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Effect of twist and rotation on vibration of functionally graded conical shells

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Abstract This paper presents the effect of twist and rotational speeds on free vibration characteristics of functionally graded conical shells employing finite element method. The objective is to study the natural frequencies, the influence of constituent volume fractions and the effects of configuration of constituent materials on the frequencies. The equation of dynamic equilibrium is derived from Lagrange's equation neglecting the Coriolis effect for moderate rotational speeds. The properties of the conical shell materials are presumed to vary continuously through their thickness with power-law distribution of the volume fractions of their constituents. The QR iteration algorithm is used for solution of standard eigenvalue problem. Computer codes developed are employed to obtain the numerical results concerning the combined effects of twist angle and rotational speed on the natural frequencies of functionally graded conical shells. The mode shapes for typical shells are also depicted. Numerical results obtained are the first known non-dimensional frequencies for the type of analyses carried out here.

Keywords Functionally graded material · Vibration · Rotational speed · Finite element · Twist angle · Conical shell

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Abbreviations

A_0, A_{-1}, A_1, A_2, A_3	Temperature coefficients
T	Temperature in Kelvin
A_i	Material property
V_f	Volume fraction
N	Material property graded index (or Power law exponent)
t	Thickness
E	Young's modulus
G	Shear modulus
υ	Poisson's ratio
ρ	Mass density
r_1 , r_2	Radius of curvature ($r_1 > r_2$)
r_x	Radius of curvature in x-direction
r_y	Radius of curvature in y-direction
r_{xy}	Radius of twist
Lo	Length
b_o	Reference width

ϕ_{ve}	Vertex angle
ϕ_o	Base subtended angle of cone
Ψ	Twist angle
L_f	Lagrangian function
Ω	Non-dimensional speed of rotation (Ω'/ω_o)
Ω'	Actual angular speed of rotation
ω_o	Fundamental natural frequency of a non-rotating shell
ω	Non-dimensional frequency parameter
λ	Non-dimensional frequency
$[M_e]$, [M]	Element mass matrix and Global mass matrix
$[C_e]$, [C]	Element Coriolis matrix and Global Coriolis matrix
$[K_e]$, [K]	Element elastic stiffness matrix and Global elastic stiffness matrix
$[K_{\sigma e}]$, $[K_\sigma]$	Element geometric stiffness matrix, Global geometric stiffness matrix
$[K_{Re}]$, $[K_R]$	Element rotational stiffness matrix and Global rotational stiffness matrix
$\{F(\Omega^2)\}$	Global vector of nodal equivalent centrifugal forces
$\{F_{ce}\}$, $\{F_e\}$, $\{F\}$	Element load vector due to centrifugal force, Element load vector due to externally applied load and Global vector of externally applied load
$\{\delta_e\}$,$\{\delta\}$	Element displacement vector and Global displacement vector
ς, χ	Local natural coordinates of the element
x, y, z	Local coordinate axes (plate coordinate system)
Lo/s	Aspect ratio
ω_n	Natural frequency of rotating shell
NDFF	Non-dimensional fundamental natural frequency
NDSF	Non-dimensional second natural frequency

1 Introduction

Functionally graded materials (FGM) have gained immense popularity as advanced engineering materials due to its high corrosion and heat resistance properties along with high strength and stiffness. Such materials are the novel inhomogeneous materials consisting of a mixture of isotropic materials in which the material properties are graded but continuous particularly along a given direction to improve the quality of performance. In general, these materials are made from a mixture of ceramic and metal to take the advantage of coupling the desirable properties such as heat and corrosion resistance of ceramics and high toughness, tensile strength and bonding capability of metals. Functionally graded shallow pretwisted conical shells could be employed as rotating turbomachinery blades in aerospace, nuclear and mechanical industries. Hence, for the purpose of design and manufacturing, the accurate evaluation of free vibration characteristics of such pretwisted shells under rotation is essential to arrest the zone of resonance. Therefore, a profound understanding of dynamic behaviour of such functionally graded (FG) conical shell is important for the designers to ensure the operational safety. The investigations on FGMs for the vibration analysis were carried out by Pradhan et al. (2000), Loy et al. (1999), Ng et al. (2001). The free vibration analysis of rotating truncated circular orthotropic conical shells has been studied by Hua (2000). However, to the best of authors' knowledge, the only available works regarding the free vibration analysis of rotating FG shells of revolution are mostly limited to FG cylindrical shells (Ahmad and Naeem 2009; Malekzadeh and Heydarpour 2012). On the other hand, the free vibration behaviours of stationary FG truncated conical shells (Bhangale et al. 2006; Sofiyev 2009; Tornabene et al. 2009; Setoodeh et al. 2012) have been investigated in the recent years. It is to be noted that these investigations are concerned with the stress and deformation analysis of the rotating FG cylindrical shells (Ghafoori and Asghari 2012; Heydarpour et al. 2012). Other studies on thermo-elastic buckling analysis of plates and shells are extensive. Many of these studies are for isotropic plates and shells which include Birman and Bert (1993), Tauchert (1991), Thornton (1993), Gotsis and Guptill (1994), Eslami et al. (1996) and Sofiyev (2007). Finite element method employed for the analysis of shells

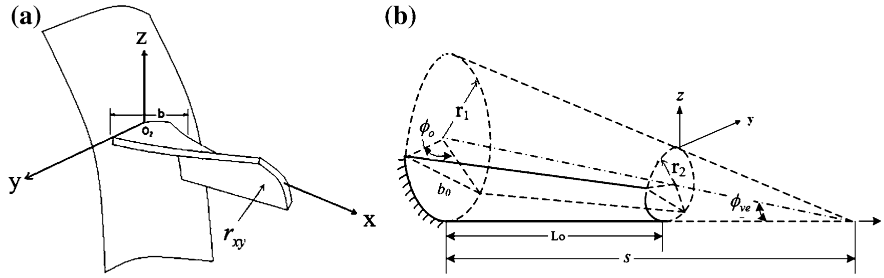

Fig. 1 **a** Turbo-machinery blade idealized as pretwisted conical shells, **b** Conical shell model

subjected to vibration was also examined by He et al. (2002), Ng et al. (2002), Liew et al. (2004), Shu (1996) and Bardell et al. (1998). Much effort has been devoted to various structural analyses of functionally graded structures, such as thermal stresses, static, buckling, and vibration analyses. Remarkable investigation is also carried out by Noda (1999), Fukui et al. (1993) and Praveen and Reddy (1998).

In the present study, two types of functionally graded materials, namely Stainless Steel/Ni FG and Ti–6Al–4V/aluminium oxide FGM are investigated to address the effects of the volume fraction and boundary condition on their natural frequencies. To bridge the research gap identified in the review of open literature, the effect of rotational speeds on free vibration analysis of functionally graded conical shells is studied. To the best of authors' knowledge and considering the review of open literature, it is revealed that the combined effect of rotation and twist on functionally graded shallow conical shell structure has not received due attention. To fill up this apparent void, present study employed a finite element based numerical approach to study the free vibration characteristics of pretwisted functionally graded rotating shallow conical shells which could be idealized as turbomachinery blades. The shell elements are developed by superimposing the stiffness of the membrane element and plate bending element. The finite element analyses are carried out using an eight-noded isoparametric quadratic element and QR iteration algorithm (Bathe 1990) is utilized to solve the standard eigenvalue problem. The non-dimensional fundamental natural frequencies are obtained considering the effects of triggering parameters like the twist angle and rotational speed.

2 Theoretical formulation

Functionally graded materials (FGM) are defined as the combination of two or more materials. Most of the functionally graded materials are employed in high-temperature environments and many of the constituent materials may possess temperature-dependent properties. The material properties (A) can be expressed as a function of temperature (Touloukian 1967) as,

$$A = A_0 + A_{-1}T^{-1} + 1 + A_1T + A_2T^2 + A_3T^3 \tag{1}$$

where A_0, A_{-1}, A_1, A_2 and A_3 are the coefficients of temperature T(K) expressed in Kelvin and are unique to the constituent materials and 'A' is a function of the material properties and volume fractions of the constituent materials, and is expressed as

$$A = \sum_{i=1}^{k} A_i \quad V_{f\,i} \tag{2}$$

where A_i and V_f are material property and volume fraction of the constituent materials i, respectively. The volume fractions of all the constituent materials can be related as

$$\sum_{i=1}^{k} V_{f\,i} = 1 \quad \text{and} \quad V_{fi} = \left[\frac{z + \frac{t}{2}}{t}\right]^N \tag{3}$$

where N is the material property graded index which is a positive real number ($0 \leq N \leq \infty$). The material properties of the FG conical shells is varied continuously and smoothly in the thickness direction, i.e. z-direction, of FG conical shell. Without loss of generality of the formulation and the method of solution in this study, the effective material properties are obtained using the power law distribution (Loy et al. 1999),

$$E(z) = E_m + (E_c - E_m)\left[(2z + t)/2t\right]^N \tag{4}$$

$$\upsilon(z) = \upsilon_m + (\upsilon_c - \upsilon_m)\,[(2z + t)/2t]^N \quad (5)$$

$$\rho(z) = \rho_m + (\rho_c - \rho_m)\,[(2z + t)/2t]^N \quad (6)$$

where E, υ and ρ denote Young's modulus, Poisson's ratio and mass density with suffix as 'c' and 'm' indicating the corresponding values at the outer surface (ceramic rich) and inner surface (metal rich) of the FG conical shell. A functionally graded conical shell panel is shown in Fig. 1, wherein a coordinate system (x, y, z) is established on the middle surface of the conical shell panel. The geometric parameters of FG conical shell are represented by length *Lo*, semi-vertex angle ϕ_{ve}, base subtended angle ϕ_o, thickness t, and the radii at the two ends r_1 and r_2 ($r_1 > r_2$). In the spanwise direction of FG conical shell, no curvature is considered (i.e., $r_x = \infty$). The cone radius at any point along its length is expressed as (Zhao and Liew 2011)

$$r_y(z) = r_2 - z\,Sin(\phi_{ve}/2) \quad (7)$$

A shallow shell is characterized by its middle surface which is defined by the equation (Leissa et al. 1984),

$$z = -0.5[(x^2/r_x) + (2xy/r_{xy}) + (y^2/r_y)] \quad (8)$$

Considering non-dimensional coordinate (ς, χ) system, the radius of twist (r_{xy}), length (*Lo*) of the shell and twist angle (*Ψ*) are related as (Liew et al. 1994)

$$\varsigma = \frac{x}{Lo}\; \chi = \frac{y}{b_o}\; f(\varsigma,\chi) = \frac{s\tan(\phi_{ve}/2)}{r_y(\varsigma,\chi)}$$
$$r_{xy} = -Lo/\tan\psi \quad (9)$$

Hamilton's principle applicable to non-conservative system can be expressed as,

$$\delta H = \int_{t_1}^{t_2} [\delta T - \delta U - \delta W]dt = 0 \quad (10)$$

where T, U and W are total kinetic energy, total strain energy and total potential of the applied load, respectively wherein $\delta W = 0$ for free vibration analysis. The Lagrange's equation of motion is given by

$$\frac{d}{dt}\left[\frac{\partial L_f}{\partial \delta_e}\right] - \left[\frac{\partial L_f}{\partial \delta_e}\right]\{F_e\} \quad (11)$$

where $\{F_e\}$ is the applied external force vector of an element and L_f is the Lagrangian function. Substituting $L_f = T - U$ in Lagrange's equation incorporating the corresponding expressions for *T* and *U*, the dynamic equilibrium equation for each element in the following form (Karmakar and Sinha 2001)

$$[M_e]\{\ddot{\delta}_e\} + [C_e]\{\dot{\delta}_e\} + ([K_e] + [K_{\sigma e}] - [K_{Re}])\{\delta_e\}$$
$$= \{F_{ce}\} + \{F_e\} \quad (12)$$

where $\{\delta_e\}$ is the element displacement vector, $[M_e]$ is the element mass matrix, $[C_e]$ is the element Coriolis matrix (skew symmetric), $[K_e]$ is the element stiffness matrix, $[K_{\sigma e}]$ is the element geometric stiffness matrix, $[K_{Re}]$ is the element rotational stiffness matrix (symmetric and positive definite), $\{F_e\}$ is the element force vector, $\{F_{ce}\}$is the element centrifugal force vector. After assembling all the element matrices and the force vectors with respect to common global coordinates, the resulting equilibrium equation of the structure becomes

$$[M]\{\ddot{\delta}\} + [C]\{\dot{\delta}\} + ([K] + [K_\sigma] - [K_R])\{\delta\}$$
$$= \{F(\Omega^2)\} + \{F\} \quad (13)$$

where $[M]$, $[C]$, $[K]$, $[K_\sigma]$ and $[K_R]$ are the respective global matrices, $\{F(\Omega^2)\}$ is the vector of nodal equivalent centrifugal forces, $\{F\}$ is the global vector of externally applied load and {δ} is the global displacement vector. For moderate rotational speed, Coriolis matrix and rotational stiffness matrix are neglected and finally the dynamic equilibrium equation in global form in absence of externally applied load ({F}) reduces to (Sreenivasamurthy and Ramamurti 1981)

$$[M]\{\ddot{\delta}\} + ([K] + [K_\sigma])\{\delta\} = \{F(\Omega^2)\} \quad (14)$$

where $[K_\sigma]$ depends on initial stress distribution and is obtained by the iterative procedure upon solving

$$([K] + [K_\sigma])\{\delta\} = \{F(\Omega^2)\} \quad (15)$$

The natural frequencies (ω_n) are determined from the standard eigenvalue problem (Bathe 1990) which is represented below and is solved by QR iteration algorithm

$$[X_{neq}]\{\delta\} = \lambda\{\delta\} \quad (16)$$

$$\text{where,}\quad [X_{neq}] = ([K] + [K_\sigma])^{-1}[M] \quad (17)$$

$$\lambda = 1/\omega_n^2 \quad (18)$$

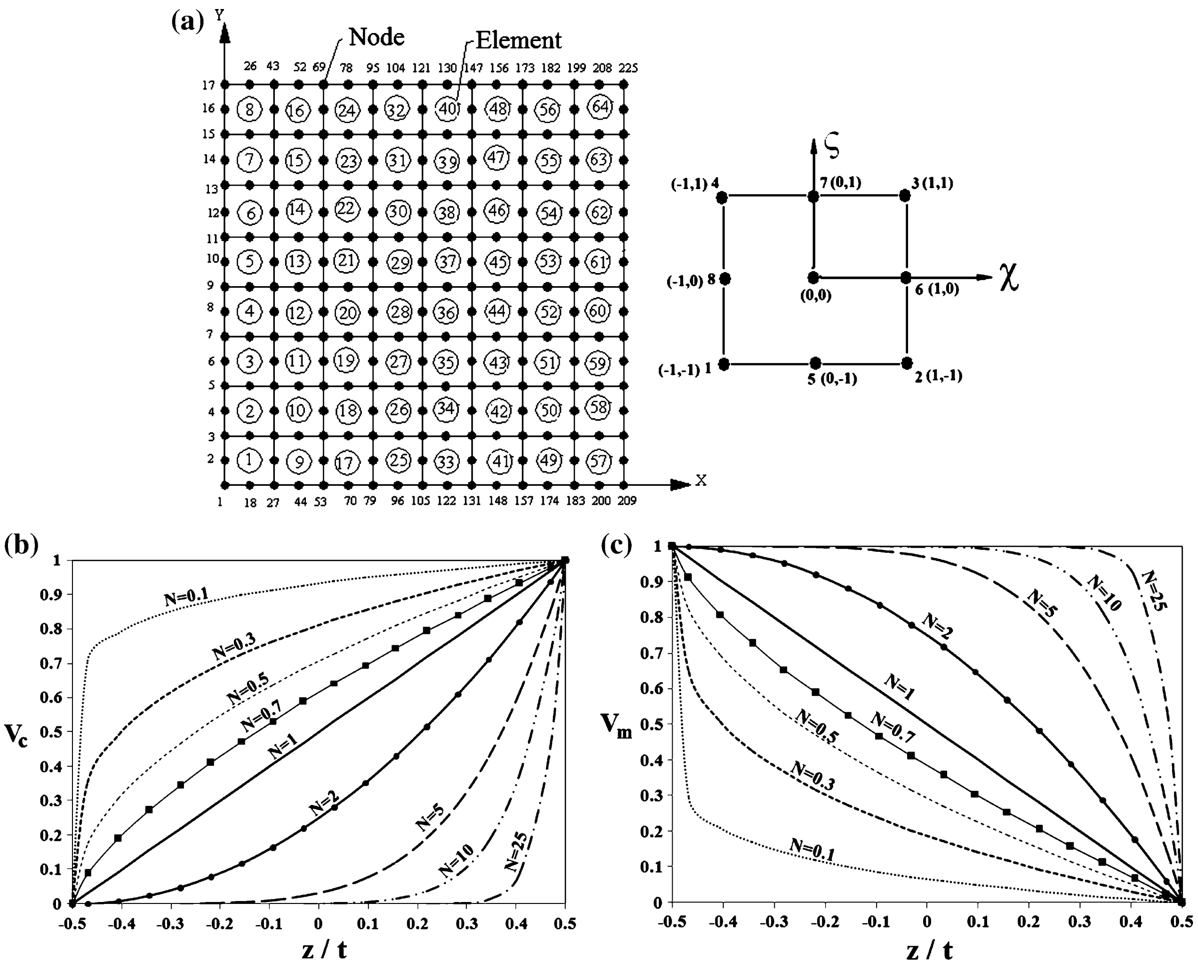

Fig. 2 **a** Finite element discretisation of 8 × 8 mesh on plan area with elements, node numbers and the natural coordinates considering isoparametric element. **b** Variation of volume fraction of ceramic (V_c) with respect to the thickness parameter (z/t) of Al–ZrO_2 FG conical shell. **c** Variation of volume fraction of metal (V_m) with respect to the thickness parameter (z/t) of Al–ZrO_2 FG conical shell

An eight noded isoparametric quadratic element with five degrees of freedom at each node (three translation and two rotations) is employed wherein the shape functions (S_j) are considered as function of local natural coordinates of the element (ς, χ). Here $\chi_j = +1$ for nodes 2, 3, 6 and $\chi_j = -1$ for nodes 1, 4, 8 and $\varsigma_j = +1$ for nodes 3, 4, 7 and $\varsigma_j = -1$ for nodes 1, 2, 5 as furnished in Fig. 2a. The variation of volume fraction through the thickness with different volume fraction exponents for Al–ZrO_2 is illustrated in Fig. 2b, c which indicates the variation of ceramic and metallic composition (V_c, V_m) across the thickness parameter (z/t) as per power law of functionally graded material as furnished in Eq. (3). By virtue of FGM properties, linear variation of volume fractions for both ceramic and metallic composition is only observed for N = 1.

3 Results and discussion

Parametric studies are carried out with respect to rotational speed, twist angle on natural frequencies of functionally graded rotating shallow conical shells. Non-dimensional frequencies for conical shells ($r_x = \alpha$) having square plan-form ($Lo/b_o = 1$), curvature ratio (b_o/r_y) of 0.5 and thickness ratio (s/t) of 1000 are obtained corresponding to non-dimensional speeds of rotation, $\Omega = (\Omega'/\omega_o) = 0.0$, 0.5 and 1.0,

Table 1 Properties of the FGM components at 300 K

Material	Properties		
	E (N/m^2)	υ	ρ (Kg/m^3)
Ti–6Al–4V	105.700×10^9	0.298000	4,429
Aluminum oxide (Al_2O_3)	320.200×10^9	0.260000	3,750
Stainless steel (SS)	207.788×10^9	0.317756	8,166
Nickel (Ni)	205.098×10^9	0.310000	8,900
Aluminum (Al)	70.000×10^9	0.300000	2,707
Zirconia (ZrO_2)	151.000×10^9	0.300000	3,000

considering three different angles of twist of conical shells, namely $\psi = 15°, 30°$ and $45°$, in addition to the untwisted one ($\psi = 0°$). The variation of volume fractions (V_c, V_m) through the thickness for Al–ZrO_2 are furnished in Fig. 2b, c. An eight noded isoparametric quadratic element with five degrees of freedom at each node is considered for finite element formulation. Material properties of functionally graded materials (Loy et al. 1999; Zhao and Liew 2011) are adopted as stipulated in Table 1.

Even though the main application of FG material is found in the area of thermal field where the temperature gradient is significantly high, the present study is aimed to address the combined effect of twist and rotational speeds on the free vibration characteristics of the functionally graded conical shell which is idealized as turbomachinery blades. The constant uniform temperature (300 K) is considered as the temperature transient of the blades during engine startup (i.e., at the beginning from uniform room temperature) and the blades are initially exposed to operate in external environment. Computer codes are developed based on present finite element method.

Table 2 Convergence study for NDFF [$\omega = \omega_n \cdot a^2 \sqrt{(\rho_c/E_c t^2)}/2\pi$, for Al/$ZrO_2$ functionally graded conical shells, considering $r_1 = 0.2$ m, $t = 0.01$ m, $Lo = 0.8$ m, $\phi_{ve} = 30°$, $\phi_o = 120°$

N	Zhao and Liew (Zhao and Liew 2011)	Present FEM		
		6 × 6	8 × 8	10 × 10
0.0	1.3666	1.3608	1.3449	1.2356
0.5	1.2486	1.2387	1.2271	1.0848
1.0	1.1893	1.1792	1.1641	1.1038
5.0	1.0737	1.0578	1.0492	0.9910
10.0	1.0404	1.0222	1.0102	0.9566

Table 3 Non-dimensional fundamental natural frequencies [$\omega = \omega_n Lo^2 \sqrt{(\rho t/D)}$] of an isotropic rotating cantilever plate, $Lo/b = 1$, $t/Lo = 0.12$, $D = Et^3/\{12(1 - \nu^2)\}$, $\nu = 0.3$

Non-dimensional speed (Ω)	Present FEM	Sreenivasamurthy and Ramamurti (Sreenivasamurthy and Ramamurti 1981)
0.0	3.4174	3.4368
0.2	3.4933	3.5185
0.4	3.7110	3.7528
0.6	4.0458	4.1287
0.8	4.4690	4.5678
1.0	4.9549	5.0916

The numerical results obtained are compared and validated with the results of published literature (Zhao and Liew 2011; Sreenivasamurthy and Ramamurti 1981) as furnished in Tables 2 and 3. Table 2 presents the validation of non-dimensional fundamental natural frequencies of functionally graded Al/ZrO_2 conical shells (Zhao and Liew 2011), while the effect of rotation on non-dimensional fundamental natural frequencies of an isotropic rotating cantilever plate (Sreenivasamurthy and Ramamurti 1981) is furnished in Table 3. The comparative study shows an excellent agreement with the previously published results and hence it demonstrates the capability of the computer codes developed and proves the accuracy of the analyses. The predictive capability of the computer programs in respect of FG conical shells with rotating and twisted models is thus confirmed. Convergence studies are also performed to determine the converged mesh size as furnished in Table 2. It is also observed from the convergence study that uniform mesh divisions of (6 × 6), (8 × 8) and (10 × 10) give closer values to the results obtained by Zhao and Liew et al. (2011) considering the complete plan-form of the shell and provide nearly equal results the difference being around one percent (1 %). To save the computer time, (6 × 6) mesh division was chosen which gave closest results to the benchmark results (Zhao and Liew 2011). It is to be noted that the numerical results depicted in Table 2 indicates that the accuracy of numerical results of increased mesh size (viz. 10 × 10) is adverse compared to that of the coarse mesh size (viz. 6 × 6). In conformity of the same, the percentage difference of the results obtained by (8 × 8) mesh and (10 × 0) mesh with respect to the benchmark values were found much more than that of

Table 4 NDFF [$\omega = \omega_n t^2 \sqrt{(\rho_m/E_m)}$] of simply supported aluminum–zirconia functionally graded plate a = b = 1 m, N = 1 (For aluminum: E_m = 70 GPa, ν_m = 0.3, and ρ_m = 2,707 kg/m^3 and for zirconia: E_c = 200 GPa, ν_c = 0.3 and ρ_c = 2,702 kg/m^3)

a/h	Present FEM	Pradyumna and Bandyopadhyay (Pradyumna and Bandyopadhyay 2008)	Matsunaga (Matsunaga 2008)	Ferreira et al. (Ferreira et al. 2006)
5	0.2235	0.2257	0.2285	0.2188
10	0.0608	0.0613	0.0618	0.0592
20	0.0156	0.0157	0.0158	0.0147

(6 × 6) mesh. Hence, (8 × 8) mesh and (10 × 10) mesh were excluded in order to achieve better accuracy of results and to reduce computational cost. The lower mesh size (6 × 6) consisting of 36 elements and 133 nodes, has been used for the analysis due to computational efficiency. The total number of degrees of freedom involved in the computation is 665 as each node of the isoparametric quadratic element is having five degrees of freedom comprising of three translations and two rotations. Table 4 represents the non-dimensional fundamental natural frequencies of simply supported aluminum-zirconia FG plate (Pradyumna and Bandyopadhyay 2008; Matsunaga 2008; Ferreira et al. 2006).

The parametric study is carried out for functionally graded (stainless steel–nickel and Ti–6Al–4V and Aluminum oxide) conical shells for different twist angles at both stationary and rotating condition, as furnished in Tables 5 and 6. At stationary condition, non-dimensional fundamental natural frequencies are identified to attain lowest value at $\psi = 0°$ and gradually are found to increase to maximum value at $\psi = 45°$. For both stationary and rotational cases, non-dimensional fundamental frequencies are found to decrease with increase of material property graded index (N), irrespective of twist angle. Due to decrease in elastic stiffness at stationary condition, minimum value of non-dimensional fundamental natural frequencies of stainless steel–nickel FG conical shells is

Table 5 NDFF and NDSF [$\omega = \omega_n Lo^2 \sqrt{(\rho/E_1 t^2)}$] of rotating stainless steel–nickel functionally graded conical shells for various twist angles considering Lo/s = 0.7, r_1 = 0.2 m, Lo = 0.8 m, t = 0.01 m, ϕ_o = 45°, ϕ_{ve} = 20°

Ψ	N	Non-dimensional natural frequencies					
		NDFF			NDSF		
		Ω = 0	Ω = 0.5	Ω = 1.0	Ω = 0	Ω = 0.5	Ω = 1.0
0°	0	0.8121	1.0097	1.4504	2.0889	2.3701	3.0542
	1	0.7916	0.9915	1.4326	2.0364	2.3212	3.0078
	5	0.7792	0.9714	1.3982	2.0046	2.2782	2.9415
	10	0.7763	0.9666	1.3899	1.9970	2.2679	2.9257
15°	0	0.8536	1.0626	1.5229	2.0891	2.3979	3.1347
	1	0.8321	1.0434	1.5041	2.0366	2.3493	3.0883
	5	0.8190	1.0222	1.4679	2.0047	2.3052	3.0194
	10	0.8159	1.0171	1.4593	1.9971	2.2946	3.0029
30°	0	1.0525	1.3198	1.8759	2.0578	2.5110	3.4848
	1	1.0260	1.2960	1.8521	2.0061	2.4642	3.4377
	5	1.0097	1.2694	1.8076	1.9747	2.4151	3.3579
	10	1.0059	1.2632	1.7972	1.9672	2.4035	3.3390
45°	0	1.4253	1.8202	2.5483	1.9931	2.7358	4.0867
	1	1.3896	1.7873	2.5146	1.9431	2.6917	4.0364
	5	1.3673	1.7504	2.4549	1.9126	2.6333	3.9389
	10	1.3622	1.7419	2.4410	1.9054	2.6196	3.9161

Table 6 NDFF and NDSF [$\omega = \omega_n L^2 \sqrt{(\rho/E_1 h^2)}$] of rotating Ti–6Al–4V and aluminum oxide (Al_2O_3) functionally graded conical shells for various twist angles considering Lo/s = 0.7, $r_1 = 0.2$ m, Lo = 0.8 m, t = 0.01 m, $\phi_o = 45°$, $\phi_{ve} = 20°$

Ψ	N	Mode					
		NDFF			NDSF		
		Ω = 0	Ω = 0.5	Ω = 1.0	Ω = 0	Ω = 0.5	Ω = 1.0
0°	0	2.6948	3.3537	4.8139	6.9362	7.8761	10.1525
	1	2.0107	2.6506	3.6314	5.1641	6.2044	7.9374
	5	1.6874	2.0767	3.0300	4.3399	4.9093	6.3692
	10	1.5967	1.9830	2.8640	4.1102	4.6651	6.0284
15°	0	2.8328	3.5291	5.0547	6.9368	7.9585	10.4209
	1	2.1254	2.9653	3.8425	5.1655	6.3297	8.0864
	5	1.7708	2.1852	3.1796	4.3401	4.9648	6.5331
	10	1.6720	2.0803	2.9963	4.1110	4.7167	6.1765
30°	0	3.4961	4.3854	6.2307	6.8332	8.3447	11.5930
	1	2.6536	3.8049	4.8216	5.0899	6.6928	8.9356
	5	2.1613	2.6968	3.8805	4.2736	5.1850	7.2339
	10	2.0289	2.5223	3.6363	4.0463	4.7324	6.8114
45°	0	4.7372	5.9051	8.4696	6.6211	9.0916	13.5957
	1	3.6219	5.2735	6.6290	4.9389	6.9492	10.6233
	5	2.9028	3.7335	5.2286	4.1297	5.6734	8.4363
	10	2.7133	3.4425	4.8727	3.9071	5.2637	7.8817

identified at N = 10 while due to increase in elastic stiffness the maximum value is found at N = 0 irrespective of twist angle.

Considering N = 0, 1, 5 and 10, the differences between maximum and minimum value of non-dimensional fundamental natural frequencies at stationary condition for stainless steel–nickel FG conical shells are found 4.41, 4.42, 4.43 and 4.44 % for $\psi = 0°$, 15°, 30° and 45°, respectively. Here, the maximum difference (in percentage) between maximum and minimum NDFF values at stationary condition for stainless steel–nickel FG conical shell is identified for $\psi = 45°$ while the minimum difference (in percentage) is observed at $\psi = 0°$.

In contrast, the differences between maximum and minimum values of non-dimensional fundamental natural frequencies at lower rotational speeds (Ω = 0.5) of stainless steel–nickel FG conical shells are identified 4.27, 4.28, 4.29 and 4.30 % for $\psi = 0°$, 15°, 30° and 45°, respectively, while at higher rotational speeds the differences between maximum and minimum NDFF values are found 4.17, 4.18, 4.20 and 4.21 % for $\psi = 0°$, 15°, 30° and 45°, respectively. Therefore, it is noted that the percentage difference between maximum and minimum value of fundamental natural frequencies obtained from the analyses of four different material property graded index (N = 0, 1, 5 and 10) for stainless steel–nickel FG conical shell decreases as the twist angle increases. Hence it leads to the fact that for untwisted conical shells, material property graded index has pronounced effect on non-dimensional natural frequencies. At rotating condition, NDFF and NDSF are found to reduce with increase of material property graded index. Table 6 stipulates the trends of NDFF and NDSF of stainless steel–nickel functionally graded conical shells for different twist angles at rotating condition. At both lower rotational speeds (Ω = 0.5) and higher rotational speeds (Ω = 1.0), non-dimensional fundamental natural frequencies and non-dimensional second natural frequencies at twisted cases are found predominantly higher compared to respective NDFF and NDSF values of untwisted cases. The trend of relative frequencies (ratio of rotating natural frequency and stationary natural frequency) for both SS–Ni and Ti–6Al–4V and Aluminum oxide FG configuration at

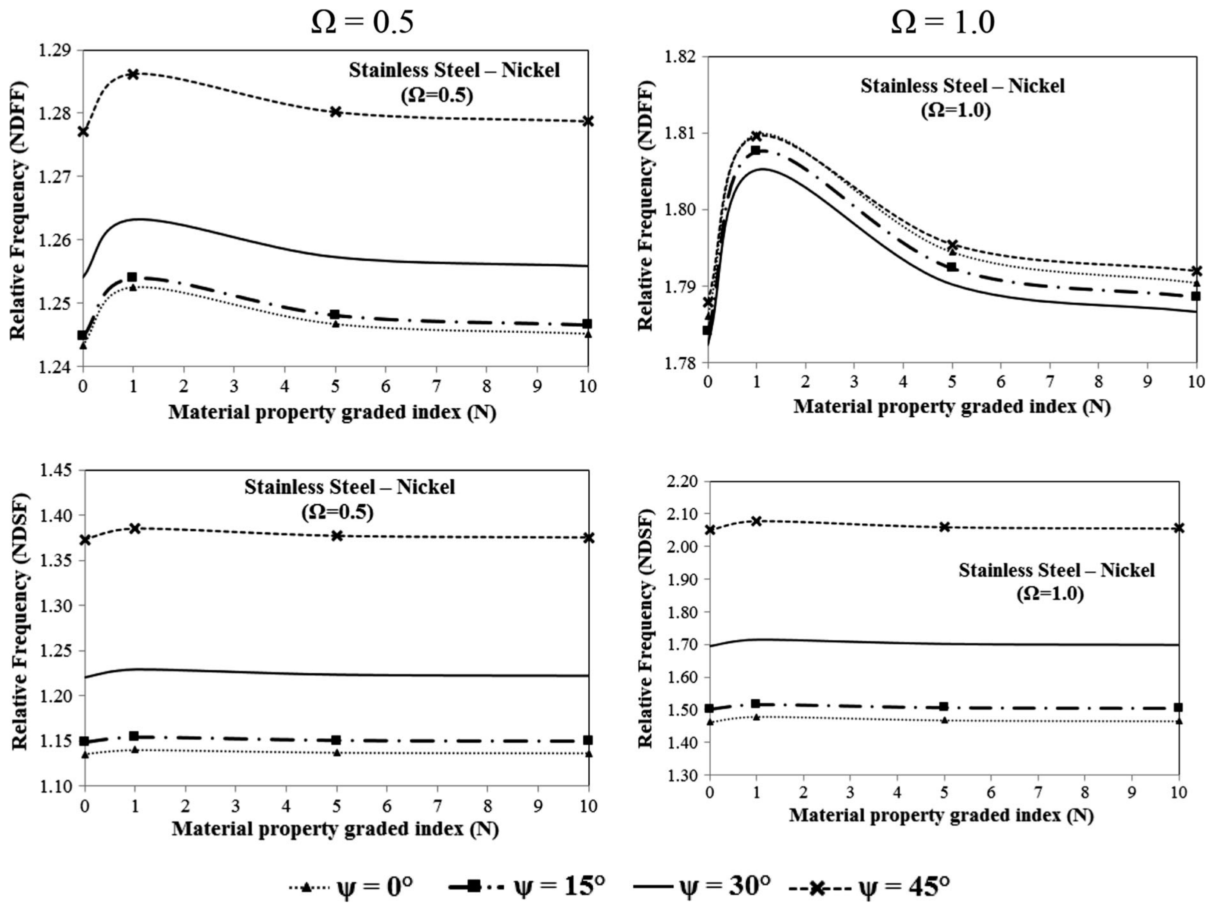

Fig. 3 Variation of relative frequencies (NDFF and NDSF) with material property graded index (N) at rotating conditions ($\Omega = 0.5$ and 1.0) for stainless steel–nickel functionally graded conical shells with various twist angles considering Lo/s = 0.7, $r_1 = 0.2$ m, Lo = 0.8 m, t = 0.01 m, $\phi_o = 45°$, $\phi_{ve} = 20°$

$\Omega = 0.5$, 1.0 are furnished in Figs. 3 and 4, respectively. The centrifugal stiffening effect (i.e., increase of structural stiffness with increase of rotational speed) is predominantly observed for both NDFF and NDSF values irrespective of twist angle for both stainless steel–nickel and Ti–6Al–4V and Aluminum oxide functionally graded conical shells. This phenomenon can be attributed to the fact that the variation in natural frequencies is occurred due to contribution of geometric stiffness matrix which changes the effective stiffness matrix of the functionally graded material. The geometric stiffness matrix depends on initial stress distribution and influences the elastic stiffness differently, yielding to different frequency parameters irrespective of twist angle.

For both FG conical shells, it is observed that relative frequencies [ratio of natural frequency at rotating condition (i.e., $\Omega = 0.5$ or 1.0) and respective natural frequency at stationary condition (i.e., $\Omega = 0.0$)] corresponding to untwisted cases are mostly in the lower range compared to those of twisted ones in respect of lower ($\Omega = 0.5$) and higher ($\Omega = 1.0$) rotational speeds. In other words, rotating effect is more pronounced for twisted shell in comparison to untwisted one. The centrifugal stiffening effect (i.e., increase of structural stiffness with increase of rotational speeds) with respect to both NDFF and NDSF are observed for Ti–6Al–4V and Aluminum oxide FG configuration irrespective of twist angle. Considering $\psi = 0°$, 15°, 30° and 45°, the differences between maximum and minimum value of non-dimensional fundamental natural frequencies at stationary condition for Ti–6Al–4V and Aluminum oxide FG conical shells are found 43.1, 44.5, 41.9 and

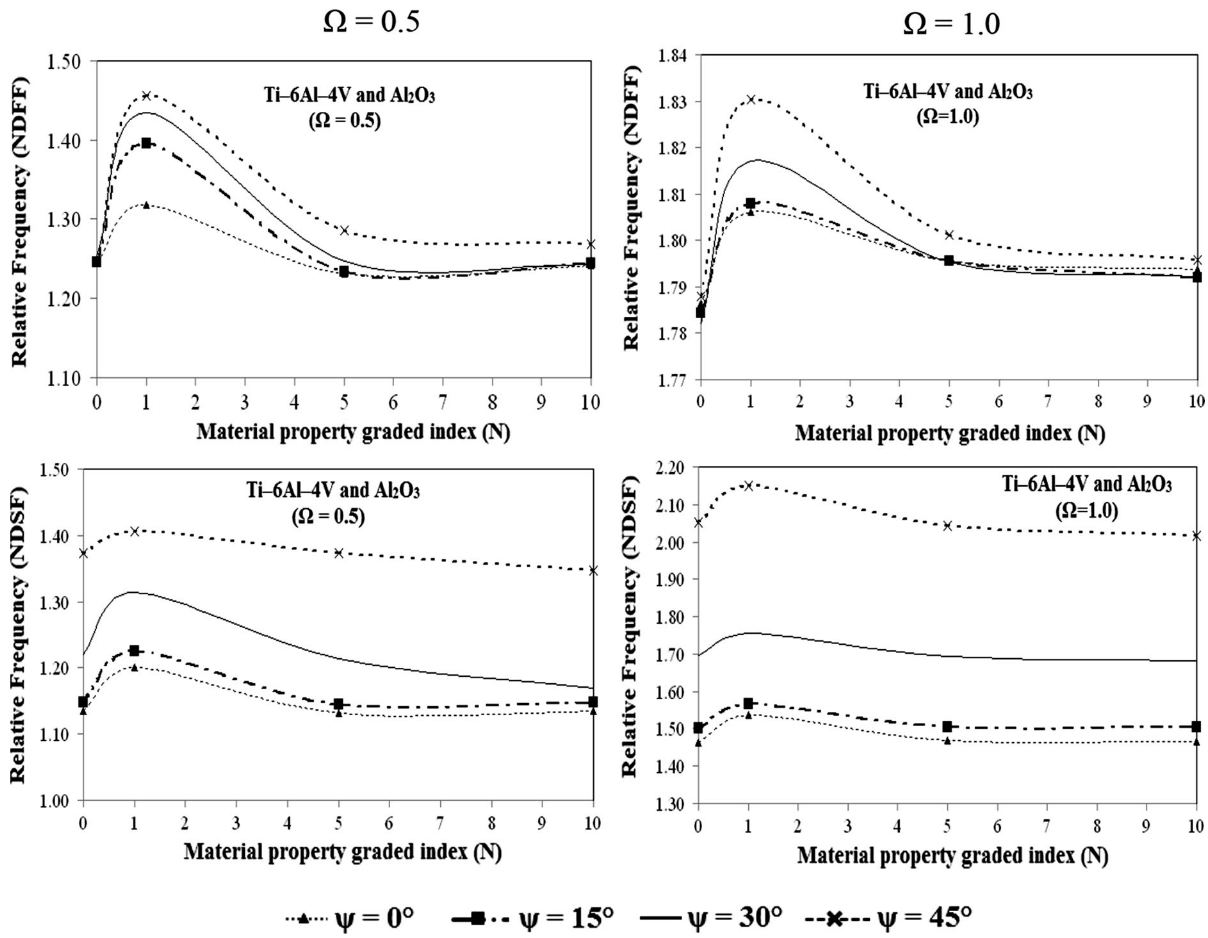

Fig. 4 Variation of relative frequencies (NDFF and NDSF) with material property graded index (N) at rotating conditions ($\Omega = 0.5$ and 1.0) for Ti–6Al–4V and Aluminum oxide (Al_2O_3) functionally graded conical shells for various twist angles considering Lo/s = 0.7, $r_1 = 0.2$ m, Lo = 0.8 m, t = 0.01 m, $\phi_o = 45°$, $\phi_{ve} = 20°$

41.2 % for N = 0, 1, 5 and 10, respectively. It is also observed that the maximum value of relative frequency with respect to both NDFF and NDSF are obtained at $\psi = 15°$ for N = 1.

Functionally graded conical shells, because of the possible different volume fractions for the constituent materials, also exhibit interesting frequency characteristics that are not seen in homogeneous isotropic conical shells. The influence of the value of material property graded index (N), which affects the constituent volume fraction, can be seen from the above tables (Tables 5, 6). As the value of 'N' increases, the natural frequency decreases irrespective of twist angle. This corroborates with the results obtained by Loy et al. (1999). Due to variation in elastic stiffness of FG conical shell panels with certain material graded index or power law exponent, the variation in NDSF has lesser pronounced effect compared to NDFF and subsequently observed to decrease the effect as the twist angle increases.

4 Mode shapes

The mode shapes corresponding to the natural frequencies are furnished for various twist angles ($\psi = 0°$, 15°, 30° and 45°) and rotational speeds ($\Omega = 0.0$, 0.5 and 1.0), considering stainless steel–nickel and Ti–6Al–4V and Aluminum oxide FG conical shells, as shown in Figs. 5 and 6, respectively. The fundamental frequency corresponds to the first torsional mode for all cases. It is identified that the

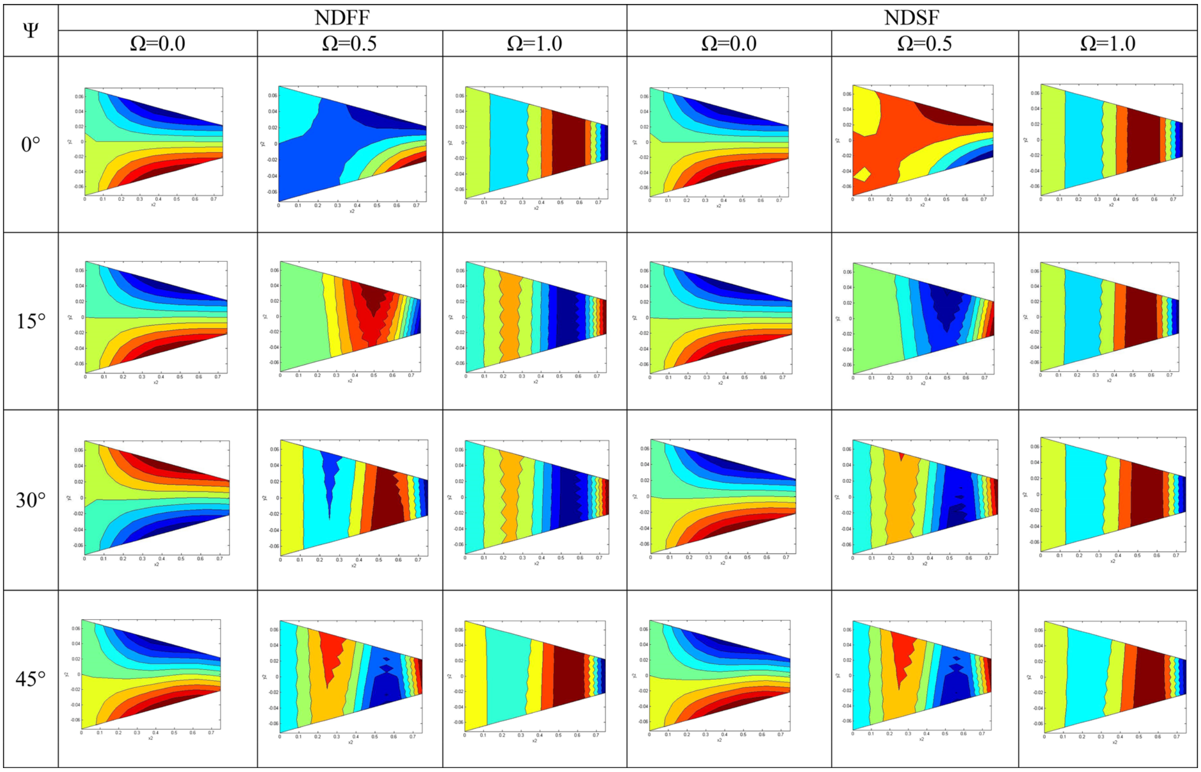

Fig. 5 Effect of twist and rotation on NDFF and NDSF for mode shapes for SS–Ni FG conical shells for different twist angles, considering $r_1 = 0.2$ m, t = 0.01 m, N = 1, s/t = 1000, Lo/s = 0.7, $\phi_o = 45°$, $\phi_{ve} = 20°$

symmetry modes are absent when twist angle is non-zero. The first span wise bending is observed at higher rotational speeds (Ω = 1.0) for stainless steel–nickel FG conical shells corresponding to both NDFF and NDSF, irrespective of twist angle. In contrast, for Ti–6Al–4V and Aluminum oxide FG conical shells, the first span wise bending is restricted to untwisted cases of both lower rotational speeds (Ω = 0.5) and higher rotational speeds (Ω = 1.0) corresponding to NDFF. Due to variation of effective elastic stiffness of FG conical shell, the centrifugal stiffening effect is found to comply with the mode shapes of both twisted and untwisted cases. At higher rotational speeds (Ω = 1.0), the influence of geometric stiffness matrix is influenced to transform the mode shape of both fundamental and second natural frequency of SS–Ni FG conical shell from torsional mode to first spanwise bending mode. In contrast, for Ti–6Al–4V and Aluminum oxide FG conical shells, due to dominant effect of twist angle say, at ψ = 30° and 45°, the mode shape of fundamental natural frequency remain under torsional mode throughout at higher rotational speeds (Ω = 1.0) while due to predominance of geometric stiffness matrix, first spanwise bending mode is identified only at twist angle ψ = 0° and 15° for higher rotational speeds.

5 Conclusions

This paper illustrated the combined effect of twist and rotational speeds for functionally graded conical shells. A validation of the analysis has been carried out by comparing results with those in the literature and has found to be accurate. Non-dimensional natural frequencies of both stainless steel–nickel and Ti–6Al–4V and Aluminum oxide FG conical shells are consistently observed to increase with rise of rotational speeds irrespective of twist angle. At both stationary and rotating condition, the NDFF and NDSF are found to decrease with increase of material property graded index (viz. N = 0, 1, 5, 10) corresponding to both twisted and untwisted cases. For both stationary and rotating condition, it is observed that NDFF and NDSF are obtained to increase with increase of twist and the

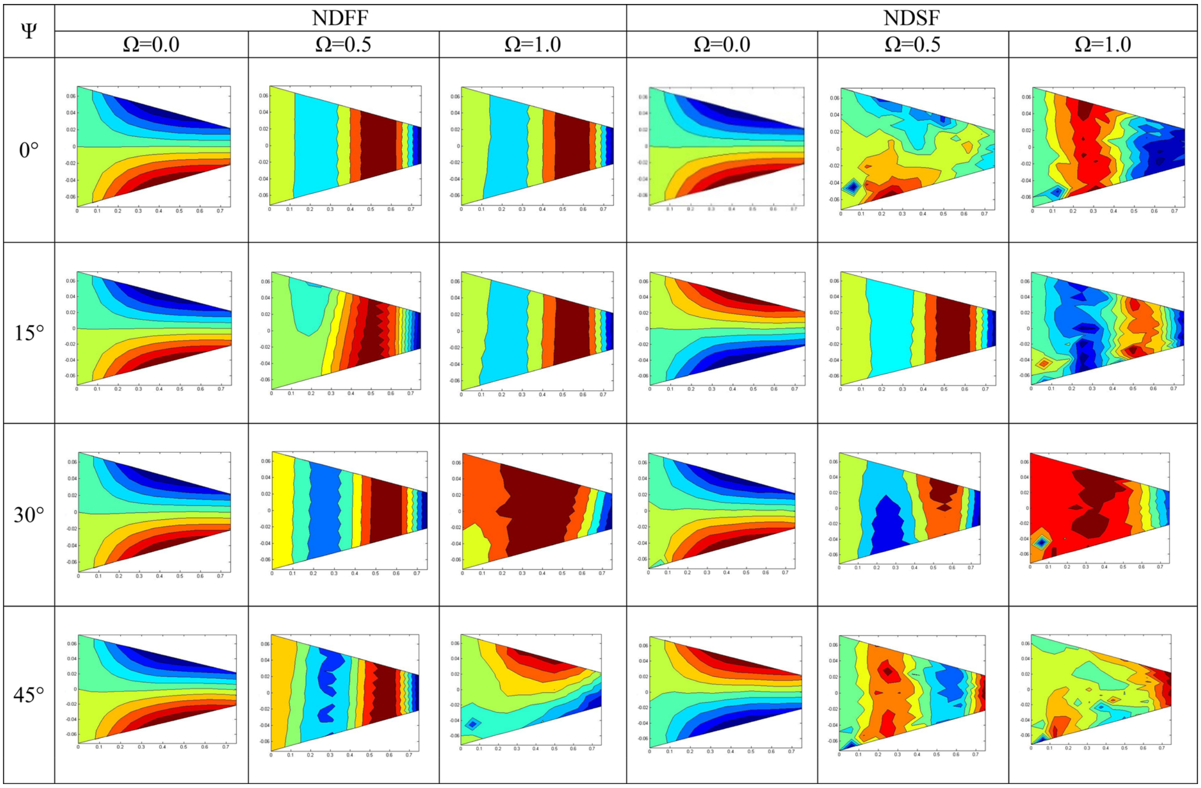

Fig. 6 Effect of twist and rotation on NDFF and NDSF for mode shapes of Ti–6Al–4V and Aluminum oxide (Al_2O_3) FG conical shells for different twist angles, considering $r_1 = 0.2$ m, $t = 0.01$ m, $N = 1$, $s/t = 1000$, $Lo/s = 0.7$, $\phi_o = 45°$, $\phi_{ve} = 20°$

rotating effect is found to be more pronounced for twisted FG conical shells compared to untwisted one. The centrifugal stiffening effect is predominantly observed for both NDFF and NDSF values irrespective of twist angle for both stainless steel–nickel and Ti–6Al–4V and aluminium oxide functionally graded conical shells. It is noted from the mode shapes of the FG conical shells that the fundamental and second natural frequencies at stationary condition corresponds to the first torsional mode for all cases irrespective of twist angle. It is identified that the symmetry modes are absent when twist angle is non-zero. The non-dimensional frequencies obtained are the first known results of the type of analyses carried out here and the results could serve as reference solutions for future investigators.

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