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Probabilistic characterisation for dynamics and stability of laminated soft core sandwich plates

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Abstract

This paper presents a generic multivariate adaptive regression splines-based approach for dynamics and stability analysis of sandwich plates with random system parameters. The propagation of uncertainty in such structures has significant computational challenges due to inherent structural complexity and high dimensional space of input parameters. The theoretical formulation is developed based on a refined C^0 stochastic finite element model and higher-order zigzag theory in conjunction with multivariate adaptive regression splines. A cubical function is considered for the in-plane parameters as a combination of a linear zigzag function with different slopes at each layer over the entire thickness while a quadratic function is assumed for the out-of-plane parameters of the core and constant in the face sheets. Both individual and combined stochastic effect of skew angle, layer-wise thickness, and material properties (both core and laminate) of sandwich plates are considered in this study. The present approach introduces the multivariate adaptive regression splines-based surrogates for sandwich plates to achieve computational efficiency compared to direct Monte Carlo simulation. Statistical analyses are carried out to illustrate the results of the first three stochastic natural frequencies and buckling load.

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Uncertainty quantification, sandwich plates, composites, stochastic natural frequency, stochastic buckling load, multivariate adaptive regression splines

Introduction

The application of sandwich structures has gained immense popularity in advanced engineering applications, especially in aerospace, marine, civil, and mechanical structures that require superior performances and outstanding properties such as lightweight, high stiffness, high structural efficiency, and durability. The construction of sandwich panels consisting of thin face sheets of high strength material separated by a relatively thick and low density material offers excellent mechanical properties such as high strength-to-weight ratio and high stiffness-to-weight ratios. The characteristic features of such structures are affected by their layered construction and variations in properties through their thickness, and therefore it is important to predict their overall response in a realistic manner considering all these features. The effect of shear deformation plays a vital role in the structural analysis of sandwich and composite constructions because of their low shear modulus compared to extensional rigidity with a large variation in material properties between the core and the face layers. Moreover, due to their special type of construction and behavior, sandwich structures possess high statistical variations in the material and geometric properties. These inherent uncertainties should be properly taken into account in the analysis in order to have more realistic and safe design. This cannot be mapped by the conventional deterministic analysis. In fact, accurate predictions of the vibration response of these structures become more challenging to the engineers in the presence of inherent scatter in stochastic input parameters consisting of both material and geometric properties. Stochastic natural frequencies of such sandwich structures consist of overall mode and localised ones or through the thickness that the classical deterministic theories lack to detect. Due to the dependency of a large number of parameters in complex production and fabrication processes, the system properties are inevitably random in nature resulting in uncertainty in the response of the sandwich plate. Therefore, there is a need for an efficient and accurate computational technique which takes into account the effects of parameter uncertainty on the structural response. In the deterministic analysis of structures, the variations in the system parameters are neglected and the mean values of the system parameters are used in the analysis. But the variations in the system parameters should not be ignored for accurate and realistic studies that require a probabilistic description in which the response statistics can be adequately achieved by considering the material and geometric properties to be stochastic in nature.

Many review articles [1–4] are published on deterministic analysis of sandwich composite plates. Several investigators [5–11] studied on deterministic bending, buckling and free vibration analysis of skew composite and sandwich plates and thereby optimising such structures. Recently, a study has been published on analytical development for free vibration analysis of sandwich panels with randomly irregular honeycomb cores [12]. Free vibration response of laminated skew sandwich plates is investigated by Garg et al. [13] using C_0 isoparametric finite element model based on HSDT. The vibration behaviour of imperfect sandwich plates with in-plane partial edge load is presented by Chakrabarti and Sheikh [14] in a deterministic framework, while free vibration analyses of sandwich plates subjected to thermo-mechanical loads is studied by Shariyat [15] using a generalised global–local higher order theory. Many literatures [16–20] are found which investigate on dynamic and stability of soft core sandwich plates by analytical or finite element method. Radial basis function is used by Roque et al. [21], Ferrera et al. [22–24] and Rodrigues et al. [25,26] to analyse bending, buckling and free vibration characteristics of composite sandwich plates. The mesh-free moving Kriging interpolation method is presented by Bui et al. [27] for analysis of natural frequencies of laminated composite plates while Yang et al. [28] studied the vibration and damping analysis of thick sandwich cylindrical shells with a viscoelastic core. There is plenty of literature found which presents buckling and free vibration analyses of sandwich plates using Rayleigh-Ritz method [29–32]. A failure analysis of laminated sandwich shells has been carried out by Kumar et al. [33]. Several recent reports investigate bending and buckling analysis of sandwich plates with functionally graded material [34,35]. Researchers reported their results using Galerkin method [36], quadrature method [37], state–space method [38,39], Levy's method [40,41], Navier's method [42] or exact solutions method [43] for buckling and free vibration analyses of laminated sandwich plates. Recently, an analytical approach has been presented to obtain equivalent elastic properties of spatially irregular honeycomb core [44–46], which is often used as the core of sandwich panels. Such equivalent elastic properties of irregular honeycomb core can be an attractive solution for stochastic analyses of honeycomb panels with honeycomb core.

Most of the investigations carried out so far concerning the analysis of sandwich composite panels are deterministic in nature that lacks to cater the necessary insight on the behaviour of different structural responses generated from inherent statistical variations of stochastic material and geometric parameters. The studies on the stochastic analysis of sandwich plates with transversely flexible core are very scarce in literature [47]. In general, Monte Carlo simulation (MCS) is commonly used for stochastic response analysis. But the traditional MCS-based stochastic analysis approach is very expensive because it requires thousands of finite element simulations to be carried out to capture the random nature of parametric uncertainty. Hence, reduced order modelling (ROM) techniques are used to reduce the computational time and cost. In past, reduced order computational models are found to be employed in stochastic analysis of structures and materials [48–50] and some of them are specifically applied in laminated composite plates and shells including the effect of noise [51–65]. In the present study, we propose a multivariate adaptive regression splines (MARS)-based efficient uncertainty quantification algorithm for composite sandwich structures. In this approach, the expensive finite element model for sandwich composite structures is effectively replaced by the computationally efficient MARS model making the overall process of uncertainty quantification much more cost-effective. Compared to other reduced order modelling techniques, the use of MARS for engineering design applications is relatively new. Sudjianto et al. [66] used MARS model to emulate a conceptually intensive complex automotive shock tower model in fatigue life durability analysis while Wang et al. [67] compare MARS to linear, second-order, and higher-order regression models for a five variable automobile structural analysis. Friedman [68] integrated MARS procedure to approximate behaviour of performance variables in a simple alternating current series circuit. Literature suggests that the major advantages of using the MARS-based reduced order modelling appears to be the accuracy and significant reduction in computational cost associated with constructing the surrogate model compared to other conventional emulators such as Kriging [69].

To the best of the authors' knowledge, no work is reported in scientific literature on the study of dynamics and stability for laminated sandwich skewed plates with random geometric and material properties based on efficient MARS approach. MARS constructs the input/output relation from a set of coefficients and basis functions that are entirely driven from the regression data. The algorithm allows partitioning of the input space into regions, each with its own regression equation. This makes MARS particularly suitable for problems with high dimensional input parameter space. As finite element models of sandwich structures normally have large number of stochastic input parameters, MARS has the potential to be an efficient mapping route for the inputs and responses of such structures. In the present study, a stochastic analysis for free vibration and buckling of laminated sandwich skewed plates is carried out by solving the random eigenvalue problem through an improved higher-order zigzag theory in conjunction with MARS following a bottom-up random variable framework. Characterisation of probabilistic distributions for natural frequencies and buckling load of sandwich plates is first attempted in this study. Subsequently, relative individual effect of different stochastic input parameters towards natural frequencies and buckling is discussed. The uncertain geometric and material properties are considered along with the effect of the transverse normal deformation of the core. The in-plane displacement fields are assumed as a combination of a linear zigzag model with different slopes in each layer and a cubically varying function over the entire thickness. The out-of-plane displacement is assumed to be quadratic within the core and constant throughout the faces. The sandwich plate model is implemented with a stochastic C^0 finite element formulation developed for this purpose. The proposed computationally efficient MARS-based approach for uncertainty quantification of sandwich composite plates is validated with direct Monte Carlo simulation. The numerical results are presented for both individual and combined layer-wise variation of the stochastic input parameters.

Theoretical formulation

Consider a laminated soft core sandwich plate (Figure 1) with thickness 't' and skew angle 'ϕ' (as shown in Figure 2), consisting of 'n' number of thin lamina, the stress–strain relationship considering plane strain condition of an orthotropic layer or lamina (say k-th layer) having any fiber orientation angle 'θ' with respect to structural axes system (X–Y–Z) can be expressed as: $\{\sigma(\tilde{\omega})\} = \{Q_k(\tilde{\omega})\}\{\varepsilon(\tilde{\omega})\}$

$$\begin{Bmatrix} \sigma_{xx}(\tilde{\omega}) \\ \sigma_{yy}(\tilde{\omega}) \\ \sigma_{zz}(\tilde{\omega}) \\ \tau_{xy}(\tilde{\omega}) \\ \tau_{xz}(\tilde{\omega}) \\ \tau_{yz}(\tilde{\omega}) \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11}(\tilde{\omega}) & \bar{Q}_{12}(\tilde{\omega}) & \bar{Q}_{13}(\tilde{\omega}) & \bar{Q}_{14}(\tilde{\omega}) & 0 & 0 \\ \bar{Q}_{21}(\tilde{\omega}) & \bar{Q}_{22}(\tilde{\omega}) & \bar{Q}_{23}(\tilde{\omega}) & \bar{Q}_{24}(\tilde{\omega}) & 0 & 0 \\ \bar{Q}_{31}(\tilde{\omega}) & \bar{Q}_{32}(\tilde{\omega}) & \bar{Q}_{33}(\tilde{\omega}) & \bar{Q}_{34}(\tilde{\omega}) & 0 & 0 \\ \bar{Q}_{41}(\tilde{\omega}) & \bar{Q}_{42}(\tilde{\omega}) & \bar{Q}_{43}(\tilde{\omega}) & \bar{Q}_{44}(\tilde{\omega}) & 0 & 0 \\ 0 & 0 & 0 & 0 & \bar{Q}_{55}(\tilde{\omega}) & \bar{Q}_{56}(\tilde{\omega}) \\ 0 & 0 & 0 & 0 & \bar{Q}_{65}(\tilde{\omega}) & \bar{Q}_{66}(\tilde{\omega}) \end{bmatrix} \begin{Bmatrix} \varepsilon_{x}(\tilde{\omega}) \\ \varepsilon_{y}(\tilde{\omega}) \\ \varepsilon_{z}(\tilde{\omega}) \\ \gamma_{xy}(\tilde{\omega}) \\ \gamma_{xz}(\tilde{\omega}) \\ \gamma_{yz}(\tilde{\omega}) \end{Bmatrix} \tag{1}$$

where $\{\sigma(\tilde{\omega})\}$, $\{\varepsilon(\tilde{\omega})\}$ and $\{Q_k(\tilde{\omega})\}$ are random stress vector, random strain vector and random transformed rigidity matrix of k-th lamina, respectively. Here the symbol $\tilde{\omega}$ indicates the stochasticity of respective input parameters. Figure 2(a) represents the in-plane displacement field. The in-plane displacement parameters are expressed as

$$\mathrm{U_x}(\tilde{\omega}) = u_o + z\theta_x + \sum_{i=1}^{n_u-1} (z - z_i^u)(\tilde{\omega})\,\Delta(z - z_i^u)\alpha_{xu}^i$$

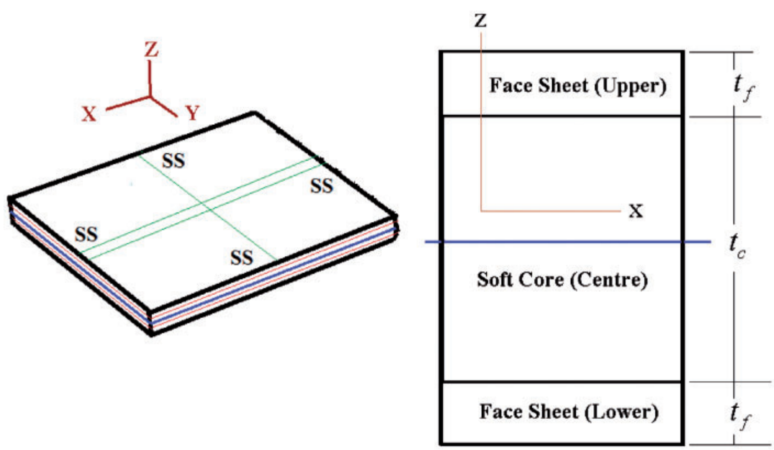

Figure 1. Simply supported soft core sandwich plate.

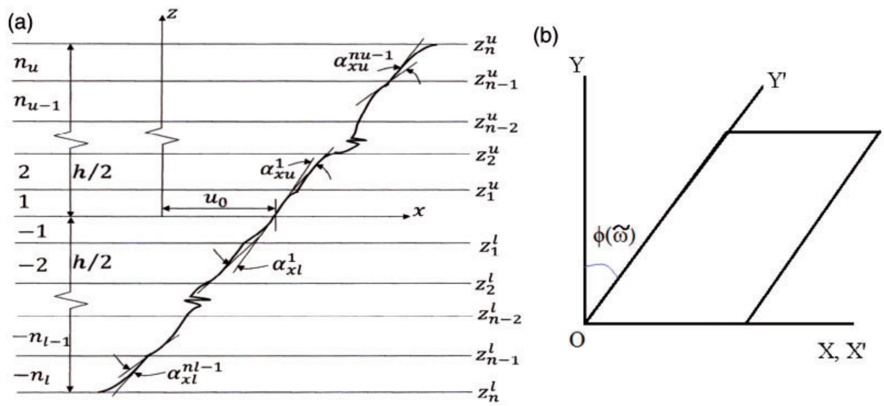

Figure 2. (a) General lamination and displacement configuration. (b) Skewed laminate geometry.

$$+\sum_{j=1}^{n_l-1}(z - z^l_j)(\tilde{\omega})\,\Delta(-z + z^l_{j_i})\,\alpha^j_{xl} + \beta_x z^2 + \chi_x z^3 \tag{2}$$

$$\begin{aligned} V_x(\tilde{\omega}) = v_o + z\theta_y + \sum_{i=1}^{n_u-1}(z - z^u_i)(\tilde{\omega})\,\Delta(z - z^u_i)\alpha^i_{yu} \\ + \sum_{j=1}^{n_l-1}(z - z^l_j)(\tilde{\omega})\,\Delta(-z + z^l_{j_i})\,\alpha^j_{yl} + \beta_y z^2 + \chi_y z^3 \end{aligned} \tag{3}$$

where u_o and v_o are the in-plane displacements of any point in the X-axis and Y′-axis on the mid-surface, θ_x and θ_y are the rotations of the normal to the middle plane about the Y-axis and X-axis respectively, n_u and n_l are the number of upper and lower layers, respectively while β_x, β_y, χ_x, and χ_y are the higher-order unknown co-efficient, α^i_{xu}, α^i_{yu}, α^j_{xl}, and α^j_{yl} are the slopes of *i*-th and *j*-th layer corresponding to upper and lower layers, respectively, and $\Delta(z - z^u_i)$ and $\Delta(-z + z^l_{j_i})$ are the unit step functions. The general lamination scheme, governing equations and displacement configuration are considered as per Pandit et al. [70]. The transverse displacements are assumed to vary quadratically through the core thickness and constant over the face sheets and it may be expressed as

$$W(\tilde{\omega}) = \frac{z(z+t_l)}{t_u(t_u+t_l)}w_u(\tilde{\omega}) + \frac{(t_l+z)(t_u-z)}{t_u\,t_l}w_o(\tilde{\omega}) + \frac{z(t_u-z)}{-t_l(t_u+t_l)}w_l(\tilde{\omega}) \quad \text{(for core)} \tag{4}$$

$$W(\tilde{\omega}) = w_u(\tilde{\omega}) \quad \text{(for upper face layers)} \tag{5}$$

$$W(\tilde{\omega}) = w_l(\tilde{\omega}) \quad \text{(for lower face layers)} \tag{6}$$

where $w_u(\tilde{\omega})$, $w_o(\tilde{\omega})$ and $w_l(\tilde{\omega})$ are the values of the transverse displacement at the top layer, middle layer and bottom layer of the core, respectively. Utilising the conditions of zero transverse shear stress at the top and bottom surfaces of the plate and imposing the conditions of the transverse shear stress continuity at the interfaces between the layers along with the conditions, $u = u_u$ and $v = v_u$ at the top and $u = u_l$ and $v = v_l$ at the bottom of the plate, the generalised displacement vector $\{\delta\}$ for the present plate model can be expressed as

$$\{\delta\} = \{u_o \ v_o \ w_o \ \theta_x \ \theta_y \ u_u \ v_u \ w_u \ u_l \ v_l \ w_l \ \}^T \text{ and } y_l = y_l' \ \cos\left[\phi(\tilde{\omega})\right] \tag{7}$$

where $\phi(\tilde{\omega})$ denotes the random skew angle (Figure 2). For the skewed plates, the elements on the inclined edges may not be parallel to the global axes ($x_g - y_g - z_g$). To determine the elemental stiffness matrix at skew edges, it becomes necessary to use edge displacements (u_o, v_o, w_o, θ_x, θ_y, u_u, v_u, w_u, u_l, v_l and w_l) in local coordinates ($x' - y' - z'$) (Figure 2). It is thus required to transform the element matrices corresponding to global axes to local axes with respect to which the elemental stiffness matrix can be conveniently determined. The relation between the global and local degrees of freedom of a node on the skew edge can be obtained through the simple transformation rules and the same can be expressed as

$$\{\delta^L(\tilde{\omega})\}^T = [T_n(\tilde{\omega})] \ \{\delta\}^T \tag{8}$$

A nine-noded isoparametric element is used for finite element formulation considering 11 degrees of freedom, where $\{\delta^L(\tilde{\omega})\}$ and $[T_n(\tilde{\omega})]$ are the displacement vector in the localised coordinate system and node transformation matrix, respectively. Using the node transformation matrix, the elemental transformation matrix $[T_{ele}(\tilde{\omega})]$ can be determined, which is used to transfer the elemental stiffness matrix of the skew edge elements from the global axes to local axis. The node transformation matrix $[T_n(\tilde{\omega})]$ and the elemental transformation matrix $[T_{ele}(\tilde{\omega})]$ are expressed as

$$[T_n(\tilde{\omega})] = \begin{bmatrix} m & -n & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ n & m & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & m & -n & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & n & m & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & m & -n & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & n & m & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & m & -n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & n & m & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \text{ and} \tag{9}$$

$$[T_{ele}(\tilde{\omega})] = \begin{bmatrix} [T_n(\tilde{\omega})] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & T_n(\tilde{\omega})] & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & T_n(\tilde{\omega})] & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & T_n(\tilde{\omega})] & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & T_n(\tilde{\omega})] & 0 & 0 & 0 & 0 & 0 \\ & & & & & T_n(\tilde{\omega})] & 0 & 0 & 0 & 0 \\ & Sym. & & & & & T_n(\tilde{\omega})] & 0 & 0 & 0 \\ & & & & & & & T_n(\tilde{\omega})] & 0 & 0 \\ & & & & & & & & T_n(\tilde{\omega})] & 0 \\ & & & & & & & & & T_n(\tilde{\omega})] \end{bmatrix} \quad (10)$$

where $m = \sin\theta(\tilde{\omega})$ and $n = \cos\theta(\tilde{\omega})$, wherein $\theta(\tilde{\omega})$ is the random fibre orientation angle.

Using linear strain–displacement relation, the strain field $\{\bar{\varepsilon}(\tilde{\omega})\}$ may be expressed in terms of unknowns (for the structural deformation) as

$$\{\bar{\varepsilon}(\tilde{\omega})\} = \left[\frac{\partial U(\tilde{\omega})}{\partial x}\frac{\partial V(\tilde{\omega})}{\partial y}\frac{\partial W(\tilde{\omega})}{\partial z}\frac{\partial U(\tilde{\omega})}{\partial x}+\frac{\partial V(\tilde{\omega})}{\partial y}\frac{\partial U(\tilde{\omega})}{\partial z}+\frac{\partial W(\tilde{\omega})}{\partial x}\frac{\partial V(\tilde{\omega})}{\partial z}+\frac{\partial W(\tilde{\omega})}{\partial x}\right]$$
$$\text{i.e. } \{\bar{\varepsilon}(\tilde{\omega})\} = [H(\tilde{\omega})]\{\varepsilon(\tilde{\omega})\} \tag{11}$$

where

$$\begin{aligned}\{\varepsilon\} = [&u_0 v_0 w_0 \theta_x \theta_y u_u v_u w_u u_l v_l w_l\,(\partial u_0/\partial x)(\partial u_0/\partial y)(\partial v_0/\partial x)(\partial v_0/\partial y)(\partial w_0/\partial x)(\partial w_0/\partial y)\\ &(\partial\theta_x/\partial x)(\partial\theta_x/\partial y)(\partial\theta_y/\partial x)(\partial\theta_y/\partial y)(\partial u_u/\partial x)(\partial u_u/\partial y)(\partial v_u/\partial x)(\partial v_u/\partial y)(\partial w_u/\partial x)\\ &(\partial w_u/\partial y)(\partial u_l/\partial x)(\partial u_l/\partial y)(\partial v_l/\partial x)(\partial v_l/\partial y)(\partial w_l/\partial x)(\partial w_l/\partial y)]\end{aligned}$$

and the elements of $[H]$ are functions of z and unit step functions. In the present problem, a nine-node quadratic element with 11 field variables ($u_{0,}$ v_0, $w_{0,}$ θ_x, θ_y, u_u, v_u, w_u, u_l, v_l and w_l) per node is employed. Using finite element method, the generalised displacement vector $\{\delta(\tilde{\omega})\}$ at any point may be expressed as

$$\{\delta(\tilde{\omega})\} = \sum_{i=1}^{n} N_i(\tilde{\omega})\delta_i(\tilde{\omega}) \tag{12}$$

where $\{\delta\} = \{u_0 v_0 w_0 \theta_x \theta_y u_u v_u w_u u_l v_l w_l\}^{\mathrm{T}}$ as defined earlier, δ_i is the displacement vector corresponding to node i, N_i is the shape function associated with the node i and n is the number of nodes per element. With the help of equation (12), the strain vector $\{\varepsilon\}$ that appeared in equation (11) may be expressed in terms of unknowns (for the structural deformation) as

$$\{\varepsilon(\tilde{\omega})\} = [B(\tilde{\omega})]\{\delta(\tilde{\omega})\} \tag{13}$$

where $[B]$ is the strain–displacement matrix in the Cartesian coordinate system.

From Hamilton's principle [71], the dynamic equilibrium equation for free vibration analysis can be expressed as [72]

$$[K(\tilde{\omega})]\ \ \{\bar{\delta}\} = \ \lambda^2\,[M(\tilde{\omega})]\{\bar{\delta}\} \tag{14}$$

where $[\lambda(\tilde{\omega})]$ is the stochastic free vibration frequencies for different modes and the global mass matrix $[M(\tilde{\omega})]$ may be formed by assembling a typical element mass matrix as shown below

$$[M(\tilde{\omega})] = \sum_{i=1}^{n_u+n_l} \iiint \rho_i(\tilde{\omega})\ [N]^T[P]^T[N][P]\mathrm{d}x\,\mathrm{d}y\mathrm{d}z = \iint [N]^T[R(\tilde{\omega})][N]\mathrm{d}x\,\mathrm{d}y \quad (15)$$

where $\rho_i(\tilde{\omega})$ is the random mass density of the i-th layer, matrix $[P]$ is of order 3×11 and contains z terms and some constant quantities, matrix $[N]$ is the shape function matrix and the matrix $[R(\tilde{\omega})]$ can be expressed as

$$[R(\tilde{\omega})] \ = \ \sum_{i=1}^{n_u+n_l} \rho_i(\tilde{\omega})\ [P]^T[P]\ \mathrm{d}z \quad (16)$$

A numerical code is developed to implement the above-mentioned operations involved in the proposed finite element model to determine the vibration response of laminated skew composite sandwich plates. The skyline technique is used to store the global stiffness matrix in a single array. Simultaneous iteration technique of Corr and Jennings [73] is used in free vibration analysis. In the present study, a nine-noded isoparametric element with 11 degrees of freedom at each node is considered for finite element formulation. The elemental potential energy can be expressed as [17]

$$\Pi_e = U_s - \ U_{ext} \quad (17)$$

where U_s and U_{ext} are the strain energy and the energy due to external in-plane load, respectively.

$$\begin{aligned}\prod\nolimits_e &= \frac{1}{2}\iint \{\delta\}^T[B(\tilde{\omega})]^T[D(\tilde{\omega})]\,[B(\tilde{\omega})]\{\delta\}\mathrm{d}x\,\mathrm{d}y \\ &\quad -\frac{1}{2}\iint \{\delta\}^T[B(\tilde{\omega})]^T[G(\tilde{\omega})][B(\tilde{\omega})]\{\delta\}\mathrm{d}x\,\mathrm{d}y \\ &= \frac{1}{2}\{\delta\}^T[K_e(\tilde{\omega})]\,\{\delta\} - \frac{1}{2}\lambda\,\{\delta\}^T[K_G(\tilde{\omega})]\,\{\delta\}\end{aligned} \quad (18)$$

where $[K_e(\tilde{\omega})] = \int [B(\tilde{\omega})]^T\, [D(\tilde{\omega})]\ [B(\tilde{\omega})]\ \mathrm{d}x$ and $[K_G(\tilde{\omega})] = \int [B(\tilde{\omega})]^T\, [G(\tilde{\omega})][B(\tilde{\omega})]\mathrm{d}x$

Here $[B(\tilde{\omega})]$ is the random strain–displacement matrix while $[K_e(\tilde{\omega})]$ and $[K_G(\tilde{\omega})]$ are the stochastic elastic stiffness matrix and geometric stiffness matrix, respectively. The equilibrium equation can be obtained by minimising Π_e as given in equation (18) with respect to $\{\delta\}$ as

$$[K_e(\tilde{\omega})]\ \{\delta\} = \lambda(\tilde{\omega})\ [K_G(\tilde{\omega})]\ \{\delta\} \quad (19)$$

where $\lambda(\tilde{\omega})$ is a stochastic buckling load factor. The skyline technique has been used to store the global stiffness matrix in a single array and simultaneous iteration technique is used for solving the stochastic buckling equation (19).

Formulation of multivariate adaptive regression splines (MARS)

MARS [66] provides an efficient mathematical relationship between input parameters and output feature of interest for a system under investigation based on few algorithmically chosen samples. MARS is a nonparametric regression procedure that makes no assumption about the underlying functional relationship between the dependent and independent variables. MARS algorithm adaptively selects a set of basis functions for approximating the response function through a forward and backward iterative approach. The MARS model can be expressed as

$$Y = \sum_{k=1}^{n} \alpha_k H_k^f(x_i) \tag{20}$$

with $H_k^f(x_1, x_2, x_3 \ldots x_n) = 1$, for $k = 1$
where α_k and $H_k^f(x_i)$ are the coefficient of the expansion and the basis functions, respectively. Thus, the first term of equation (20) becomes α_1, which is basically an intercept parameter. The basis function can be represented as

$$H_k^f(x_i) = \prod_{i=1}^{i_k} \left[z_{i,k}(x_{j(i,k)} - t_{i,k})\right]_{tr}^q \tag{21}$$

where i_k is the number of factors (interaction order) in the k-th basis function, $z_{i,k} \pm 1, x_{j(i,k)}$ is the j-th variable, $1 \leq j(i,k) \leq$ n, and $t_{i,k}$ is a knot location on each of the corresponding variables. q is the order of splines. The approximation function Y is composed of basis functions associated with k sub-regions. Each multivariate spline basis function $H_k^f(x_i)$ is the product of univariate spline basis functions $z_{i,k}$, which is either order one or cubic, depending on the degree of continuity of the approximation. The notation '*tr*' means the function is a truncated power function.

$$[z_{i,k}(x_{j(i,k)} - t_{i,k})]_{tr}^k = [z_{i,k}(x_{j(i,k)} - t_{i,k})]^q \text{ for } [z_{i,k}(x_{j(i,k)} - t_{i,k})] < 0 \tag{22}$$

$$[z_{i,k}(x_{j(i,k)} - t_{i,k})]_{tr}^q = 0 \quad \text{Otherwise} \tag{23}$$

Here each function is considered as piecewise linear with a trained knot '*tr*' at each $x_{(i,k)}$. By allowing the basis function to bend at the knots, MARS can model functions that differ in behaviour over the domain of each variable. This is applied to interaction terms as well. The interactions are no longer treated as global across the entire range of predictors but between the sub-regions of every basis function generated. Depending on fitment, the maximum number of knots to be considered, the minimum number of observations between knots, and the highest order of interaction terms are determined. The screening of automated variables occur as a result of using a modification of the generalised cross-validation (GCV) model fit criterion, developed by Craven and Wahba [74]. MARS finds the location and number of the needed spline basis functions in a forward or backward stepwise fashion. It starts by over-fitting a spline function through each knot, and then by removing the knots that least contribute to the overall fit of the model as determined by the modified GCV criterion, often completely removing the most insignificant variables. The equation depicting the lack-of-fit (L_f) criterion used by MARS is

$$L_f(Y_{\tilde{k}}) = G_{cv}(\tilde{k}) = \frac{\frac{1}{n}\sum_{i=1}^{n}\left[Y_i - Y_{\tilde{k}}(x_i)\right]^2}{\left[1 - \frac{\tilde{c}(\tilde{k})}{n}\right]^2} \tag{24}$$

where $\tilde{c}(\tilde{k}) = c(\tilde{k}) + M.\tilde{k}$

Here 'n' denotes the number of sample observations, $\tilde{c}(\tilde{k})$ is the number of linearly independent basis functions, $\tilde{k}$ is the number of knots selected in the forward process, and 'M' is a cost for basis-function optimisation as well as a smoothing parameter for the procedure. Larger values of 'M' result in fewer knots and smoother function estimates. The best MARS approximation is the one with the highest GCV value. Thus, MARS is also compared with parametric and nonparametric approximation routines in terms of its accuracy, efficiency, robustness, model transparency, and simplicity and it is found suitable methodologies because it is more interpretable than most recursive partitioning, neural and adaptive strategies wherein it distinguishes well between actual and noise variables. Moreover, the MARS are reported [75] to work satisfactorily in terms of computational cost irrespective of dimension (low–medium–high) and noise.

Random input representation

The layer-wise random input parameters such as ply-orientation angle, skew angle, thickness and material properties (e.g. mass density, elastic modulus, Poisson's ratio) of both core and face sheet are considered for sandwich plates. It is assumed that the random uniform distribution of input parameters exists within a certain band of tolerance with their deterministic values. The individual and combined cases wherein the input variables considered in both soft core and each layer of face sheet of sandwich are as follows

(Case-a) Variation of ply-orientation angle only: $\theta(\tilde{\omega}) = [\{\theta_1\ \theta_2\ \theta_3 \ldots \theta_i \ldots \theta_l\}](\tilde{\omega})$

(Case-b) Variation of thickness only: $t_{tot}(\tilde{\omega}) = [\{t_c\}, \{t_{fs(1)}\ t_{fs(2)} \ldots t_{s(l)}\}](\tilde{\omega})$
(Case-c) Variation of mass density only: $\rho(\tilde{\omega}) = [\{\rho_{fs(1)}\ \rho_{c(2)}\}](\tilde{\omega})$
(Case-d) Variation of skew angle only: $\varphi(\tilde{\omega})$
(Case-e) Variation of material properties

$$P(\tilde{\omega}) = [\, E_{x(fs,c)}, E_{y(fs,c)}, E_{z(fs,c)}, G_{12(fs,c)}, G_{13(fs,c)}, G_{23(fs,c)}, \mu_{12(fs,c)}, \mu_{21(fs,c)}, \ldots \mu_{13(fs,c)}, \mu_{23(fs,c)}, \mu_{32(fs,c)}, \rho_{(fs,c)}](\tilde{\omega})$$

(Case-f) Combined variation of ply orientation angle, thickness, mass density, skew angle, elastic moduli, shear moduli, Poisson ratios and mass density for both core and face sheet (total 63 numbers of random input variables):

$$C(\tilde{\omega}) = [\,\theta(\tilde{\omega}),\ t_{tot}(\tilde{\omega}),\ \rho(\tilde{\omega}),\ \varphi(\tilde{\omega}),\ P(\tilde{\omega})]$$

where θ, t, ρ and φ are the ply orientation angle, thickness, mass density and skew angle, respectively. The subscripts c and fs are used to indicate core and face sheet, respectively. 'l' denotes the number of layer in the laminate, where $i = 1, 2, \ldots, l$. Six different cases are considered for the analysis: layer-wise stochasticity in ply orientation angle ($\theta(\tilde{\omega})$), combined effect for thickness of core and face sheet ($t_{tot}(\tilde{\omega})$), combined effect for mass density of core and face sheet ($\rho(\tilde{\omega})$), skew angle ($\varphi(\tilde{\omega})$), combined effect for material properties of core and face sheet ($P(\tilde{\omega})$) and combined variation of all parameters ($C(\tilde{\omega})$). In the present study, $\pm 5^\circ$ variation for ply orientation angle and skew angle and $\pm 10\%$ variation in material properties are considered from their respective deterministic values unless mentioned otherwise for some analyses. Figure 3 presents the flowchart of proposed stochastic frequency analysis using MARS model for the laminated soft core sandwich structure.

Results and discussion

The present study considers a sandwich composite plate with soft core (both upper and lower as 0°) and two facesheets with four-layered cross-ply ($90^\circ/0^\circ/90^\circ/0^\circ$) laminate covering the core in both top and bottom side. A nine-noded isoparametric plate bending element is considered for finite element formulation. For the analysis, the dimensions and boundary conditions considered for the sandwich composite plate are as follows: length $(L) = 1$ m, width $(b) = 0.5$ m and thickness $(t) = L/10$, with simply supported boundary conditions (unless otherwise mentioned). The considered material properties of the sandwich plate are provided in Table 1.

The present MARS model is employed to find a predictive and representative surrogate model relating each natural frequency to a number of stochastic input variables. The MARS-based surrogate models are used to determine the first three natural frequencies corresponding to a set of input variables, instead of time-consuming and expensive finite element analysis. The probability density function

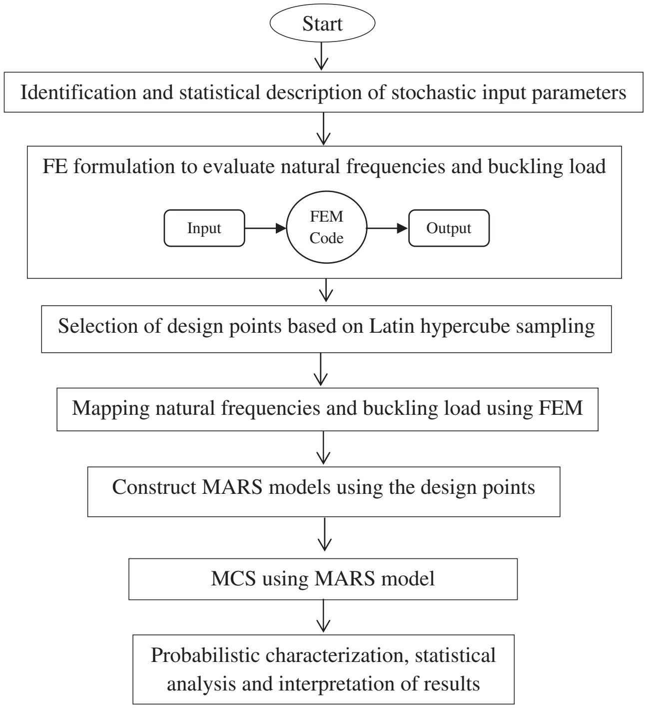

Figure 3. Flowchart of stochastic analysis using MARS model.

Table 1. Material properties for core and face sheet of sandwich plate.

Material properties	Core	Face sheet
E_1	0.5776 GPa	276 GPa
E_2 and E_3	0.5776 GPa	6.9 GPa
G_{12} and G_{13}	0.1079 GPa	6.9 GPa
G_{23}	0.2221 GPa	6.9 GPa
ν_{12} and ν_{13}	0.25	0.25
ν_{21} and ν_{31}	0.00625	0.00625
ν_{23} and ν_{32}	0.25	0.25
ρ	1000 kg/m^3	681.8 kg/m^3

is plotted as the benchmark of bottom line results. The variation of geometric and material properties is considered to fluctuate within the range of lower and upper limit (tolerance limit) as $\pm 10\%$ with respective mean values while for ply orientation angles and skew angles as within $\pm 5^\circ$ fluctuation (as per industry standard) with respect to their deterministic mean values. A layer-wise random variable approach is employed for generating the set of random input variables which are considered for surrogate-based numerical finite element iteration to obtain the respective set of random output parameters accordingly. The transverse shear stresses vanish only at the top and bottom surfaces of the laminate irrespective of the considered boundary conditions, e.g. for clamped boundary condition, all the kinematic variables vanish at clamped edges. Results are presented for stochastic natural frequencies and stochastic buckling load for the sandwich plate.

Stochastic natural frequency analysis

Mesh convergence and validation of the finite element model for the sandwich plate is conducted first considering a deterministic analysis. The optimum mesh size is finalised on the basis of a mesh convergence study as presented in Figure 4, wherein a mesh size of (14×14) is found to be adequate. The non-dimensional natural frequencies ($\varpi = 100 \times \omega L\sqrt{\frac{\rho_c}{E_{2f}}}$, where ρ_c is the density of the core layer) for the first two modes based on the present model are obtained for various skew angles and are tabulated in Table 2 along with the previous results obtained by Wang et al. [76] and Kulkarni and Kapuria [77]. Table 3 presents the results for non-dimensional natural frequencies of a four-layered clamped symmetric ($0^\circ/90^\circ/90^\circ/0^\circ$) laminated composite plate obtained from present analysis for various aspect ratios with respect to the previous analyses reported by Kulkarni and Kapuria [76] and Khandelwal et al. [78]. The results corroborate good agreement of the deterministic natural frequencies obtained using the present finite elememt model with respect to previous works. The validation of the MARS model as a surrogate of the actual finite elemet model is presented using scatter plots and probability density function plots (refer to Figures 5 and 6). The low deviation of points from the diagonal line in the scatter plot (Figure 5) corroborates the high accuracy of prediction capability of the MARS model with respect to finite element model for all the random input parameter sets (combined effect of 63 numbers of random input parameters). The probability density function plots presented in Figure 6 show a negligible deviation between MARS model and original MCS model indicating validity and high level of precision for the present surrogate-based approach further. It is noteworthy that the proposed MARS-based approach requires 256 numbers of original finite element simulations for the layer-wise individual variation of stochastic input parameters, while due to increment in number of input variables, 512 finite element simulations are found to be adequate for combined random variation of input parameters. Here, although the same sample size as in direct MCS (10,000 samples) is considered for characterising the probability distributions of natural frequencies, the number of actual finite element simulations is much less compared to direct MCS approach. Hence, the computational time and effort expressed in terms of expensive finite element simulations is reduced significantly compared to full scale direct MCS. This provides an efficient affordable way for simulating the uncertainties in natural frequency. The optimum number of finite

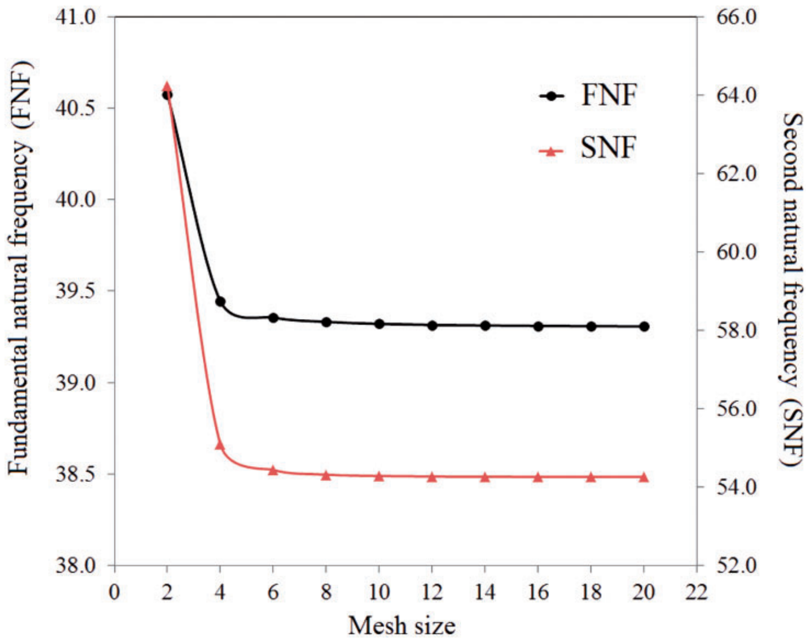

Figure 4. Mesh convergence study of finite element analysis with different mesh sizes with respect to fundamental and second natural frequencies of sandwich skewed plates
FNF: first natural frequency; SNF: second natural frequency.

Table 2. Non-dimensional natural frequencies of a four-layered (0°/90°/0°/90°) anti-symmetric composite plate.

Skew angle	Mode	Present analysis	Wang et al. [76]	Kulkarni and Kapuria [77]
30°	1	1.8889	1.9410	1.9209
	2	3.4827	2.9063	3.5353
45°	1	2.5806	2.6652	2.6391
	2	3.7516	3.2716	4.1810

element simulations (i.e. the number of design points in Latin hypercube sampling) required to construct the MARS models is decided based on a convergence study as presented in Table 4.

In the present analysis, all the layer-wise individual cases of stochasticity are studied as described in the Random input representation section. It is, however, noticed that skew angle, mass density and transverse shear modulus are the three most sensitive factors for first three stochastic natural frequencies (refer to

Table 3. Non-dimensional natural frequencies of a four-layered clamped symmetric (0°/90°/90°/0°) laminated composite plate.

Aspect ratio	Mode	Present analysis	Kulkarni and Kapuria [77]	Khandelwal *et al.* [78]
10	1	18.0843	18.2744	17.9550
	2	28.9441	28.9047	28.9674
20	1	23.4534	24.1130	23.9339
	2	37.0587	36.7473	37.0614

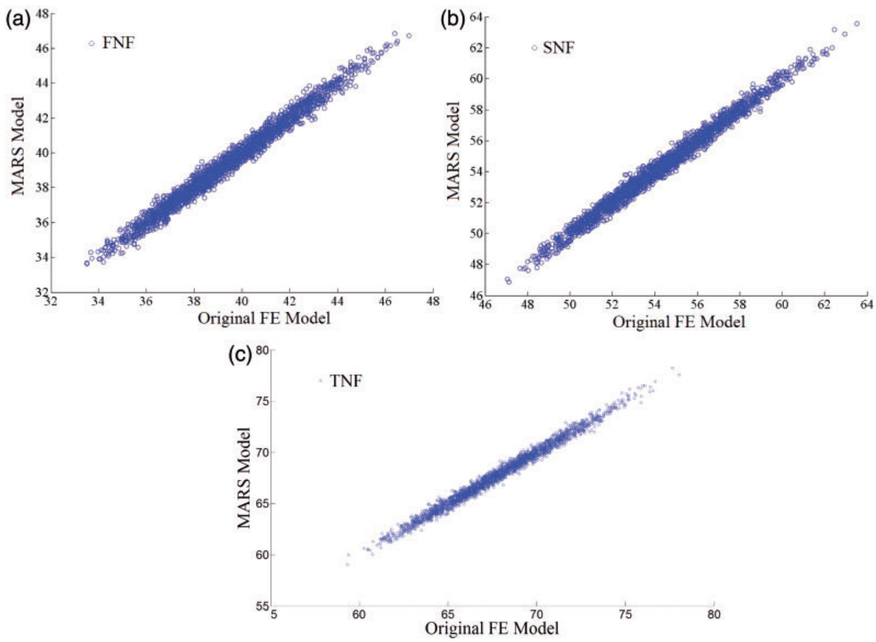

Figure 5. Scatter plot of finite element (FE) model with respect to MARS model for (a) fundamental natural frequency (FNF), (b) second natural frequency (SNF) and (c) third natural frequency (TNF) of simply supported sandwich skewed plates considering combined variation (total 63 numbers of random input variables) for $\phi(\tilde{\omega}) = 45°$.

Figure 7) by analysing the relative coefficient of variations [64]. Relative combined effect of the other parameters are (logitudinal and transverse elastic modulus, ply orientation angle, thickness, longtudinal shear modulus and Poisson ratio) also shown in Figure 7 for the first three natural frequencies. As the effect of other parameters has neglegible sensitivity on stochastic natural frequencies,

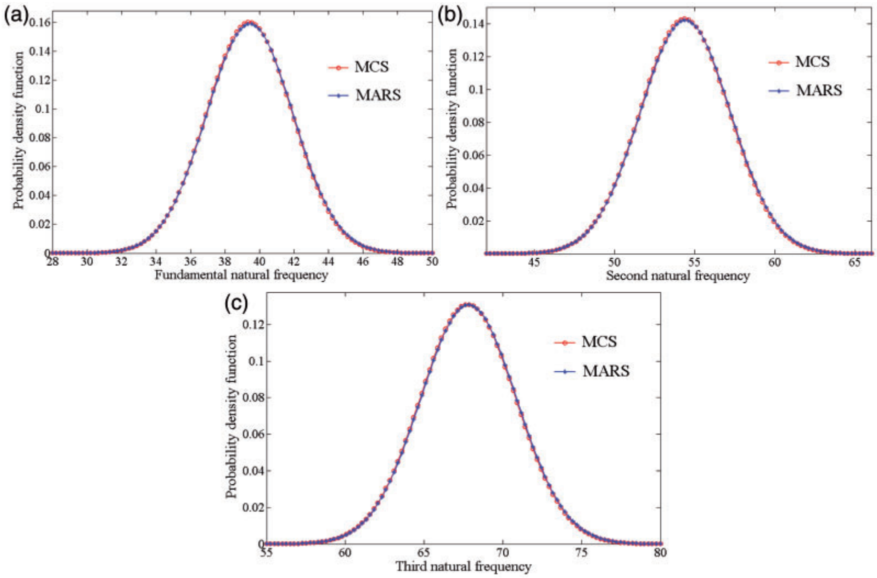

Figure 6. Probability density function for MCS as well as MARS model for the first three natural frequencies of simply supported compsoite sandwich skewed plates considering combined variation (a total of 63 random input variables) for $\phi(\tilde{\omega}) = 45^{\circ}$.

representative results are furnished for stochastic effect of two most effective parameters (skew angle and mass density) for analysis of individual cases.

Probability distributions for first three stochastic natural frequencies of a simply supported composite sandwich plate due to only variation in skew angles are furnished in Figure 8. As the skew angle increases, the mean of stochastic natural frequencies is also found to increase, while probability distributions corresponding to different skew angles vary consideranly. Figure 9 presents the stochastic first three natural frequencies of a simply supported sandwich composite skewed plate (for skew angle $\phi(\tilde{\omega}) = 45^{\circ}$) due to only variation of mass density (layer-wise) with different degree of stochasticity. As the percentage of stochasticity in mass density increases, the response bounds are found to increase accordingly, while the mean does not change for different percentage of variation in mass density. The effect of combined stochasticity in all input parameters (referred as $C(\tilde{\omega})$ in the Random input representation section) is also analysed for different skew angles in addition to individual effect of the input parameters for stochastic natural frequnencies of sandwich plates. In Figure 10, the stochastic first three natural frequencies are presented for simply supported sandwich composite plates with different skew angles considering combined variation of input parameters $C(\tilde{\omega})$ (total 63 numbers random input variables), wherein a general trend is noticed that the mean and response bounds increase with the increase in skew angle. Response bounds of the first three natural frequencies due to combined variation are noticed to be

Table 4. Convergence study of first three modes due to individual and combined variation of inputs for simply supported sandwich plates.

Individual variation	Value	f_1 MCS (10,000)	f_1 MARS (Sample run) 64	f_1 MARS 128	f_1 MARS 256	f_2 MCS (10,000)	f_2 MARS (Sample run) 64	f_2 MARS 128	f_2 MARS 256	f_3 MCS (10,000)	f_3 MARS (Sample run) 64	f_3 MARS 128	f_3 MARS 256
$\theta(\tilde{\omega})$	Max	39.6407	39.6614	39.6527	39.6493	54.6218	54.7123	54.6531	54.6338	67.8206	67.9845	67.9214	67.8584
	Min	38.9549	38.9420	38.9456	38.9485	53.8200	53.8064	53.8119	53.8168	67.4350	67.4220	67.4307	67.4321
	Mean	39.3039	39.3115	39.3101	39.3092	54.2480	54.2311	54.2342	54.2381	67.6402	67.6998	67.6831	67.6587
	SD	0.1186	0.1194	0.1195	0.1196	0.1458	0.1473	0.1468	0.1461	0.0668	0.0735	0.0698	0.0681
$t_s(\tilde{\omega})$	Max	39.6858	39.7564	39.7313	39.7257	54.7669	54.8214	54.7917	54.7764	68.4852	68.6154	68.5978	68.5432
	Min	38.8943	38.8021	38.8264	38.8409	53.7134	53.3116	53.3982	53.4918	66.8631	66.8167	66.8227	66.8497
	Mean	39.3044	39.3164	39.3114	39.3081	54.2644	54.4951	54.3718	54.3083	67.6574	67.6831	67.7952	67.7098
	SD	0.1159	0.1173	0.1176	0.1179	0.1527	0.1584	0.1562	0.1552	0.2391	0.2487	0.2423	0.2416
$\rho(\tilde{\omega})$	Max	40.3206	41.1121	40.9821	40.5127	55.6577	56.1064	55.9561	55.7473	69.3890	69.9983	69.8134	69.4835
	Min	38.4011	38.3942	38.3964	38.9873	53.0081	52.6876	52.7942	52.9264	66.0856	65.8421	65.9226	65.9928
	Mean	39.3299	39.6734	39.5154	39.4212	54.2901	54.5876	54.5083	54.4221	67.6840	67.8674	67.8050	67.7213
	SD	0.4905	0.6154	0.5584	0.5129	0.6772	0.7954	0.7054	0.6997	0.8444	0.9533	0.8624	0.8517
$\varphi(\tilde{\omega})$	Max	41.7226	42.2134	42.0219	41.8687	56.7749	57.1259	56.9641	56.7963	70.1767	70.9897	70.6245	70.3516
	Min	37.2399	36.8276	36.9893	37.0867	52.1238	51.7383	51.9767	52.1013	65.5452	64.9984	65.1137	65.4194
	Mean	39.3663	39.6124	39.5483	39.4468	54.3251	54.6682	54.5437	54.4198	67.7213	67.9457	67.8438	67.7438
	SD	1.2740	1.3130	1.3030	1.2991	1.3218	1.3356	1.3286	1.3264	1.3164	1.3552	1.3487	1.3258

Combined variation	Value	f_1 MCS (10,000)	f_1 MARS (Sample run) 128	f_1 MARS 256	f_1 MARS 512	f_2 MCS (10,000)	f_2 MARS (Sample run) 128	f_2 MARS 256	f_2 MARS 512	f_3 MCS (10,000)	f_3 MARS (Sample run) 128	f_3 MARS 256	f_3 MARS 512
$C(\tilde{\omega})$	Max	46.59067	47.00219	46.9832	46.9265	62.80511	63.51472	63.2164	63.05806	77.35152	78.02273	77.9516	77.8462
	Min	33.0163	33.50135	33.4134	33.3372	46.86064	47.0558	46.9671	46.9493	59.25218	59.32715	59.2971	59.2832
	Mean	39.43564	39.39828	39.4002	39.4138	54.4066	54.36105	54.3883	54.3921	67.8260	67.77762	67.7884	67.7935
	SD	2.5081	2.4872	2.4889	2.4992	2.8085	2.7907	2.7921	2.7944	3.0515	3.0439	3.0476	3.0497

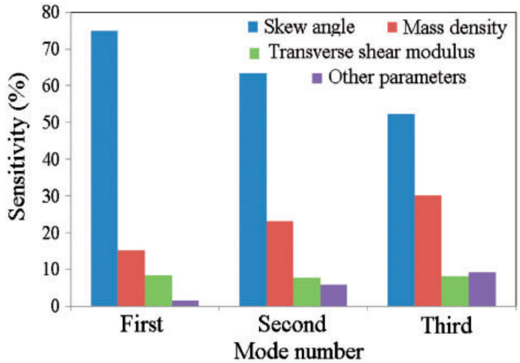

Figure 7. Sensitivity for first three natural modes for simply supported sandwich plates.

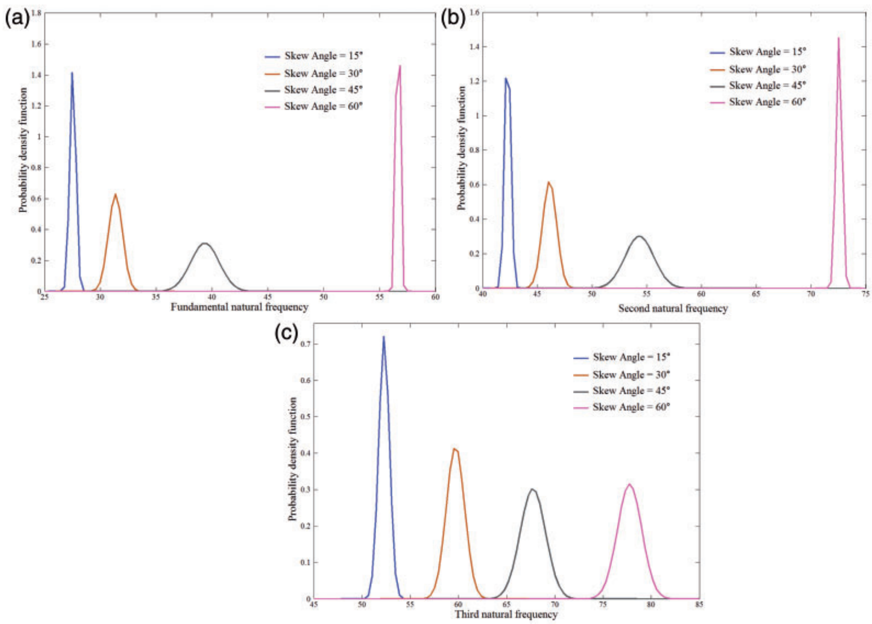

Figure 8. Stochastic first three natural frequencies (rad/s) of simply supported composite sandwich plates due to only variation of skew angles.

higher than individual variation of input parameters in all cases. The stochastic first three natural frequencies of sandwich composite skewed plates with different boundary conditions (C-Clamped, S-Simply supported, F-Free) are shown in Figure 11 considering combined variation of input parameters to investigate the

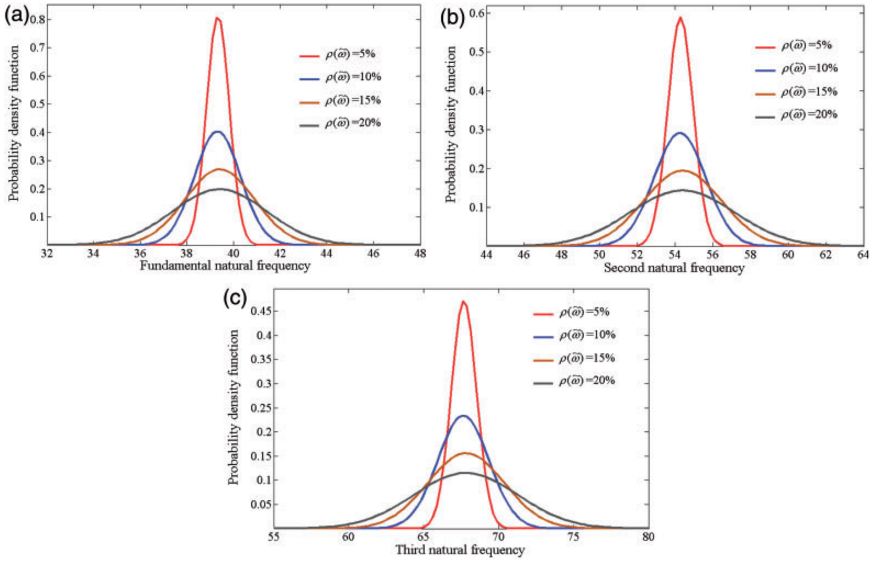

Figure 9. Stochastic first three natural frequencies (rad/s) of simply supported sandwich composite skewed plates for $\phi(\tilde{\omega}) = 45^{\circ}$due to only variation of mass density with different degree of stochasticity.

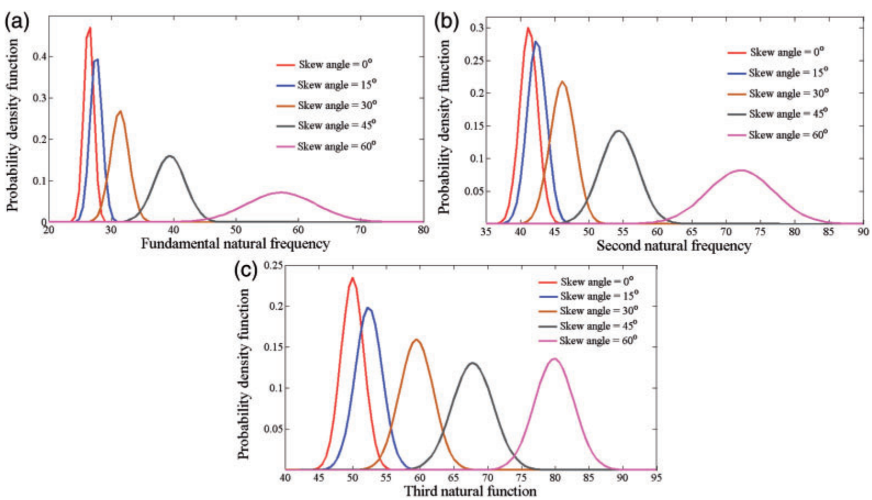

Figure 10. Stochastic first three natural frequencies (rad/s) of simply supported sandwich composite plates for different skew angles considering combined variation of input parameters (a total of 63 random input variables).

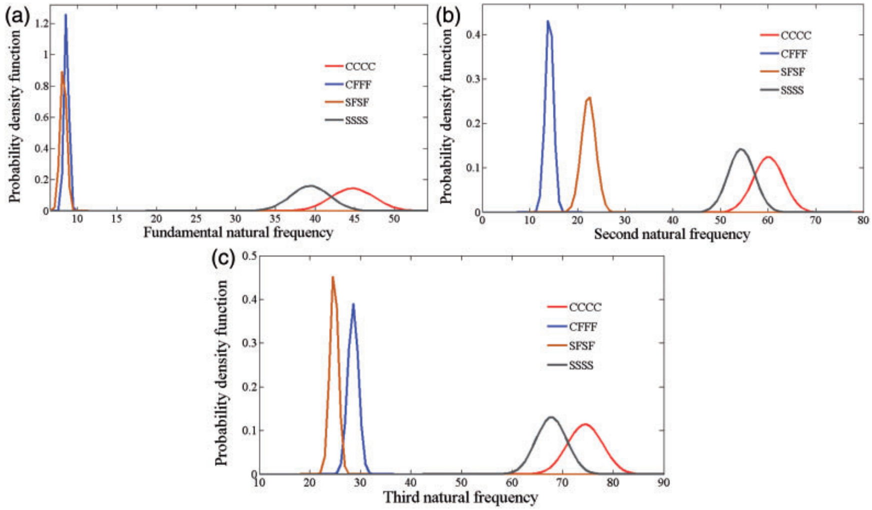

Figure 11. Stochastic first three natural frequencies (rad/s) of sandwich composite skewed plates for $\phi(\tilde{\omega}) = 45^\circ$ with different boundary conditions considering combined variation of input parameters (a total of 63 random input variables) (C: clamped; S: simply supported; F: free).

effect of boundary conditions. The probability distributions are found to vary significantly depending on the considered boundary condition. Both mean and standard deviation of CCCC boundary condition are found to be highest for combined variation of all input parameters.

Stochastic buckling load analysis

Mesh convergence and validation of the finite elemnt model for deterministic buckling load is presented in Figure 12. The convegence study on finite element mesh size is conducted to obtain the optimum mesh size. In the present study, the results of buckling load corresponding to different mesh sizes are found to be convergent as depicted in Figure 12, wherein the mesh convergence study is carried out to compare the critical bi-axial buckling load for laminated sandwich plates with different boundary conditions such as CCCC, SCSC and SSSS (where S – simply supported, C – clamped, indicating boundary condition of four sides). As the computational iteration time increases with the increase of mesh size, a (14×14) optimal mesh size is considered in the present study. The present buckling load are also validated with the results obtained by Liew and Huang [79]. The results corroborate good agreement of the buckling load obtained using the present finite elememt model with respect to previous works of Liew and Huang irrespective of imposed bounary conditions. Further, the MARS model that is employed to achieve computational efficiency is validated with traditional Monte Carlo simulation (MCS). Representative results are furnished for combined variation of all

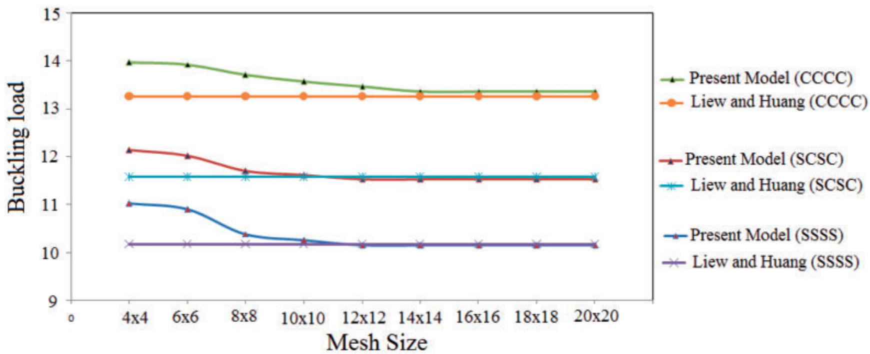

Figure 12. Mesh convergence study and validation for comparison of non-dimensionalised critical bi-axial buckling load [$\bar{\lambda} = (\lambda l^2) / (h^2 E_{Tf})$where λ, l, h and E_{Tf} are the buckling load factor, depth of the plate and transverse modulus of elasticity of face layer, respectively] for laminated sandwich plates with different boundary conditions.

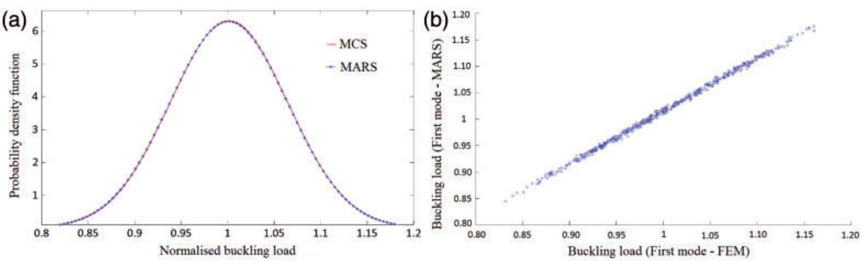

Figure 13. Probability density function and Scatter plot for buckling load of sandwich plates considering combined variation of all input parameters ($C(\tilde{\omega})$).

input parameters (512 samples) using probability density function plots and scatter plot as shown in Figure 13. The figures indicate high degree of precision and accuracy of the present MARS model with respect to original finite element model. The results for buckling load are presented hereafter (Figure 13 to 19) as a ratio of stochastic buckling load and deterministic buckling load to provide a clear and direct interpretation for stochasticity in different input parameters.

The effects of variation of core thickness and face sheet thickness on stochastic normalised buckling load of sandwich plates are shown in Figures 14 and 15, respectively. It is found that as the percentage of variation of both core and face sheet thickness increases, the response bound of stochastic buckling load also increases, while the mean does not vary. The sparsity of stochastic normalised buckling load due to variation of core thickness is observed to be significantly higher than that of the same due to variation of face sheet thickness. The effect of variation of all core material properties on stochastic buckling load of sandwich plates is furnished in Figure 16, while Figure 17 presents the effect of ply

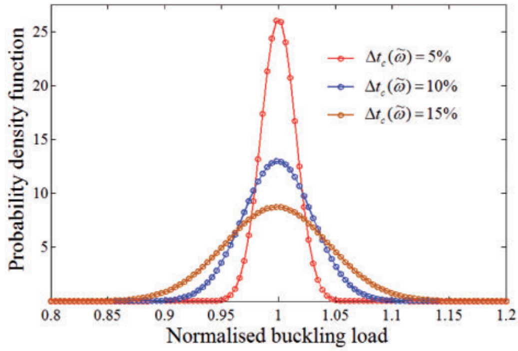

Figure 14. Effect of variation of core thickness on normalised buckling load of sandwich plates with SCSC (S – simply supported, C – clamped).

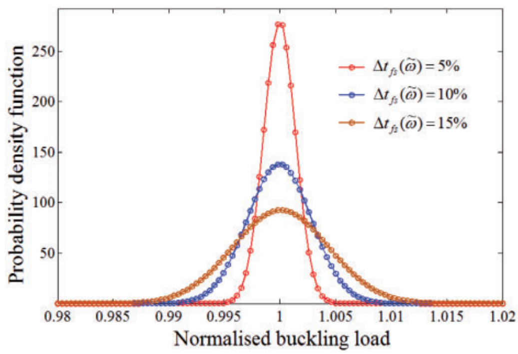

Figure 15. Effect of variation of face sheet thickness on normalised buckling load of sandwich plates with SCSC (S – simply supported, C – clamped).

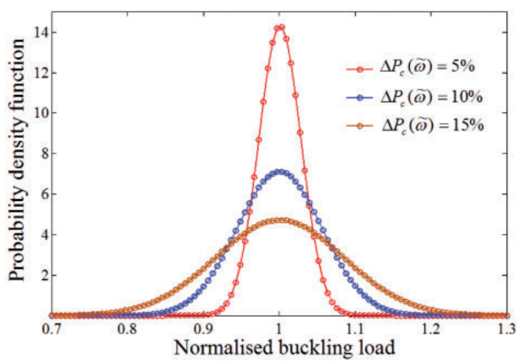

Figure 16. Effect of variation in material properties of core on normalised buckling load of sandwich plates with SCSC (S – simply supported, C – clamped).

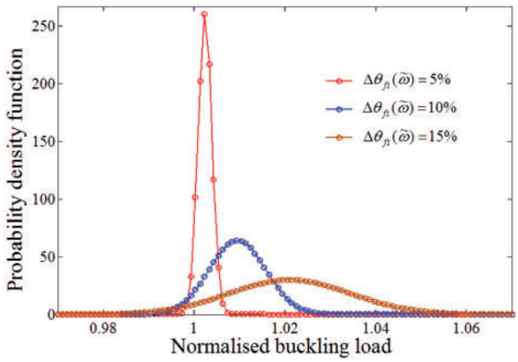

Figure 17. Effect of variation of ply orientation angle of face sheet on normalised buckling load of sandwich plates with SCSC (S – simply supported, C – clamped).

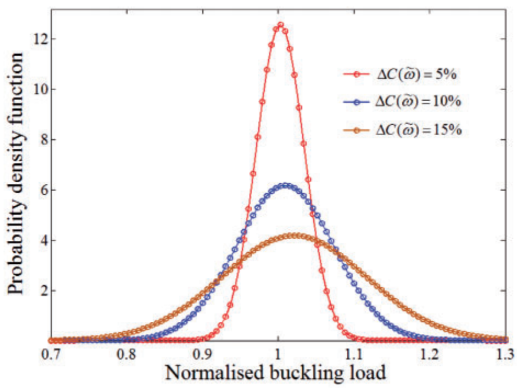

Figure 18. Effect of combined variation of stochastic input parameters (core and face sheet thickness, ply-orientation angle of face sheet and material properties) on normalised buckling load of sandwich plates.

orientation angle of face sheet on stochastic normalised buckling load of sandwich plates. Besides variation of core and face sheet thickness (Figures 14 and 15), the mean value for stochastic buckling load remains unaltered with different degrees of stochasticity in core material properties, while the standard deviation increases with increase in degree of stochasticity. In contrast, both mean and standard deviation of stochastic buckling load are found to increase with increasing degree of stochasticity in ply orientation angle. The variation in buckling load due to stochasticity of core material properties (Figure 16) is found to be higher than the other three individual cases (Figures 14,15 and 17). However, the maximum variation in normalised buckling load is observed in case of combined stochasticity of core and face sheet thickness, ply-orientation angle of face sheet and material properties (Figure 18). The effect of different boundary conditions (CCCC,

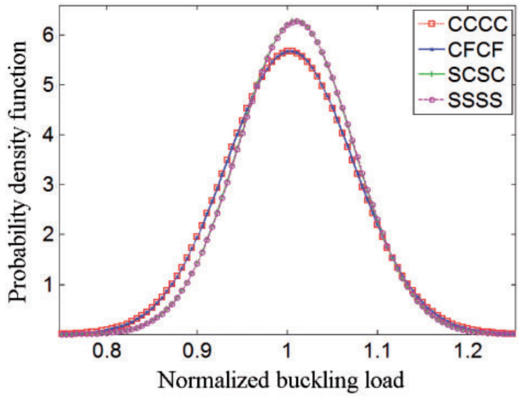

Figure 19. Effect of boundary condition on normalised buckling load of sandwich plates.

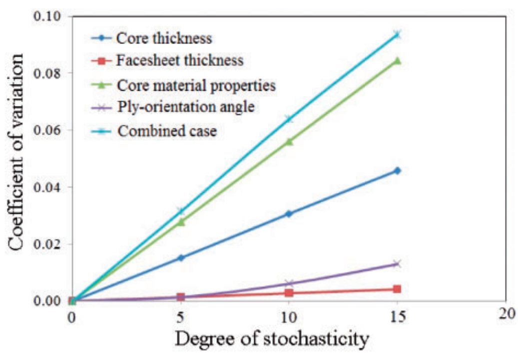

Figure 20. Coefficient of variation on buckling load with respect to degree of stochasticity of input parameters for simply supported sandwich plates.

CFCF, SCSC and SSSS; where S – simply supported, C – clamped and F – fixed end condition) on normalised stochastic buckling load of sandwich plates is presented in Figure 19. Even though the response bounds for different boundary conditions for normalised buckling load does not vary, the probability distributions for buckling loads in actual values will vary significantly depending on their deterministic values. The coefficient of variation corresponding to different degrees of stochasticity for different cases considered in this study is plotted in Figure 20. From the figure it is evident that the effect on buckling load due to stochastic variation of different input parameters in a decreasing order is: combined variation of all stochastic input parameters, core material properties, core thickness, ply orientation angle and face sheet thickness. The slope of the curves for different parameters corresponding to different degrees of stochasticity provides a clear interpretation about their relative sensitivity towards buckling load.

Conclusions

This article illustrates the layer-wise propagation of uncertainties in sandwich skewed plates in an efficient surrogate based bottom-up framework. The probability distributions of first three natural frequencies and buckling load are analysed considering both individual and combined stochasticity in input parameters. A multivariate adaptive regression splines (MARS)-based approach is developed in conjunction with finite element modelling to map the variation of first three natural frequencies and buckling load caused due to uncertain input parameters, wherein it is found that the number of finite element simulations is exorbitently reduced compared to direct Monte Carlo simulation without compromising the accuracy of results. The computational expense is reduced by (1/78) times (individual effect of stochasticity) and (1/39) times (combined effect of stochasticity) of direct Monte Carlo simulation. The skew angle is found to be most sensitive to the frequencies corresponding to the first three modes. The mass density and transverse shear modulus are other two effective factors for the first three natural frequencies among the considered stochastic input parameters, respectively. The combined effect of the material properties of soft-core has the most sensitivity for buckling load, followed by core thickness, ply orientation angle and face sheet thickness, respectively.

Novelty of the present study includes probabilistic characterisation of natural frequencies and buckling load for laminated sandwich plates following an efficient MARS-based uncertainty propagation algorithm. The numerical results presented in this article shows that stochasticity in different material and geometric properties of laminated sandwich plates has considerable influence on the dynamics and stability of the structure. Thus, it is of prime importance to incorporate the effect of stochasticity in subsequent analyses, design and control of such structures. The proposed MARS-based uncertainty quantification algorithm can be extended further to explore other stochastic systems in future course of research.

Declaration of Conflicting Interests

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