UNCERTAINTY QUANTIFICATION IN NATURAL FREQUENCY OF COMPOSITE PLATES - AN ARTIFICIAL NEURAL NETWORK BASED APPROACH

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ABSTRACT
This paper presents the stochastic natural frequency for laminated composite plates by using artificial neural network (ANN) model. The ANN model is employed as a surrogate and is trained by using Latin hypercube sampling. Subsequently the stochastic first two natural frequencies are quantified with ANN based uncertainty quantification algorithm. The convergence of the proposed algorithm for stochastic natural frequency analysis of composite plates is verified and validated with original finite element method (FEM) in conjunction with Monte Carlo simulation. Both individual and combined variation of stochastic input parameters are considered to address the influence on the output of interest. The sample size and computational cost are reduced by employing the present approach compared to traditional Monte Carlo simulation.

Keywords: Composite, artificial neural network, random natural frequency, uncertainty quantification

1. INTRODUCTION

Composite materials are extensively applied in aerospace, civil, naval and other high-performance engineering applications because of their significant application specific attributes such as light weight, high specific strength, high stiffness-to-weight ratios, excellent fatigue strength and most importantly the design flexibility. The capability to predict the structural response and enable a better understanding and characterization of the actual behaviour of laminated composite plates when subjected to combined load is of prime interest to structural analysis. Such types of structures have more uncertainties and variabilities in the structural properties than conventional structures due to involvement of many structural parameters and complex manufacturing processes. In order to have more exact and realistic analysis, they must be modelled as stochastic structures i.e., structures with uncertain system parameters. The complete control of the stochastic input variables associated with manufacturing and fabrication of the laminated composite plate is very difficult due to involvement of large number of inter-dependent parameters and hence variations in the lamina properties are inherent in nature. Besides these, the uncertainties in modelling and determination of foundation are also inherent in nature. In practice, the uncertainty in the system parameters finally results in dispersion in the stochastic response behaviour of the structures. Considerable researches based on deterministic analysis have been carried out on free vibration analysis of composite plates [1-3]. Reddy et al. have presented an ANN based approach for predicting natural frequencies of composite plates in a deterministic framework [4]. Some recent analyses of composite structures with stochastic characteristics are reported in the literature; see [5-9] for example. This is not the case, however, for the analysis of composite structures with a comprehensive implementation of uncertainties. Sources of uncertainties range from the statistical nature of the material properties of the constituents, to the inevitable fabrication randomness in layup and curing. To implement the effects of material and manufacturing uncertainties, a set of random variables representing laminate mechanical properties, density and orientation angles is chosen. The randomness in these variables can be quantified either experimentally or using simulation codes. To the best of the authors' knowledge, there is no literature covering the uncertainty quantification of natural frequencies in laminated composite structures using the artificial neural network model considering both individual and combined variation of random input parameters.

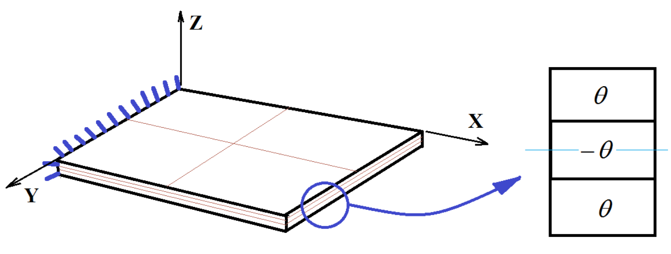


Fig. 1: Laminated composite cantilever plate

2. MATHEMATICAL FORMULATION

In present study, a laminated composite cantilever plates with uniform thickness '*t*' is considered as shown in Fig. 1. Based on the first-order shear deformation theory, the displacement can be expressed as:

$$\begin{aligned} u(x, y, z) &= u^0(x, y) - z\,\theta_x(x, y) \\ v(x, y, z) &= v^0(x, y) - z\,\theta_y(x, y) \\ w(x, y, z) &= w^0(x, y) = w(x, y) \end{aligned} \quad (1)$$

where, u^0, v^0, and w^0 are displacements of the reference plane and θ_x and θ_y are rotations of the cross section relative to x and y axes, respectively. Each of the thin fibre of laminae can be oriented at an arbitrary angle ‘θ’ with reference to the x-axis. The constitutive equations [10] are given by:

$$\{F\} = [D(\varpi)]\ \{\varepsilon\} \tag{2}$$

where

Force resultant $\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$

$$\{F\} = \left[\int_{-h/2}^{h/2} \{\sigma_x, \sigma_y, \tau_{xy}, \sigma_{xz}, \sigma_{yz}, \tau_{xyz}, \tau_{xz}, \tau_{yz}\}\, dz\right]^T$$

and strain $\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \varepsilon_{xy}, k_x, k_y, k_{xy}, \gamma_{xz}, \gamma_{yz}\}^T$

The elasticity matrix of the laminated composite plate is given by,

$$[D'(\varpi)] = \begin{bmatrix} A_{ij}(\varpi) & B_{ij}(\varpi) & 0 \\ B_{ij}(\varpi) & D_{ij}(\varpi) & 0 \\ 0 & 0 & S_{ij}(\varpi) \end{bmatrix} \tag{3}$$

Where:

$$[A_{ij}(\varpi),\ B_{ij}(\varpi),\ D_{ij}(\varpi)] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\{\bar{Q}_{ij}(\varpi)\}_{on}]_k\ [1,\ z,\ z^2]\, dz \qquad i,j = 1,2,6$$

$$[S_{ij}(\varpi)] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha_s\ [\bar{Q}_{ij}]_k\ dz \qquad i,j = 4,5$$

where ϖ indicates the stochastic representation and α_s is the shear correction factor (=5/6) and $[\bar{Q}_{ij}]$ are elements of the off-axis elastic constant matrix which is given by:

$$[\bar{Q}_{ij}]_{off} = [T_1(\varpi)]^{-1}\, [\bar{Q}_{ij}]_{on}\, [T_1(\varpi)]^{-T} \quad \text{for } i,j = 1,2,6$$
$$[\bar{Q}_{ij}]_{off} = [T_2(\varpi)]^{-1}\, [\bar{Q}_{ij}]_{on}\, [T_2(\varpi)]^{-T} \quad \text{for } i,j = 4,5 \tag{4}$$

$$\text{where } [T_1(\varpi)] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2-n^2 \end{bmatrix} \text{ and}$$
$$[T_2(\varpi)] = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \tag{5}$$

in which $m = \sin\theta(\varpi)$ and $n = \cos\theta(\varpi)$, wherein $\theta(\varpi)$ is random ply orientation angle.

$$[Q_{ij}(\varpi)]_{on} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{12} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \text{for } i,j = 1,2,6$$
$$[\bar{Q}_{ij}(\varpi)]_{on} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \text{for } i,j=4,5 \tag{6}$$

where:

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}},\quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}},\quad Q_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}},$$
$$Q_{66} = G_{12},\ Q_{44} = G_{23}\ and\ Q_{55} = G_{13}$$

An eight nodded isoparametric quadratic element with five degrees of freedom at each node (three translations and two rotations) is considered in finite element formulation. Using Hamilton’s principle [11] and Lagrange’s equation, the dynamic equilibrium equation for the equation of motion of free vibration system with n degrees of freedom can be expressed as:

$$[M(\varpi)][\ddot{\delta}] + [K(\varpi)]\{\delta\} = 0 \tag{7}$$

In the above equation, $M(\varpi) \in R^{n\times n}$ is the mass matrix, $[K(\varpi)]$ is the elastic stiffness matrix and $\{\delta\} \in R^n$ is the vector of generalized coordinates. The governing equations are derived based on Mindlin’s theory incorporating transverse shear deformation. For free vibration, the random natural frequencies $[\omega_n \varpi]$ are determined from the standard eigenvalue problem [12] using QR iteration algorithm. The composite plate is assumed to be lightly damped and the natural frequencies of the system are obtained as:

$$\omega_j^2(\varpi) = \frac{1}{\lambda_j(\varpi)} \quad \text{where } j = 1,2,3,\ldots\ldots, n_{mode} \tag{8}$$

Here $\lambda_j(\varpi)$ is the j-th eigenvalue of matrix $A = K^{-1}(\varpi)\, M(\varpi)$ and n_r indicates the number of modes retained in this analysis.

3. ARTIFICIAL NEURAL NETWORK (ANN) MODEL

The fundamental processing element of ANN is an artificial neuron (or simply a neuron). A biological neuron receives inputs from other sources, combines them, generally performs a non-linear operation on the result, and then outputs the final result [13]. In the present study, the stochastic natural frequencies can be determined due to variability of input parameters. The ability of the ANNs, to recognize and reproduce the cause-effect relationships through training for the multiple input-output systems makes them efficient to represent even the most complex systems [14]. The main advantages of ANN as compared to response surface method (RSM) include:

- ANN does not require any prior specification of suitable fitting function, and
- It also has a universal approximation capability to approximate almost all kinds of non-linear functions including quadratic functions, whereas RSM is generally useful for quadratic approximations [15].

A multi-layer perceptron (MLP) based feed-forward

ANN, which makes use of the back propagation learning algorithm, was applied for computational modelling. The network consists of an input layer, one hidden layer and an output layer. Each neuron acts firstly as a adding junction, summing together all incoming values. After that, it is filtered through an activation transfer function, the output of which is forwarded to the next layer of neurons in the network. The hyperbolic tangent was used as the transfer function for the input and hidden layer nodes. The reason behind employing the transfer function as logistic function or hyperbolic tangent (*tanh*) can be described as the logistic function generates the values nearer to zero if the argument of the function is substantially negative. Hence, the output of the hidden neuron can be made close to zero, and thus lowering the learning rate for all subsequent weights. Thus, it will almost stop learning. The tanh function, in the similar fashion, can generate a value close to -1.0, and thus will maintain learning. The algorithm used to train ANN in this study is quick propagation (QP). This algorithm is belonging to the gradient descent back-propagation. It has been reported in the literature that quick propagation learning algorithm can be adopted for the training of all the ANN models [16]. The performance of the ANNs are statistically measured by the root mean squared error (RMSE), the coefficient of determination (R2) and the absolute average deviation (AAD) obtained as follows:

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i - Y_{id})^2} \tag{9}$$

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(Y_i - Y_{id})^2}{\sum_{i=1}^{n}(Y_{id} - Y_m)^2} \tag{10}$$

$$AD = \left[\frac{1}{n}\sum_{i=1}^{n}\left|\frac{(Y_i - Y_{id})}{Y_{id}}\right|\right] \times 100 \tag{11}$$

where n is the number of points, Y_i is the predicted value, Y_{id} is the actual value, and Y_m is the average of the actual values.

4. STOCHASTIC APPROACH USING ANN MODEL

The layer-wise stochasticity in material and geometric properties are considered as input parameters for stochastic natural frequency analysis of composite plates. The individual and combined cases of layer-wise random variations considered in the present analysis are as follows:

(a) Variation of ply-orientation angle only:

$$\theta(\varpi) = \{\theta_1\ \theta_2\ \theta_3 ....\theta_i ......\theta_l\ \}$$

(b) Combined variation of ply orientation angle, elastic and shear modulus and mass density:

$$g\{\theta(\varpi), E_1(\varpi), E_2(\varpi), G_{12}(\varpi), G_{23}(\varpi), \mu_{12}(\varpi), \rho(\varpi)\}$$
$$= \{\ \Phi_1(\theta_1..\theta_l), \Phi_2(E_{1(1)}..E_{1(l)}),.....\Phi_3(E_{2(1)}..E_{2(l)}),$$
$$\Phi_4(G_{12(1)}..G_{12(l)}), \Phi_5(G_{23(1)}..G_{23(l)}), \Phi_6(\mu_{12(1)}..\mu_{12(l)}), \Phi_7(\rho_1..\rho_l)\}$$

where θ_i , $E_{1(i)}$, $E_{2(i)}$, G_{12i} , $G_{23(i)}$, $\mu_{12(i)}$ and ρi are the ply orientation angle, elastic modulus (longitudinal direction), elastic modulus (transverse direction), shear modulus (longitudinal direction), shear modulus (transverse direction), Poisson ratio and mass density, respectively and 'l' denotes the number of layer in the laminate. In past, stochastic natural frequencies are found to be portrayed by using different other surrogate models [17-22] but ANN model is not yet encountered for laminated composite plates. In the present study, it is assumed that the distribution for randomness of input parameters exists within a certain band of tolerance with their deterministic values. 5^o for ply orientation angle and 10% tolerance for material properties and thickness from deterministic values are considered. The flowchart of the proposed stochastic natural frequency analysis using ANN model is shown in Fig. 2. Latin hypercube sampling is employed for generating sample points to ensure the representation of all portions of the vector space.

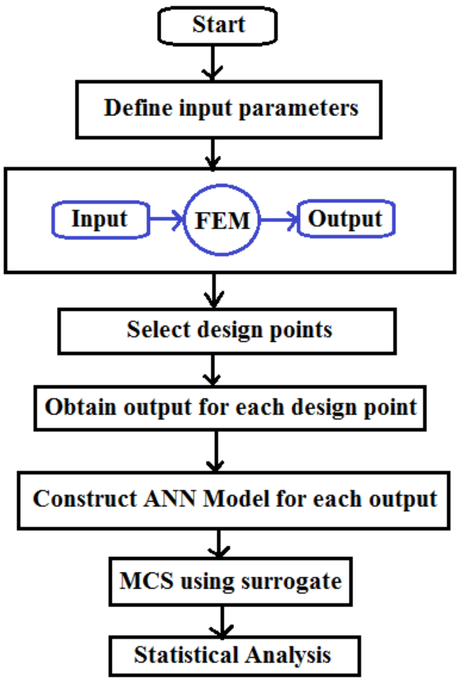


Fig. 2: Flowchart of stochastic natural frequency analysis using ANN model

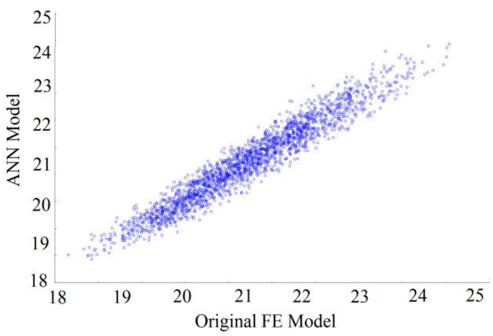
5. RESULTS AND DISCUSSION

In this study, three layered graphite-epoxy symmetric angle-ply laminated composite cantilever shallow plates are considered. The length, width and thickness of the composite laminate considered in the present analysis are 1 m, 1 m and 5 mm, respectively. Material properties of graphite–epoxy composite [23] considered with deterministic mean value as E_1 = 138.0 GPa, E_2 = 8.96 GPa, G_{12} = 7.1 GPa, G_{13} = 7.1 GPa, G23 = 2.84 GPa, μ = 0.3, ρ=1600 kg/

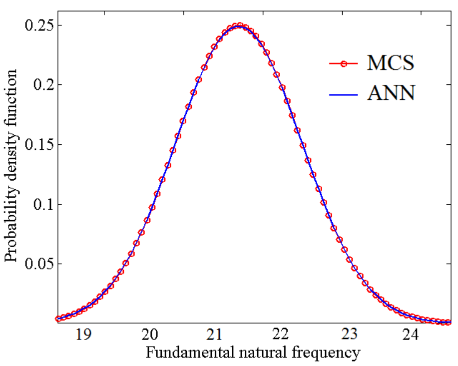
m3. A discretization of (6 × 6) mesh on plan area with 36 elements 133 nodes with natural coordinates of an isoparametric quadratic plate bending element are considered for the present FEM approach. The finite element mesh size is finalized using a convegence study as shown in Table 1. For full scale MCS, the number of original finite element analysis is same as the sampling size. The random response in terms of natural frequencies of the composite structure can only be evaluated numerically at the end of a structural analysis procedure such as the finite element method which is often time-consuming and computationally expensive. The present ANN method is employed to develop a predictive and representative surrogate model relating each natural frequency to a number of input variables. Thus the ANN model represents the result of structural analysis encompassing every possible combination of all stochastic input variables.

Table 1: Convergence study for non-dimensional fundamental natural frequencies [$\omega=\omega_n L^2 \sqrt{(\rho/E_1 t_2)}$] of three layered (θ°/-θ°/θ°) graphite-epoxy untwisted composite plates, a/b=1, b/t=100, considering E_1 = 138 GPa, E_2 = 8.96 GPa, G12 = 7.1 GPa, ν_{12} = 0.3.

Ply angle, θ	Present FEM (4 × 4)	Present FEM (6 × 6)	Present FEM (8 × 8)	Present FEM (10 × 10)	Qatu and Leissa [24]
0°	1.0112	1.0133	1.0107	1.004	1.0175
90°	0.2553	0.2567	0.2547	0.2542	0.2590



(a)



(b)

Fig. 3: (a) Scatter plot and (b) Probability density function plot for stochastic fundamental natural frequency (rad/s) using ANN approach for combined variation of ply angle, elastic modulus, shear modulus, Poisson ratio and mass density of angle-ply (45°/-45°/45°) composite cantilever plate

Table 2: Convergence study of natural frequencies corresponding to first two modes due to combined effect of stochasticity

Type	Mode	Method	Sample size	Maximum	Minimum	Mean	Standard deviation
Combined Variation	First	MCS	10,000	24.9356	18.4329	21.3748	1.0304
		Present method	128	23.8486	19.3301	22.5235	1.1823
			256	24.0731	19.1009	22.0653	0.9761
			512	24.7771	19.0203	21.8591	0.9964
			1024	24.9776	18.7541	21.3849	1.0151
	Second	MCS	10,000	70.3451	58.3114	63.6933	1.5675
		Present method	128	67.4915	60.3312	64.7537	1.7419
			256	68.9886	59.2077	64.4981	2.9044
			512	69.4864	59.0384	64.0893	2.1205
			1024	70.0649	58.7656	63.5095	2.0900

A convergence study of sample size for ANN model formation with respect to direct MCS is shown in Table 2 for the first two stochastic natural frequencies due to individual (ply-orientation angle) and combined variation (ply-orientation angle, elastic moduli, shear moduli, poission ratio and mass density). By analysing the statistical parameters presented in the Table 2 it is evident that sample size of 512 and 1024 are adequate for the ANN model formation corresponding to individual and combined cases, respectively. The probability density function (PDF) is plotted as the benchmark of bottom line results in this article. To illustrate the validation of results of the proposed ANN based approach with respect to direct MCS, natural frequency corresponding to first mode is considered for combined variation case. The scatter plot validating the present ANN model with respect to finite element model and the probability density function plot comparing the result of ANN approach and direct MCS approach for first mode of vibration is presented in Fig. 3. It is evident that the results of the proposed ANN based approach are in good agreement with that of direct MCS simulations corroborating accuracy of the proposed approach. Further to check the accuracy of the fitted ANN model, different criteria such as root mean squared error (RMSE), coefficient of determination (R^2) and the absolute average deviation (AAD) are checked. The RMSE and AAD values are found to be less than 0.005, while R^2 values are close to 1. Such values of checking criteria indicate high level of precision of the constructed ANN models.

Fig. 4(a) and 4(b) show probability distribution of the first two natural frequencies with different degree of stochasticity (δ), wherein it is evident that the response bounds of the frequencies increse with higher degree of stochasticity in the input parameters. Change in coefficient of variation (ratio of standard deviation and mean for a distribution) with degree of stochasticity is plotted in Fig. 5, which shows that fundamental natural frequency is more sensitive than the second natural frequency to increasing degree

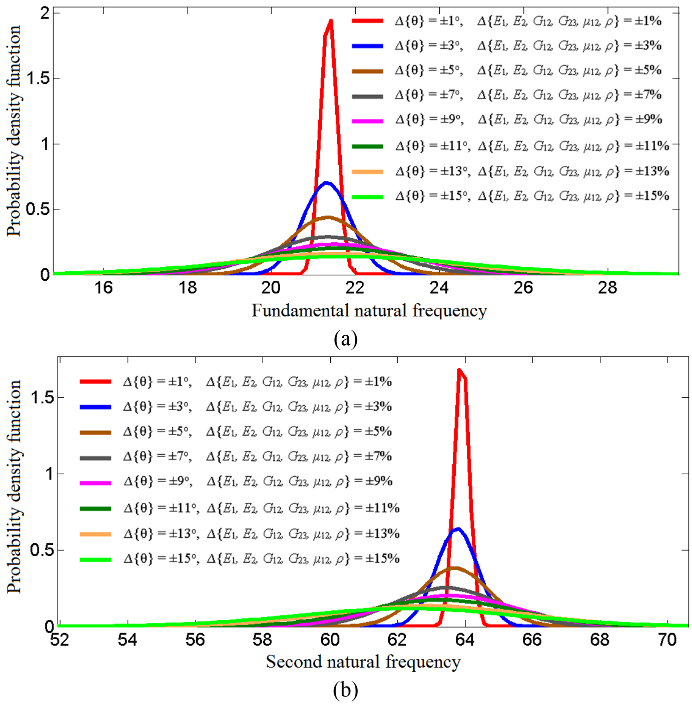


Fig. 4: Probability density function plots corresponding to different degree of stochasticity

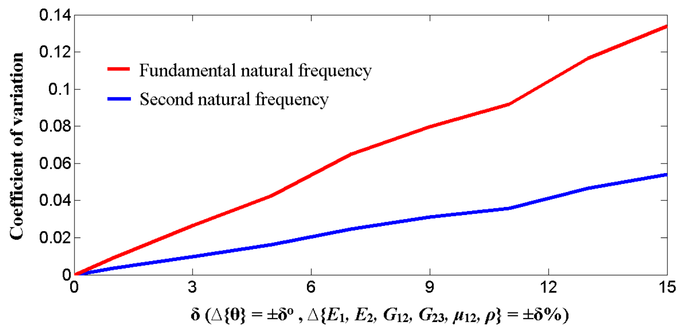


Fig. 5: Coefficient of variation for different degree of stochasticity (δ)

of stochasticity.

6. CONCLUSIONS

The novelty of the present study includes incorporation of artificial neural network based uncertainty propagation algorithm in laminated composite plates. Stochastic natural frequencies are analysed considering layer-wise variation of individual as well as combined cases for random input parameters. In this study, the uncertainty quantification of natural frequencies with uniform random input variables (such as ply orientation and material properties) is formulated implicitly using finite element method and thereby ANN approach is incorporated to achieve computational efficiency. The computational time and cost is reduced by using the present ANN approach compared to traditional Monte Carlo simulation method. This ANN based approach for uncertainty quantification presented in this article can be extended to deal with more complex systems in future.

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