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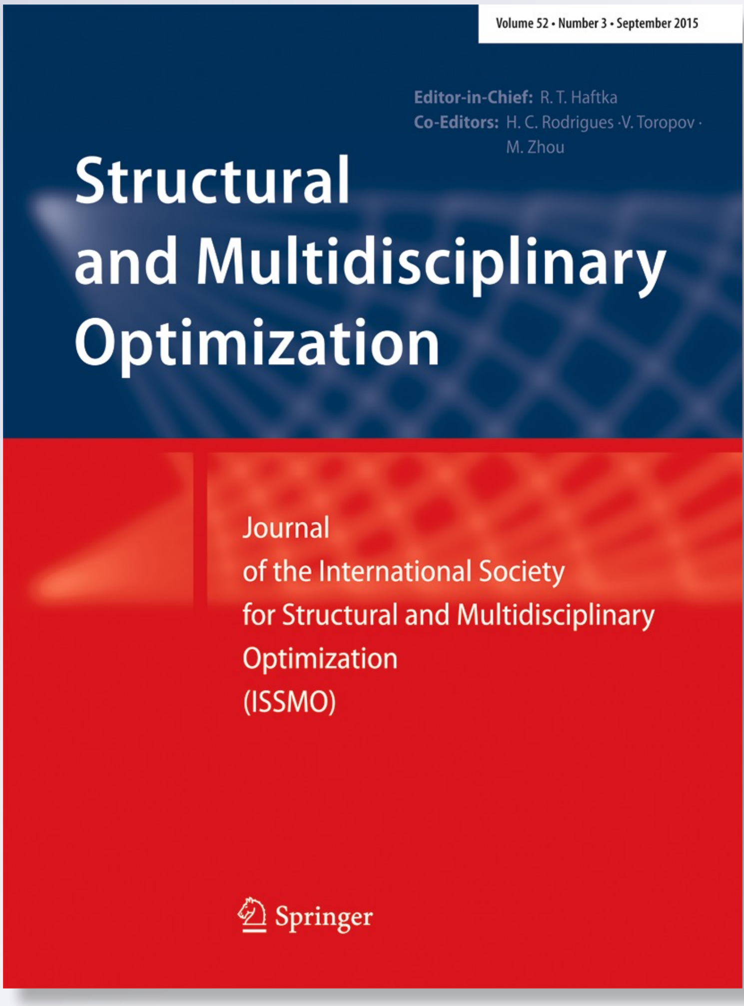

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RESEARCH PAPER

Optimum design of FRP bridge deck: an efficient RS-HDMR based approach

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Abstract A novel efficient hybrid method based on random sampling-high dimensional model representations (RS-HDMR) and genetic algorithm coupled with a local unconstrained multivariable minimization function is proposed in this study for optimization of FRP composite web core bridge deck panels. The optimization is performed for lightweight design of FRP composite bridge deck panels based on deflection limit, stresses, buckling and failure criteria and subsequently the representative design curves are developed considering normal as well as skew configurations of FRP bridge decks. Sensitivity analysis is also performed to study the effect of variation in geometry of the bridge deck to its deflection, stress and buckling behaviours. High level of computational efficiency can be achieved without compromising the accuracy of results for optimization of high dimensional systems following the proposed approach.

Keywords FRP bridge deck · lightweight design · RS-HDMR · Genetic algorithm · Sensitivity analysis · Optimization

Nomenclature

F	Applied stress
f_a	Permissible stress
N	Applied buckling load
N_{cr}	Permissible load
δ	Observed deflection
δ_a	Permissible deflection
L	Deck length
D	Depth of web core
W	Width of the deck
t_b	Thickness of bottom face plate
t_t	Thickness of top face plate
t_w	Thickness of web webs
n	Number of webs in the core

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1 Introduction

Fiber reinforced polymer (FRP) composite laminates are extensively used in various civil engineering applications especially in bridge deck. Composite structures have high specific strength, specific stiffness, corrosion resistance, enhanced fatigue life, non-magnetic properties, controllable thermal properties, lower life-cycle costs, faster field installation in addition to application specific tailorable mechanical properties. Such useful features of FRP composites over conventional materials motivate their use in structures for new construction as well as for rehabilitation purpose. A review of the existing literature pertaining to the use of FRP composites reveals that FRP has a wide range of application such as various structural forms like reinforcement bars, jacketing of girders and piers, FRP layer sandwiched between concrete slabs, cables for cable stayed bridges, cellular decks adhesively bonded to steel girders, bridge columns made of concrete-filled FRP tubes, structural stay-in-place form, slabs for bridges and many more (Bakis et al. (2002),Keller (2002)). Over the last few decades, many theoretical and experimental (both laboratory and in-situ) investigations have been performed to study the behaviour of FRP laminated web core/sandwich decks/panels for new construction as well as retrofitting of old bridge decks. (Plunkett (1997), Qiao et al. (2000), Davalos et al. (2001) Salim et al. (2006) Keller et al. (2007), Aref et al. (2001), Aref and Alampalli (2001)). These studies are mainly focused on analysing different type of FRP bridge deck configurations and their static as well as dynamic behaviour under various constituent material properties and ply orientations. In last few decades short span pedestrian bridges constructed using FRP only are becoming increasingly popular. These constructions, consisting of a lightweight core and stiff laminated FRP face sheets provides high mechanical performance with minimum weight, which have enhanced the potential of FRP as an alternative to the conventional reinforced concrete bridges. Reducing the weight of decks/panels presents the opportunity of rapid replacement and reduction in dead load which also raises the live load rating of the structure. Thus the main challenge remains in developing lightweight cost-effective FRP deck systems that can take advantage of the unique material characteristics of FRP composites necessitating study on optimization of FRP bridge decks. The optimization of structural components and material properties of FRP bridge decks is a long and complicated process, which involves a large number of design variables and constraints. Though several literatures pertaining to the optimization of structures for other applications are available but resources available related to optimization of FRP bridge decks are still very limited. Important contributions pertaining to optimization of FRP bridge decks are briefly discussed next. Cohn and Dinovitzer (1994) have presented the purpose of structural optimization in real application and the theories behind it in their paper by conducting studies on the structural optimization in practical applications and surveyed the extent of practical use of optimization in structural engineering. The authors have suggested that optimization could become more attractive to practising designers if more concrete examples of its application were available, especially for realistic structures, loading conditions and limit states after examining the possible causes for its rather limited and slow adoption. Optimization studies related to traditional concrete bridges are found in Zordan et al. (2010); Briseghella et al. (2013), Aydın and Ayvaz (2010), Nieto et al. (2009), Ohkubo et al. (2008) and Barros et al. (2012) . Optimization of FRP bridge deck with cellular and stiffened box geometries have been presented by Burnside et al. (1993). Salem (2000) has developed an optimum design algorithm for precast FRP bridges by minimizing the cost with the help of Excel Solver based on a nonlinear optimization technique. Kovacs et al. (2004) have analysed fibre-reinforced composite Structures using experimental and numerical data and carried out optimization for minimizing the cost of the sandwich structure. Kim et al. (2003) have devised an optimization procedure based on modified genetic algorithm (GA) which was used in conjunction with a commercial FE analysis engine to obtain optimum design for GFRP bridge deck. A three dimensional numerical model has been developed by Park et al. (2005) to optimize the geometry of bridge decks and properties of FRP materials. The proposed optimization technique is capable of handling more complex objective functions and constraints. Uddin and Abro (2008) have demonstrated a unique approach for the design and manufacturing of structurally efficient low cost thermoplastic composite bridge superstructures. Qiao et al. (2008) have conducted a combined homogenization and multi-objective technique to study and optimize the properties of thin-walled sinusoidal honeycomb cores implementing a sequential quadratic programming code with the help of tools available in MATLAB.

Gradient based method (GBM) of optimization has been studied for FRP web core bridge deck system by Dey et al. (2013). This method is found to be iterative in nature, thus time consuming and computationally intensive. Optimization of FRP web core bridge deck using GBM needs to run computationally expensive finite element simulations several times and the post processing work followed by finite element simulations is also tedious. To overcome this lacuna a novel hybrid technique based on random sampling-high dimensional model representations (RS-HDMR) and genetic algorithm coupled with a local unconstrained multivariable minimization function is proposed in this paper for optimization of FRP web core bridge deck. In this approach, the actual finite element model is replaced by a surrogate model based on RS-HDMR, making the overall process computationally much more efficient and cost effective as the number of actual finite element simulations required in this case becomes minimal. High-dimensional model representations (HDMR) is a quantitative model assessment and analysis tool for meta-model formation, which maps high-dimensional input–output system relationship very efficiently (Rabitz and Alıs (1999)). When data are randomly sampled for constructing the meta-model, a RS (random sampling)-HDMR can be constructed (Li et al. (2002a); Li et al. (2006); Ziehn and Tomlin (2009)). Over the years, HDMR has been successfully applied in many different fields (Li et al. (2002b); Ziehn and Tomlin (2008a); Ziehn and Tomlin (2008b); Miller et al. (2012)), but literature related to its application in the area of structural mechanics is still very scarce. Cut-HDMR has been recently applied in the field of uncertainty quantification and stochastic analysis of structural systems (Chowdhury and Rao (2009); Chowdhury et al. (2009a); Chowdhury et al. (2009b); Balu and Rao (2011); Balu and Rao (2012)). Application of RS-HDMR in the area of structural mechanics is found only in sensitivity analysis of unreinforced masonry structure (Mukherjee et al. (2011)) and stochastic free vibration analysis of angle-ply composite plates (Dey et al. (2015a)). RS-HDMR has not been applied in the realm of optimization yet. Literature related to the application of RS-HDMR in the field of FRP composite structures is very scanty, in spite of the fact that it has a great potential to be used as surrogate of expensive finite element simulation/experimentations of repetitive nature, particularly for composite structures for its capability of efficiently handling large number of input parameters, which is commonly encountered in case of composite materials. Moreover RS-HDMR possesses some of the critically desirable features for optimization such as good prediction capability of the meta-model in the whole design space including the tail regions. Prime novelty of the present work lies in application of RS-HDMR approach in the area of FRP bridge deck optimization to achieve high level of computational efficiency. Furthermore Application of genetic algorithm coupled with RS-HDMR for optimization of a system is also the first attempt of its kind to the best of authors' knowledge. The present paper is organized as, Section 1: Introduction, Section 2: Description of the FRP web core bridge deck optimization problem, Section 3: Detailed description of the proposed RS-HDMR based hybrid optimization algorithm, Section 4: Description of the considered FRP web core bridge deck, Section 5: Finite element modelling and validation, Section 6: Result and discussion and section 7: Conclusion.

2 Description of the FRP bridge deck optimization problem

Optimization can be carried out at structural, geometrical and material levels for FRP bridge decks. In this section, design objective function and the design constraints for the present optimization problem are discussed.

2.1 Design objective function

In the present research, a typical optimization has been carried out for structural components. Objective function has been chosen to minimize the volume of the FRP bridge decks to thrive for the recent trend of lightweight structures as discussed in the previous section. The following objective functions provide the volume of the FRP material to be used:

For normal bridge deck configurations

$$V = [L \times t_b + L \times t_t + n \times t_w \times (D - t_b - t_t)] \times W \tag{1}$$

For skew bridge deck configurations

$$V = [L \times t_b \times cos\theta + L \times t_t \times cos\theta + n \times t_w \times (D - t_b - t_t)] \times W \tag{2}$$

where L, D, W, θ, t_b, t_t, t_w, and n are the deck length, depth, width, skew angle, thickness of bottom plate, thickness of top plate, thickness of web and the number of webs respectively (Fig. 1).

2.2 Design constraints

The design constraints are varied depending upon the applicability of the structural components. The constraints considered in this present study for design requirements and serviceability are normal and shear stresses in longitudinal and transverse direction, buckling strength, deflection due to live load obtained following the design provisions specified by Kim et al. (2003) Table 1), in addition to different failure criteria such as Tsai-Hill, Tsai-Wu, Azzi-Tsai-Hill and Maximum stress theory (Tsai(1987), Tsai(1965), Tsai and Wu(1971), Azzi and Tsai (1965), Hinton et al. (2004)).

The deflection limit considered for the present work is *L/800* based upon AASTHO (1998) specification. In Table 1, δ, f, N represents the maximum deflection, stresses, and buckling load respectively. Subscripts 'a' and 'cr' denote the allowable and critical values of these parameters.

3 Optimization methodology

Optimization is about finding out the maximum or minimum of an objective function having either linear or nonlinear nature. Various optimization techniques are

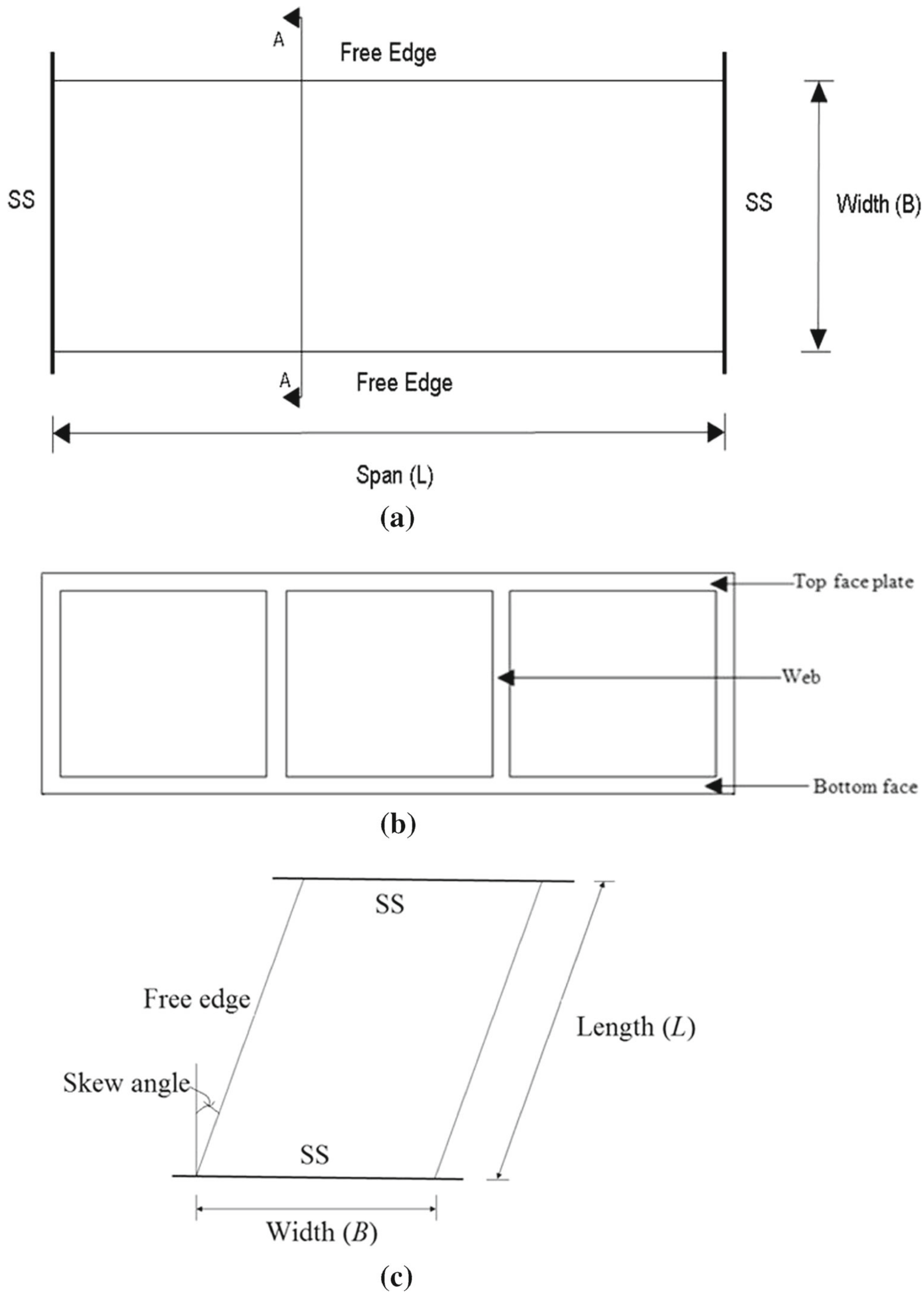


Fig. 1 **a** Plan and **b** Cross-sectional view of typical FRP bridge deck system having normal configuration **c** Plan of a typical FRP bridge deck system having skew configuration (SS denotes simply supported)

Table 1 Constraints for design requirements and serviceability

Descriptions	Conditions
Maximum stress	$f/f_a-1<0$
Local bucking load	$N/N_{cr}-1<0$
Deflection due to live load	$\delta/\ \delta_a-1<0$

available for single objective function with linear as well as nonlinear design constraint. In this section, RS-HDMR, genetic algorithm and details of the proposed hybrid optimization algorithm for FRP composite bridge deck have been described.

3.1 Surrogate modelling based on RS-HDMR

There are many sampling techniques as well as surrogate model formation methods available as furnished

Table 2 Sampling techniques and surrogate modelling methods

Sampling techniques	Surrogate modelling methods
- Classical methods (Design of experiments)	- Polynomial regression method
• Factorial designs	- Moving least squares method
• Central composite design	- Kriging model
• Optimal designs	- Artificial neural networks
• Taguchi's orthogonal array design	- Radial basis function
• Plackett-Burman design	- Polynomial chaos expansion
• Koshal design	- Multivariate adaptive regression splines
• Box-Behnken design	- Support vector regression
- Space filling methods	- High dimensional model representation
• Latin hypercube sampling	- Hybrid models
• Sobol sequence	
• Uniform designs	
• Simple Grids	
• Hammersley sequence	
- Hybrid methods	
- Random or human selection	
- Importance sampling	
- Sequential or adaptive methods	

in Table 2 (Montgomery (1991), Santner et al. (2003), Dey et al. (2015b, 2015c), Koehler and Owen (1996), Forrester et al. (2008)). Sampling technique and surrogate modelling method for a particular problem should be chosen depending on the complexity of the model, presence of noise in sampling data, nature and dimension (number) of input parameters, desired level of accuracy and computational efficiency. Many comparative studies have been performed over the years to guide the selection of surrogate model types (Jin et al. (2001), Kim et al. (2009), Li et al. (2010), Mukhopadhyay et al. (2015)). RS-HDMR is a new approach for surrogate model formation and it has shown very promising characteristics in the application of mainly Chemical and Environmental sciences as mentioned in Section 1. The application of RS-HDMR for surrogate model formation in the area of structural engineering and mechanics (specifically in FRP composite structures) is very scarce. There are few literatures available dealing with application of RS-HDMR only in the fields of uncertainty quantification and sensitivity analysis (Mukherjee et al. (2011), Dey et al. (2015a)). However, successful applications of RS-HDMR in these fields motivated the authors to explore its performance in the problem of optimization. In the present study, RS-HDMR method has been chosen after rigorous checking of its prediction capability following different criteria as discussed later in this section.

RS-HDMR method can provide a straight forward approach to develop the input–output mapping of a high dimensional model without requiring a large number of samples (Li et al. (2002a); Li et al. (2006)). The mapping between the input variables $x_1, x_2, \ldots, x_n$ and the output variables $f(X)=f(x_1, x_2, \ldots, x_n)$ in the domain R^n can expressed in the following form:

$$f(X) = f_o + \sum_{i=1}^{n} f_i(x_i) + \sum_{1 \le i \le j \le n} f_{ij}(x_i, x_j) + \ldots + f_{12\ldots n}(x_1, x_2, \ldots, x_n) \quad (3)$$

Each term in the representation reflects the independent and cooperative contributions of the inputs upon the output. Here f_o is a constant (zeroth order) denoting the mean of all outputs. The function $f_i(x_i)$ is a first order term giving the effect of variable x_i acting independently upon the output $f(X)$. The function $f_{ij}(x_i,x_j)$ is a second order term describing the cooperative effects of the variables x_i and x_j upon the output $f(X)$. The higher order terms reflect the cooperative effects of increasing numbers of input variables acting together to influence the output $f(X)$. The last term $f_{12\ldots n}(x_1,x_2,\ldots, x_n)$ contains any residual n^{th} order correlated contribution of all input variables. If there is no interaction between the input variables, then only the zeroth order term and the first order terms will appear in the RS-

HDMR expansion. In most of the cases a RS-HDMR expression up to second order provides satisfactory results (Li et al. (2002a)). The component functions of RS-HDMR have the following forms:

$$
\begin{aligned}
f_o &= \int_{K^n} f(x)dx \\
f_i(x_i) &= \int_{K^{n-1}} f(x)dx^i - f_o \\
f_{ij}(x_i, x_j) &= \int_{K^{n-2}} f(x)dx^{ij} - f_i(x_i) - f_j(x_j) - f_o
\end{aligned} \tag{4}
$$

where dx^i stands for the product $dx_1dx_2\ldots dx_n$ without dx_i and dx^{ij} stands for the same product without dx_i and dx_j. The last term $f_{12\ldots n}(x_1, x_2, \ldots, x_n)$ is evaluated from the difference between $f(X)$ and all the other component functions.

RS-HDMR expansion can be constructed when sample points are random in nature in a domain R^n. All the input variables are rescaled such a way that $0 \leq x_i \leq 1$ for all i. The output response function is thus defined in the domain of an unit hypercube $K^n = \{(x_1, \ldots, x_n), i = 1, \ldots, n\}$. Generation of random sample points is a very important aspect for forming the RS–HDMR surrogate model. The quality of sample points governs the convergence rate of Monte Carlo simulation, which is used in formation of the component functions of RS–HDMR (Ziehn and Tomlin 2008a). Quasi-random sequences (Niederreiter (1992)) (e.g., Halton sequence (Halton (1960)), Sobol' sequence (Sobol' (1967)), Faure sequence (Faure (1992))) having low discrepancy are generally used to generate random sample points. This ensures a more uniform distribution of sample points in the input domain compared to pseudo-random sample points resulting faster convergence. This property of RS-HDMR, which corroborates good prediction and identification capability of the surrogate model throughout the entire domain including the tail regions, could be potentially quite useful in optimization problems. In the present work, Sobol' sequence has been used for generating the input sample set as it exhibits better convergence than either Faure or Halton sequences (Galanti and Jung (1997)).

The zeroth order term f_o in the RS-HDMR expression can be calculated by the average value of all $f(X)$ in a set of samples. The determination of the higher order component functions involve high-dimensional integrals that may be approximately calculated by Monte Carlo (MC) integration. Because the direct determination of high-order RS-HDMR component functions by Monte Carlo integration is prohibitively expensive, analytical basis functions, such as orthonormal polynomials, cubic B spline functions, and polynomials can be employed to approximate RS-HDMR component functions. With such approximations, only one set of random samples is needed to determine all RS-HDMR component functions making the sampling effort quite reasonable. Approximation using orthonormal polynomials provides the best accuracy among the different analytical basis functions (Li et al. (2002a)). Thus the first and second order component functions are expressed in the following form:

$$
f_i(x_i) \approx \sum_{r=1}^{k} \alpha_r^i \varphi_r(x_i) \; f_{ij}(x_i, x_j) \approx \sum_{p=1}^{l} \sum_{q=1}^{l'} \beta_{pq}^{ij} \varphi_p(x_i) \varphi_q(x_j) \tag{5}
$$

where k, l, l' represent the order of the polynomial expansion, α_r^i and β_{pq}^{ij} are the constant coefficients which are determined by a minimization process and MC

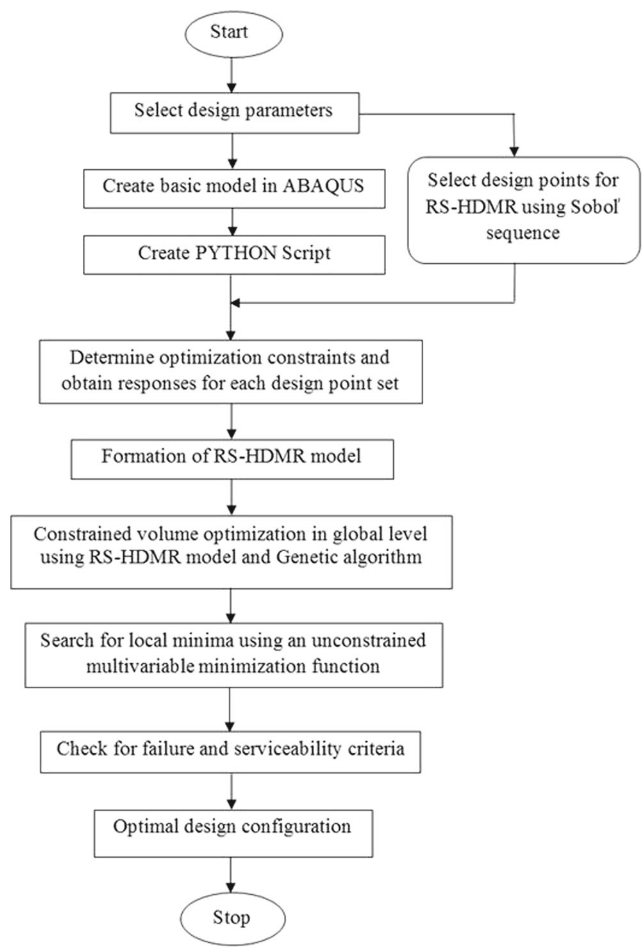


Fig. 2 Flowchart for proposed optimization of FRP composite bridge deck algorithm

integration. For a sample set $X^{(s)}=(x_1^{(s)}, x_2^{(s)},\ldots,x_n^{(s)})$, $s=1, 2,\ldots,N$ these constant coefficients are evaluated as:

$$\alpha_r^i \approx \frac{1}{N}\sum_{S=1}^{N} f\left(X^{(s)}\right)\varphi_r\left(x_i^{(s)}\right);\ \beta_{pq}^{ij}\approx\frac{1}{N}\sum_{S=1}^{N} f\left(X^{(s)}\right)\varphi_p\left(x_i^{(s)}\right)\varphi_q\left(x_j^{(s)}\right) \tag{6}$$

$\varphi_r(x_i), \varphi_p(x_i)$ and $\varphi_q(x_j)$ are the orthonormal basis functions. An orthonormal basis function $\varphi_k(x)$ has the following properties in a domain $[a,b]$

$$\int_a^b \varphi_k(x)dx = 0 \quad k = 1,2,\ldots \quad \text{(i.e. zero mean)} \tag{7}$$

$$\int_a^b \varphi_k(x)\varphi_l(x)dx = 0 \quad k\neq l \text{(i.e. mutually orthogonal)}$$

$$\int_a^b \varphi_k^2(x)dx = 1 \quad k = 1,2,\ldots \text{(i.e. unit norm)}$$

From the above conditions, the orthonormal polynomials can be readily constructed in the domain [0, 1] as:

$$\begin{aligned}\varphi_1(x) &= \sqrt{3}(2x-1)\\ \varphi_2(x) &= 6\sqrt{5}(x^2-x+\frac{1}{6})\\ \varphi_3(x) &= 20\sqrt{7}(x^3-\frac{3}{2}x^2+\frac{3}{5}x-\frac{1}{20})\\ &\vdots\end{aligned} \tag{8}$$

Thus the final equation of RS-HDMR up to second order component functions takes the following form:

$$f(X) = f_o + \sum_{i=1}^{n}\sum_{r=1}^{k}\alpha_r^i\varphi_r(x_i) + \sum_{1\le i\le j\le n}\sum_{p=1}^{l}\sum_{q=1}^{l'}\beta_{pq}^{ij}\varphi_p(x_i)\varphi_q\left(x_j\right) \tag{9}$$

The error of the MC integration for calculating the expansion coefficients α and β controls the accuracy of the RS-HDMR expansion. Variance reduction methods can be applied to improve the accuracy of the MC integration without increasing the sample size. Correlation method (Li et al. (2003)) and ratio control variate method (Li and Rabitz (2006)) have been successfully applied for this purpose. In both cases the determination of the expansion coefficients is an iterative process and requires an analytical reference function $h(X)$ which has to be similar to $f(X)$. A truncated RS-HDMR expansion can be used as a reference function by calculating its expansion coefficients using direct MC integration.

As the HDMR component functions are independent, the order of the polynomial approximation can be chosen separately for each component function to improve the accuracy of the final meta-model. For highly nonlinear input–output relationship, higher-order polynomials may be used. But unnecessary use of higher order polynomials may lead to poor ap-

Table 3 Design bridge deck configurations

Case	Length (m)	Width (m)	Depth (mm)	Skew angle (Degree)	Number of webs	Number of configurations
1	2.5	1	150	0	5, 7, 11	12
			175	0	5, 7, 11	
			200	0	5, 7, 11	
			225	0	5, 7, 11	
2	2	1	150	0	5, 7, 11	36
				30	5, 7, 11	
				45	5, 7, 11	
			175	0	5, 7, 11	
				30	5, 7, 11	
				45	5, 7, 11	
			200	0	5, 7, 11	
				30	5, 7, 11	
				45	5, 7, 11	
			225	0	5, 7, 11	
				30	5, 7, 11	
				45	5, 7, 11	
3	1.5	1	150	0	5, 7, 11	12
			175	0	5, 7, 11	
			200	0	5, 7, 11	
			225	0	5, 7, 11	

proximation, especially if a small sample size N is used to form the meta-model. The reason behind this is, higher order polynomials have more number of terms than lower-order polynomials, and each term has its own MC integration error. If the input–output relationship of a component function is only linear, then it is sufficient to use first-order polynomial only. Sometimes if the contribution of a certain RS-HDMR component function to the overall function value is close to zero, then it can be excluded from the RS-HDMR expansion function completely. A computationally efficient optimization technique based on least square method has been developed to choose the best polynomial order for each of the component functions (Ziehn and Tomlin (2008b)). The idea behind this optimization technique is to calculate the sum of the square errors using the results of the full model runs of sample size N and the approximation of the component functions by either first, second or third-order polynomials or excluding the component function. The best approximation order for the corresponding component function is indicated by the smallest sum of square error. Further, a threshold has also been introduced (Ziehn and Tomlin (2008a)) to exclude unimportant terms from the RS-HDMR expansion for the systems having high number of input parameters.

The RS-HDMR model constructed should be checked for the R^2 (coefficient of determination) value, which should be close to 1.

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}\left(0 \le R^2 \le 1\right) \tag{10}$$

Where, $SS_T = SS_E + SS_R$ is the total sum of square, SS_R and SS_E are regression sum of squares and residual sum of squares respectively. The quality of the surrogate model obtained using RS–HDMR can also be determined by Relative Error (RE), computed as follows

$$RE(\%) = \frac{|F - F'|}{F} \times 100 \tag{11}$$

where F is the actual response and F' is the approximated response using HDMR. It is stated by how many of the tested samples (in percentage) are within a range of 1, 5 and 10 % of the relative error. Further, two graphical plots can also be obtained, a scatter plot showing the relationship between the original model and the HDMR meta-model and a comparison of the empirical probability distribution functions (pdf) for both, the original model and the HDMR meta-model. These plots are very useful in judging the quality of the constructed meta-model.

Table 4 Elastic properties (compressive) (Aref and Alampalli (2001))

Composite laminate elastic properties of FRP glass/vinlyester (QM6408)	
E_L	23 GPa
E_T	18 GPa
G_{LT}	9 GPa
ν_{LT}	0.25
Density	1826 kg/m^3

Table 5 Ultimate strength properties (compressive) (Aref and Alampalli (2001))

Composite laminate elastic properties of FRP glass/vinlyester (QM6408)	
σ_{LU}	483 MPa
σ'_{LU}	427 MPa
σ_{TU}	331 MPa
σ'_{TU}	324 MPa
σ_{LTU}	179 MPa

3.2 Genetic algorithm coupled with an unconstrained multivariable minimization function

Charles Darwin first stated the process of natural selection and evaluation. This philosophy of survival of the fittest facilitates to solve numerical optimization problems, where natural evaluation and adaptation to environmental variation is simulated mathematically by using genetic algorithm (GA). This robust algorithm works based on an iterative procedure consisting of a constant-sized population of individuals, usually encoded as binary strings (chromosomes), representing candidate solutions in a given search space comprising of all the possible solutions to the optimization problem (Goldberg (1989); Deb (2001)). The initial population of individuals is generated randomly. It is then encouraged to evolve over generations to produce new better or fitter generations using genetic operators until the problem is satisfactorily solved. An elitist selection scheme is used to obtain the new generation taking organisms from the current population and from the children population just created. This process is repeated until the convergence criterion is met. The three fundamental genetic operators are selection (this is according to the fitness of individual solutions so that the number of times an individual is selected is dependent on its relative performance in the

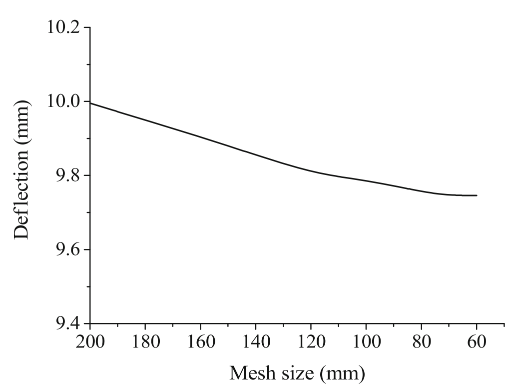


Fig. 3 Convergence study for mesh divisions

Table 6 Validation of model with Aref et al. (2001)

Dimensions (mxm)	Skew angle	Deflection(mm)			
		Central		Maximum	
		Present model	Aref et al. (2001)	Present model	Aref et al. (2001)
7.807×10.072	0	8.9253	8.504	9.7796	9.447
	30	5.8762	5.637	7.800	7.228

population.) crossover (this is performed to form new individuals by exchanging chromosome between two selected individuals segments) and mutation (this prevents premature convergence by randomly changing part of one selected individual's chromosome). Some of the applications related to GA in the area of structural engineering can be found in Roy and Chakraborty (2009), António (2009) and Woon et al. (2001).

An unconstrained multivariable minimization function has been coupled with genetic algorithm in this study in order to improve the value of the fitness function. Genetic algorithm searches the results globally first and after the GA terminates a local search is employed with the end results of GA as its initial point to search for the local minimum using the unconstrained multivariable minimization function *fminunc* in Matlab Version 8.2.0.701 (R2013b) (2013) (Fletcher (1980); Wu et al. (2014)).

3.3 Detailed optimization algorithm for FRP bridge deck

Optimization of a practical FRP bridge deck problem includes many continuous discrete variables, discontinuous and non-convex design spaces. Optimization can be performed for geometry as well as material property. The proposed method can be applied to all types of optimization. Different structural components of FRP bridge deck like thicknesses of top plate (t_t), bottom plate (t_b), web core (t_w), length (L) and depth (d) of bridge deck, number of webs have been considered for optimization in the present study subjected to the constraints as discussed in section 2.2. A flowchart of the proposed optimization algorithm is provided in Fig. 2. The steps that have been followed for the optimization process are summarized below:

Step 1: Design variables, objective function and optimization constraints are identified first. In the present study thicknesses of top plate, bottom plate and web core have been optimized for different combinations of length, depth and number of webs to obtain the minimum volume of FRP bridge deck.

Step 2: Finite element model of FRP bridge and ultimately Python script has been developed using graphical user interface (GUI) of ABAQUS CAE 6.8 (2008) in the next step. Then Python script has been parameterized to obtain the responses (optimization constraints) of a FRP bridge deck corresponding to different set of top plate thickness, bottom plate thickness, and web core thickness, length of bridge deck and overall depth of bridge deck. Thus the modified Python script is capable of performing multiple runs taking different set of input values (design points) of the above mentioned structural design parameters.

Step 3: In this step the governing constraining criteria are identified for the optimization problem and then RS-HDMR model is formed for the chosen criteria (response) of the FRP composite bridge deck in terms of top plate thickness, bottom plate thickness and web core thickness by using the modified Python script obtained in the previous step.

Step 4: Once the RS-HDMR models corresponding to a particular structural combination (length, depth of bridge deck and number of webs) for the chosen constraining criteria are constructed, the final step is to perform the constrained optimization. Thus overall optimization problem for each set of length, depth and number of webs combination, considered in this study can be described as follows:

$$\min_{t_t,t_b,t_w} V(t_t,t_b,t_w), \quad \text{such that} \quad \begin{array}{l} \delta(t_t,t_b,t_w)\le\delta_a \\ f(t_t,t_b,t_w)\le f_a \\ N(t_t,t_b,t_w)\le N_{cr} \\ FOS(t_t,t_b,t_w)\ge 1 \\ lb\le t_t,t_b,t_w\le ub \end{array} \tag{12}$$

Here δ, f, N represents the maximum deflection, stresses, and buckling load respectively. Subscripts 'a' and 'cr' denote the allowable and critical values of these parameters. FOS represents the factor of safety value considering different failure criteria (Tsai-Hill, Tsai-Wu, Azzi-Tsai-Hill and Maximum stress Theory). lb and ub represent respective lower bound and upper bound of top plate, bottom plate and web core thickness.

For solving the optimization problem described in (12), GA coupled with an unconstrained multivariable minimization function has been employed in this study. Here fitness function is the expression of volume in terms of top plate thickness, bottom plate thickness

Table 7 Optimization results for different normal configurations of FRP bridge deck

Bridge deck dimension(m)	Depth(mm)	Optimization technique	Number of Webs											
			5				7				11			
			Optimized plate thickness (mm)			Volume (m^3)	Optimized plate thickness (mm)			Volume (m^3)	Optimized plate thickness (mm)			Volume (m^3)
			Web	Top plate	Bottom plate		Web	Top plate	Bottom plate		Web	Top plate	Bottom plate	
2.5 m x 1 m	150	GBM	18	16	18	0.309	18	15	17	0.305	18	14	15	0.304
		Present method	17.1	17.3	16.35	0.307	15.22	16.82	17.74	0.304	15.22	15.44	15.20	0.303
	175	GBM	15	15	14	0.270	14	12	15	0.261	15	11	13	0.264
		Present method	14.94	14.54	13.96	0.270	12.95	13.54	14.46	0.259	12.35	12.59	13.37	0.262
	200	GBM	12	13	11	0.228	12	11	12	0.230	11	11	11	0.235
		Present method	12.31	11	13	0.227	11.28	11	12.81	0.229	10.71	10.63	11.64	0.235
	225	GBM	12	11	9	0.204	12	9	9	0.199	12	8	9	0.214
		Present method	11.31	9	11.02	0.202	10.06	8.84	10.18	0.197	9.15	9.21	9.69	0.209
2 m x 1 m	150	GBM	15	13	15	0.208	14	12	14	0.202	14.5	11	12	0.203
		Present method	15	11.88	13.29	0.206	15	13.52	14.23	0.205	14.25	11	12.20	0.203
	175	GBM	14	11	11.1	0.179	12	11	11	0.178	11.25	10	10	0.179
		Present method	11.89	11	11	0.178	13.38	11.85	11	0.179	11.19	10	10	0.179
	200	GBM	12.21	10	9.21	0.153	12	8.2	8.4	0.157	10	8.25	10.25	0.171
		Present method	11.41	8.14	8.52	0.150	10.82	11.19	9	0.160	10.21	8.95	9.11	0.170
	225	GBM	10.2	8	8	0.135	10	7.25	7.5	0.138	8	6.5	9	0.147
		Present method	9.82	7.65	7.22	0.138	11	8.75	7	0.140	8.47	7.33	7.42	0.146
1.5 m x 1 m	150	GBM	13	9	10.5	0.119	12	8.5	8.5	0.113	11	7	7	0.109
		Present method	13	9.07	10.44	0.119	12	8.24	8.69	0.112	10.89	7	7	0.109
	175	GBM	12	8	9	0.110	11	7	7	0.102	9	7	6	0.102
		Present method	12	8.27	8.06	0.107	10.75	7	7.02	0.097	9.65	6.02	6	0.101
	200	GBM	10	7	6.75	0.094	10	6	6	0.094	9	5	5	0.095
		Present method	9.89	7.66	6	0.093	9.17	6	6	0.091	8.69	5.21	5.11	0.095
	225	GBM	10	7	6	0.094	7	5	7	0.085	7	5	5	0.089
		Present method	8.27	7.80	6	0.092	8	5.04	5.05	0.081	6.96	4.82	5.02	0.088

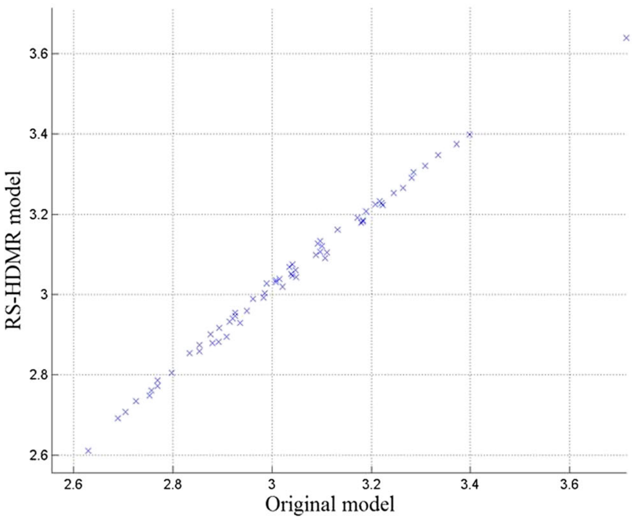
Table 8 Optimized configuration of FRP bridge deck having dimensions 2 m x 1 m (skew configuration)

Numbers of webs	Depth (mm)	Skew angle (degree)	Optimized Configuration (mm)			Volume (m^3)
			Web thickness	Top plate thickness	Bottom plate thickness	
5	150	0	15	11.88	13.29	0.206
		30	13.56	10.28	9.89	0.161
		45	9.9	8.3	6.98	0.103
	175	0	11.89	11	11	0.178
		30	13.20	10.10	9.81	0.126
		45	10.54	6.25	5.89	0.094
	200	0	11.41	8.14	8.52	0.150
		30	11	7.60	7.61	0.116
		45	8.23	5.88	5.12	0.083
	225	0	9.82	7.65	7.22	0.138
		30	10.76	6	6.12	0.116
		45	7.53	5.09	4.33	0.076
7	150	0	15	13.52	14.23	0.205
		30	15.2	11.8	11.71	0.161
		45	12.1	8.2	8.23	0.102
	175	0	13.38	11.85	11	0.179
		30	13.16	10.11	9.82	0.145
		45	10.6	7.1	6.8	0.092
	200	0	10.82	11.19	9	0.160
		30	11.4	9.23	8.23	0.132
		45	9.15	6.3	6.2	0.087
	225	0	11	8.75	7	0.140
		30	11.37	9.20	8.27	0.118
		45	8.25	6.1	5.31	0.077
11	150	0	14.25	11	12.20	0.203
		30	13.32	10.11	9.7	0.160
		45	9.21	7.31	6.6	0.101
	175	0	11.19	10	10	0.179
		30	11.5	8.4	7.9	0.146
		45	9.24	5.6	5.15	0.092
	200	0	10.21	8.95	9.11	0.170
		30	9.12	7.87	7.64	0.136
		45	7.15	5.34	5.20	0.087
	225	0	8.47	7.33	7.42	0.146
		30	8.44	6.12	6.33	0.122
		45	6.79	4.11	4.41	0.080

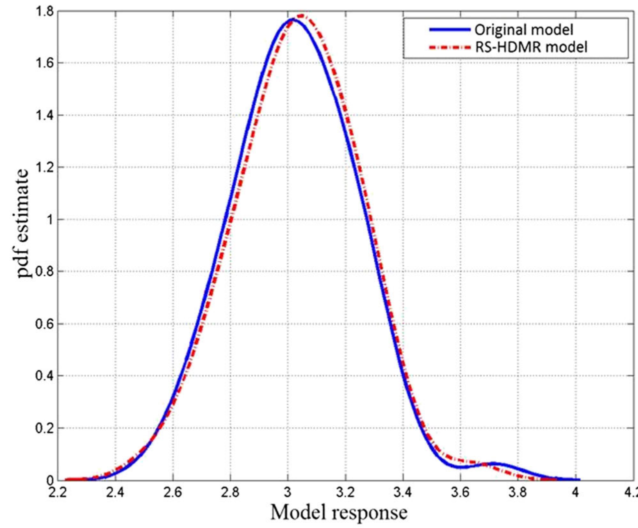
Table 9 Convergence study for coefficient of determination (R^2) and the relative error (*RE*) of a typical 2.5 m normal bridge deck having 7 number of webs and 200 mm depth

Sample size	R^2	*RE* (1 %)	*RE* (5 %)	*RE* (10 %)
32	70.23 %	71.21 %	79.32 %	86.54 %
64	99.27 %	94.06 %	100 %	100 %
128	99.34 %	95.20 %	100 %	100 %

and web core thickness. Initial values of different design variables to commence the optimization process are decided based on deflection criterion considering equal thickness for top plate, bottom plate and webs. The population size has been decided based on a convergence study of the value of the fitness function. A stochastic uniform selection criterion has been adopted for choosing parents to create the next generations. Two fittest individuals have been guaranteed to survive for the next generation after each iteration following the



(a)



(b)

Fig. 4 **a** Typical scatter plot representing the original model vs. RS-HDMR model responses showing its capability of predicting outputs (deflection) **b** Typical probability distribution function plot for the responses (deflection) of original model and meta-model in case of the 2.5 m normal bridge deck having 7 webs

elitist selection scheme. An adaptive mutation strategy has been employed that can randomly generate directions which are adaptive with respect to the last successful or unsuccessful generation. The direction and step length are chosen to satisfy the bounds and constraints of the optimization problem in this algorithm. The crossover operation is applied by generating a random number to define the crossover point with an aim to create new better organisms (children) in the reproduction process by combining genetic information taken from a pair of organisms (parents) selected from the current population (Almeida and Awruch (2009)) in this study. Global solution is obtained using GA following the above specifications. A local search function is then employed with the end results of GA as its initial point to search for the local minimum in order to improve the solution further.

4 Design problem of FRP web core bridge deck

The web core bridge decks considered in this study are made up of FRP composite layers. The bridge deck panel systems consist of unidirectional web core between two face sheets. Normal as well as skew bridge deck configurations have been considered in this study. In case of normal configuration, three sets of deck panels have been considered for the present investigation having span of length 2.5, 2 and 1.5 m while the width is taken as 1 m for all three panels. In case of skew configuration, the length and width of the bridge deck considered are 2 and 1 m respectively having three different skew angles (0, 30 and 45 degree). Such composite decks can be used for constituting deck panels of a long span bridge as well as for short span bridges/culverts itself. The depth of web core has been varied as per the practical application. Different depths considered in this study are 150, 175, 200 and 225 mm for each set of deck panels. The numbers of webs taken into consideration are 5, 7 and 11 respectively (Table 3). The face plates contain three layers (0/90/0) and the webs contain two layers (45 / −45°) of laminated composite sheets with various overall thicknesses to be optimised.

A symmetric laminate configuration with respect to mid-plane of the bridge deck section has been considered in this study to avoid the bending-extension coupling (He and Aref (2003). Since the web core in the FRP sandwich deck is provided for transferring shear stresses and to resist local buckling, it is quite reasonable to assume the webs to have only 45° fiber orientation angle. The thickness of each layer has been considered to be same for the two face plates (consisting of three layers each) and the webs (consisting of two layers each) following standard manufacturing practice. The material properties and geometric properties such as ply orientation angle and number of layers for the present bridge deck problem have been considered based on the work by He and Aref (2003) and Aref et al. (2005), who carried out parametric studies using iterative schemes to arrive at the optimum number of layers and their orientations for FRP bridge deck panels. Therefore, the results obtained in the aforementioned two articles have been assumed regarding number of layers and ply orientations for the present problem to study the effect of other remaining parameters for reducing the computational efforts.

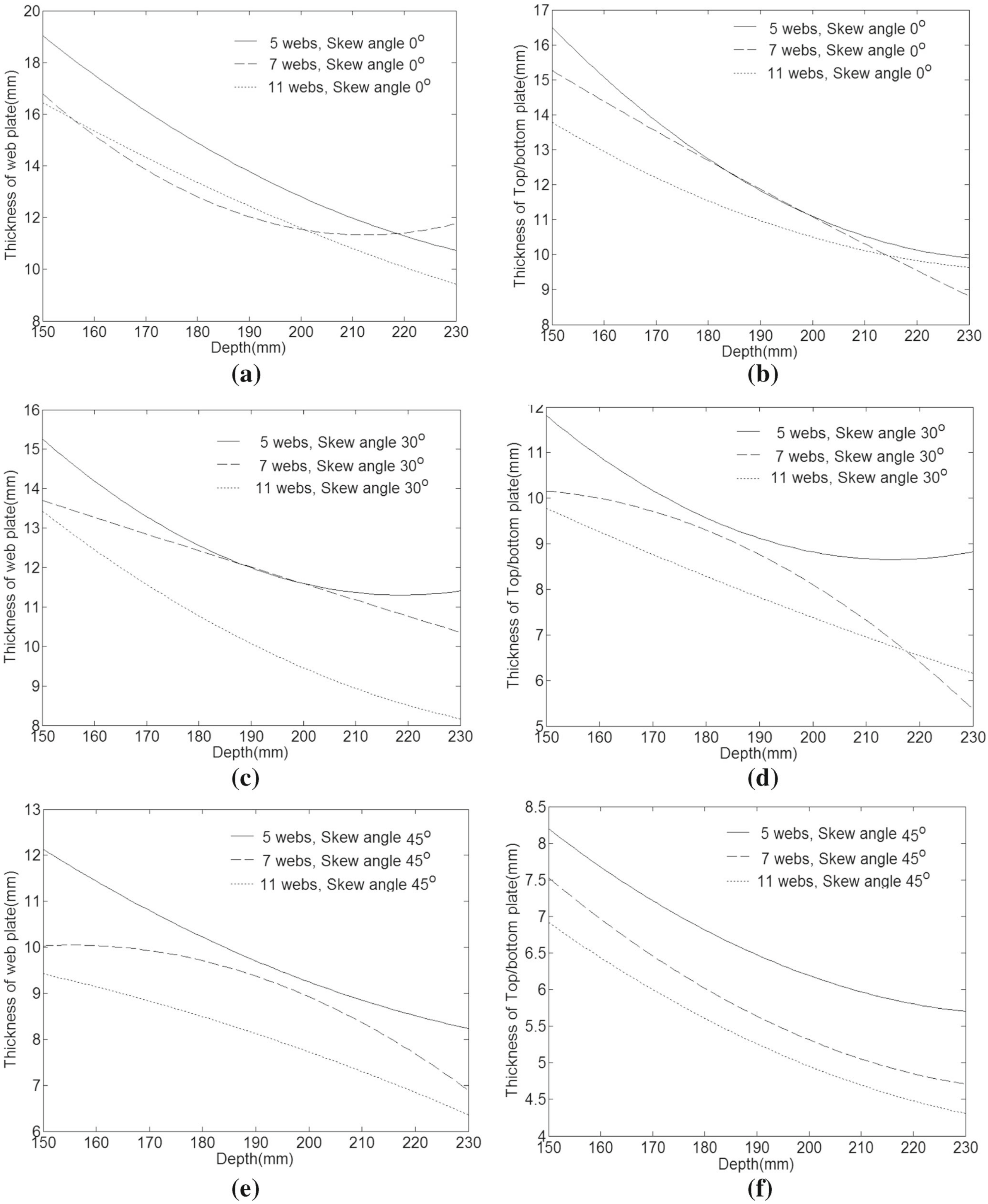
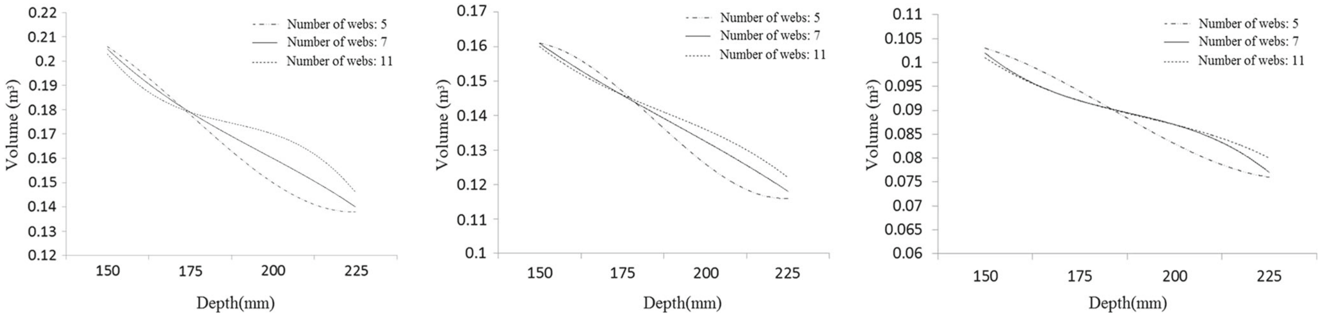


Fig. 5 Design curve for web plate thickness and top/bottom plate thickness with varying depth, skew angle and number of webs for a typical 2 m long skew bridge deck

The main emphasis in this article has been laid to establish the proposed optimum design methodology based on RS-HDMR algorithm, which is capable of handling any number of design variables quite efficiently. However, it should be noted that the



(a) Skew angle 0° (b) Skew angle 30° (c) Skew angle 45°

Fig. 6 Variation of optimized volume with depth for different configurations of FRP bridge deck having 2 m length

computational effort will be proportionately higher with the increase in number of design variables. The material properties of FRP Composites (Aref et al. (2001), Aref and Alampalli (2001)) used in the present problem are shown in Tables 4 and 5. In the present study, two opposite edges are considered as simply supported and other two edges are free (Fig. 1). The bridge deck is accommodated with a patch load of IRC 70R track vehicle loading as per IRC: 6 (2000) with impact effect of 25 % in addition to the self-weight. The structural system is subjected to a patch load of 0.11 N/mm^2.

5 Detailed finite element modelling and validation

A parametric model has been developed by Python scripting language using Graphical User Interface (GUI) of ABAQUS. The structural components of bridge deck system are top face sheet, bottom face sheet, web core and patch area. The parts of the bridge deck have been modeled using conventional shell elements (i.e., S4R: Conventional Stress/Displacement 3D Shell, 4-node, Reduced Integration) having three displacements and three rotational degrees of freedom at each node.

The shape of the shell elements are considered to be quadrilateral having linear element geometry. In mesh module, the top and bottom face plates are meshed by 70 by 70 divisions and the rib core is meshed with a global seed size of 70 mm. The mesh sizes have been finalized using convergent studies of mesh division as shown in Fig. 3. In the convergence study, the mesh divisions are varied unless the response gets saturated. The interaction between the common nodes of the individual components is defined by using tie elements and the analysis is done by general static loading option. The reason behind the usage of conventional tie members is to avoid the slip between two adjacent surfaces. The loading of 70R tracked vehicle as per IRC: 6 (2000) has been considered with the effect of impact along with the self weight. The created model has been further validated with different bridge cases having same properties as specified by Aref et al. (2001). The validations of the results are shown in the Table 6 for normal and 30° skew bridge. The python script generated in GUI environment of ABAQUS has been modified to make the parametric model.

6 Results and discussion

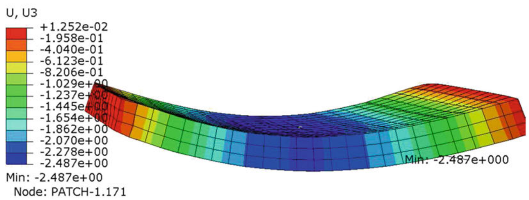
6.1 Optimization

The different structural components considered for optimization are top plate thickness, bottom plate thickness and web plate thickness for several configurations of the FRP bridge deck having various length, depth and number of webs.

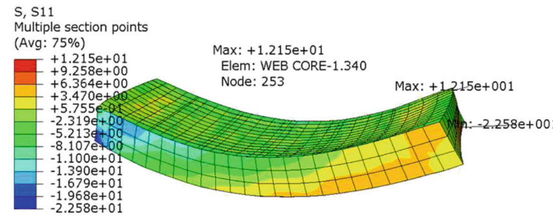
Optimization results for different normal configurations of bridge deck obtained using the present method has been

Table 10 Structural analysis results for 2 m bridge deck

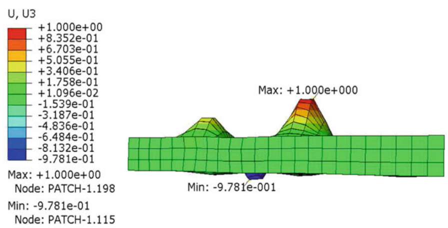
Description	Design limits	Results of analysis	Factor of safety
Stress in fiber direction (MPa)	427	12.15	35.13
Stress in transverse direction (MPa)	324	10.62	30.51
In-plane shear stress (MPa)	179	10.25	17.47
Tsai-Hill failure criteria	<1.0	0.08	12.92
Tsai-Wu failure criteria	<1.0	0.09	11.56
Azzi-Tsai-hill failure	<1.0	0.08	12.92
Maximum stress theory failure	<1.0	0.06	13.34
Buckling strength (N/sq. mm)	4.71	0.11	42.82
Maximum deflection of depth (L/800, mm)	2.5	2.49	1



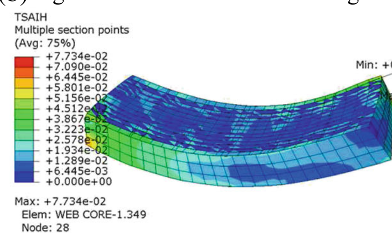
(a) Deflection contour under general static load of the bridge deck system



(b) Fig. 5. Stress contours of the bridge deck system



(c) Mode shape under buckling load of the bridge deck system



(d) Failure contours under Tsai-Hill theory of the bridge deck system

Fig. 7 Typical deflection, stress, buckling mode shape and failure contours for a 2 m long bridge deck

compared with the results using GBM, which was followed in Dey et al. (2013).Optimization results obtained using the present methodology are found to be in good agreement with the results of GBM (Table 7). For skew configurations of the bridge deck new results have been presented using the proposed method (Table 8). It is worthwhile to mention here that the number of numerical simulations needed for a typical GBM based optimization of FRP bridge can be expressed as $N=n^k$, Where N is the total number of trials, k is the number of optimization variables and n is the total numbers of terms considered for optimization from the mean value. In most of the cases, optimization in primary level was carried out considering $k=3$ and $n=5$, whereas further optimization in second level was performed considering $k=3$ and $n=7$ in Dey et al. (2013). Though the value for n may vary for second level optimization process depending on the gradient pattern of the primary optimization plot, the number of FE simulations needed in case of GBM for a typical case stated above is 468, whereas the number of FE simulations needed according to the present optimization methodology in this study is only 64 (i.e., the number of samples needed for constructing the RS-HDMR model). The prediction capability of the constructed RS-HDMR models have been checked following different criteria as mentioned in section 3.1 and they have been found out to be satisfactory. A convergence study is presented showing the coefficient of determination (R^2) and the relative error (*RE*) values for different sample sizes in Table 9. A typical scatter plot and a pdf plot for the 2.5 m normal bridge deck having 7 number of webs and 200 mm depth have been shown in Fig. 4. The low scatterness of the points around the diagonal line in Fig. 4a and the low deviation between the pdf estimations of original and RS-HDMR responses in Fig. 4b corroborates that RS-HDMR metamodels are formed with desired level of accuracy.

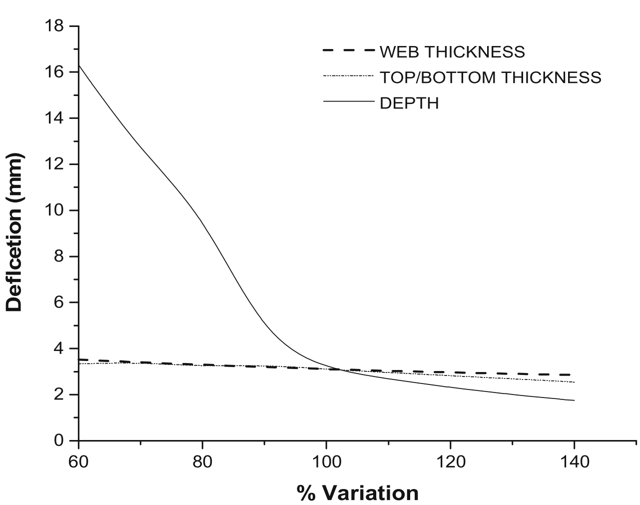


Fig. 8 Typical sensitivity analysis plot of web thickness, top/bottom plate thickness and overall depth for maximum deflection of a 2.5 m long FRP bridge deck having 5 numbers of web

The results of the present method have been considered for drawing design curves of various structural components for different configurations. For the skew bridge deck configurations having length of 2 m, web plate thickness and top/bottom plate thickness to be provided for various depths and skew angles having different number of webs are shown in Fig. 5. Due to the identical configuration of top and bottom plates having symmetrical ply orientation, the effect of the thickness of either of these components is interchangeable. Therefore, the design curves in Fig. 5 have been drawn considering equal and average thickness of the top and bottom plates. Typical variation of optimized volume for different configurations of FRP bridge deck having 2 m length with overall depth has been presented in Fig. 6. From design curves (Figs. 5 and 6), a general tendency is evident that, as the overall depth increases, web as well as face plate thickness decreases resulting decrease in the total volume of the FRP bridge deck. From Fig. 6, critical values of overall depth can be identified for all three skew angles, below which 11 web configurations produce least volume and above which five web configurations produce least volume. Such design curves presented in Figs. 5 and 6 will provide a clear guideline to the designer in deciding the geometrical dimensions of different components of FRP bridge deck having different length for lightweight design.

After optimization of various design parameters, several analyses have been performed considering the optimized dimensions of the bridge deck to check the safety against different criteria such as allowable stress, buckling and various failure criteria (Tsai-Hill, Tsai-Wu, Azzi-Tsai-Hill and Maximum stress Theory).

Factor of safety values corresponding to different failure criteria have been calculated for different optimized configurations of the bridge deck. Factor of safety against deflection has been noticed to be minimum for all configurations of bridge deck. Thus for the present study, deflection of the FRP bridge deck is the governing criterion. This is expected as the FRP materials have generally a low value of elastic modulus as compared to many conventional materials. Factor of safety values for a typical 2 m normal FRP bridge deck configuration are presented in Table 10. Typical deflection profile and the stress contours under general static load obtained in this analysis are shown in Fig. 7a and b respectively. A typical buckling mode shape for the optimized bridge deck system is also shown in Fig. 7c for the applied load. The result

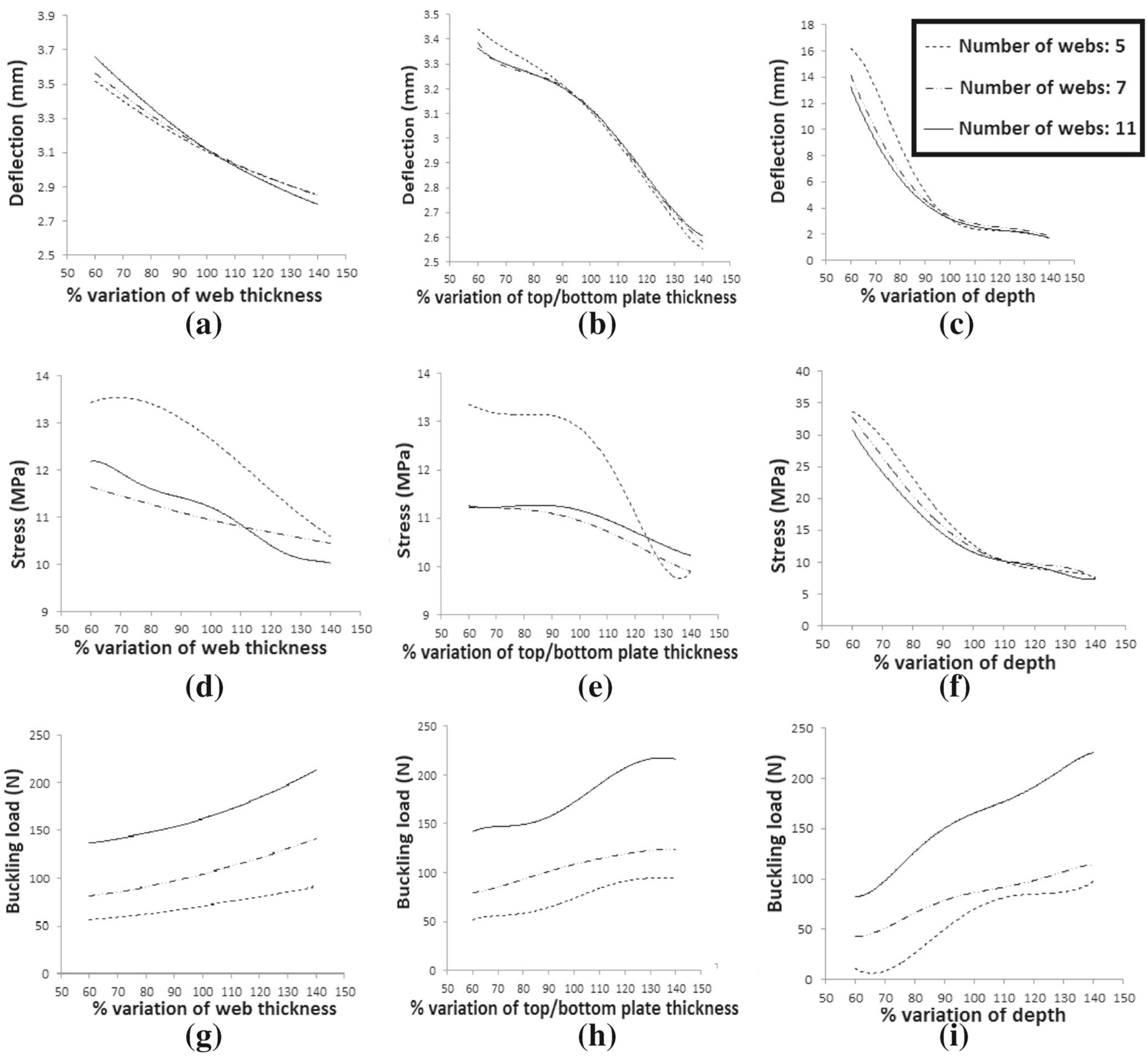


Fig. 9 Effect of variations in design parameters towards the global deflection, stress and buckling behaviour for a typical 2.5 m long FRP bridge deck

of a typical failure analysis carried out on the optimized configuration is presented in Fig. 7d.

6.2 Sensitivity analysis

Sensitivity analysis has been carried out for different configurations of FRP bridge deck having different span lengths to study the effect of variations in web thickness, face plate thickness and overall depth towards the deflection, in-plane normal stress and buckling behaviour of the bridge deck.

Figure 8 shows the effect of variation in web thickness, top/bottom plate thickness and overall depth within a range of ±40 % of the optimized configuration of a typical 2.5 m long FRP bridge deck towards its maximum deflection. This figure clearly indicates that overall depth is more sensitive towards maximum deflection than the other two parameters. Another plot of sensitivity analysis for deflection having different number of webs with the variation of web thickness, face plate thickness and overall depth have been shown in Fig. 9a-c, which reveals that as the overall depth, web thickness and face plate thickness increase, the deflection of the bridge deck has a general tendency to decrease. However, considering the ranges of variation in deflection for same percentage change in the three design parameters, it can be concluded that overall depth is more sensitive towards global deflection than the other two parameters. Figure 9d-f show the sensitivity of different structural components towards maximum stress.

These figures interpret that, stress has a similar tendency as deflection to decrease with the increase in overall depth and web as well as face plate thickness, overall depth being the most sensitive parameter. Figure 9g-i show the sensitivity of different structural components towards buckling load. These figures reveal that the buckling load increases with the increase in overall depth as well as web/face plate thickness. Figure 9g-i clearly reveal that capacity against buckling load increases with the increase in number of webs. These collective sensitivity plots presented in Fig. 9 provide a complete idea regarding the contribution of various structural components towards different aspects of design considerations.

6.3 Summary of results

It is observed that the lightweight design of FRP bridge decks is governed by deflection criteria. Optimum number of webs is found to be related with overall depth in minimizing the volume of the bridge deck. A critical value of overall depth can be identified, below which lesser number of webs produce minimum volume and above which higher number of webs produce minimum volume of the FRP bridge deck. For practical purpose to decide the optimum number of webs in order to achieve least volume of the bridge deck, such study is very helpful. Overall depth of the bridge deck is found to be the most sensitive factor in minimizing the volume of FRP bridge deck. As the overall depth of the bridge deck is increased, the required thickness of web and face plate decrease, which results in subsequent reduction of total volume of the FRP bridge deck. Though the objective of the present study is to minimize weight of the bridge deck and that can be achieved by increasing the overall depth, it is not pragmatic to increase overall depth beyond a certain limit depending on various practical design considerations. Thus optimum overall depth of FRP web core bridge deck can be decided using the design curves furnished in this article and based on practical design and aesthetic aspects. In the present study, results have been obtained for overall depth ranging between 150 mm to 225 mm.

7 Conclusion

In this article a novel efficient optimization algorithm based on RS-HDMR and GA coupled with a local unconstrained multivariable minimization function has been proposed for optimum design of FRP bridge decks using different normal as well as skew configurations. These composite decks can be used for constituting deck panels of a long span bridge as well as for short span bridges/culverts. In the proposed approach, a global optimization is carried out first using GA and then the end results of GA are further improved following a local minimization algorithm. Top/ bottom plate thickness, web thickness and number of webs for various dimensions and skew angles of the panel have been optimized under critical design constraints to achieve minimum weight of the composite structure. Prime novelty of the present study lies in introducing RS-HDMR approach for optimization of FRP bridge decks to achieve high level of computational efficiency without compromising the accuracy of results compared to traditional optimization solutions like GBM. Application of such surrogate-modelling technique in the present context eliminates the need of running computationally expensive finite element simulations repetitively for obtaining desired outputs. Actual number of FE analysis required in the present approach is found to be only 64, which is seven times lesser than that required in GBM. The optimization results, design curves and sensitivity analysis plots for different configurations of FRP bridge deck obtained in this study are the first known results of the type of analysis carried out here and these results could serve as reference for future investigators in this field. The optimization algorithm proposed in this paper can be extended to more complex configurations of FRP bridge decks as well as for optimizing material properties in addition to geometry in future. Furthermore the novel optimization methodology based on hybrid genetic algorithm coupled with RS-HDMR can be applied to many other fields of science and engineering involving optimization problems with high dimensional systems using the knowledge shared in this article.

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