Acta Mech 226, 2537–2553 (2015)
DOI 10.1007/s00707-015-1316-4

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Stochastic natural frequency of composite conical shells

Received: 15 October 2014 / Revised: 20 December 2014 / Published online: 21 March 2015

Abstract The present study portrays the stochastic natural frequencies of laminated composite conical shells using a surrogate model (D-optimal design) approach. The rotary inertia and transverse shear deformation are incorporated in probabilistic finite element analysis with uncertainty due to variation in angle of twist. A sensitivity analysis is carried out to address the influence of different input parameters on the output natural frequencies. Typical fiber orientation angle and material properties are randomly varied to obtain the stochastic natural frequencies. The sampling size and computational cost are exorbitantly reduced by employing the present approach compared to direct Monte Carlo simulation. Statistical analysis is presented to illustrate the results. The stochastic natural frequencies obtained are the first known results for the type of analyses carried out here.

1 Introduction

Composite materials have gained immense popularity due to their weight-sensitivity and cost-effectiveness along with high specific stiffness and strength. An accelerated growth in demand of composite materials is observed across a wide range of industries. Laminated composite materials are generally fabricated from unidirectional plies of given thickness and with fiber orientation angles. Because of its inherent complexity, a laminated composite shell is difficult to manufacture accurately according to its exact design specifications, resulting in undesirable uncertainties. Fiber and matrix are fabricated with three basic steps, namely tape, layup, and curing. The development of composite structures in the production process is always subject to large variability due to manufacturing imperfection and operational factors. Hence, the uncertainties incurred during the manufacturing process are due to the misalignment of ply orientation, intra-laminate voids, incomplete curing of the resin, excess resin between plies, excess matrix voids, and porosity resulting from machine, human, and process inaccuracy. Moreover, composite structures are often pretwisted due to design and operational needs. The variation in twist angle inadvertently provides effects structural; for example, the wing twist of an aircraft is provided to maintain optimum angle of attack preventing negative lift or thrust and maximizing the aerodynamic efficiency. In addition, the natural frequency of such structures made of composites shows a variability from its average value. In general, an additional factor of safety is incorporated by the designer to account for such unpredictable frequency responses that may lead to either an ultraconservative or an unsafe design. Thus, the structural stability is subjected to a considerable element of risk. Such risk is involved as the variability in the estimated output (say stochastic natural frequency) resulting from the uncertainties in the values of random input material properties and fiber parameters in each layer of laminated composites. Uncertainties from one input parameter may propagate and influence another, through linking parameters, and

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the final system output may have a significant cascading effect due to the accumulation of risk. This variability can result in significant deviations from the expected output. Therefore, it is essential to estimate the variability in natural frequencies together with the expected performance characteristic value to ensure the operational safety.

The Monte Carlo simulation technique is commonly utilized to generate the variable output frequency dealing with large sample size. Although the uncertainty in material and geometric properties can be computed by direct Monte Carlo Simulation, it is inefficient and incurs high computational cost. To mitigate this lacuna, D-optimal design technique [1–5] is employed in the present analysis to map the inadvertent uncertainties efficiently. Its purpose is to estimate response quantities in a cost-effective manner based on randomness considered in input parameters (ply orientation angle, mass density, shear modulus, and elastic modulus). The proposed procedure is based on constructing response surfaces that represent an estimate for the relationship between the input parameters of the finite element model and response quantities of interest. The D-optimality criterion was found to provide the best rational means among all other optimality criteria for creating experimental designs in case of irregularly shaped response surfaces [6]. A multiplicative method was utilized for computing D-optimal stratified designs of experiments by Radoslav [7]. The dynamic stability of uncertain laminated beams subjected to subtangential loads was studied by Goyal and Kapania [8], and subsequently, the stochastic finite element analysis of the free vibration of laminated composite plates was studied by Shaker et al. [9]. On the other hand, Fang and Springer [10] studied on the design of composite laminates by a Monte Carlo method, while Sasikumar et al. [11] investigated on stochastic finite element analysis of layered composite beams with spatially varying non-Gaussian inhomogeneities. Ankenman et al. [12] employed the Kriging approach for stochastic simulation by meta-modelling. Park et al. [13] studied stochastic finite element method for laminated composite structures while Ganesan and Kowda [14] investigated on free vibration of composite beam-columns with stochastic material and geometric properties subjected to random axial loads. The multi-response linear models with a qualitative factor were studied by Yue et al. [15] using D-optimal design, while the optimal sampling frequency was investigated for high frequency using a finite mixture model by Choi et al. [16]. Xu et al. [17] studied the uncertainty propagation in statistical energy analysis for structural–acoustic coupled systems with non-deterministic parameters, while a finite element-based modal method was employed for the determination of plate stiffness considering uncertainties by Kuttenkeuler [18]. The stochastic finite element method [19–21] was employed in conjunction with laminated composites and other applications.

Even though the assessment of the natural frequency of composite structures has gained a wide spectrum of attention and applications, the treatment of uncertainties with a twist to quantify and analyze the same for composite shells has received little attention. The novelty in the present study includes quantifying the uncertainty of the first three natural frequencies of laminated composite conical shells. Moreover, to the best of the authors' knowledge, the application of D-optimal design is the first attempt of its kind in the realm of stochastic analysis of laminated composites considering randomness in the twist angle. In the present study, an algorithm is developed to quantify the stochastic natural frequencies of cantilever composite conical shells using D-optimal design technique, and its efficacy is compared with the direct Monte Carlo simulation (MCS) technique. A sensitivity analysis is also carried out to map the effect of individual input parameters on output responses. An eight-noded isoparametric quadratic element with five degrees of freedom at each node is considered incorporating rotary inertia and transverse shear deformation in the finite element formulation.

2 Governing equations

The present study is carried out as an extension of the previous work [22,23] to investigate the effect of stochasticity in twist angle, ply orientation angle, and material properties applied to laminated composite conical shells. The pretwisted composite cantilever conical shell with length Lo, reference width b, thickness t, vertex angle ϕ_{ve}, and base subtended angle of cone ϕ_o is furnished in Fig. 1. The component of radius of curvature in the chordwise direction $r_y(x, y)$ is a parameter varying both in the x- and y- directions. The variation in x-direction is considered to be linear, and there is no curvature along the spanwise direction ($r_x = \infty$). The cantilever shell is clamped along $x = 0$ with radius of twist r_{xy}, which is assumed to vary depending on the angle of twist. Thus, a shallow conical shell of uniform thickness, made of laminated composite, is considered. A shallow shell is characterized by its middle surface and is defined by the equation [24],

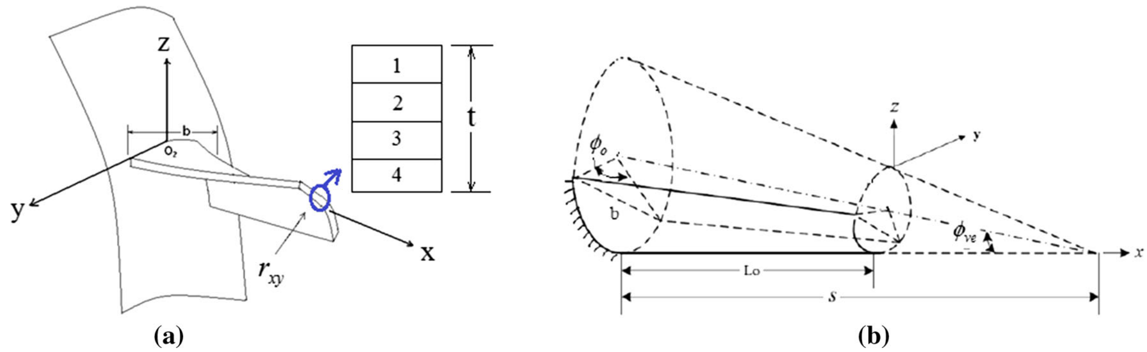

Fig. 1 **a** Pretwisted composite cantilever shell, **b** conical shell model

$$z(\bar{\omega}) = -0.5\left[\left(x^2/r_x\right) + \left\{2xy/r_{xy}(\bar{\omega})\right\} + \left(y^2/r_y\right)\right]. \tag{1}$$

The radius of twist (r_{xy}), length (Lo) of the shell, and twist angle (Ψ) are expressed as [25]

$$r_{xy}(\bar{\omega}) = -Lo/\tan\psi(\bar{\omega}), \tag{2}$$

where $\psi(\bar{\omega})$ denotes the random angle of twist, wherein the symbol $(\bar{\omega})$ indicates the stochasticity of parameters. The constitutive equations for composite conical shell are given by [26]

$$\{F\} = [D(\bar{\omega})]\,\{\varepsilon\}, \tag{3}$$

where {F} is the internal force resultant vector expressed as

$$\begin{aligned} \{F\} &= \left\{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\right\}^{\mathrm{T}}, \\ \{F\} &= \left[\int_{-h/2}^{h/2} \{\sigma_x, \sigma_y, \tau_{xy}, \sigma_x z, \sigma_y z, \tau_{xy} z, \tau_{xz}, \tau_{yz}\}\, z\mathrm{d}z\right]^{\mathrm{T}}, \\ \{\varepsilon\} &= \{\varepsilon_{\mathrm{x}}, \varepsilon_y, \varepsilon_{xy}, k_x, k_{\mathrm{y}}, k_{xy}, \gamma_{xz}, \gamma_{yz}\}^{\mathrm{T}}, \end{aligned} \tag{4}$$

$$[D(\bar{\omega})] = \begin{bmatrix} A_{ij}(\bar{\omega}) & B_{ij}(\bar{\omega}) & 0 \\ B_{ij}(\bar{\omega}) & D_{ij}(\bar{\omega}) & 0 \\ 0 & 0 & S_{ij}(\bar{\omega}) \end{bmatrix}. \tag{5}$$

The elements of the elastic stiffness matrix $[D(\bar{\omega})]$ are expressed as [26]

$$\begin{aligned} \left[A_{ij}(\bar{\omega})\right] &= \sum_{kk=1}^{nl} \left[\bar{Q}_{ij}(\bar{\omega})\right]_{kk} (z_{kk} - z_{kk-1}) \quad \text{for } i, j = 1, 2, 6, \\ \left[B_{ij}(\bar{\omega})\right] &= \frac{1}{2}\sum_{kk=1}^{nl} \left[\bar{Q}_{ij}(\bar{\omega})\right]_{kk} \left(z_{kk}^2 - z_{kk-1}^2\right) \quad \text{for } i, j = 1, 2, 6, \\ \left[D_{ij}(\bar{\omega})\right] &= \frac{1}{3}\sum_{kk=1}^{nl} \left[\bar{Q}_{ij}(\bar{\omega})\right]_{kk} \left(z_{kk}^3 - z_{kk-1}^3\right) \quad \text{for } i, j = 1, 2, 6, \\ \left[S_{ij}(\bar{\omega})\right] &= \alpha_s \sum_{kk=1}^{nl} \left[\bar{Q}_{ij}(\bar{\omega})\right]_{kk} (z_{kk} - z_{kk-1}) \quad \text{for } i, j = 4, 5, \end{aligned} \tag{6}$$

where nl is the number of the laminate layer and α_{s} is the shear correction factor and is assumed as 5/6. $[A_{ij}(\bar{\omega})]$, $[B_{ij}(\bar{\omega})]$, and $[D_{ij}(\bar{\omega})]$ are the extension, bending–extension coupling, and bending stiffness coefficients, respectively. $[\bar{Q}_{ij}]$ is the off-axis elastic constant matrix, which is given by

$$\left[\bar{Q}_{ij}(\bar{\omega})\right]_{\text{off}} = [T_{\text{m}}(\bar{\omega})]^{-1}\left[\bar{Q}_{ij}\right]_{\text{on}}[T_{\text{m}}(\bar{\omega})]^{-\text{T}} \quad \text{for } i, j = 1, 2, 6,$$
$$\left[\bar{Q}_{ij}(\bar{\omega})\right]_{\text{off}} = \left[\bar{T}_{\text{m}}(\bar{\omega})\right]^{-1}\left[\bar{Q}_{ij}\right]_{\text{on}}\left[\bar{T}_{\text{m}}(\bar{\omega})\right]^{-\text{T}} \quad \text{for } i, j = 4, 5, \tag{7}$$

where

$$[T_{\text{m}}(\bar{\omega})] = \begin{bmatrix} \sin^2\theta(\bar{\omega}) & \cos^2\theta(\bar{\omega}) & 2\sin\theta(\bar{\omega})\cos\theta(\bar{\omega}) \\ \cos^2\theta(\bar{\omega}) & \sin^2\theta(\bar{\omega}) & -2\sin\theta(\bar{\omega})\cos\theta(\bar{\omega}) \\ -\sin\theta(\bar{\omega})\cos\theta(\bar{\omega}) & 2\sin\theta(\bar{\omega})\cos\theta(\bar{\omega}) & \sin^2\theta(\bar{\omega}) - \cos^2\theta(\bar{\omega}) \end{bmatrix}, \tag{8}$$
$$\text{and } \left[\bar{T}_{\text{m}}(\bar{\omega})\right] = \begin{bmatrix} \sin\theta(\bar{\omega}) & -\cos\theta(\bar{\omega}) \\ \cos\theta(\bar{\omega}) & \sin\theta(\bar{\omega}) \end{bmatrix},$$

in which $\theta(\bar{\omega})$ is the random ply orientation angle.

$$\left[Q_{ij}(\bar{\omega})\right]_{\text{on}} = \begin{bmatrix} Q_{11}(\bar{\omega}) & Q_{12}(\bar{\omega}) & 0 \\ Q_{12}(\bar{\omega}) & Q_{22}(\bar{\omega}) & 0 \\ 0 & 0 & Q_{66}(\bar{\omega}) \end{bmatrix} \quad \text{for } i, j = 1, 2, 6, \tag{9}$$

$$\left[\bar{Q}_{ij}(\bar{\omega})\right]_{\text{on}} = \begin{bmatrix} Q_{44}(\bar{\omega}) & Q_{45}(\bar{\omega}) \\ Q_{45}(\bar{\omega}) & Q_{55}(\bar{\omega}) \end{bmatrix} \quad \text{for } i, j = 4, 5, \tag{10}$$

where

$$Q_{11}(\bar{\omega}) = \frac{E_1(\bar{\omega})}{1 - \nu_{12}\nu_{21}}, \quad Q_{22}(\bar{\omega}) = \frac{E_2(\bar{\omega})}{1 - \nu_{12}\nu_{21}} \quad \text{and} \quad Q_{12}(\bar{\omega}) = \frac{\nu_{12}E_2(\bar{\omega})}{1 - \nu_{12}\nu_{21}},$$
$$Q_{66}(\bar{\omega}) = G_{12}(\bar{\omega}), \quad Q_{44}(\bar{\omega}) = G_{23}(\bar{\omega}) \quad \text{and} \quad Q_{55}(\bar{\omega}) = G_{13}(\bar{\omega}).$$

The stochastic mass per unit area for each element of the conical shell is expressed as [27]

$$P_{\text{e}}(\bar{\omega}) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho_{\text{e}}(\bar{\omega})\,\text{d}z, \tag{11}$$

where the suffix "*e*" represents the respective parameters for an element, and $\rho_{\text{e}}(\bar{\omega})$ denotes the stochastic density parameter at each element. The element mass matrix is expressed as

$$[M_{\text{e}}(\bar{\omega})] = \int_{V} [N][P_{\text{e}}(\bar{\omega})][N]\text{d}V, \tag{12}$$

where V is the volume, and $[N]$ is the shape function matrix. The random stiffness matrix is given by

$$[K(\bar{\omega})] = \int_{-1}^{1}\int_{-1}^{1} [B(\bar{\omega})]^T [D(\bar{\omega})] [B(\bar{\omega})]\,\text{d}\xi\,\text{d}\eta, \tag{13}$$

where ξ and η are the local natural coordinates of the element. Applying Hamilton's principle [28], the dynamic equilibrium equation of each element for free vibration is expressed as [29]

$$[M_{\text{e}}(\bar{\omega})][\ddot{\delta}_{\text{e}}] + [K_{\text{e}}(\bar{\omega})]\{\delta_{\text{e}}\} = 0. \tag{14}$$

After assembling all the element matrices and the force vectors with respect to the common global coordinates, the equation of motion of a free vibration system with n_{dof} degrees of freedom can be expressed [30] as

$$[M(\bar{\omega})]\left[\ddot{\delta}\right] + [K(\bar{\omega})]\{\delta\} = 0. \tag{15}$$

In the above equation, $M(\bar{\omega}) \in R^{n\times n}$ is the mass matrix, $[K(\bar{\omega})] \in R^{n\times n}$ is the stiffness matrix, while $\{\delta\} \in R^n$ is the vector of generalized coordinates. The governing equations are derived based on Mindlin's theory [27] incorporating rotary inertia and transverse shear deformation. For free vibration, the stochastic natural frequencies $[\omega_n(\bar{\omega})]$ are determined from the standard eigenvalue problem [30] which is solved by the QR iteration algorithm.

3 D-optimal design method

In general, a statistical measure of goodness of a model obtained by least-squares regression analysis is the minimum generalized variance in the estimates of the model coefficients [4]. The D-optimal design method is employed to provide a mathematical and statistical approach in portraying the input–output mapping by the construction of meta-model with small representative number of samples. Considering the problem of estimating the coefficients of a linear approximation is modelled by least-squares regression analysis

$$Y = X\beta + \varepsilon, \tag{16}$$

where Y is a vector of observations of sample size, ε is the vector of errors having normal distribution with zero mean, X is the design matrix, and β is a vector of unknown model coefficients and can be estimated by using the least-squares method as

$$\beta = \left(X^{\mathrm{T}}X\right)^{-1} X^{\mathrm{T}}Y. \tag{17}$$

A measure of accuracy of the column of estimators, β, is the variance–covariance matrix that is defined as

$$V(\beta) = \sigma^2 \left(X^{\mathrm{T}}X\right)^{-1}, \tag{18}$$

where σ^2 is the variance in the error. The matrix $V(\beta)$ is a statistical measure of the goodness of the fit. $V(\beta)$ is a function of $(X^{\mathrm{T}}X)^{-1}$, and therefore, one would want to minimize $(X^{\mathrm{T}}X)^{-1}$ to improve the quality of the fit. If X denotes the design matrix as a set of value combinations of coded parameters and X^{T} is the transpose of X, then D-optimality is achieved if the determinant of $(X^{\mathrm{T}}X)^{-1}$ is minimal. The letter "D" stands for the determinant of the $(X^{\mathrm{T}}X)$ matrix associated with the model. In the present study, the constructed meta-models provide an approximate meta-model equation that relates the input random parameters x_i (say ply orientation angle and elastic modulus of each layer of laminate) and output Y (say natural frequency) for a particular system. The response surface model developed is actually an approximate mathematical model representing a certain inherent property of a physical system, and it maps the input parameters x_i to the corresponding responses Y by an explicit function

$$Y = f(x_1, x_2, x_3, x_4, \ldots x_i \ldots x_k) + \varepsilon, \tag{19}$$

where f denotes the approximate response function, ε is the statistical error term having a normal distribution with null mean value, and k is the number of input variables. The input variables are usually coded as dimensionless variables with zero as mean value and a standard deviation of x_i. The first-order and second-order polynomials are expressed as [4]

$$\text{First-order model (interaction):} \quad Y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j + \varepsilon, \tag{20}$$

$$\text{Second-order model:} \quad Y = \beta_o + \sum_{i=1}^{k} \beta_i x_i + \sum_{i=1}^{k} \sum_{j>i}^{k} \beta_{ij} x_i x_j + \sum_{i=1}^{k} \beta_{ii} x_i^2 + \varepsilon, \tag{21}$$

where β_o, β_i, β_{ij}, and β_{ii} are the coefficients of the polynomial equation. The meta-model is employed to fit approximately for a set of points in the design space using a multiple regression fitting scheme. The position of design points is chosen algorithmically according to the selected number of input variables and their range of variability. Hence, the design points are not considered at any specific positions; instead, they are selected in such a fashion that it meets the optimality criteria. It provides the good accuracy of approximation compared to direct Monte Carlo simulation method. In D-optimal design, the total sample size (n) is the summation of the minimum number of design points $[n_{\mathrm{d}} = \frac{1}{2}(k+1)(k+2)]$, additional model points ($n_{\mathrm{a}} = k$), and lack-of-fit points (n_{l}). (i.e., $n = n_{\mathrm{d}} + n_{\mathrm{a}} + n_{\mathrm{l}}$) where k is the number of stochastic input parameter. For model construction in the present study, an overdetermined D-optimal design [31,32] (number of additional samples n_{a}, along with the minimum point design and $n_{\mathrm{l}} = 10$ samples to estimate the lack of fit) has been used. The output feature (natural frequency) is selected in such a way that it is found to be sensitive enough corresponding to chosen input features. The insignificant input features are screened out and not considered in the model formation. A quantitative evaluation for the effect of each parameter on the total model variance is carried out using analysis of variance (ANOVA) method according to its F test value

$$F_{\mathrm{p}} = \frac{n-k-1}{k}\left(\frac{\mathrm{SS_R}}{\mathrm{SS_E}}\right), \tag{22}$$

where F_{p} denotes the F test value of any input parameter p, while n, $\mathrm{SS_E}$, and $\mathrm{SS_R}$ are the number of samples used in the design procedure, sum of squares due to the model, and the residual error, respectively. The metamodel constructed is checked by three basic criteria such as coefficient of determination or R^2 term (measure of the amount of variation around the mean explained by the model) and R^2_{adj} term (measure of the amount of variation with respect to the mean value explained by the model, adjusted for the number of terms in the model), and R^2_{pred} (measure of the prediction capability of the response surface model), which are expressed as

$$R^2 = \left(\frac{\mathrm{SS_R}}{\mathrm{SS_T}}\right) = 1 - \left(\frac{\mathrm{SS_E}}{\mathrm{SS_T}}\right) \qquad \text{where } 0 \le R^2 \le 1, \tag{23}$$

$$R^2_{\mathrm{adj}} = 1 - \frac{\left(\frac{\mathrm{SS}_E}{n-k-1}\right)}{\left(\frac{\mathrm{SS_T}}{n-1}\right)} = 1 - \left(\frac{n-1}{n-k-1}\right)\left(1-R^2\right) \qquad \text{where } 0 \le R^2_{\mathrm{adj}} \le 1, \tag{24}$$

$$R^2_{\mathrm{pred}} = 1 - \left(\frac{\mathrm{PRESS}}{\mathrm{SS_T}}\right) \qquad \text{where } 0 \le R^2_{\mathrm{pred}} \le 1, \tag{25}$$

where $\mathrm{SS_T} = \mathrm{SS_E} + \mathrm{SS_R}$ is the total sum of square and *PRESS* (predicted residual error sum of squares) measures the quality of the fit of the model to chosen samples in the design space.

There are many sampling techniques, such as factorial designs, central composite design, optimal design, Taguchi's orthogonal array design, Plackett–Burman design, Koshal designs, Box–Behnken design, Latin hypercube sampling, Sobol sequence [33–35], and surrogate model formation methods, such as polynomial regression method, moving least-squares method, Kriging, artificial neural networks, radial basis function, multivariate adaptive regression splines, support vector regression, and high-dimensional model representation [36]. The sampling method and surrogate modelling technique for a particular problem should be chosen depending on the complexity of the model, the presence of noise in sampling data, nature and dimension (number) of input parameters, desired level of accuracy, and computational efficiency. Many comparative studies have been performed over the years to guide the selection of surrogate model types [37–39]. The application of D-optimal design for surrogate model formation in the area of structural engineering and mechanics (specifically in FRP composite structures) is still very scarce, in spite of the fact that it has an immense potential to be used as surrogate of expensive finite element simulation or experimentations of repetitive nature. The D-optimal design method is suitable to handle a large number of input parameters efficiently for composite structures compared to other methods, as the present model under consideration is linear in nature, and all the data used are noise-free.

4 Stochastic approach using D-optimal design

The stochasticity in material properties of laminated composite conical shells, such as elastic modulus, mass density, and geometric properties such as ply orientation angle, twist angle (Fig. 2) as input parameters, is considered depending on the boundary condition for natural frequency analysis of composite conical shells. In the present study, the frequency domain feature (first three natural frequencies) is considered as output. It is assumed that the distribution of randomness of input parameters exists within a certain band of tolerance with their central deterministic values. The cases wherein the random variables considered in each layer of laminate are investigated for

(a) Variation in ply orientation angle only: $\theta(\bar{\omega}) = \{\theta_1\, \theta_2\, \theta_3 \ldots \theta_i \ldots \theta_l\}$
(b) Variation in mass density only: $\rho(\bar{\omega}) = \{\rho_1\, \rho_2\, \rho_3 \ldots \rho_i \ldots \rho_l\}$
(c) Variation in longitudinal shear modulus: $G_{12}(\bar{\omega}) = \{G_{12(1)}\, G_{12(2)}\, G_{12(3)} \ldots G_{12(i)} \ldots G_{12(l)}\}$

(d) Variation in transverse shear modulus: $G_{23}(\bar{\omega}) = \{G_{23(1)}\ G_{23(2)}\ G_{23(3)} \ldots G_{23(i)} \ldots G_{23(l)}\}$
(e) Variation in elastic modulus (longitudinal): $E_1(\bar{\omega}) = \{E_{1(1)}\ E_{1(2)}\ E_{1(3)} \ldots E_{1(i)} \ldots E_{1(l)}\}$
(f) Combined variation in ply orientation angle, mass density, shear modulus along longitudinal and transverse direction, and elastic modulus (longitudinal):

$$g_1\{\theta(\bar{\omega}), \rho(\bar{\omega}), G_{12}(\bar{\omega}), G_{23}(\bar{\omega}), E_1(\bar{\omega})\}$$
$$= \{\Phi_1(\theta_1 \ldots \theta_l), \Phi_2(\rho_1 \ldots \rho_l), \Phi_3(G_{12(1)} \ldots G_{12(l)}), \Phi_4(G_{23(1)} \ldots G_{23(l)}), \Phi_5(E_{1(1)} \ldots E_{1(l)})\}$$

(g) Combined variation in ply orientation angle, mass density, shear modulus along longitudinal and transverse direction, elastic modulus (longitudinal), and angle of twist

$$g_2\{\theta(\bar{\omega}), \rho(\bar{\omega}), G_{12}(\bar{\omega}), G_{23}(\bar{\omega}), E_1(\bar{\omega})\}$$
$$= \{\Phi_1(\theta_1 \ldots \theta_l), \Phi_2(\rho_1 \ldots \rho_l), \Phi_3(G_{12(1)} \ldots G_{12(l)}), \Phi_4(G_{23(1)} \ldots G_{23(l)}), \Phi_5(E_{1(1)} \ldots E_{1(l)}), \Phi_6(\psi_1 \ldots \psi_l)\}$$

where θ_i, ρ_i,$G_{12(i)}$, $G_{23(i)}$, $E_{1(i)}$, and ψ_i are the ply orientation angle, mass density, shear modulus along longitudinal direction, shear modulus along transverse direction, elastic modulus along longitudinal direction, and angle of twist, respectively, and "l" denotes the number of layer in the laminate. The tolerances of $\pm 5°$, $\pm 10°$, and $\pm 15°$ for ply orientation angle with subsequent ± 10, ± 20, and $\pm 30\,\%$ for material properties, respectively, from deterministic mean value are considered in the present study. In addition to the above, the uncertainty in natural frequency due to random angle of twist $[\psi(\bar{\omega})]$ ($\pm 5°$ variation considered for $\psi = 0°$, 15°, 30°, and 45°) is also investigated for a typical laminate.

5 Results and discussion

In the present study, four-layered graphite–epoxy symmetric angle-ply and cross-ply laminated composite cantilever shallow conical shells are considered with aspect ratio of 0.7 and thickness ratio of 280. The parameters of graphite–epoxy composite cantilever conical shells [40] are considered with deterministic value as $E_1 = 138$ GPa, $E_2 = 8.9$ GPa, $G_{12} = G_{13} = 7.1$ GPa, $G_{23} = 2.84$ GPa, $\rho = 3202$ kg/m^3, $t = 0.002$ m, $\nu = 0.3$, $Lo/s = 0.7$, $\phi_o = 45°$, $\phi_{ve} = 20°$. A typical discretization of a (6 × 6) mesh on plan area with 36 elements and 133 nodes with natural coordinates of an isoparametric quadratic element is considered for the present finite element method. For full-scale MCS, number of original FE analysis is same as the sampling size. In general for complex composite structures, the performance function is not available as an explicit function of the random design variables. The random response in terms of natural frequencies of the composite structure can only be evaluated numerically at the end of a structural analysis procedure such as the finite element method which is often time-consuming. The present D-optimal design methodology is employed to find a predictive and representative meta-model relating each natural frequency to a number of input variables. The meta-models are used to determine the first three natural frequencies corresponding to given values of input variables, instead of time-consuming deterministic FE analysis. The probability density function is plotted as the benchmark of bottom line results. Due to the scarcity of space, only a few important representative results are furnished.

5.1 Validation

The present computer code is validated with the results available in the open literature. Table 1 presents the non-dimensional fundamental frequencies of graphite–epoxy composite twisted plates with different ply orientation angle [41]. The present formulation is validated for non-dimensional first three natural frequencies for composite conical shells as furnished in Table 2. The differences between the results by Liew et al. [25] and the present FEM approach can be attributed to the consideration of transverse shear deformation and rotary inertia in the present FEM approach. The comparative study depicts an excellent agreement with the previously published results, and hence, it demonstrates the capability of the computer codes developed and insures the accuracy of analyses. In the present D-optimal design method, a sample size of 29 is considered

Table 1 Non-dimensional first three natural frequencies [$\omega = \omega_n L^2 \sqrt{(\rho/E_1 h^2)}$] of three-layered [$\theta, -\theta, \theta$] graphite–epoxy twisted plates, $L/b = 1$, $b/h = 20$, $\psi = 30°$

Fiber orientation angle (θ)	f_1		f_2		f_3	
	Present FEM	Qatu and Leissa [41]	Present FEM	Qatu and Leissa [41]	Present FEM	Qatu and Leissa [41]
15°	0.8618	0.8759	2.6551	2.6840	3.7661	3.8113
30°	0.6790	0.6923	2.5594	2.5952	3.7464	3.7969
45°	0.4732	0.4831	2.2287	2.2560	2.7493	2.7938
60°	0.3234	0.3283	1.7797	1.8088	1.9528	2.0015

Table 2 First three non-dimensional natural frequencies [$\omega = \omega_n b^2 \sqrt{(\rho h/D)}$, $D = Eh^3/12(1-\nu^2)$] for the untwisted shallow conical shells with $\nu = 0.3$, $s/t = 1000$, $\phi_o = 30°$, $\phi_{ve} = 15°$

Aspect ratio (L/s)	f_1		f_2		f_3	
	Present FEM	Liew et al. [25]	Present FEM	Liew et al. [25]	Present FEM	Liew et al. [25]
0.6	0.3552	0.3599	1.1889	1.2037	1.4602	1.4840
0.7	0.3013	0.3060	0.8801	0.8963	1.4090	1.4323
0.8	0.2731	0.2783	0.6984	0.7122	1.3795	1.4065

for each layer's individual variation in ply orientation angle, mass density, longitudinal and transverse shear modulus and longitudinal elastic modulus and twist angle, respectively. Due to the increased number of input variables for combined random variation in ply orientation angle, mass density, longitudinal and transverse shear modulus, and elastic modulus, the subsequent sample size of 261 is adopted to meet the convergence criteria. Figure 3 depicts a representative plot describing the relationship between the original FE model and the constructed D-optimal design meta-model for fundamental natural frequencies signifying the accuracy of the present meta-model. In contrast, Fig. 4 shows a sample comparison of the probability density functions (PDF) for both original MCS and D-optimal design meta-model using a sample size of 10,000 corresponding to the first three natural frequencies. The low scatterness of the points found around the diagonal line in Fig. 3 and the low deviation obtained between the probability density function estimations of original MCS and D-optimal design responses in Fig. 4 corroborate the fact that D-optimal design meta-models are accurately formed. These two plots are checked and are found in good agreement ensuring the efficiency and accuracy of the present constructed meta-model. While evaluating the statistics of responses through full-scale MCS, computational time is exorbitantly high because it involves number of repeated FE analysis. However, in the present method, MCS is conducted in conjunction with the D-optimal design model. Here, although the same sampling size as in direct MCS (with sample size of 10,000) is considered, the number of actual FE analysis is much less compared to original MCS and is equal to number representative sample required to construct the D-optimal design meta-model. The D-optimal design meta-model is formed on which the full sample size of direct MCS is conducted. Hence, the computational time and effort expressed in terms of FE calculation is reduced compared to full-scale direct MCS. Hence, in order to save computational time, the present constructed D-optimal design methodology is employed instead of traditional Monte Carlo simulation. This provides an efficient affordable way for simulating the uncertainties in natural frequency. The sensitivity of a given material or geometric property to each random variable is also quantified in the present meta-model context. The representative probability density function for angle-ply and cross-ply laminate with respective stochastic input parameters is compared with the results predicted by a Monte Carlo simulation (10,000 samples) wherein a good agreement is observed as furnished in Fig. 4. In the present analysis, the values of R^2, R^2_{adj} and R^2_{pred} are found to be close to one ensuring the best fit. The difference between R^2_{adj} and R^2_{pred} is found less than 0.2, which indicates that the model can be used for further prediction. In addition to above, another check is carried out, namely adequate precision which compares the range of the predicted values at the design points to the average prediction error. For all cases of present D-optimal design meta-model, its value is consistently found greater than four, which indicates the present model is adequate to navigate the design space. The computational time required in the present study is observed to be around (1/345) times (for individual variation in inputs) and (1/38) times (for combined variation in inputs) of direct Monte Carlo simulation.

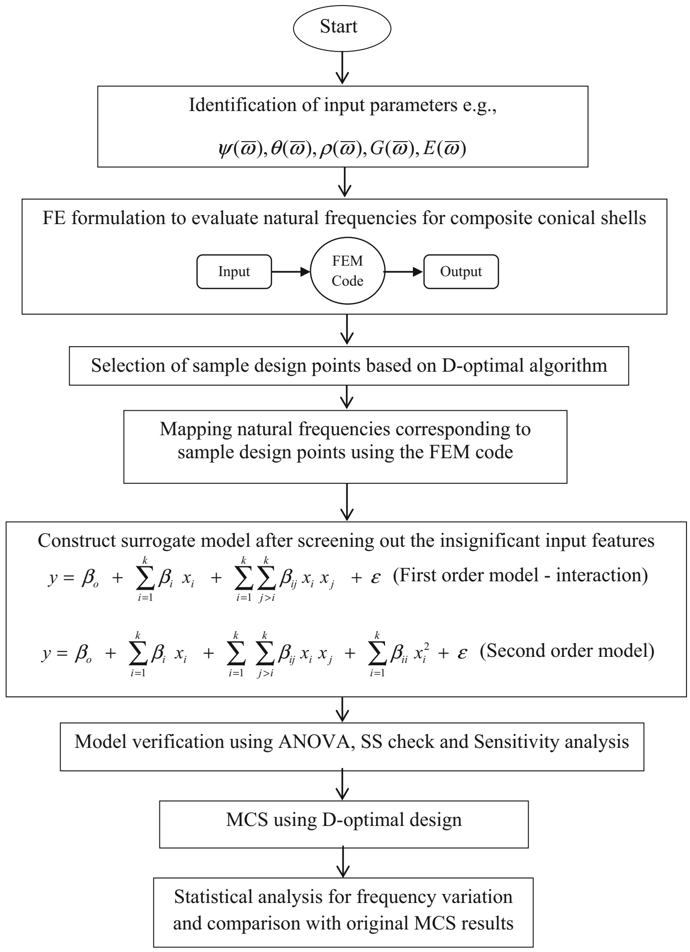

Fig. 2 Flowchart of stochastic natural frequency using D-optimal design

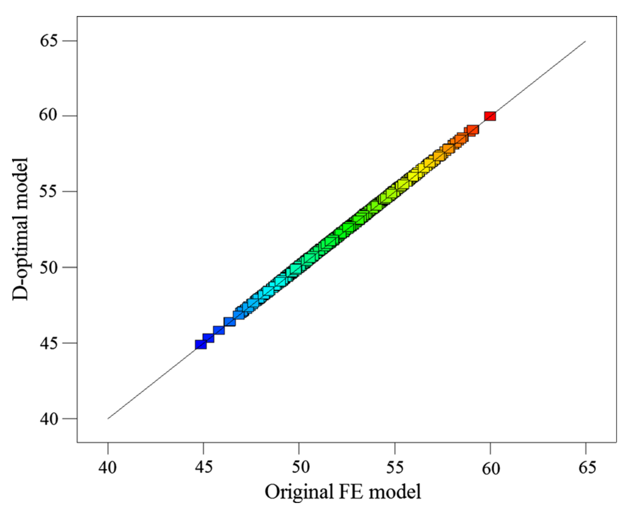

Fig. 3 Surrogate model validation for fundamental natural frequencies for individual variation in ply orientation angle of angle-ply (45°/−45°/−45°/45°) composite cantilever conical shells

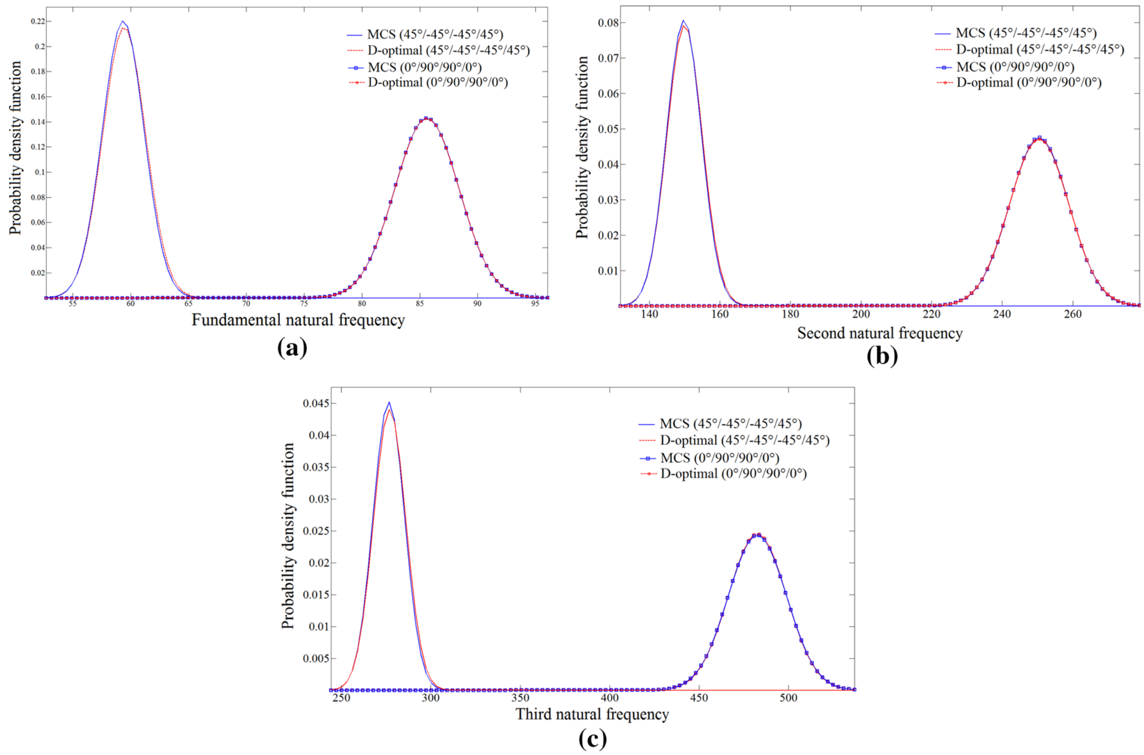

Fig. 4 a–c Comparative study by PDF plot of original MCS and D-optimal design results with respect to first three natural frequencies considering combined variation $[\rho(\bar{\omega}), G_{12}(\bar{\omega}), G_{23}(\bar{\omega}), E_1(\bar{\omega})]$ for angle-ply and cross-ply composite conical shells

5.2 Statistical analysis

The material properties such as mass density $[\rho(\bar{\omega})]$, shear modulus (longitudinal and transverse) $[G_{12}(\bar{\omega}), G_{23}(\bar{\omega})]$, and elastic modulus in longitudinal direction $[E_1(\bar{\omega})]$ are assumed to be varied randomly in each layer of the laminate. The composite conical shell is scaled randomly with uniform distribution of input variables in the range having the lower and the upper limit as $\pm 10\,\%$ variability with respective mean values, while for twist angle $[\psi(\bar{\omega})]$ and ply orientation angle $[\theta(\bar{\omega})]$, the bounds are considered as within $\pm 5^\circ$ fluctuation with respective mean values of each layer of laminates. Although the uniform distribution of stochastic input variables is considered, the stochastic outputs (i.e., natural frequencies) are very close to a Gaussian distribution. The D-optimal design meta-models are formed to generate the first three natural frequencies for angle-ply and cross-ply composite conical shells. The natural frequencies of the tested angle-ply $(45^\circ/-45^\circ/-45^\circ/45^\circ)$ laminate are found to be lower than that of the same for cross-ply $(0^\circ/90^\circ/90^\circ/0^\circ)$ laminate irrespective of stochasticity considered in random input parameters. Table 3 presents the comparative study between MCS and D-optimal design for maximum values, minimum values, and percentage of deviation for first three natural frequencies obtained due to individual stochasticity in each layer due to randomness considered in only ply orientation angle for angle-ply and cross-ply composite cantilever conical shells. In this case, the percentage of deviation between original MCS and D-optimal design of maximum value, minimum value, mean value, and standard deviation for fundamental natural frequencies of angle-ply is found to be higher than that of the same for cross-ply composite conical shells. Similar to the previous case, the percentage of deviation between original MCS and D-optimal design of maximum value, mean value, and standard deviation for fundamental natural frequencies of the angle-ply is found to be higher than that of the same for cross-ply composite conical shells expect for the minimum value, where a reverse trend is identified. All the results obtained using original MCS and D-optimal design are observed to be in good agreement. Due to the cascading effect resulting from combined stochasticity considered in five input parameters in each layer, the bandwidth of variation in natural frequency is found to be higher than the stochasticity considered for variation in any single input parameter.

Table 3 Comparative study between MCS (10,000 samples) and D-optimal design (29 samples) results for maximum values, minimum values, and percentage of deviation for first three natural frequencies obtained due to individual stochasticity in ply orientation angle [$\theta(\bar{\omega})$] for angle-ply (45°/−45°/−45°/45°) and cross-ply (0°/90°/90°/0°) composite conical shells

Parameter	Type of analysis	Angle-ply (45°/−45°/−45°/45°)			Cross-ply (0°/90°/90°/0°)		
		f_1	f_2	f_3	f_1	f_2	f_3
Max. value	MCS	41.0610	108.3828	185.9936	52.4799	156.4330	287.6360
	D-optimal	41.1975	108.9131	186.8028	52.4771	156.4974	287.9516
	Deviation (%)	−0.332	−0.489	−0.435	0.005	−0.041	−0.110
Min. value	MCS	37.0048	94.8742	163.5500	47.7011	133.0286	242.0915
	D-optimal	36.9809	94.7998	163.4015	47.6972	133.1006	242.2256
	Deviation (%)	0.065	0.078	0.091	0.008	−0.054	−0.055
Mean value	MCS	39.0706	101.7120	174.9002	49.9388	144.9036	265.9599
	D-optimal	39.0839	101.7428	174.9548	49.9392	144.9268	265.9982
	Deviation (%)	−0.034	−0.030	−0.031	−0.001	−0.016	−0.014
Standard deviation (SD)	MCS	0.6866	2.2887	3.7995	0.9905	4.9971	9.6838
	D-optimal	0.6756	2.2998	3.8160	0.9920	4.9112	9.5052
	Deviation (%)	1.602	−0.485	−0.434	−0.151	1.719	1.844

Considering individual stochasticity in elastic modulus shear modulus and mass density, the parametric studies are carried out for maximum values, minimum values, means, and standard deviations of the first three natural frequencies as depicted in Table 4, where the maximum and minimum volatility in natural frequency is observed due to individual randomness of mass density and transverse shear modulus, respectively, while the longitudinal shear modulus trailed by the elastic modulus is found to be intermediate. The sensitivity analysis using D-optimal design is performed for significant input parameter screening. The effect of stochasticity of ply orientation angle of each and individual layer separately for angle-ply composite conical shells on respective fundamental natural frequency is shown in Fig. 5g wherein for two outermost layers (i.e, layer 1 and layer 4), it is found to have inverse relationship with ply orientation angle, but in due contrast to negative value of ply orientation angle for two inner layers (i.e., layer 2 and layer 3), the reverse trend is observed. The variation in elastic stiffness due to the randomness of ply orientation angle predominantly influences the fundamental natural frequency. The interaction effect corresponding to randomness in ply orientation angle of respective pair of layers for four-layered angle-ply laminate is furnished in Fig. 5a–f wherein two outermost layers are found to act predominant role to influence the trend of volatility of fundamental natural frequency. The bandwidth of fundamental natural frequency of combined ply angle variation in all four layers at a time is found in lower range compared to the range found corresponding to randomness considered in separate each and individual layer of the angle-ply laminate. Figure 6 presents the probability density function plot with stochastic angle of twist (±5° variation) for the first three natural frequencies of angle-ply (45°/−45°/45°/−45°) conical shells. Needless to mention that as the twist angle increases, due to the reduction in stochastic elastic stiffness, the stochastic first three natural frequencies are found to decrease subsequently. As the degree of variation in input parameters increases, the respective mean natural frequency values are found to reduce, but the volatility (standard deviation) in stochastic natural frequencies is found to increase. In conformity to the same, the probability density function plots with respect to first three natural frequencies are furnished considering combined variation for cross-ply composite conical shells (Fig. 7).

The sensitivity index obtained from material and geometric properties to each random variable at the constituent and ply levels is used as a guide to increase the structural reliability or to reduce the cost. Furthermore, the probabilistic structural analysis and risk assessment is performed once the uncertain laminate properties are computationally simulated. The sensitivity of different random variables for the laminate frequencies of different modes may be different. In case of combined random variation in all input parameters, the percentage contribution to the sensitivity of each and individual input parameter influencing output responses of angle-ply and cross-ply composite conical shells is shown in Figs. 8 and 9, respectively, wherein the first three natural frequencies are considered. From Fig. 8, it is observed that the predominant effect on the sensitivity of the tested natural frequencies is influenced by the ply orientation angle followed by the mass density, trailed by longitudinal shear modulus, elastic modulus, and the least contribution is observed for the transverse shear modulus. In contrast to cross-ply composite conical shells, the sensitivity effect of the tested natural frequencies is found to be dominated by the ply orientation angle followed by the elastic modulus of the two outmost layers (null sensitivity effect is identified for the inner two layers due to the coupling effect), while subsequently sensitivity contribution is trailed by the mass density followed by the longitudinal shear modulus, and null sensitivity effect is identified for the transverse shear modulus.

Table 4 Maximum value, minimum value, mean value, and standard deviation (SD) of first three natural frequencies obtained by D-optimal design method (29 samples) due to individual stochasticity in $[\rho(\bar{\omega})]$, $[G_{12}(\bar{\omega})]$, $[G_{23}(\bar{\omega})]$, and $[E_1(\bar{\omega})]$ in each layer for four-layered graphite–epoxy angle-ply (45°/−45°/−45°/45°) and cross-ply (0°/90°/90°/0°) composite conical shells

Laminate	Parameter	$\rho(\bar{\omega})$			$G_{12}(\bar{\omega})$			$G_{23}(\bar{\omega})$			$E_1(\bar{\omega})$		
		f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3	f_1	f_2	f_3
Angle-ply	Max.	56.6011	147.7441	263.9452	55.4453	139.1648	258.7947	54.0149	135.2645	251.8812	54.6998	136.7561	254.8680
	Min.	51.6930	129.4530	241.0577	52.4898	131.1424	244.5731	54.0064	135.0492	251.8499	53.2482	133.5960	248.5535
	Mean	54.0328	135.3123	251.9686	53.9973	135.2167	251.7860	54.0100	135.2569	251.8656	53.9938	135.2197	251.7920
	SD	0.7842	1.9639	3.6571	0.4765	1.3220	2.4287	0.0013	0.0025	0.0051	0.2567	0.5659	1.1415
Cross-ply	Max.	90.1346	264.7853	510.6328	86.9052	253.8686	489.5655	86.0162	252.6815	487.2938	89.0553	263.1691	507.5916
	Min.	82.4120	242.0987	466.8847	85.0697	251.3858	484.8193	86.0010	252.6479	487.2215	82.8033	241.6712	465.9084
	Mean	86.0389	252.7533	247.4294	85.9966	252.6479	487.2274	86.0086	252.6647	487.2580	85.9591	252.4768	486.8506
	SD	1.2491	3.6695	7.0761	0.3493	0.4525	0.8575	0.0031	0.0069	0.0148	1.2723	4.3821	8.4763

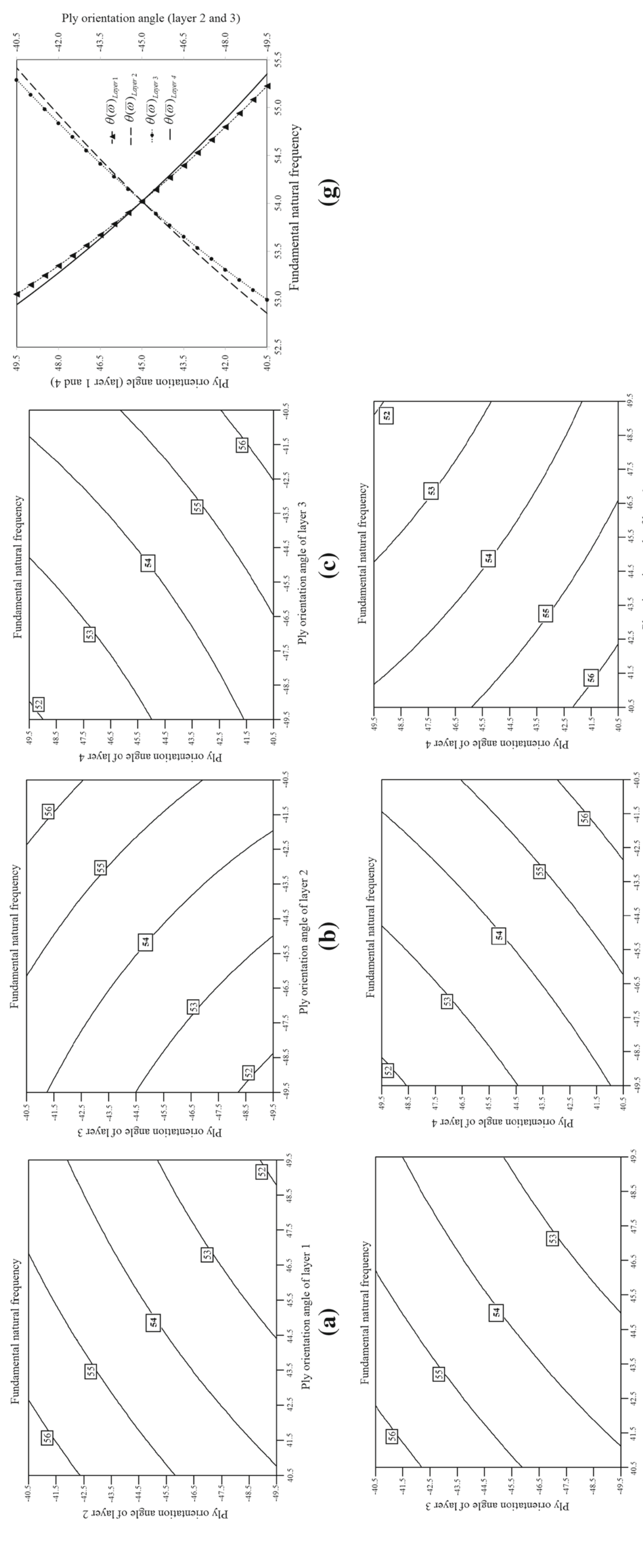

Fig. 5 Variation in fundamental natural frequency with interaction effect (sample size = 29) due to variation in ply orientation angle of angle-ply (45°/−45°/−45°/45°) composite conical shells

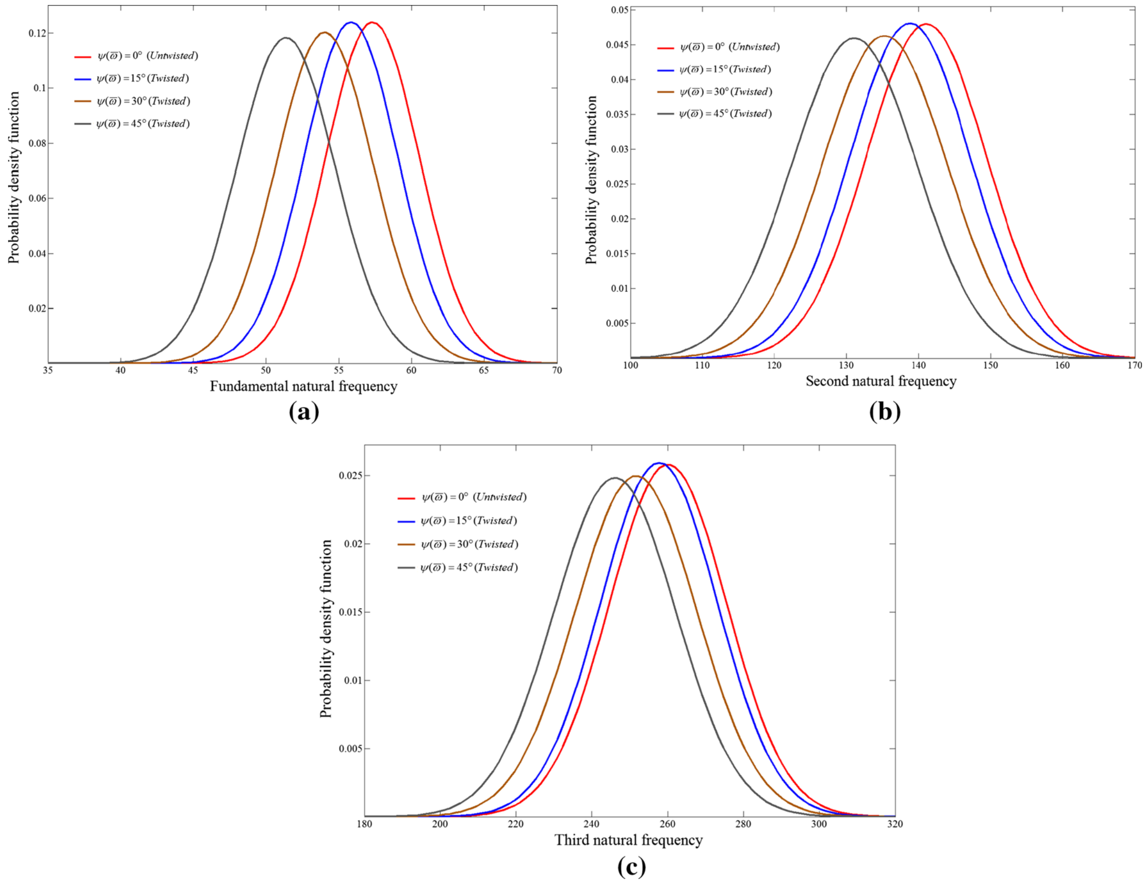

Fig. 6 Probability density function of first three natural frequencies (sample size = 261) of angle-ply (45°/−45°/45°/−45°) composite conical shells considering variation in angle of twist [$\psi(\bar{\omega})$] (with ±5° variation)

6 Conclusions

The present approach includes the uncertainty quantification of the natural frequency considering the stochastic effect of twist angle, ply orientation, and material properties for composite conical shells. D-optimal design is first attempted for stochastic analysis of composite structures in the present study. The meta-model formed from a small set of samples is found to establish the accuracy and computational efficacy. The results obtained employing D-optimal design meta-models are compared with the results of the direct Monte Carlo simulation method. The ply orientation angle is found to be most sensitive among all tested input parameters, while the transverse shear modulus is identified as least sensitive to the first three stochastic natural frequencies. As the stochastic twist angle increases, the stochastic first three natural frequencies are found to decrease subsequently due to reduction in the stochastic elastic stiffness. From sensitivity analysis of the combined input variation case, the ply orientation angles of two outer surface layers are found to dominate the variation in natural frequencies. For the combined input variation case of the cross-ply laminate, null sensitivity effect is identified at two inner layers for ply orientation angle and elastic modulus, while in contrast, the sensitivity of the ply orientation angle for the angle-ply laminate at two inner layers is found to be predominant. As the degree of variation in input parameters increases, the stochastic mean natural frequency values are found to reduce, but the volatility in stochastic natural frequencies is found to increase for both angle-ply and cross-ply laminates. For the angle-ply laminate, the predominant effect on sensitivity on tested natural frequencies is influenced by the ply orientation angle followed by mass density, trailed by subsequently longitudinal shear modulus, elastic modulus and least contribution is observed for the transverse shear modulus. For the cross-ply laminate, the prime sensitivity effect on tested natural frequencies is influenced by the ply orientation angle followed by elastic modulus, trailed by mass den-

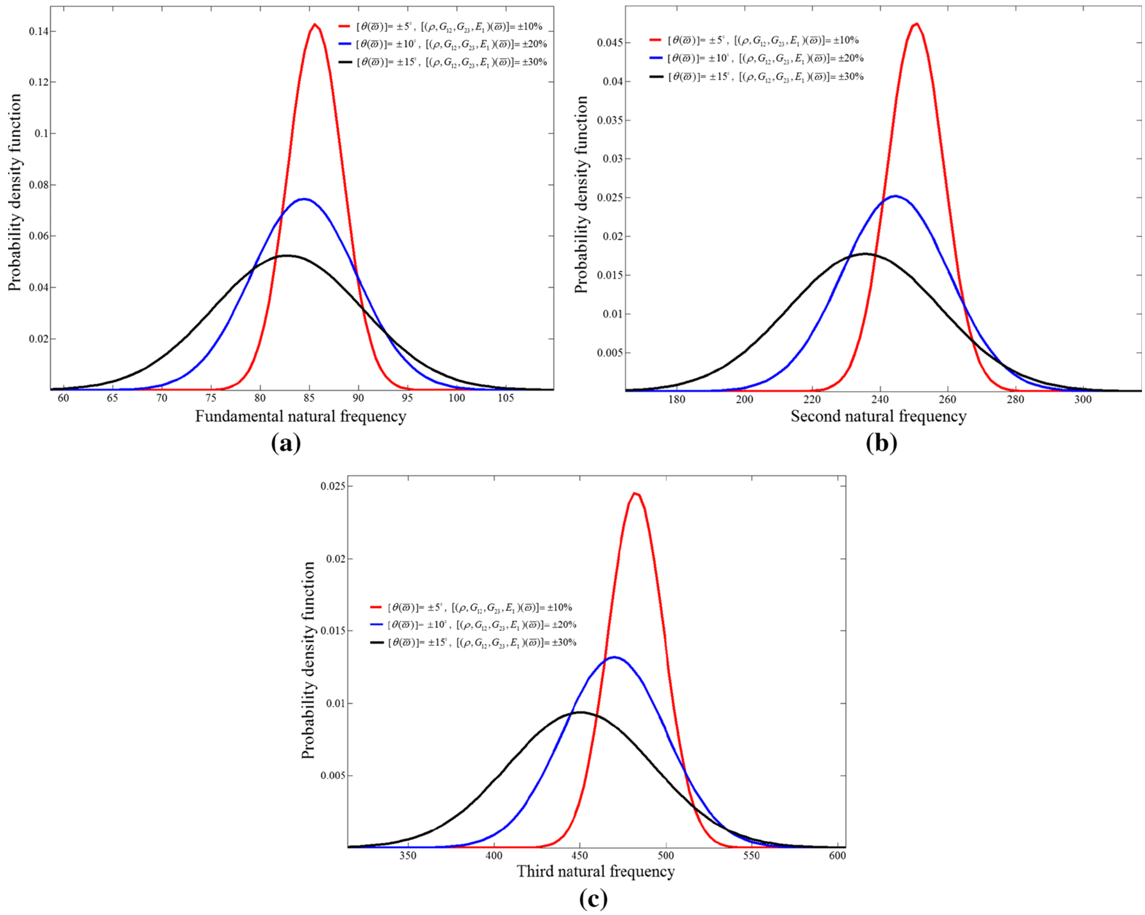

Fig. 7 Probability density function with respect to first three natural frequencies (sample size = 261) due to combined variation for cross-ply (0°/90°/90°/0°) conical shells

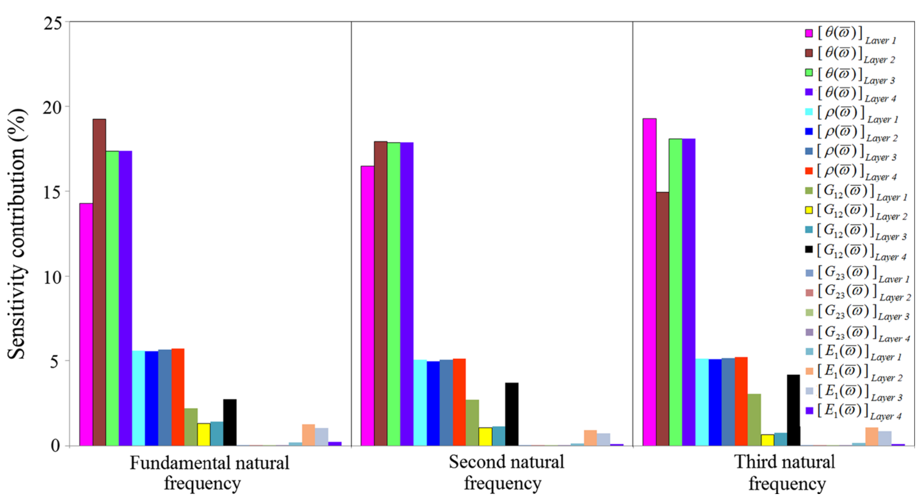

Fig. 8 Sensitivity contribution in percentage for combined variation in $[\theta(\bar{\omega}), \rho(\bar{\omega}), G_{12}(\bar{\omega}), G_{23}(\bar{\omega}), E_1(\bar{\omega})]$ (sample size = 261) for angle-ply (45°/−45°/−45°/45°) composite conical shells

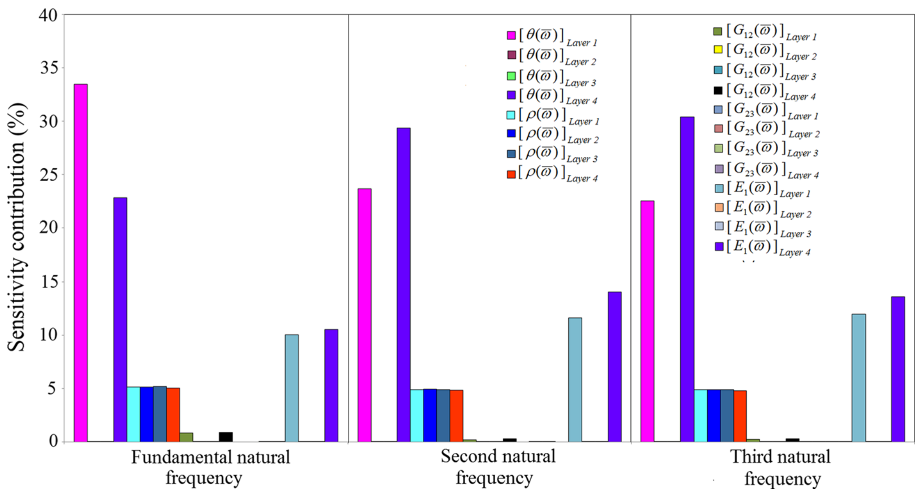

Fig. 9 Sensitivity contribution in percentage for combined variation in $[\theta(\bar{\omega}), \rho(\bar{\omega}), G_{12}(\bar{\omega}), G_{23}(\bar{\omega}), E_1(\bar{\omega})]$ (sample size = 261) for cross-ply (0°/90°/90°/0°) composite conical shells

sity and longitudinal shear modulus, while the minimum effect is identified for the transverse shear modulus. More complex systems of laminated composite structures can be explored using the present generic approach.

References

1. Mitchell, T.J.: An algorithm for the construction of D-optimal experimental designs. Technometrics **16**(2), 203–210 (1974)
2. Michael, J.B., Norman, R.D.: On minimum-point second-order designs. Technometrics **16**(4), 613–616 (1974)
3. Craig, J.A.: D-Optimal Design Method: Final Report and User's Manual. USAF Contract F33615-78-C-3011, FZM-6777, General Dynamics. Forth Worth Div. (1978)
4. Montgomery, D.C.: Design and Analysis of Experiments. Wiley, New Jersey (1991)
5. Unal, R., Stanley, D.O., Lepsch, R.A.: Parametric modeling using saturated experimental designs. J. Parametr. **XVI**(1), 3–18 (1996)
6. Giunta, A.A, Balabanov, V., Haim, D., Grossman, B., Mason, W.H., Watson, L.T.: Wing design for high-speed civil transport using DOE methodology, USAF/NASA/ ISSMO Symposium, AIAA Paper 96-4001 (1996)
7. Radoslav, H.: Multiplicative methods for computing D-optimal stratified designs of experiments. J. Stat. Plan. Inference **146**, 82–94 (2014)
8. Goyal, V.K., Kapania, R.K.: Dynamic stability of uncertain laminated beams subjected to subtangential loads. Int. J. Solids Struct. **45**(10), 2799–2817 (2008)
9. Shaker, A., Abdelrahman, W.G., Tawfik, M., Sadek, E.: Stochastic finite element analysis of the free vibration of laminated composite plates. Comput. Mech. **41**, 495–501 (2008)
10. Fang, C., Springer, G.S.: Design of composite laminates by a Monte Carlo method. Compos. Mater. **27**(7), 721–753 (1993)
11. Sasikumar, P., Suresh, R., Gupta, S.: Stochastic finite element analysis of layered composite beams with spatially varying non-Gaussian inhomogeneities. Acta Mech. **225**, 1503–1522 (2014)
12. Ankenmann, B., Nelson, B.L., Staum, J.: Stochastic kriging for simulation metamodeling. Oper. Res. **58**(2), 371–382 (2010)
13. Park, J.S., Kim, C.G., Hong, C.S.: Stochastic finite element method for laminated composite structures. J. Reinf. Plast. Compos. **14**(7), 675–693 (1995)
14. Ganesan, R., Kowda, V.K.: Free vibration of composite beam-columns with stochastic material and geometric properties subjected to random axial loads. J. Reinf. Plast. Compos. **24**(1), 69–91 (2005)
15. Yue, R.-X., Liu, X., Chatterjee, K.: D-optimal designs for multi-response linear models with a qualitative factor. J. Multivar. Anal. **124**, 57–69 (2014)
16. Choi, H., Kang, M.: Optimal sampling frequency for high frequency data using a finite mixture model. J. Korean Stat. Soc. **43**(2), 251–262 (2014)
17. Xu, M., Qiu, Z., Wang, X.: Uncertainty propagation in SEA for structural—acoustic coupled systems with non-deterministic parameters. J. Sound Vib. **333**(17), 3949–3965 (2014)
18. Kuttenkeuler, J.: A finite element based modal method for determination of plate stiffnesses considering uncertainties. J. Compos. Mater. **33**(8), 695–711 (1999)
19. Ghanem, R.G., Spanos, P.D.: Stochastic Finite Elements—A Spectral Approach. Revised. Dover Publications Inc., NY (2002)
20. Kishor, D.K., Ganguli, R., Gopalakrishnan, S.: Uncertainty analysis of vibrational frequencies of an incompressible liquid in a rectangular tank with and without a baffle using polynomial chaos expansion. Acta Mech. **220**(1-4), 257–273 (2011)
21. Shaker, A., Abdelrahman, W.G., Tawfik, M., Sadek, E.: Stochastic finite element analysis of the free vibration of laminated composite plates. Comput. Mech. **41**(4), 493–501 (2008)

22. Dey, S., Mukhopadhyay, T., Adhikari, S.: Stochastic free vibration analysis of angle-ply composite plates–a RS-HDMR approach. Compos. Struct. **122**, 526–536 (2015)
23. Dey, S., Mukhopadhyay, T., Adhikari, S.: Stochastic free vibration analyses of composite doubly curved shells—A Kriging model approach. Compos. Part B Eng. **70**, 99–112 (2015)
24. Dey, S., Karmakar, A.: Finite element analyses of bending stiff composite conical shells with multiple delamination. J. Mech. Mater. Struct. **7**(2), 213–224 (2012)
25. Liew, K.M., Lim, C.M., Ong, L.S.: Vibration of pretwisted cantilever shallow conical shells. I. J. Solids Struct. **31**, 2463–2474 (1994)
26. Jones, R.M.: Mechanics of Composite Materials. McGraw-Hill Book Co., NY (1975)
27. Cook, R.D., Malkus, D.S., Plesha, M.E.: Concepts and Applications of Finite Element Analysis. Wiley, New York (1989)
28. Meirovitch, L.: Dynamics and Control of Structures. Wiley, New York (1992)
29. Karmakar, A., Sinha, P.K.: Failure analysis of laminated composite pretwisted rotating plates. J. Reinf. Plast. Compos. **20**, 1326–1357 (2001)
30. Bathe, K.J.: Finite Element Procedures in Engineering Analysis. PHI, New Delhi (1990)
31. Carpenter, W.C.: Effect of design selection on response surface performance. NASA Contractor Report 4520 (1993)
32. Mukhopadhyay, T., Dey, T.K., Dey, S., Chakrabarti, A.: Optimization of fiber reinforced polymer web core bridge deck—a hybrid approach. Struct. Eng. Int. IABSE. **24**(2), (2015). doi:10.2749/101686614X14043795570778
33. Giunta, A.A., Wojtkiewicz, S.F., Eldred, M.S.: Overview of modern design of experiments methods for computational simulations. In: Proceedings of the 41st American Institute of Aeronautics and Astronautics Aerospace Sciences Meeting and Exhibit, Paper AIAA 2003–0649 Reno, NV (2003)
34. Santner, T.J., Williams, B., Notz, W.: The Design and Analysis of Computer Experiments. Springer, Heidelberg (2003)
35. Koehler, J.R., Owen, A.B.: Computer experiments. In: Ghosh, S., Rao, C.R. (eds.) Handbook of Statistics, vol.13, pp. 261–308. Elsevier Science B.V., Amsterdam (1996)
36. Koziel, S., Yang, X.-S. (eds.) Computational Optimization, Methods and Algorithms. Springer, Berlin, ISBN: 978-3-642-20858-4, (Print) 978-3-642-20859-1
37. Jin, R., Chen, W., Simpson, T.: Comparative studies of metamodelling techniques under multiple modelling criteria. Struct. Multidiscip. Optim. **23**(1), 1–13 (2001)
38. Kim, B.S., Lee, Y.B., Choi, D.H.: Comparison study on the accuracy of metamodeling technique for non-convex functions. J. Mech. Sci. Techn. **23**(4), 1175–1181 (2009)
39. Mukhopadhyay, T., Dey, T.K., Chowdhury, R., Chakrabarti, A.: Structural damage identification using response surface-based multi-objective optimization: A comparative study. Arab. J. Sci. Eng. (2015). doi:10.1007/s13369-015-1591-3
40. Qatu, M.S., Leissa, A.W.: Natural frequencies for cantilevered doubly-curved laminated composite shallow shells. Compos. Struct. **17**, 227–255 (1991)
41. Qatu, M.S., Leissa, A.W.: Vibration studies for laminated composite twisted cantilever plates. Int. J. Mech. Sci. **33**(11), 927–940 (1991)