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Short Communication

On quantifying the effect of noise in surrogate based stochastic free vibration analysis of laminated composite shallow shells

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ABSTRACT

This paper presents the effect of noise on surrogate based stochastic natural frequency analysis of composite laminates. Surrogate based uncertainty quantification has gained immense popularity in recent years due to its computational efficiency. On the other hand, noise is an inevitable factor in every real-life design process and structural response monitoring for any practical system. In this study, a novel algorithm is developed to explore the effect of noise in surrogate based uncertainty quantification approaches. Representative results have been presented for stochastic frequency analysis of spherical composite shallow shells considering Kriging based surrogate model. The finite element formulation for laminated composite shells has been developed based on Mindlin's theory considering transverse shear deformation. The proposed approach for quantifying the effect of noise is general in nature and therefore, it can be extended to explore other surrogates under the influence of noise.

1. Introduction

Composite materials and structures have gained immense attention from the research community initiated from industrial needs over the last few decades. Such structures are extensively used in aerospace, marine, construction and other industries because of their application specific tailorable material properties along with weight sensitivity and cost-effectiveness. Due to the inherent complexity, laminated composite materials are difficult to manufacture accurately according to its exact design specifications, resulting undesirable uncertain responses due to random material and geometric properties, which in turn affect the vibration characteristics of the structure. Moreover, due to different forms of damages and defects, effective material properties may vary considerably from the specified values. Many researchers have studied free vibration characteristics of composite plates and shells following deterministic framework [1–10] in last couple of decades. Recent studies on doubly curved composite shells include vibration analysis considering elastic restraints and including the effects of shear deformation, rotary inertia and initial curvature [6], non-linear free vibration analysis [7] and large amplitude in free vibration analysis of thermally post-buckled composite doubly curved panel [8]. Tornabene et al. [9] have studied free vibration of laminated composite doubly-curved shells and panels by using Radial Basis Function (RBF). Versino et al. [10] have proposed a four-node shell element based on the Refined Zigzag Theory for doubly curved multilayered composites. To tackle the effect of stochasticity, in recent years researches have concentrated to quantify uncertainties associated with different output responses of laminated composites [11–13]. One of the most popular approach is Monte Carlo simulation (MCS) based uncertainty quantification [14,15]. However the major drawback of the traditional MCS method for uncertainty quantification is its computational intensiveness as thousands of finite element simulations are needed to be carried out in this approach. To overcome this lacuna, surrogate based uncertainty quantification approach has been proposed [16–19], which has gained huge attention in last couple of years.

In surrogate based approach of uncertainty quantification, a virtual mathematical model is formed for the response quantity of interest that effectively replaces the actual expensive finite element model. The surrogate model is built up on the basis of information acquired regarding the behavior of the response quantity throughout the entire design space utilizing few algorithmically chosen design points. There remains another inevitable source for second phase of uncertainty associated with the information acquired

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through the design points that needs further attention (refer to Fig. 2). In the present paper, this second source of uncertainty associated with the surrogate model formation has been addressed by developing a novel algorithm to account it in the form of random noise. The effect of such simulated noise can be regarded as considering other sources of uncertainty besides conventional material and geometric uncertainties, such as error in measurement of responses, error in modeling and computer simulation and various other epistemic uncertainties involved with the system. Noise effects are found to be accounted in several other studies in available literature [20–23] dealing with deterministic analysis. In the present article an algorithm has been developed to quantify the effect of noise for Kriging based stochastic analysis of doubly curved composite shells (refer to Fig. 1(a)). Kriging model has been successfully applied for uncertainty quantification recently and this has been found to be one of the most effective methods for surrogate based uncertainty quantification of laminated composites [16,24]. To the best of authors' knowledge, this work is the first attempt of its kind for assessing any surrogate based uncertainty propagation algorithm under the effect of noise. The results presented for different levels of noise have been compared by using probability density function to provide a comprehensive idea about stochastic structural responses under influence of simulated noise.

2. Mathematical formulation for laminated composite shell

In present study, a composite cantilever shallow spherical shells with uniform thickness 't' and principal radii of curvature R_x and R_y along x- and y-direction respectively is considered as furnished in Fig. 1. Based on the first-order shear deformation theory, the displacement field of the shells can be described as

$$\begin{aligned} u(x,y,z) &= u^0(x,y) - z\theta_x(x,y) \\ v(x,y,z) &= v^0(x,y) - z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) = w(x,y) \end{aligned} \quad (1)$$

where u^0, v^0, and w^0 are displacements of the reference plane and θ_x and θ_y are rotations of the cross section relative to x and y axes, respectively. Each of the thin fiber of laminae can be oriented at an arbitrary angle 'θ' with reference to the x-axis. The constitutive equations [25] for the shell are given by

$$\{F\} = [D(\bar{\omega})]\{\varepsilon\} \quad (2)$$

where Force resultant $\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$

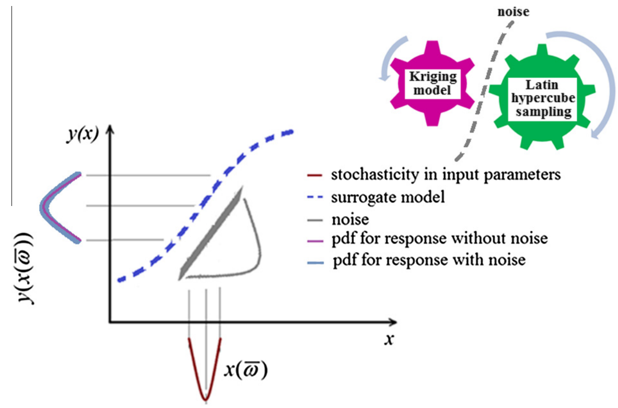

Fig. 2. Surrogate based analysis of stochastic system under the influence of noise.

$$\{F\} = \left[\int_{-h/2}^{h/2} \{\sigma_x,\ \sigma_y,\ \tau_{xy},\ \sigma_{xz},\ \sigma_{yz},\ \tau_{xyz},\ \tau_{xz},\ \tau_{yz}\}\ dz\right]^T$$

and strain $\{\varepsilon\} = \{\varepsilon_x, \varepsilon_y, \varepsilon_{xy}, k_x, k_y, k_{xy}, \gamma_{xz}, \gamma_{yz}\}^T$.

The elements of elastic stiffness matrix $[D(\bar{\omega})]$ can be expressed as

$$[A_{ij}(\bar{\omega}), B_{ij}(\bar{\omega}), D_{ij}(\bar{\omega})] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\{\bar{Q}_{ij}(\bar{\omega})\}_{on}]_k [1, z, z^2]dz \quad i, j = 1, 2, 6$$
$$[S_{ij}(\bar{\omega})] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha_s [\bar{Q}_{ij}]_k dz \quad i, j = 4, 5 \quad (3)$$

where $\bar{\omega}$ indicates the stochastic representation and α_s is the shear correction factor (=5/6) and $[\bar{Q}_{ij}]$ are elements of the off-axis elastic constant matrix which is given by

$$\begin{aligned} [\bar{Q}_{ij}]_{off} &= [T_1(\bar{\omega})]^{-1}[\bar{Q}_{ij}]_{on}[T_1(\bar{\omega})]^{-T} \quad \text{for } i,\ j = 1,2,6 \\ [\bar{Q}_{ij}]_{off} &= [T_2(\bar{\omega})]^{-1}[\bar{Q}_{ij}]_{on}[T_2(\bar{\omega})]^{-T} \quad \text{for } i,\ j = 4,5 \end{aligned} \quad (4)$$

$$[T_1(\bar{\omega})] = \begin{bmatrix} m^2 & n^2 & 2mn \\ n^2 & m^2 & -2mn \\ -mn & mn & m^2 - n^2 \end{bmatrix} \text{ and } [T_2(\bar{\omega})] = \begin{bmatrix} m & -n \\ n & m \end{bmatrix} \quad (5)$$

in which $m = Sin\theta(\bar{\omega})$ and $n = Cos\theta(\bar{\omega})$, wherein $\theta(\bar{\omega})$ is random fiber orientation angle.

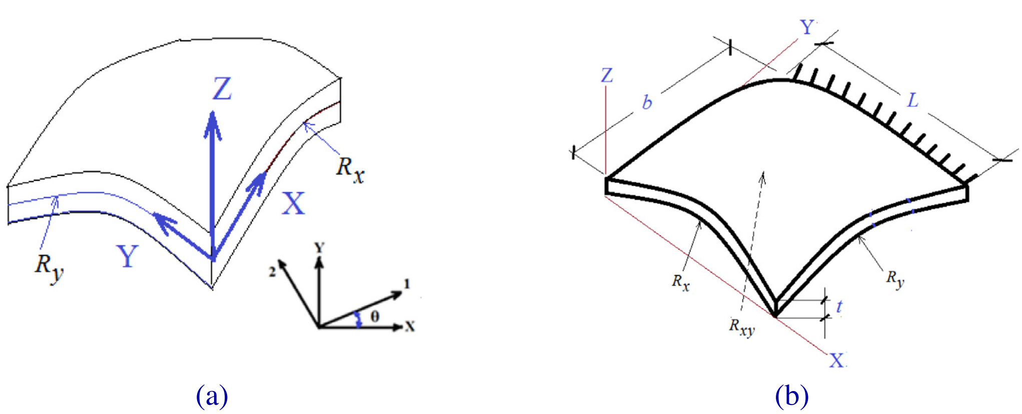

Fig. 1. (a, b) Laminated composite doubly curved spherical shallow shell model.

$$[Q_{ij}(\bar{\omega})]_{on} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{12} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \text{ for } i,\ j = 1,\ 2,\ 6$$

$$[\bar{Q}_{ij}(\bar{\omega})]_{on} = \begin{bmatrix} Q_{44} & Q_{45} \\ Q_{45} & Q_{55} \end{bmatrix} \text{ for } i,\ j = 4, 5 \tag{6}$$

where

$$Q_{11} = \frac{E_1}{1 - \nu_{12}\nu_{21}},\ Q_{22} = \frac{E_2}{1 - \nu_{12}\nu_{21}},\ Q_{12} = \frac{\nu_{12}\ E_2}{1 - \nu_{12}\nu_{21}},$$
$$Q_{66} = G_{12},\ Q_{44} = G_{23} \text{ and } Q_{55} = G_{13}.$$

In case of doubly curved shallow shell elements are employed to model the middle-surface geometry more accurately. An eight noded isoparametric quadratic element with five degrees of freedom at each node (three translations and two rotations) is considered. The mass per unit area for composite spherical shell is expressed as

$$P(\bar{\omega}) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho(\bar{\omega}) dz \tag{7}$$

The mass matrix can be expressed as

$$[M(\bar{\omega})] = \int_{Vol} [N][P(\bar{\omega})][N] d(vol) \tag{8}$$

The stiffness matrix is given by

$$[K(\bar{\omega})] = \int_{-1}^{1} \int_{-1}^{1} [B(\bar{\omega})]^T\ [D(\bar{\omega})][B(\ \bar{\omega})] d\varsigma d\chi \tag{9}$$

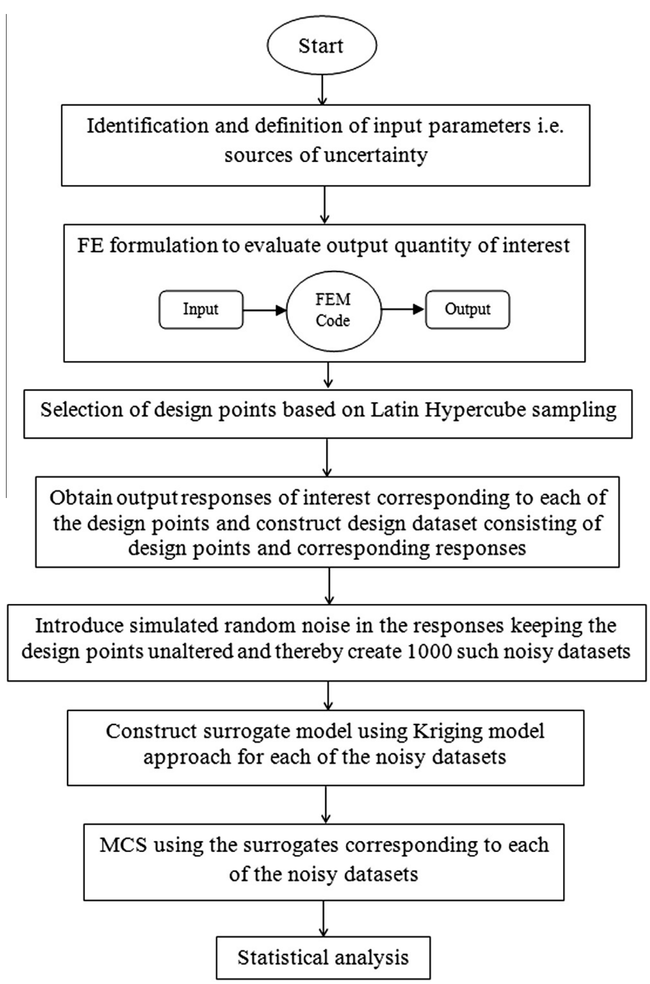

Fig. 3. Flowchart for analyzing the effect of noise on surrogate based uncertainty quantification.

Using Hamilton's principle [26] and Lagrange's equation, the dynamic equilibrium equation for the equation of motion of free vibration system with n degrees of freedom can be expressed as

$$[M(\bar{\omega})][\ddot{\delta}] + [K(\bar{\omega})]\{\delta\} = 0 \tag{10}$$

In the above equation, $M(\bar{\omega})$ and $[K(\bar{\omega})]$ are the mass and elastic stiffness matrices, respectively while $\{\delta\}$ denotes the vector of generalized coordinates. The governing equations are derived based on Mindlin's theory incorporating transverse shear deformation. For free vibration, the random natural frequencies $[\omega_n(\bar{\omega})]$ are determined from the standard eigenvalue problem using QR iteration algorithm [27].

3. Kriging based surrogate modeling

A surrogate is a mathematical and statistical approximation of the input–output relationship that is implied by the underlying simulation model. Kriging is a Gaussian process based surrogate modeling method, which is compact and cost effective for computation. Kriging model [28] for simulation of required output is expressed as,

$$y(x) = y_0(x) + Z(x) \tag{11}$$

where $y(x)$ is the unknown function of interest, x is an m dimensional vector (m design variables), $y_0(x)$ is the known approximation (usually polynomial) function and $Z(x)$ represents is the realization of a stochastic process with mean zero, variance, and nonzero covariance. The $y_0(x)$ term is similar to a polynomial response surface, providing global model of the design space [29]. In present study, $y_0(x)$ globally approximates the design space, $Z(x)$ creates the localized deviations so that the Kriging model interpolates the p-sampled data points for composite shallow spherical shells. The covariance matrix of $Z(x)$ is given as

$$Cov[Z(x^i),\ Z(x^j)] = \sigma^2 R[R(x^i,\ \ x^j)] \tag{12}$$

where R is a $(p \times p)$ correlation matrix and $R(x^i, x^j)$ is the correlation function between any two of the p-sampled data points x^i and x^j. R is an $(p \times p)$ symmetric matrix with ones along the diagonal. The correlation function $R(x^i, x^j)$ is specified by the user, and a variety of correlation functions exist. Using Gaussian correlation function

$$R(x^i, x^j) = \exp\left[-\sum_{k=1}^{n} \theta_k |x_k^i - x_k^j|^2\right] \tag{13}$$

where n is the number of design variables, θ_k is the unknown correlation parameters used to fit the model, and $x_k^i$and x_k^j are the k-th components of the sample points x^i and x^j, respectively. The predicted estimates, $\hat{y}$ of the response $y(x)$ at random values of x are defined as Kriging predictor

$$\hat{y}(x) = \hat{\beta} + r^T(x)\ R^{-1}\ [y - f\hat{\beta}] \tag{14}$$

where y is the column vector of length p that contains the sample values of the frequency responses and f is a column vector of length p that is filled with ones when $y_0(x)$is taken as constant. Now, $r^T(x)$ is the correlation vector of length p between the random x and the sample data points $\{x^1,\ x^2, \ldots x^p\}$

$$r^T(x) = [R(x, x^1),\ R(x, x^2),\ R(x, x^3) \ldots R(x, x^p)]^T \tag{15}$$

$$\hat{\beta} = (f^T R^{-1} f)^{-1}\ f^T R^{-1} y \tag{16}$$

An estimate of the variance between underlying global model $\hat{\beta}$ and y is estimated by

$$\hat{\sigma}^2 = \frac{1}{p}(y - f\hat{\beta})^T R^{-1}(y - f\hat{\beta}) \tag{17}$$

Table 1

Non-dimensional fundamental frequencies $\left[\omega=\omega_n a^2\sqrt{\frac{12\rho(1-\nu_{12}^2)}{E_1 t^2}}\right]$ of isotropic, corner point-supported shells considering $a/b = 1$, $a'/a = 1$, $a/t = 100$, $a/R = 0.5$, $\nu_{12} = 0.3$.

R_x/R_y	Shell type	Present FEM	Leissa and Narita [2]	Chakravorty et al. [25]
1	Spherical	50.74	50.68	50.76

Now the model fitting is accomplished by maximum likelihood (i.e., best guesses) for θ_k. The maximum likelihood estimates (i.e., "best guesses") for the θ_k in Eq. (13) used to fit a Kriging model are obtained as

$$Max.\Gamma(\theta_k) = -\frac{1}{2}[p\ln(\hat{\sigma}^2) + \ln|R|] \tag{18}$$

where the variance σ^2 and $|R|$ are both functions of θ_k, is solved for positive values of θ_k as optimization variables. For determining the quality of fitted model, maximum mean square error (*MMSE*) and maximum error (*ME*) are calculated as,

$$MMSE = \max\left[\frac{1}{k}\sum_{i=1}^{k}(\bar{y}_i - y_i)^2\right] \tag{19}$$

$$ME(\%) = Max\left[\frac{y_{i,MCS} - y_{i,Kriging}}{Y_{i,MCS}}\right] \tag{20}$$

where $y_i$and $\bar{y}_i$ are the vector of the true values and the vector corresponding to i-th prediction, respectively. Latin hypercube sampling [30] has been employed in this study for generating random sample points to ensure the well distributed representation throughout the vector space.

4. Effect of noise on surrogate based uncertainty quantification

In this section, the algorithm for quantifying effect of noise in surrogate based stochastic analysis of composite laminates is discussed. The finite element formulation for laminated composite shells (Section 2) has been coupled with Kriging based surrogate modeling approach (Section 3) for stochastic analysis of the first three natural frequencies including the effect of noise. The combined effect of layer wise stochasticity has been considered in material and geometric properties of the laminated composite shell as follows:

$$\begin{aligned} g\{\theta(\bar{\omega}), \rho(\bar{\omega}), G_{12}(\bar{\omega}), G_{23}(\bar{\omega}), E_1(\bar{\omega})\} \\ = \{\Phi_1(\theta_1 \,..\theta_l),\ \Phi_2(\rho_1..\rho_l)\ ,\ \Phi_3(G_{12(1)}..G_{12(l)}), \\ \Phi_4(G_{23(1)}..G_{23(l)}),\ \Phi_5(E_{1(1)}..E_{1(l)}),\ \Phi_6(E_{2(1)}..E_{2(l)})\} \end{aligned} \tag{21}$$

where θ_i, ρ_i, $G_{12(i)}$, $G_{23(i)}$, $E_{1(i)}$ and $E_{2(i)}$ are the ply orientation angle, mass density, shear modulus along longitudinal direction, shear modulus along transverse direction and elastic modulus along longitudinal and transverse direction, respectively and 'l' denotes the number of layer in the laminate. $\bar{\omega}$ is the stochastic representations for input parameters. It has been assumed that the distribution of randomness of input parameters exists within a certain band of tolerance ($\pm 10°$ for ply orientation angle and $\pm 10\%$ for material properties) with their central deterministic mean values following a normal distribution.

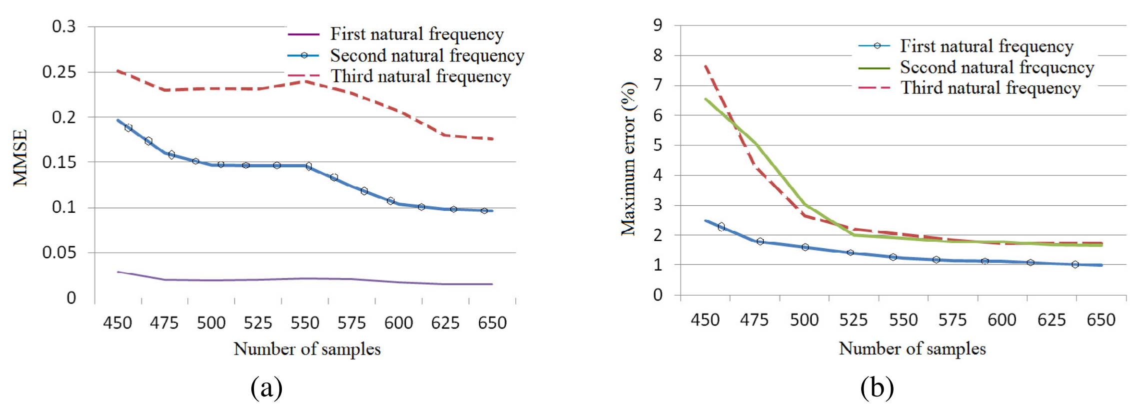

Fig. 4. Convergence study of sample size for first three natural frequencies following maximum mean square error (MMSE) and maximum error without effect of noise by Kriging and MCS.

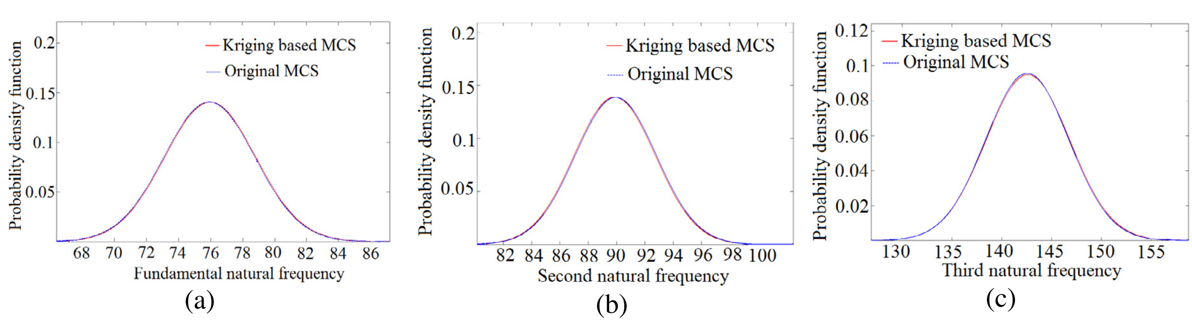

Fig. 5. Probability density function obtained by original MCS and Kriging model with respect to first three natural frequencies for combined variation of 28 stochastic input parameters for a sample size of 600 without considering the effect of noise.

A second source of uncertainty has been identified in the surrogate based uncertainty propagation approaches besides the conventionally considered uncertainties ($g\{\bar{\omega}\}$) as discussed in Section 1. Focus of the present article is to analyze effect of the same through introducing simulated noise in the system. To portray the effect of noise on surrogate based uncertainty quantification algorithm, different levels of noise has been introduced in the responses of design points while constructing the surrogate models as described in Fig. 3. In the proposed approach,

Gaussian white noise with a specific level (p) has been introduced in the set of output responses (natural frequencies), which is subsequently used for surrogate model formation:

$$F_{ijN} = f_{ij} + p \times \xi_{ij} \tag{22}$$

where f denotes natural frequency with the subscript i and j as frequency number and sample number in the design point set, respectively. ξ_{ij} is a function that generates normally distributed random numbers. Subscript N is used here to indicate the noisy frequency.

Thus simulated noisy dataset (i.e. the sampling matrix for surrogate model formation) is formed by introducing pseudo random noise in the responses, while the input design points are kept unaltered. Subsequently for each dataset, surrogate based MCS is carried out to quantify the uncertainty of composite laminates following a non-intrusive approach as described in Fig. 2. From the flowchart it can be understood that quantification of the effect of noise involves carrying out 1000 surrogate based MCS requiring formation of such surrogates 1000 times for analyzing each noise level. The kind of analysis carried out here will provide a comprehensive idea about the robustness of surrogate based uncertainty quantification algorithm under noisy data.

5. Results and discussion

A four layered graphite-epoxy symmetric angle-ply (45°/−45°/−45°/45°) laminated composite cantilever shallow doubly curved spherical ($R_x = R_y = R$ and $R_{xy} = \infty$) shell has been considered for the analysis. The length, width and thickness of the composite laminate considered in the present analysis are 1 m, 1 mm and 5 mm, respectively. Material properties of graphite–epoxy composite [1] considered with deterministic mean value as E_1 = 138.0 GPa,

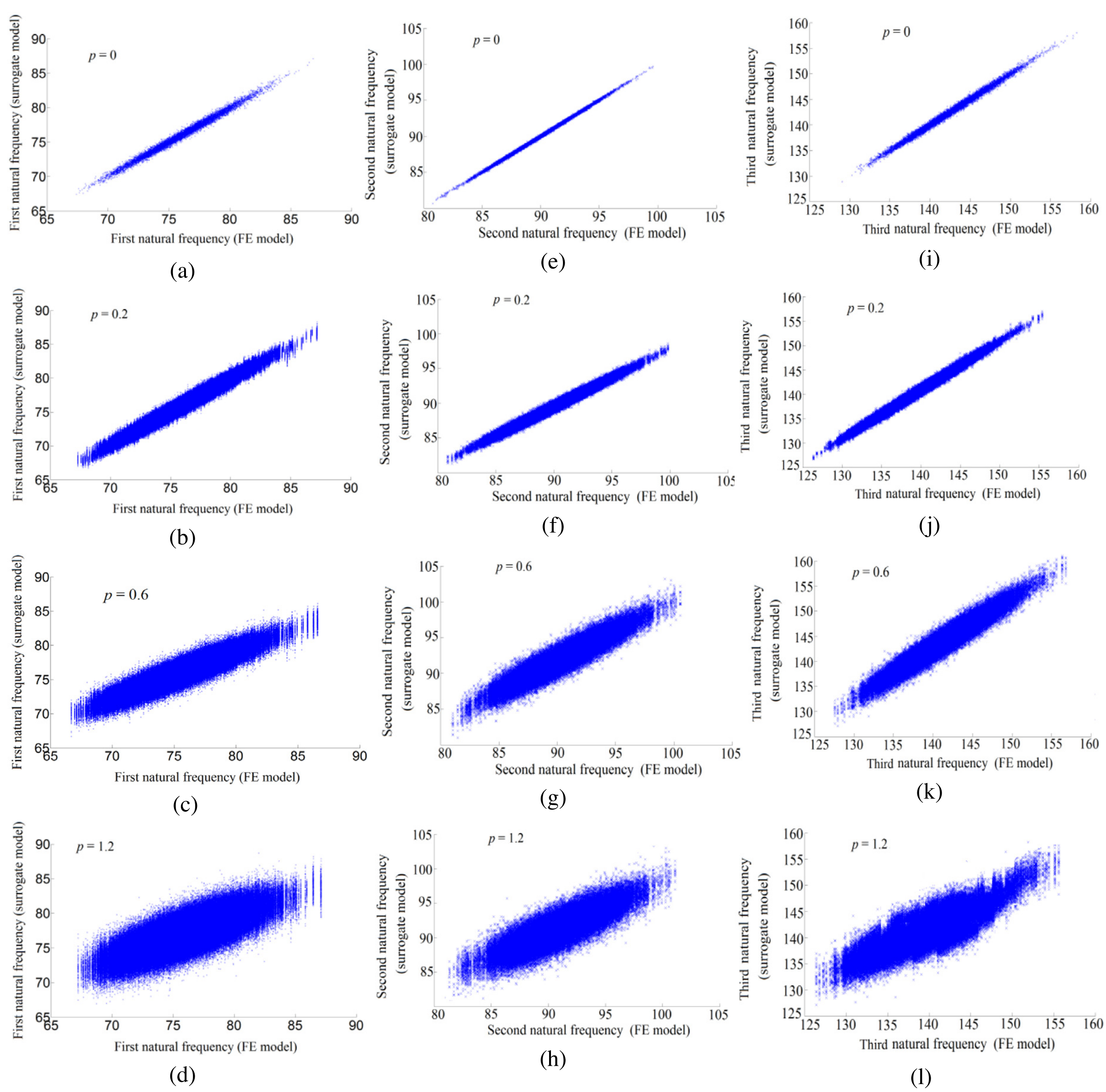

Fig. 6. Effect of noise on prediction capability of Kriging based surrogate model for first three natural frequencies for noise level (p) = 0, 0.2, 0.6 and 1.2.

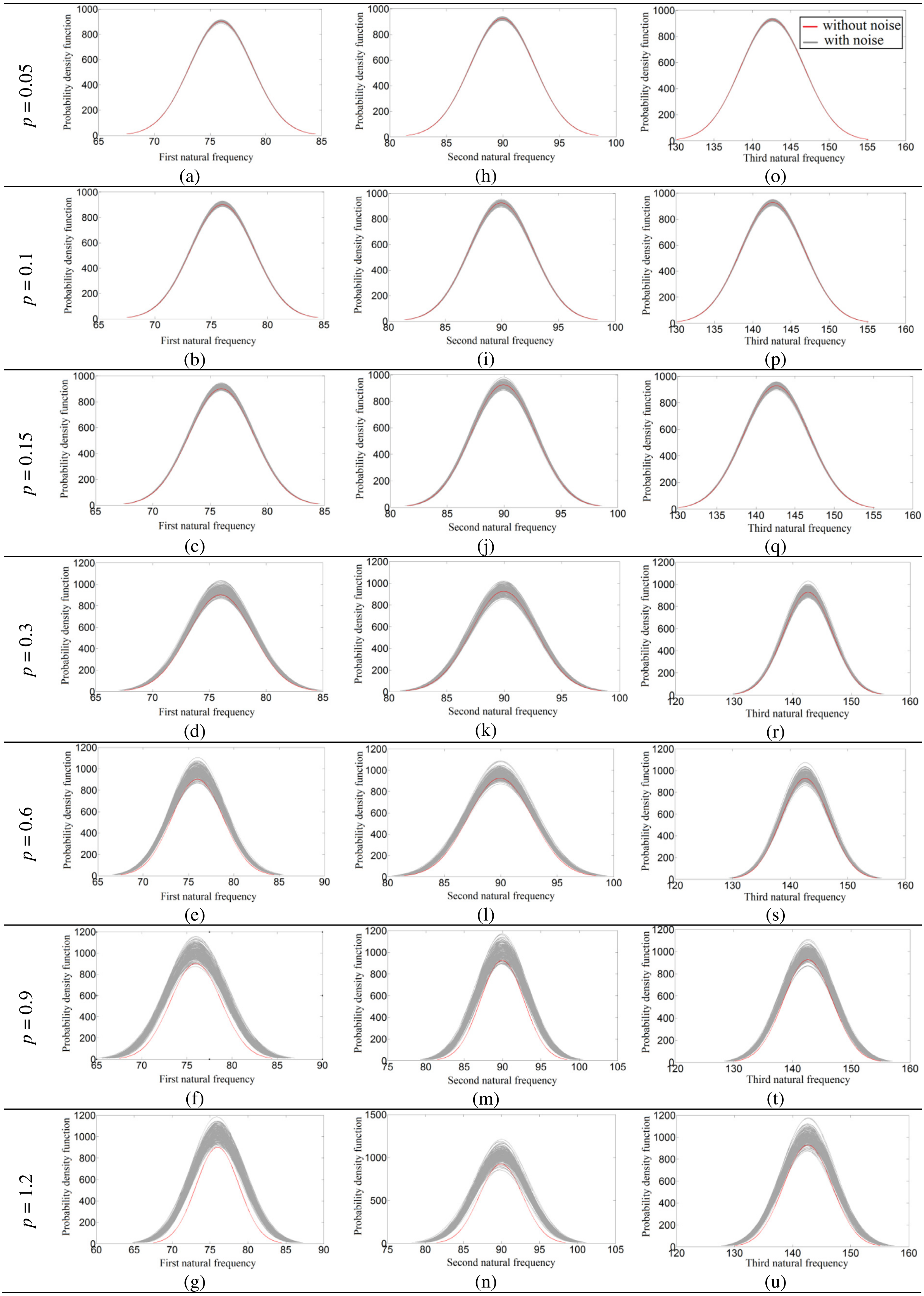

Fig. 7. Probability density function plots of first three natural frequencies for different levels of noise.

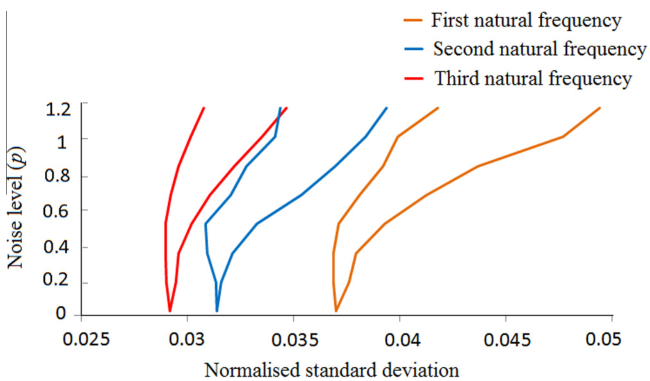

Fig. 8. Variation of normalized standard deviation with noise level for natural frequencies.

$E_2 = 8.96$ GPa, $\nu_{12} = 0.3$, $G_{12} = 7.1$ GPa, $G_{13} = 7.1$ GPa, $G_{23} = 2.84$ GPa, $\rho = 3202$ kg/m^3. A typical discretization of (6×6) mesh on plan area with 36 elements and 133 nodes with natural coordinates of an isoparametric quadratic plate bending element has been considered for the present FEM approach and the results for deterministic analysis has been validated with available literature (Table 1).

In the present study, total 28 numbers of stochastic input parameters have been considered for combined effect of uncertainty in natural frequencies. The number of samples for constructing the Kriging model has been finalized using a convergence study as shown in Fig. 4. It has been found that 600 samples provide adequate level of accuracy for all three natural frequencies. Fig. 5 shows comparative results for original MCS (10,000 FE simulations) and Kriging based MCS (600 FE simulations) without the effect of noise ($p = 0$) corroborating validity of the Kriging based surrogate model. Fig. 5 represents the stochastic natural frequencies due to conventionally considered material and geometric uncertainties. Scatter plot bounds depicting the effect of simulated random noise on the prediction of Kriging based surrogates are presented in Fig. 6 for different levels of noise using 1000 simulations for each case. From the Fig. 6, it is evident that as the level of noise increases, the deviation of the points from diagonal line also becomes more, indicating higher influence of noise on surrogate predictions. Fig. 7 presents probability density function plots for first three natural frequencies showing the effect of noise with different levels. It can be noticed from the figures that the response bounds increase with increasing level of noise for all three natural frequencies. Normalized standard deviations (with respect to deterministic values of respective natural frequencies) for the first three natural are presented in Fig. 8. The figure shows that the bounds of normalized standard deviation decreases for higher modes of frequencies indicating subsequent reduction in sensitivity of noise.

6. Conclusion

In this article, another prospective source of uncertainty has been identified in the surrogate based uncertainty propagation approaches besides the conventionally considered uncertainties and the effect of same has been analyzed through introducing simulated noise in the system. A novel algorithm for quantifying the effect of noise on surrogate based uncertainty propagation approach has been developed. The effect of such simulated noise can be regarded as inclusion of other sources of uncertainty beside the conventionally considered stochastic material and geometric parameters, such as error in measurement of responses, error in modeling and computer simulation and various other epistemic uncertainties involved with the system. The kind of analysis presented in this paper provides a thorough insight on the stochastic responses under investigation. The representative results have been presented for laminated composite spherical shallow shell based on Kriging approach considering different levels of noise, wherein it is evident that the simulated noise has considerable effect on stochastic natural frequencies of the system. Consideration of the effect of such noise is thus an important criterion for robust and comprehensive analysis of stochastic systems. Though we have concentrated on Kriging based analysis of spherical composite shells only, the proposed algorithm for quantifying effect of noise in stochastic analysis is general in nature. Thus it can be extended to other structures and to analyze other surrogates in future.

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