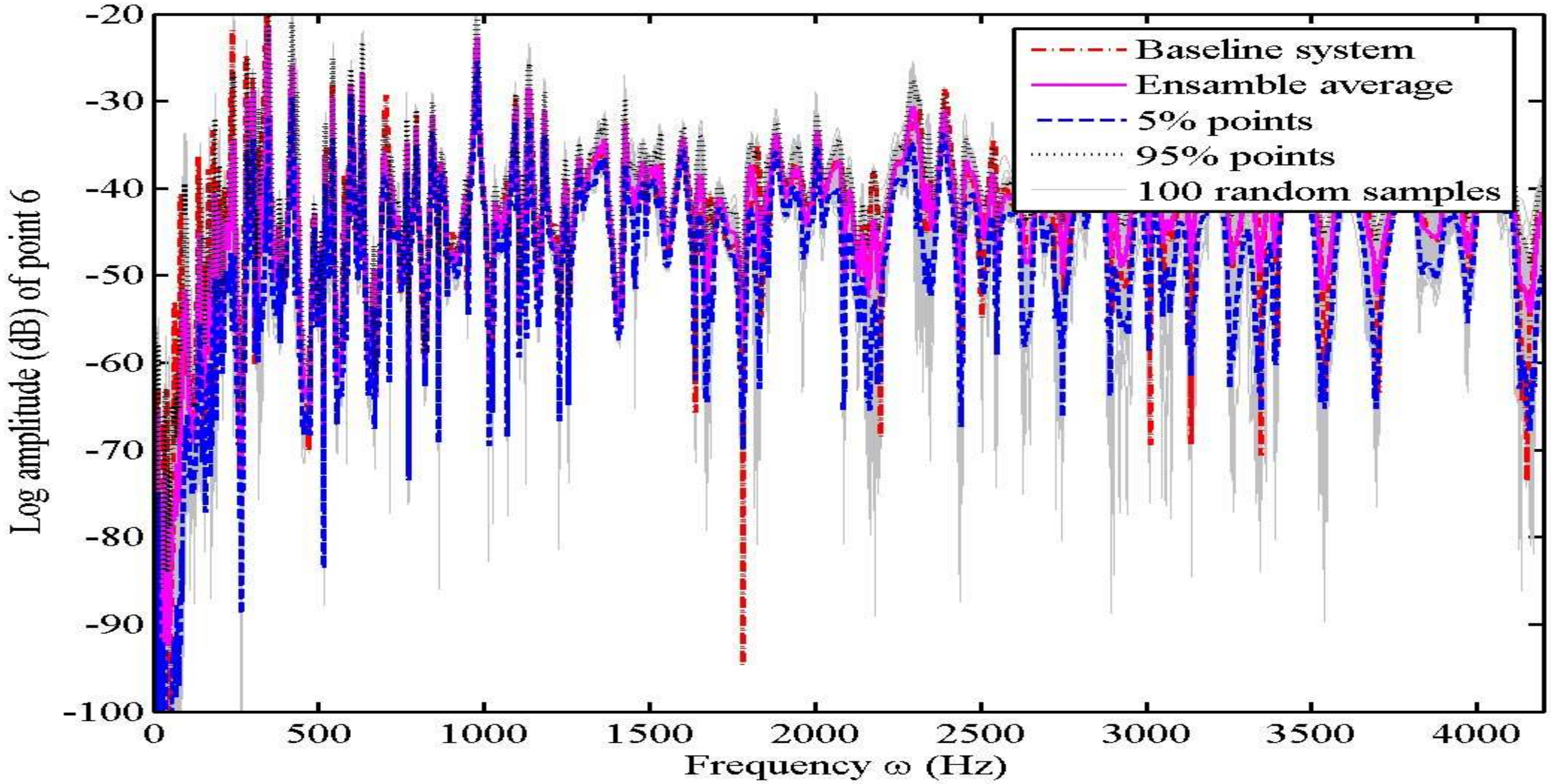
A cantilever plate: Length: 998 mm, Width: 530 mm, Thickness: 3 mm,

Density: 7860 kg/m3, Young's Modulus: 200 GPa

Unmodelled dynamics

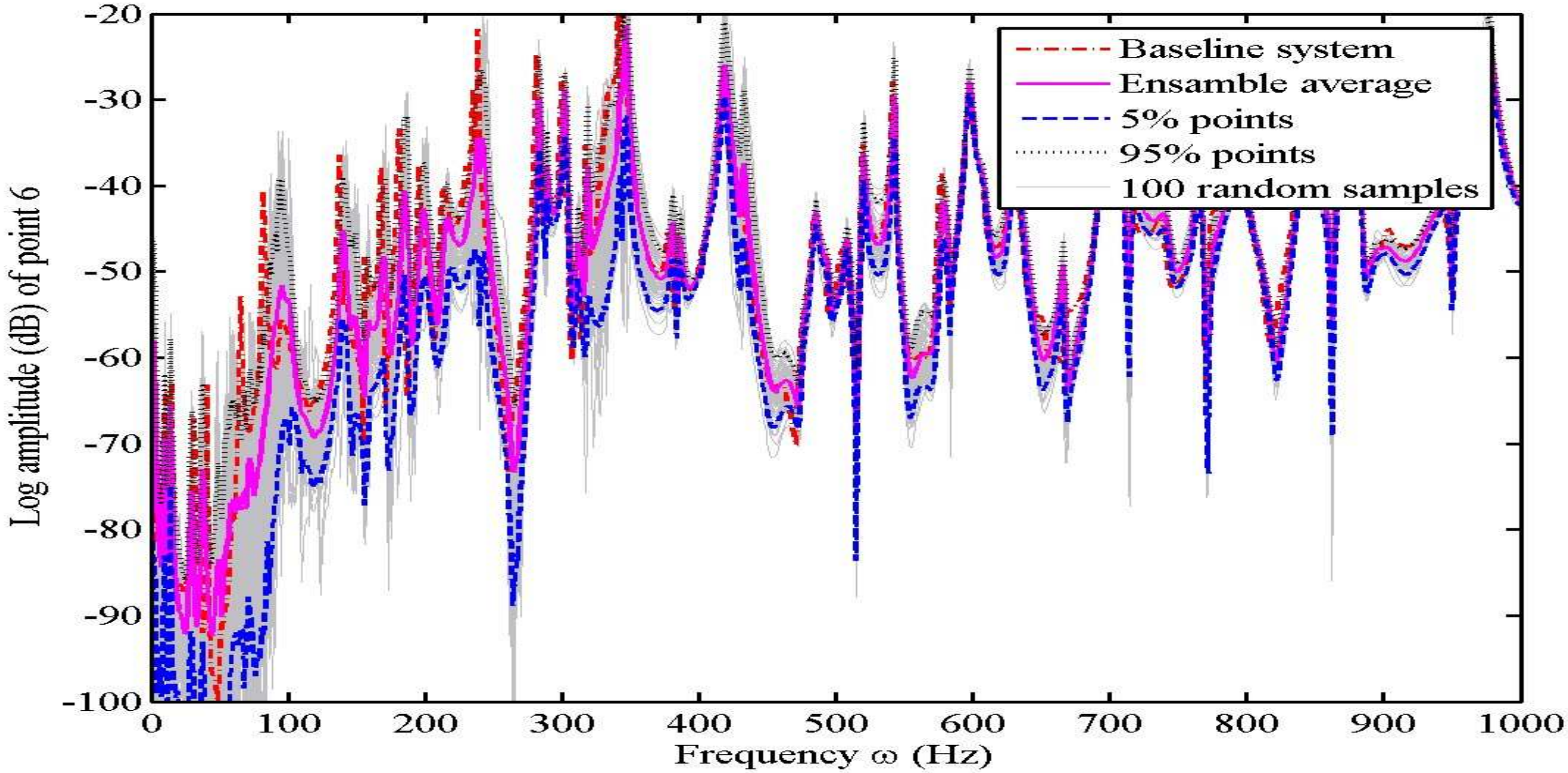
10 randomly placed oscillator; oscillatory mass: 121.4 g, fixed mass: 2 g, spring stiffness vary from 10 - 12 KN/m

FRF Variability: complete spectrum



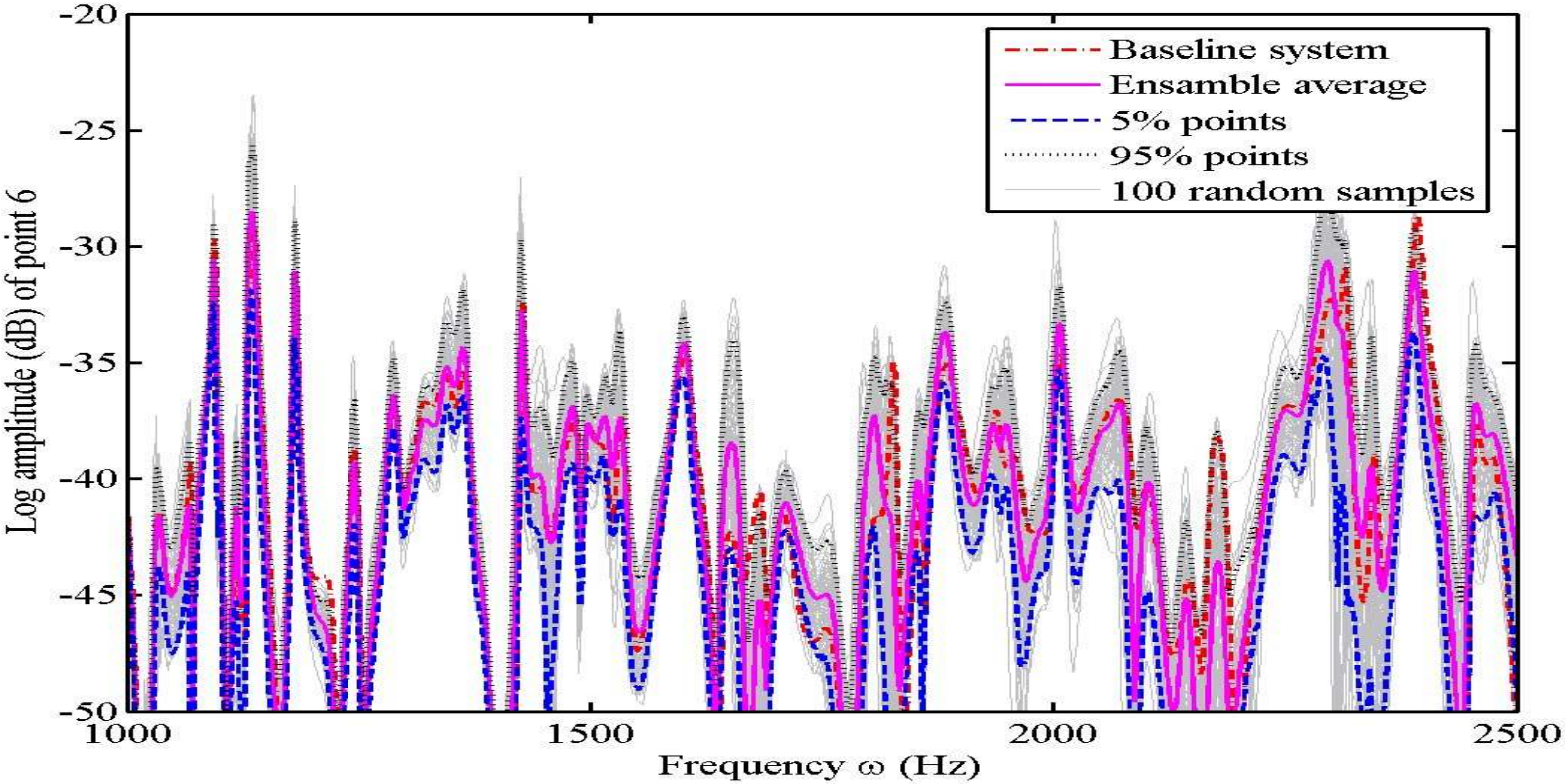
Variability in the amplitude of a cross-FRF of the plate.

FRF Variability: Low Freq



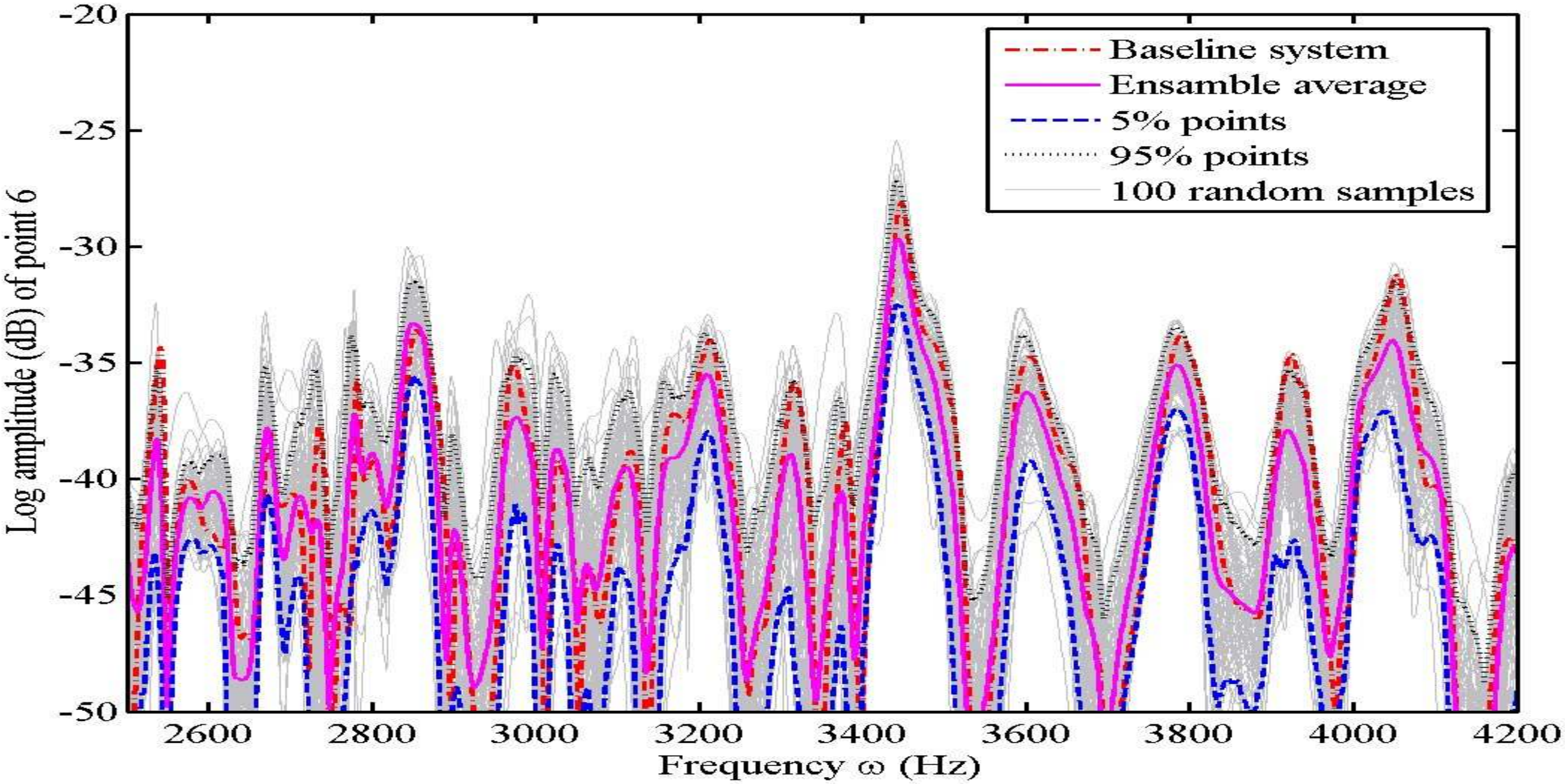
Variability in the amplitude of a cross-FRF of the plate.

FRF Variability: Mid Freq



Variability in the amplitude of a cross-FRF of the plate.

FRF Variability: High Freq



Variability in the amplitude of a cross-FRF of the plate.

Conclusions

- Experimental results involving stochastic dynamical systems is required for the uncertainty quantification and validation of numerical models of complex systems.
- Two experimental studies are described which may be used for this purpose.
- The fixed-fixed beam is easy to model and the results of a 100 sample experiment with randomly placed masses were described in this paper.
- The cantilever plate is ‘perturbed’ by 10 randomly placed oscillators. Again, a 100-sample test is conducted and the results are described.

Conclusions

- Special care has been taken so that the uncertainty in the response only arises from the randomness in the mass locations.
- Statistics of the frequency response function measured at three points of the beam were obtained for low, medium and high frequency ranges.
- It is expected that this data can be used for model validation and uncertainty quantification of dynamical systems. Data presented here will be available in the www.

Open Issues

- How one may validate/update a stochastic dynamical model when experimental data of stochastic nature (such as described here) is available?
- What can be done about the limited sample size (often only one!) in experimental results?