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Bottom up surrogate based approach for stochastic frequency response analysis of laminated composite plates

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ABSTRACT

This paper presents an efficient uncertainty quantification (UQ) scheme for frequency responses of laminated composite plates. A bottom up surrogate based approach is employed to quantify the variability in free vibration responses of composite cantilever plates due to uncertainty in ply orientation angle, elastic modulus and mass density. The finite element method is employed incorporating effects of transverse shear deformation based on Mindlin's theory in conjunction with a random variable approach. Parametric studies are carried out to determine the stochastic frequency response functions (SFRF) along with stochastic natural frequencies and modeshapes. In this study, a surrogate based approach using General High Dimensional Model Representations (GHDMR) is employed for achieving computational efficiency in quantifying uncertainty. Subsequently the effect of noise is investigated in GHDMR based UQ algorithm. This paper also presents an uncertainty quantification scheme using commercial finite element software (ANSYS) and thereby comparative results of stochastic natural frequencies are furnished for UQ using GHDMR approach and ANSYS.

1. Introduction

Managing the risk of uncertainties associated with composite structures has become increasingly important for aircraft and spacecraft industries in recent years with advancement in lightweight designs. The extensive application of such materials not only in aerospace industry, but also in civil, mechanical and marine structures, has prompted many researchers to analyze its performance in depth. The prime reasons of popularity of composites are because of its light-weight, cost-effectiveness, high specific stiffness and application specific tailorable stiffness in different directions. In general, uncertainties are broadly classified into three categories, namely aleatoric (due to variability in the system parameters), epistemic (due to lack of knowledge of the system) and prejudicial (due to absence of variability characterization). The total uncertainty of a system is the combination of these three types of uncertainties. The performance of composite structures is influenced by quality control procedures, as well as operating conditions and environmental effects. It may be observed that there can be uncertainties in input forces, system description, model calibration and computation. Laminated composite plates are typically made from different combinations of polymer prepregs. In the serial production of a composite plate, any small changes in fibre orientation angle and differences in bonding of layers may affect critical responses such as modal vibration characteristics of the composite structure. New aircraft developments (e.g., reusable launch vehicle, high-speed civil transport) are departing dramatically from traditional environments. Application of historical uneconomic uncertainty factors may not be sufficient to provide adequate safety. Conversely, the trend to design for all possible unfavorable events occurring simultaneously could produce an unacceptable dynamic response. Moreover composite materials have more intrinsic variables than metals due to their heterogeneity and are subjected to more manufacturing process sources of variation. For composite materials, the properties of constituent material vary statistically due to lack of precision and accuracy to maintain the exact properties for each layer of the laminate. Hence the uncertainties incurred during manufacturing process are due to the misalignment of ply-orientation, intralaminate voids, incomplete curing of resin, excess resin between plies, excess matrix voids and porosity resulting from machine, human and process inaccuracy. As a result, free vibration responses of such laminated composite shells show volatility from its deterministic

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Nomenclature

L, b, h	length, width and thickness of composite plate, respectively
E_1, E_2	elastic moduli along 1 and 2 axes
G_{12}, G_{13}, G_{23}	shear moduli along 1–2, 1–3 and 2–3 planes, respectively
ν	Poisson's ratio
ρ	mass density
$[M]$	global mass matrix
$[C]$	global Coriolis matrix
$[K]$	elastic stiffness matrix
$[K_\sigma]$	geometric stiffness matrix
$[K_R]$	global rotational stiffness matrix
$\{\delta\}$	global displacement vector
ω_n	natural frequency of composite plate
λ	non-dimensional frequency
u_j, v_j, w_j	nodal displacements
θ_x, θ_y	rotation about x and y axes
$\{Q\}$	transverse shear resultants
$[A]$	extension coefficient
$[B]$	bending-extension coupling coefficient
$[D]$	bending stiffness coefficients
$[B']$	strain-displacement matrix
$[D']$	elasticity matrix
$[\rho]$	inertia matrix
$[N]$	shape function matrix
$\{k\}$	curvature vector
$\{N\}$	in-plane stress-resultants
$\{M\}$	moment resultants
$\{\varepsilon\}$	strain vector
η, ξ	local natural coordinates of the element
x, y, z	local coordinate axes (plate coordinate system)
n	number of layers
ω	non-dimensional frequency parameter

mean value. Because of its inherent complexity, laminated composite structures can be difficult to manufacture accurately according to its exact design specification which results undesirable uncertainties in responses. The design and analysis of conventional materials is easier than that of composites because for conventional materials both materials and most geometric properties have either little or well-known variation from their nominal value. In contrast, the same does not hold good for design of structures made of laminated composites. Hence, the uncertainty calibration for structural reliability of such composite structures is essential to ensure operational safety by means of safe as well as economic design. The prime sources of random structural uncertainty considered in this study are material properties and fiber orientation of the individual constituent laminae. Because of the randomness in these input parameters, the mass matrices and the stiffness matrices of the composite structure become stochastic in nature. Thus it causes the statistical variation in the eigenvalues and eigenvectors and subsequently the dynamic response as well. Therefore a realistic analysis of composite laminated plates is presented in this article to quantify the uncertainties in dynamic responses arising from the randomness in the variation of parameters like ply-orientation angle, elastic modulus and mass density. A brief literature review on uncertainty quantification (UQ) of laminated composite plates is presented in the next paragraph.

The free vibration characteristics of laminated composites have been extensively considered in the literature. The pioneering work using finite element method (FEM) in conjunction with laminated composite plates is reviewed by Reddy [1]. The deterministic analyses of free vibration for laminated plate structures, incorporating a wide spectrum of approaches are reported in the open literature [2–8]. The vibration analysis of rectangular laminated composite plates is carried out by Wang et al. [9] employing first order shear deformation theory (FSDT) meshless method while the mesh-free method for static and free vibration analysis of for shear deformable laminated composite plates introduced by Dai et al. [10] and successive investigation carried out by Liu et al. [11]. The uncertain frequency responses in composite plates can be developed primarily due to variabilities in material, geometric laminate parameters, environmental and operational factors. The natural frequencies of composite plates with redial basis function (RBF)-pseudo spectral method studied by Ferreira and Fasshauer [12] while free vibrations of uncertain composite plates via stochastic Rayleigh–Ritz approach by Venini and Mariani [13]. The free vibrations of composite cylindrical panels with random material properties are studied [14–16]. The stochastic analysis of vibration of laminated composites with uncertain random material properties is investigated in [17–18]. There have actually been a good number of studies reported involving stochastic modelling of uncertainties considered in composites structures [19–20]. The natural frequencies and vibration modes of laminated composite plates with arbitrary curvilinear fiber shape paths was investigated by Honda and Narita [21]. Sepahvand et al. [22] studied the stochastic free vibration of orthotropic plates using generalized polynomial chaos expansion. Atamturktura et al. [23] studied the uncertainty quantification in model verification and validation applied to large scale historic masonry monuments while António and Hoffbauer [24] studied uncertainty analysis of composite structures followed based on global sensitivity indices. The random failure analysis of fibre composites designs based on deterministic material properties can overestimate the reliability of composite structures significantly. Not surprisingly, there is continued interest in implementing stochastic concepts in material characterization, in structural response assessment, and in developing rational design and effective utilization procedures for laminated composites. Considered as a broad area within stochastic mechanics, such analyses require the identification of uncertainties and the selection of appropriate techniques for uncertainty propagation up to different modelling scales, depending on the response of interest.

Monte Carlo simulation (MCS) technique in conjunction with FEM is found to be widely used for quantifying uncertainties of laminated composite structures, wherein thousands of finite element simulations are needed to be carried out. Thus this approach is of limited practical value due to its computational intensiveness unless some form of model-based extrapolation can be used to make the method more efficient. In view of above, the present investigation attempts to quantify the uncertainty in free vibration responses of laminated composite plates using a bottom up surrogate based approach, where the computationally expensive finite element model can be effectively replaced by an efficient mathematical model. In this approach the effect of uncertainty (such as ply orientation angle, variation in material and geometric properties etc.) is accounted in the elementary level first and then this effect is propagated towards the global responses via surrogates of the actual finite element model. A generalized high dimensional model representation (GHDMR) [25] is employed for surrogate model formation, wherein diffeomorphic modulation under observable response preserving homotopy (D-MORPH) regression is utilized to ensure the hierarchical orthogonality of high dimensional model representation component functions. Other forms of high dimensional model representation approaches (cut-HDMR and RS-HDMR) are found to be recently utilized in the area of reliability analysis, optimization and sensitivity analysis of different structures [26–29]. The significant studies are also carried out on the stochastic modal analysis, validation and updating by using response surface for structural applications [30–33]. But the application of GHDMR in the field of laminated composite structures is very scarce, in spite of the fact that it possesses some of the critically desirable factors for uncertainty quantification of laminated composite structures such as capability to deal with high dimensional problems and good prediction capability throughout the entire domain including the tail region. In this article new results are presented for stochastic natural frequencies, frequency response functions and mode-shapes using the proposed approach. A novel scheme is presented subsequently to explore the performance of GHDMR based UQ algorithm under the effect of simulated noise. Another limitation of the studies on UQ of laminated composites as presented in the literature review section is that most of the investigations are based on finite element codes written in scientific programming languages like FORTRAN [34] and MATLAB [35]. This restricts application of such uncertainty quantification methods to large-scale complex structures, for which commercially available finite element modelling packages are commonly used in industry. In this article, we present a useful industry oriented uncertainty quantification scheme using commercial finite element software (ANSYS [36]) in conjunction with MATLAB and thereby comparative results of stochastic natural frequencies are furnished for uncertainty quantification using GHDMR approach and ANSYS. This paper hereafter is organized as follows, Section 2: detail theoretical formulation for development of finite element code using FORTRAN; Section 3: formulation of GHDMR; Section 4: proposed surrogate based bottom up stochastic approach and uncertainty quantification scheme using ANSYS; Section 5: results and discussion; and Section 6: conclusion.

2. Theoretical formulation

A rectangular composite laminated cantilever plate of length L, width b, and thickness t is considered having three plies located in a three-dimensional Cartesian coordinate system (x, y, z), where the x–y plane passes through the middle of the plate thickness with its origin placed at the corner of the cantilever plate as shown in Fig. 1. An eight noded isoparametric plate bending element is considered for finite element formulation. The composite plate is considered with uniform thickness with the principal material axes of each layer being arbitrarily oriented with respect to mid-plane. If the mid-plane forms the x–y plane of the reference plane, then the displacements can be computed as

$$\begin{aligned} u(x,y,z) &= u^0(x,y) - z\theta_x(x,y) \\ v(x,y,z) &= v^0(x,y) - z\theta_y(x,y) \\ w(x,y,z) &= w^0(x,y) = w(x,y), \end{aligned} \tag{1}$$

Assuming u, v and w are the displacement components in x-, y- and z-directions, respectively and u^0, v^0 and w^0 are the mid-plane displacements, and $\theta_{\boldsymbol{x}}$ and $\theta_{\boldsymbol{y}}$ are rotations of cross-sections along the x- and y-axes. The strain–displacement relationships for small deformations can be expressed as

$$\begin{aligned} &\varepsilon_{xx} = \varepsilon_x^0 + zk_x, \quad \varepsilon_{yy} = \varepsilon_y^0 + zk_y \\ &\gamma_{xy} = \gamma_{xy}^0 + zk_{xy}, \quad \gamma_{xz} = w_{,x}^0 - \theta_x, \quad \gamma_{yz} = w_{,y}^0 - \theta_y, \end{aligned} \tag{2}$$

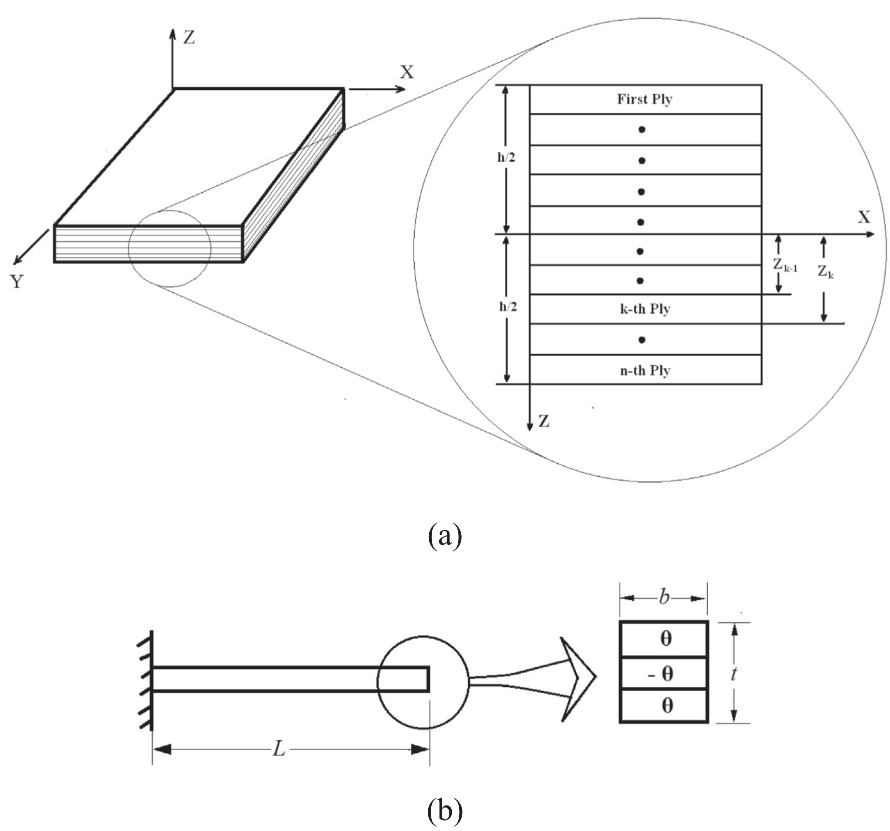

Fig. 1. Laminated composite plate.

where mid-plane components are given by

$$\varepsilon_x^0 = u_{,x}^0, \quad \varepsilon_y^0 = u_{,y}^0 \quad \text{and} \quad \gamma_{xy}^0 = u_{,y}^0 + v_{,x}^0 \tag{3}$$

and plate curvatures are expressed as

$$\begin{aligned} k_x &= -\theta_{x,x} = -w_{,xx} + \gamma_{xz,x} \\ k_y &= -\theta_{y,y} = -w_{,yy} + \gamma_{yz,y} \\ k_{xy} &= -(\theta_{x,y} + \theta_{y,x}) = -2w_{,xy} + \gamma_{xz,y} + \gamma_{yz,x} \end{aligned} \tag{4}$$

Therefore the strains in the k-th lamina can be expressed in matrix form

$$\{\varepsilon\}^k = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^0 \\ k_y^0 \\ k_{xy}^0 \end{Bmatrix} = \{\varepsilon^0\} + z\{k\} \quad \text{and} \quad \{\gamma\}^k = \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = \{\gamma\} \tag{5}$$

In general, the force and moment resultants of a single lamina are obtained from stresses as

$$\{F\} = \{N_x\ N_y\ N_{xy}\ M_x\ M_y\ M_{xy}\ Q_x\ Q_y\}^T = \int_{-t/2}^{t/2} \{\sigma_x\ \sigma_y\ \tau_{xy}\ \sigma_x z\ \sigma_y z\ \tau_{xy} z\ \tau_{xz}\ \tau_{yz}\}^T dz \tag{6}$$

In matrix form, the in-plane stress resultant $\{N\}$, the moment resultant $\{M\}$, and the transverse shear resultants $\{Q\}$ can be expressed as

$$\{N\} = [A]\{\varepsilon^0\} + [B]\ \{k\} \quad \text{and} \quad \{M\} = [B]\{\varepsilon^0\} + [D]\ \{k\} \tag{7}$$

$$\{Q\} = [A^*]\{\gamma\} \tag{8}$$

Here $[A_{ij}^*] = \int_{-t/2}^{t/2} \overline{Q}_{ij} dz$ where $i, j = 4, 5$

$$[\overline{Q}_{ij}(\varpi)] = \begin{bmatrix} m^4 & n^4 & 2m^2n^2 & 4m^2n^2 \\ n^4 & m^4 & 2m^2n^2 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & (m^4+n^4) & -4m^2n^2 \\ m^2n^2 & m^2n^2 & -2m^2n^2 & (m^2-n^2) \\ m^3n & mn^3 & (mn^3 - m^3n) & 2(mn^3 - m^3n) \\ mn^3 & m^3n & (m^3n - mn^3) & 2(m^3n - mn^3) \end{bmatrix} [Q_{ij}]$$

Here $m = Sin\theta(\varpi)$ and $n = Cos\theta(\varpi)$, wherein $\theta(\varpi)$ is the random fibre orientation angle where the symbol (ϖ) indicates the stochasticity of parameters. However, laminate consists of a number of laminae wherein $[Q_{ij}]$ and $[\overline{Q}_{ij}(\varpi)]$ denotes the on-axis elastic constant matrix and the off-axis elastic constant matrix, respectively. The elasticity matrix of the laminated composite plate is given by,

$$[D'(\varpi)] = \begin{bmatrix} A_{ij}(\varpi) & B_{ij}(\varpi) & 0 \\ B_{ij}(\varpi) & D_{ij}(\varpi) & 0 \\ 0 & 0 & S_{ij}(\varpi) \end{bmatrix}$$

where

$$[A_{ij}(\varpi),\ B_{ij}(\varpi),\ D_{ij}(\varpi)] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}(\varpi)]_k [1, z, z^2] dz \quad i,j = 1,2,6$$

$$[S_{ij}(\varpi)] = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \alpha_s [\overline{Q}_{ij}(\varpi)]_k dz \quad i,j = 4,5$$

where α_s is the shear correction factor and is assumed as 5/6.

Now, mass per unit area is denoted by P and is given by

$$P(\varpi) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho(\varpi)\ dz \tag{9}$$

where $\rho(\varpi)$ denotes the stochastic density parameter at each element. The mass matrix is expressed as

$$[M(\varpi)] = \int_{Vol} [N][P(\varpi)][N] d(vol) \tag{10}$$

The stiffness matrix is given by

$$[K(\varpi)] = \int_{-1}^{1} \int_{-1}^{1} [B\ (\varpi)]^T [D(\varpi)]\ [B(\ \varpi)]\ d\xi\ d\eta \tag{11}$$

where ξ and η are the local natural coordinates of the element.

2.1. Governing equations

The Hamilton's principle [37] is employed to study the dynamic nature of the composite structure. The principle used for the Lagrangian which is defined as

$$L_f = T - U - W \tag{12}$$

where T, U and W are total kinetic energy, total strain energy and total potential of the applied load, respectively. The Hamilton's principle applicable to non-conservative system can be expressed as,

$$\delta H = \int_{t_i}^{t_f} [\delta T - \delta U - \delta W] dt = 0 \tag{13}$$

The energy functional for Hamilton's principle is the Lagrangian (L_f) which includes kinetic energy (T) in addition to potential strain energy (U) of an elastic body. The expression for kinetic energy of an element is given by

$$T = \frac{1}{2}\{\dot{\delta}_e\}^T [M_e(\varpi)] + [C_e(\varpi)]\{\delta_e\} \tag{14}$$

The potential strain energy for an element of a plate can be expressed as,

$$U = U_1 + U_2 = \frac{1}{2}\{\delta_e\}^T [K_e(\varpi)]\{\delta_e\} + \frac{1}{2}\{\delta_e\}^T [K_{\sigma e}(\varpi)]\{\delta_e\} \tag{15}$$

The Langrange's equation of motion is given by

$$\frac{d}{dt}\left[\frac{\partial L_f}{\partial \delta_e}\right] - \left[\frac{\partial L_f}{\partial \delta_e}\right] = \{F_e\} \tag{16}$$

where $\{F_e\}$ is the applied external element force vector of an element and L_f is the Lagrangian function. Substituting $L_f = T - U$, and the corresponding expressions for T and U in Lagrange's equation, one obtains the dynamic equilibrium equation for each element in the following form [38]

$$[M(\varpi)]\{\ddot{\delta}_e\} + [C]\{\dot{\delta}_e\} + ([K_e(\varpi)] + [K_{\sigma e}(\varpi)]\{\delta_e\} = \{F_e\} \tag{17}$$

After assembling all the element matrices and the force vectors with respect to the common global coordinates, the resulting equilibrium equation is obtained. For the purpose of the present study, the finite element model is developed for different element types and finite element discretization and nodal positions of the driving point and measurement point. Considering randomness of input parameters like ply-orientation angle, elastic modulus and mass density etc., the equation of motion of a linear damped discrete system with n degrees of freedom can expressed as

$$[M(\varpi)]\ddot{\delta}(t) + [C]\dot{\delta}(t) + [K(\varpi)]\delta(t) = f(t) \tag{18}$$

where $[C]$ is the damping co-efficient matrix and $[K(\varpi)]$ is the stiffness matrix wherein $[K(\varpi)] = [K_e(\varpi)] + [K_{\sigma e}(\varpi)]$ in which $K_{\mathbf{e}}(\varpi) \in R^{n \times n}$ is the elastic stiffness matrix, $K_{\boldsymbol{\sigma}\mathbf{e}}(\varpi) \in R^{n \times n}$ is the geometric stiffness matrix while $M(\varpi) \in R^{n \times n}$ is the mass matrix, $\boldsymbol{\delta}(t) \in R^n$ is the vector of generalized coordinates and $f(t) \in R^n$ is the forcing vector. The governing equations are derived based on Mindlin's Theory incorporating rotary inertia, transverse shear deformation. The equation represents a set of coupled second-order ordinary-differential equations. The solution of this equation also requires the knowledge of the initial conditions in terms of displacements and velocities of all the coordinates. The initial conditions can be specified as

$$\delta(0) = \delta_0 \in R^n \quad \text{and} \quad \dot{\delta}(0) = \dot{\delta}_0 \in R^n \tag{19}$$

The Eq. (18) is considered to solve together with the initial conditions of Eq. (19) using modal analysis. The present study assumes that all initial conditions are zero and the forcing function is a harmonic excitation applied at a particular point of composite cantilever plate.

2.2. Modal analysis and dynamic response

Lord Rayleigh [39] showed that undamped linear systems are capable of so-called natural motions. This essentially implies that all the system coordinates execute harmonic oscillation at a given frequency and form a certain displacement pattern. The oscillation frequency and displacement pattern are called natural frequencies and normal modes, respectively. The natural frequencies (ω_j) and the mode shapes ($\mathbf{x}_j$) are intrinsic characteristic of a system and can be obtained by solving the associated matrix eigenvalue problem

$$[K(\varpi)]x_j = \omega_j^2[M(\varpi)]x_j; \quad \text{where}, j = 1, 2 \ldots n \tag{20}$$

It can be also shown that the eigenvalues and eigenvectors satisfy the orthogonality relationship that is

$$x_i^T[M(\varpi)]x_j = \lambda_{ij} \quad \text{and} \quad x_i^T[K(\varpi)]x_j = \omega_j^2\lambda_{ij}, \quad \text{where } i,j = 1, 2 \ldots n \tag{21}$$

Note that the Kroneker delta functions is given by $\lambda_{ij} = 1$ for $i = j$ and $\lambda_{ij} = 0$ for $i \neq j$. The property of the eigenvectors in (21) is also known as the mass orthonormality relationship. The solution of undamped eigenvalue problem is now standard in many finite element packages. There are various efficient algorithms available for this purpose [40]. This orthogonality property of the undamped modes is very powerful as it allows to transform a set of coupled differential equations to a set of independent equations. For convenience, we construct the matrices

$$\Omega(\varpi) = \text{diag}[\omega_1, \omega_2, \omega_3 \ldots \omega_n] \in R^{n\times n} \quad \text{and} \quad nX(\varpi) = [x_1, x_2 \ldots x_n] \in R^{n\times n} \tag{22}$$

where the eigenvalues are arranged such that $\omega_1 < \omega_2$, $\omega_2 < \omega_3$,.... $\omega_k < \omega_{k+1}$. The matrix X is known as the undamped modal matrix. Using these matrix notations, the orthogonality relationships (21) can be rewritten as

$$x_i^T[M(\varpi)]x_j = \lambda_{ij} \quad \text{and} \quad x_i^T[K(\varpi)]x_j = \omega_j^2\lambda_{ij} \quad \text{where } i,j = 1, 2 \ldots n \tag{23}$$

$$X^T[M(\varpi)]X = I \quad \text{and} \quad X^T[K(\varpi)]X = \Omega^2 \tag{24}$$

where I is a $(n \times n)$ identity matrix. We use the following coordinate transformation (as the modal transformation)

$$\delta(\varpi)(t) = Xy(t) \tag{25}$$

Using the modal transformation in Eq. (25), pre-multiplying Eq. (18) by $\mathbf{X}^T$ and using the orthogonality relationships in (24), equation of motion of a damped system in the modal coordinates may be obtained as

$$\ddot{y}(t) + X^T\ C\ X\ \dot{y}(t) + \Omega^2\ y(t) = \widetilde{f}(t) \tag{26}$$

Clearly, unless $X^T[C]X$ is a diagonal matrix, no advantage can be gained by employing modal analysis because the equations of motion will still be coupled. To solve this problem, it is common to assume proportional damping. With the proportional damping assumption, the damping matrix $[C]$ is simultaneously diagonalizable with $[M(\varpi)]$ and $[K(\varpi)]$. This implies that the damping matrix in the modal coordinate can be expressed as

$$C' = X^T[C]X \tag{27}$$

where $\mathbf{C}'$ is a diagonal matrix. This matrix is also known as the modal damping matrix. The damping factors ζ_j are defined from the diagonal elements of the modal damping matrix as

$$C'_{jj} = 2\zeta_j\omega_j \quad \text{where} \quad j = 1, 2, \ldots n \tag{28}$$

Such a damping model, introduced by Lord Rayleigh allows analyzing damped systems in very much the same manner as undamped systems since the equation of motion in the modal coordinates can be decoupled as

$$\ddot{y}_j(t) + 2\zeta_j\omega_j\dot{y}_j(t) + \omega_j^2 y(t) = \widetilde{f}_j(t) \quad \text{where} \quad j = 1, 2, \ldots n \tag{29}$$

The generalized proportional damping model expresses the damping matrix as a linear combination of the mass and stiffness matrices, that is

$$C(\varpi) = \alpha_1 M(\varpi) f(M^{-1}(\varpi)K(\varpi))$$
$$\text{where} \quad \alpha_1 = 0.005 \text{ is constant damping factor} \tag{30}$$

The transfer function matrix of the system can be obtained as

$$H(i\omega)(\varpi) = X[-\omega^2 I + 2i\omega\zeta\Omega + \Omega^2]^{-1}X^T = \sum_{j=1}^{n} \frac{X_j X_j^T}{-\omega^2 + 2i\omega\zeta_j\omega_j + \omega_j^2} \tag{31}$$

Using this, the dynamic response in the frequency domain with zero initial conditions can be conveniently represented as

$$\bar{\delta}(i\omega)(\varpi) = H(i\omega)\bar{f}(i\omega) = \sum_{j=1}^{n} \frac{X_j^T \bar{f}(i\omega)}{-\omega^2 + 2i\omega\zeta_j\omega_j + \omega_j^2} X_j \tag{32}$$

Therefore, the dynamic response of proportionally damped system can be expressed as a linear combination of the undamped mode shapes.

3. Formulation of GHDMR

The General High Dimensional Model Representation (GHDMR) can construct a proper model for prediction of the output (say natural frequency) corresponding to a stochastic input domain. The present approach can treat both independent and correlated input variables, and includes independent input variables as a special case. The role of D-MORPH is to ensure the component functions' orthogonality in hierarchical manner. The present technique decomposes the function $\Pi(\widetilde{I})$ with component functions by input parameters, $\widetilde{I} = (\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_{ni})$. As the input parameters are independent in nature, the component functions are specifically projected by vanishing condition. Hence, it has limitation for general formulation. In contrast, a novel numerical analysis with component functions is portrayed in the problem of present context wherein a unified framework for general HDMR dealing with both correlated and independent variables are established. For different input parameters, the output is calculated as [25]

$$\Pi(\widetilde{I}) = \Pi_0 + \sum_{i=1}^{ni}\Pi_i(\widetilde{I}_i) + \sum_{1\leqslant i<j\leqslant ni}\Pi_{ij}(\widetilde{I}_i, \widetilde{I}_j) + \cdots + \Pi_{12\ldots ni}(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_{ni}) \tag{33}$$

$$\Pi(\widetilde{I}) = \sum_{u \subseteq ni} \Pi_u(\widetilde{I}_u) \tag{34}$$

where Π_0 (zeroth order component function) represents the mean value. $\Pi_i(\widetilde{I}_i)$ and $\Pi_{ij}(\widetilde{I}_i, \widetilde{I}_j)$ denote the first and second order component functions, respectively while $\Pi_{1,2...,ni}(\widetilde{I}_1, \widetilde{I}_2, \ldots, \widetilde{I}_{ni})$ indicates the residual contribution by input parameters. The subset $u \subseteq \{1, 2, \ldots, ni\}$ denotes the subset where $u \subseteq ni$ for simplicity and empty set, $\Gamma \in u$. As per Hooker's definition, the correlated variables are expressed as,

$$\{\Pi_u(\widetilde{I}_u | u \subseteq ni)\} = Arg \min_{\{g_u \in L^2(R^u), u \subseteq ni\}} \int \left(\sum_{u \subseteq k} g_u(\widetilde{I}_u) - \Pi(\widetilde{I}) \right)^2 w(\widetilde{I}) d\widetilde{I} \tag{35}$$

$$\forall u \subseteq ni, \quad \forall i \in u, \quad \int \Pi_u(\widetilde{I}_u) w(\widetilde{I}) d\widetilde{I}_i d\widetilde{I}_{-u} = 0 \tag{36}$$

and

$$\forall v \subset u, \quad \forall g_v : \int \Pi_u(\widetilde{I}_u) g_v(\widetilde{I}_v) w(\widetilde{I}) d\widetilde{I} = \langle \Pi_u(\widetilde{I}_u), g_v(\widetilde{I}_v) \rangle = 0 \tag{37}$$

The function $\Pi(\widetilde{I})$ can be obtained from sample data by experiments or by modelling. To minimize the computational cost, the reduction of the squared error can be realized easily. Assuming H in Hilbert space is expanded on the basis $\{h_1, h_2, \ldots, h_{ni}\}$, the bigger subspace $\overline{H}(\supset H)$ is expanded by extended basis $\{h_1, h_2, \ldots, h_{ni}, h_{ni+1}, \ldots, h_m\}$. Then $\overline{H}$ can be decomposed as

$$\overline{H} = H \oplus H^{\perp} \tag{38}$$

where $H^{\perp}$ denotes the complement subspace (orthogonal) of H [41] within $\overline{H}$. In the pastwork [42,43], the component functions are calculated from basis functions. The component functions of Second order HDMR expansion are estimated from basis functions $\{\varphi\}$ as [44]

$$\Pi_i(\widetilde{I}_i) \approx \sum_{r=1}^{ni} \alpha_r^{(0)i} \varphi_r^i(\widetilde{I}_i) \tag{39}$$

$$\Pi_{ij}(\widetilde{I}_i, \widetilde{I}_j) \approx \sum_{r=1}^{ni} \left[\alpha_r^{(ij)i} \varphi_r^i(\widetilde{I}_i) + \alpha_r^{(ij)j} \varphi_r^j(\widetilde{I}_j) \right] + \sum_{p=1}^{l} \sum_{q=1}^{l} \beta_{pq}^{(0)ij} \varphi_p^i(\widetilde{I}_i) \varphi_q^j(\widetilde{I}_j) \tag{40}$$

i.e., the basis functions of $\Pi_{ij}(\widetilde{I}_i, \widetilde{I}_j)$ contain all the basis functions used in $\Pi_i(\widetilde{I}_i)$ and $\Pi_j(\widetilde{I}_j)$. The HDMR expansions at N_{samp} sample points of $\widetilde{I}$ can be represented as a linear algebraic equation system

$$\Gamma J = \hat{R} \tag{41}$$

where Γ denotes a matrix $(N_{samp} \times \widetilde{t})$ whose elements are basis functions at the N_{samp} values of $\widetilde{I}$; J is a vector with $\widetilde{t}$ dimension of all unknown combination coefficients; $\hat{R}$ is a vector with N_{samp}-dimension wherein l-th element is $\Pi(\widetilde{I}^{(l)}) - \Pi_0$. $\widetilde{I}^{(l)}$ denotes the l-th sample of $\widetilde{I}$, and Π_0 represents the average value of all $\Pi(\widetilde{I}^{(l)})$. The regression equation for least squares of the above equation can be expressed as

$$\frac{1}{N_{samp}} \Gamma^T \Gamma J = \frac{1}{N_{samp}} \Gamma^T \hat{R} \tag{42}$$

Due to the use of extended bases, some rows of the above equation are identical and can be removed to give an underdetermined algebraic equation system

$$\overline{A} J = \hat{V} \tag{43}$$

It has many of solutions for J composing a manifold $Y \in \Re^{\widetilde{t}}$. Now the task is to find a solution J from Y to force the HDMR component functions satisfying the hierarchical orthogonal condition. D-MORPH regression provides a solution to ensure additional condition of exploration path represented by differential equation

$$\frac{dJ(l)}{dl} = \chi v(l) = (P_t - \overline{A}^+ \overline{A}) v(l) \tag{44}$$

wherein χ denotes orthogonal projector ensuring

$$\chi^2 = \chi \quad \text{and} \quad \chi^T = \chi \tag{45}$$

$$\chi = \chi^2 = \chi^T \chi \tag{46}$$

The free function vector may be selected to ensure the wide domain for $J(l)$ as well as to simultaneously reduce the cost $\kappa(J(l))$ which can be expressed as

$$v(l) = -\frac{\partial \kappa(J(l))}{\partial J} \tag{47}$$

Then we obtain

$$\frac{\partial \kappa(J(l))}{\partial l} = \left(\frac{\partial \kappa(J(l))}{\partial J} \right)^T \frac{\partial J(l)}{\partial l} = \left(\frac{\partial \kappa(J(l))}{\partial J} \right)^T$$
$$P v(l) = -\left(P \frac{\partial \kappa(J(l))}{\partial J} \right)^T \left(P \frac{\partial \kappa(J(l))}{\partial J} \right) \leqslant 0 \tag{48}$$

The cost function can be expressed in quadratic form as

$$\kappa = \frac{1}{2} J^T \overline{B} J \tag{49}$$

where $\overline{B}$ denotes the positive definite symmetric matrix and J_∞ can be expressed as

$$J_\infty = \overline{V}_t \left(\overline{U}^T_{\widetilde{t}-r} \overline{V}_{\widetilde{t}-r} \right)^{-1} \overline{U}^T_{\widetilde{t}-r} \overline{A}^+ \hat{V} \tag{50}$$

where the last columns $(\widetilde{t} - r)$ of $\overline{U}$ and $\overline{V}$ are denoted as $\overline{U}_{\widetilde{t}-r}$ and $\overline{V}_{\widetilde{t}-r}$ which can found by decomposition of $\chi\overline{B}$ [34]

$$\chi \overline{B} = \overline{U} \begin{bmatrix} \overline{I}_r & 0 \\ 0 & 0 \end{bmatrix} \overline{V}^T \tag{51}$$

This unique solution J_∞ in Y indicates the minimized cost function. D-MORPH regression is used to find the J which ensures the HDMR component functions' orthogonality in hierarchical manner. The construction of the corresponding cost function κ can be found in previous literature [25].

4. Bottom-Up stochastic approach

In the present study, the following cases are considered wherein the random variables in each layer of laminate are investigated:

(a) Variation of ply-orientation angle only: $g\{\theta(\overline{\omega})\} = \{\theta_1 \; \theta_2 \; \theta_3 \ldots \theta_i \ldots \theta_l\}$
(b) Variation in elastic modulus: $g\{E_1(\overline{\omega})\} = \{E_{1(1)} \; E_{1(2)} \; E_{1(3)} \ldots E_{1(i)} \ldots E_{1(l)}\}$
(c) Variation of mass density only: $g\{\rho(\overline{\omega})\} = \{\rho_1 \; \rho_2 \; \rho_3 \ldots \rho_i \ldots \rho_l\}$
(d) Combined variation of ply orientation angle, elastic modulus and mass density:

$$g\{\theta(\overline{\omega}), E_1(\overline{\omega}), \rho(\overline{\omega})\} = \{\Phi_1(\theta_1 \ldots \theta_l), \Phi_2(E_{1(1)} \ldots E_{1(l)}), \Phi_3(\rho_1 \ldots \rho_l)\}$$

where θ_i, $E_{1(i)}$ and ρ_i are the ply orientation angle, elastic modulus along longitudinal direction and mass density of i-th layer, respectively and 'l' denotes the number of layer in the laminate. The stochastic responses investigated in this article corresponding to layer-wise variations of the above sources of uncertainties are natural frequency, frequency response function and mode shape. Monte Carlo Simulation (MCS) based random variable approach is employed in this study for uncertainty quantification. Though this technique is computationally expensive but it is a quite versatile technique which is well established in many fields for uncertainty quantification over a long period of time. Monte Carlo Simulation methods are based on the use of random numbers and probability statistics to investigate the problems. The results obtained by MCS are generally used as benchmarking results to compare the results obtained from other methods. In the present study, a random number generator is employed to generate the possible ply orientation angle, elastic modulus and mass density variation with a standard range of ±10% from respective deterministic mean values for the material properties and ±5° for ply orientation angle. Larger the number of sample size, more the confidence in the results obtained for MCS approach. The number of samples for MCS analysis is generally selected based on convergence study of standard deviation of the performance parameters. In the present analysis ten thousand realizations (i.e. sample size of 10,000) are performed in all the analyses.

The flowchart for uncertainty quantification of laminated composite plates based on direct MCS using finite element FORTRAN code is presented in Fig. 2. As mentioned above uncertainty quantification using MCS is generally computationally very expensive because it requires the expensive finite element code to run thousands of times. To mitigate this lacuna GHDMR is employed in the present study. In this approach few finite element simulations are generally needed to be carried out corresponding to algorithmically chosen design points (Sobol sequence [28]). On the basis of the information captured in the design domain through the design points, a fully functional mathematical model is constructed using GHDMR approach as discussed in Section 3. Once the computationally efficient surrogate is constructed, it is then used for subsequent analysis of the structure. A flowchart describing GHDMR based uncertainty quantification scheme is furnished in Fig. 3.

As mentioned in the introduction section, this study includes an uncertainty quantification scheme using commercial finite element software ANSYS. For that purpose the ANSYS Parametric Design Language (APDL) script generated after modelling the composite plate in ANSYS environment is integrated with MATLAB. A fully automated MATLAB code is developed capable of rewriting the APDL script in each iteration containing the random values of stochastic input parameters, then running the APDL script to obtain desired outputs (refer to flowchart presented in Fig. 4) and saving the results for each sample. Thus MCS can be carried out using ANSYS in conjunction with MATLAB for any number of samples following the proposed approach. The comparative results are presented in the preceding section for stochastic analysis of free vibration responses using FORTRAN code developed according

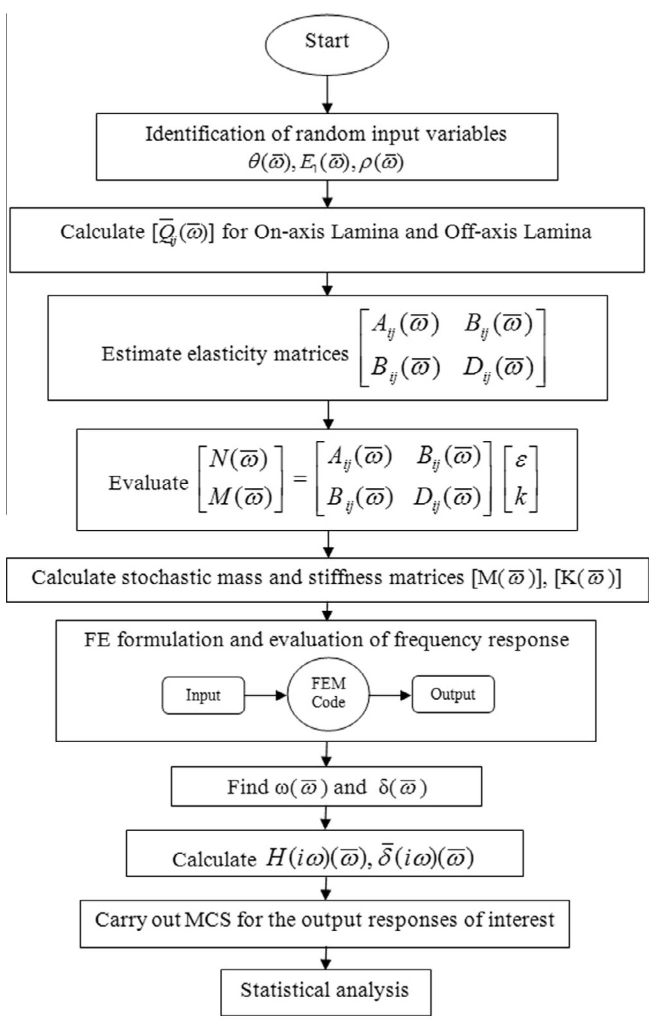

Fig. 2. Propagation of uncertainty in bottom up approach using direct MCS.

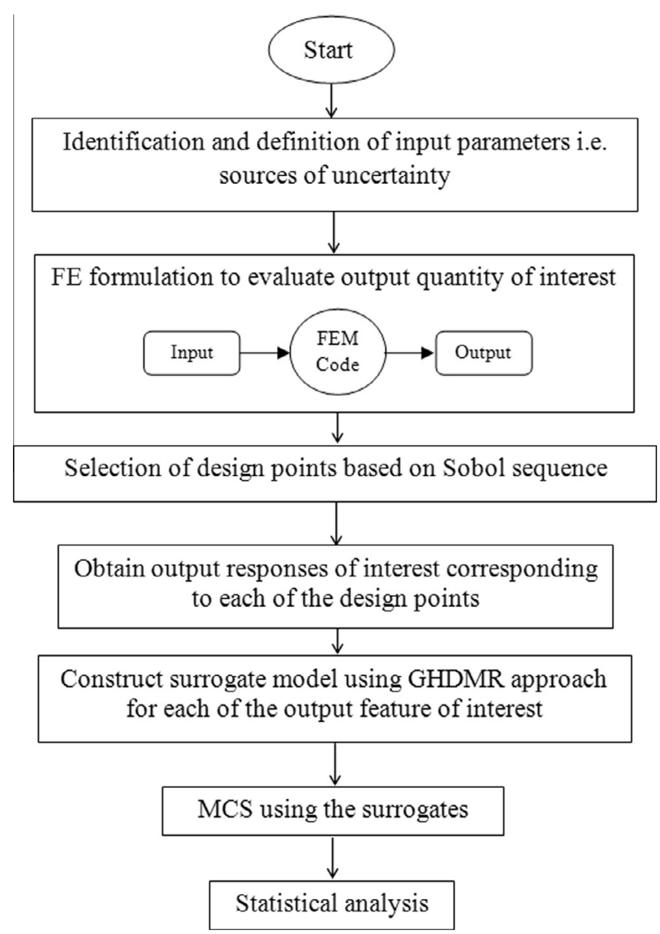

Fig. 3. Uncertainty quantification scheme using GHDMR approach.

to the formulation presented in Section 2 and UQ scheme using ANSYS.

A novel computational analysis is carried out in this article to explore the effect of simulated noise on GHDMR based uncertainty quantification scheme for fibre reinforced plastic composite plates. In the proposed approach, Gaussian white noise with a specific variance is introduced in the set of output responses, which is used for GHDMR model formation. Thus simulated noisy dataset (i.e. the sampling matrix for GHDMR model formation) is formed by introducing pseudo random noise in the responses, while the input design points are kept unaltered. Subsequently for each dataset, GHDMR based MCS is carried out to quantify uncertainty of composite plates as described in Fig. 5. Effect of noise are found to be accounted in several other studies in available literature [45–47] dealing with deterministic analysis. Assessment of any surrogate based uncertainty propagation algorithm under the effect of noise is the first attempt of its kind to the best of authors' knowledge. The effect of such simulated noise can be regarded as considering other sources of uncertainty such as error in measurement of responses, error in modelling and computer simulation and various

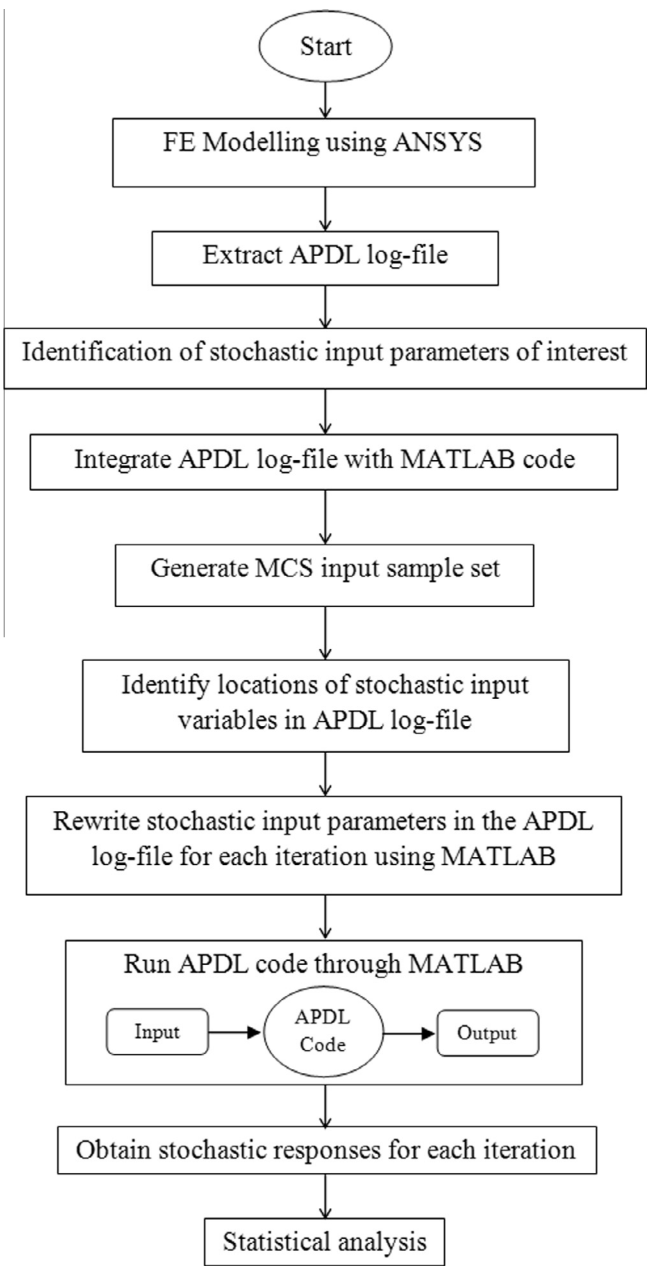

Fig. 4. Uncertainty quantification scheme using ANSYS.

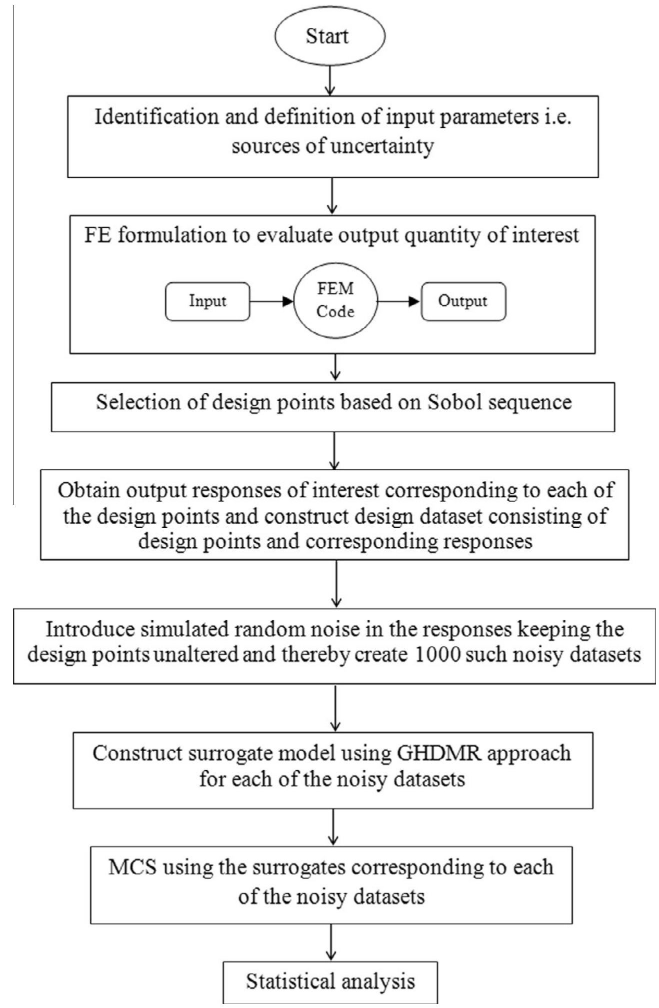

Fig. 5. Flowchart for analyzing effect of noise on GHDMR based UQ algorithm.

Table 1
Non-dimensional fundamental natural frequencies [$\omega = \omega_n L^2 \sqrt{(\rho/E_1 h^2)}$] of graphite-epoxy three layered angle-ply [$\theta/-\theta/\theta$] composite plates, considering $L/b = 1$, $b/t = 20$, $\psi = 30°$.

Fibre orientation angle, θ (°)	Present FEM	Qatu and Leissa [48]
15	0.8618	0.8759
30	0.6790	0.6923
45	0.4732	0.4831
60	0.3234	0.3283

Table 2
Fundamental natural frequencies [$\omega = \omega_n L^2 \sqrt{(\rho/E_1 h^2)}$] of three layered [90°, −90°, 90°] graphite-epoxy plates, $L = 1$ m, $h = 0.004$ m, $L/b = 1$, $E_1 = 138$ GPa, $E_2 = 8.9$ GPa, $G_{12} = G_{13} = 7.1$ GPa, $G_{23} = 2.84$ GPa.

Frequency number	Present FEM	ANSYS
1	2.1629	2.1630
2	7.3953	7.3955
3	13.5623	13.5623
4	24.9111	24.9112
5	38.2231	38.2231

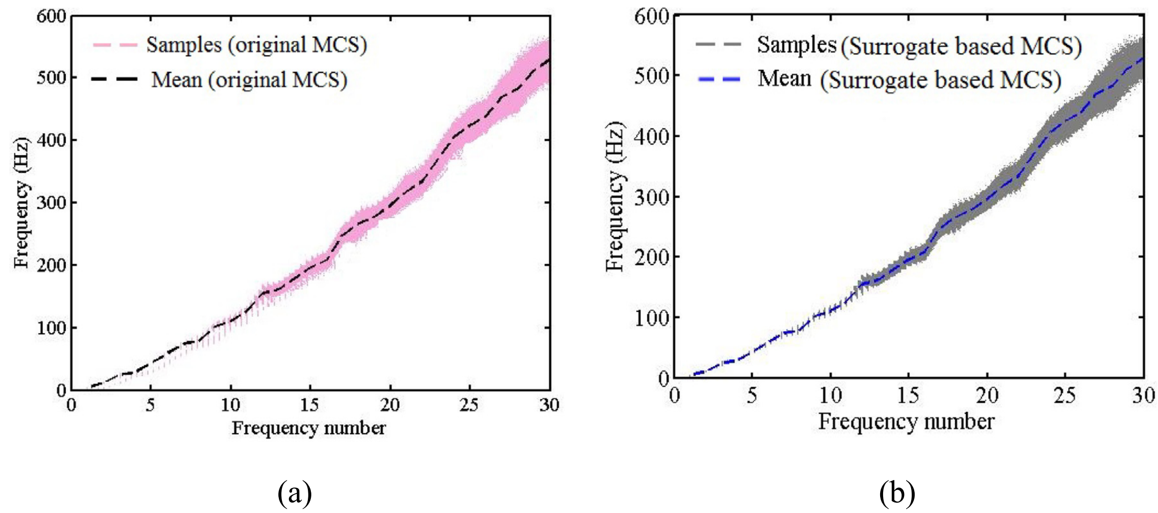

Fig. 6. Comparative plots for frequency responses of first thirty natural frequencies using direct MCS and surrogate based (GHDMR) approach for combined variation of stochastic input parameters of a angle-ply (45°/−45°/45°) composite cantilever plate.

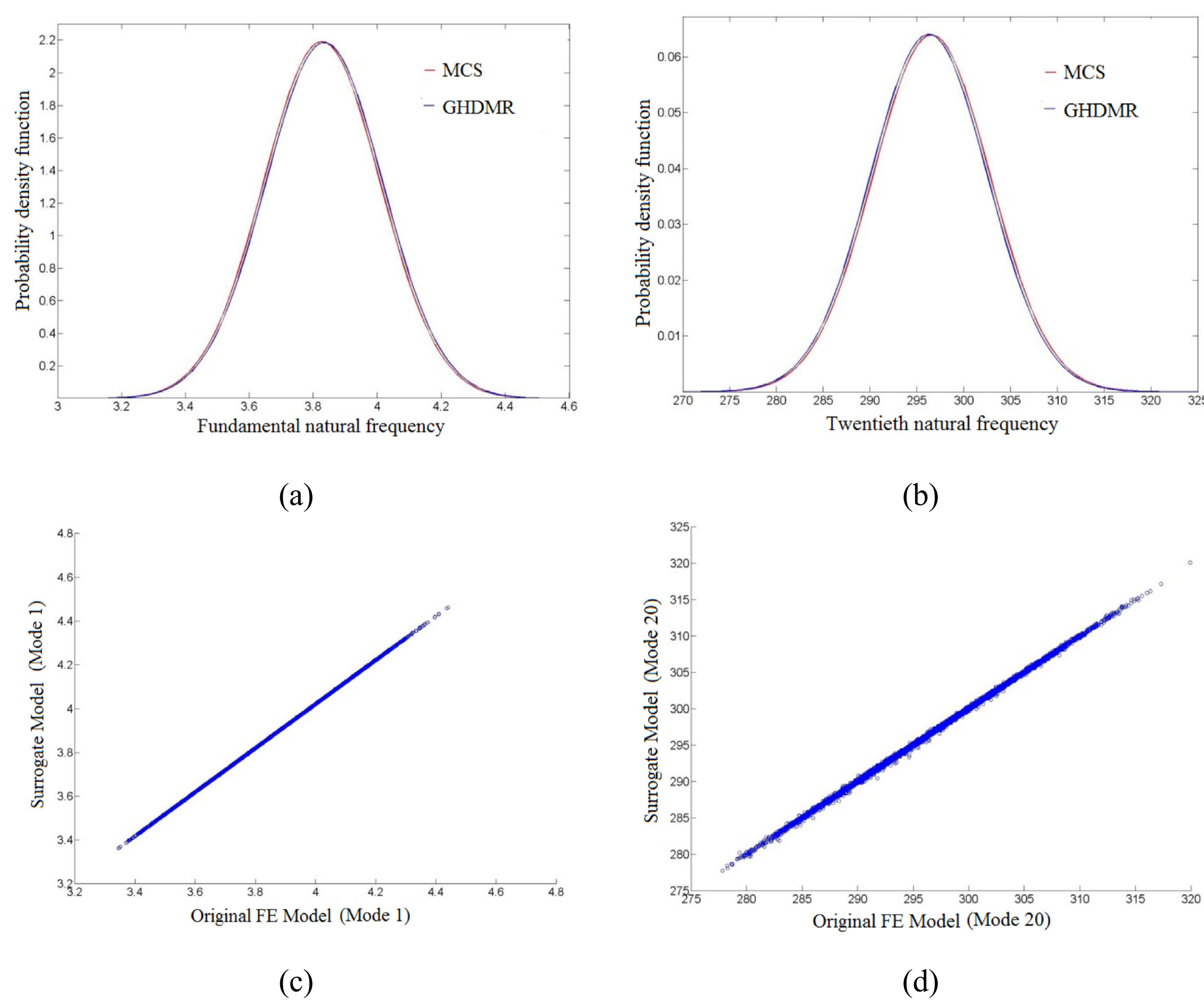

Fig. 7. (a and b) Probability density function and (c and d) Scatter plot for fundamental and twentieth modes with respect to direct MCS and GHDMR based approach for angle-ply (45°/−45°/45°) composite cantilever plates.

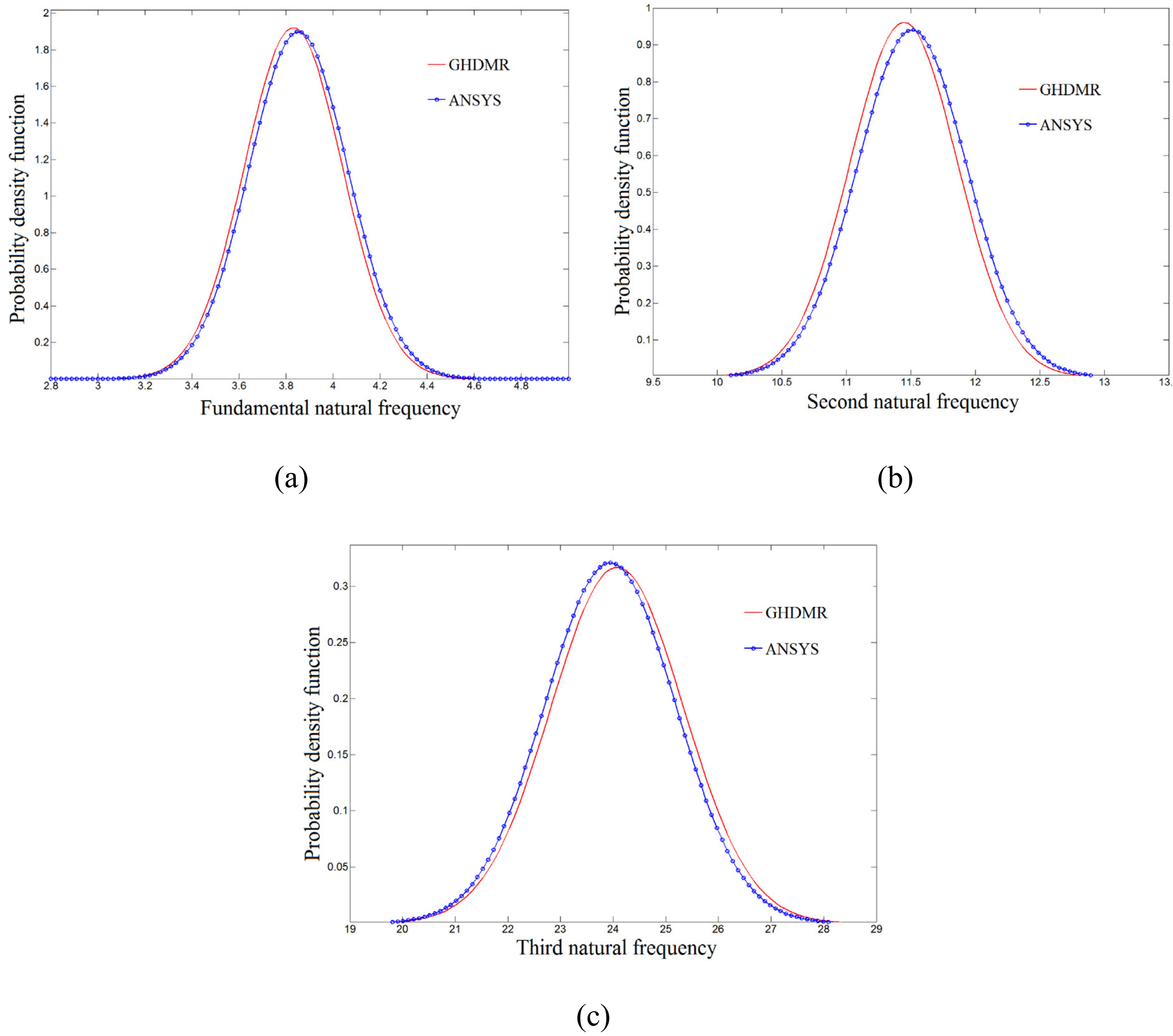

Fig. 8. Probability density function of first three stochastic natural frequencies using surrogate based approach (GHDMR) and ANSYS for combined variation of ply angle, longitudinal elastic modulus and mass density of an angle-ply (45°/−45°/45°) composite cantilever plate.

other epistemic uncertainties involved with the system. Thus the kind of analysis carried out here will provide a comprehensive idea about the robustness of GHDMR based UQ algorithm under noisy data.

5. Results and discussion

In the present study, the uncertainty analysis of laminated composite plate is carried out for ply orientation angle, elastic modulus and inertia (mass density) properties of laminates. An eight nodded isoparametric plate bending element with five degrees of freedom at each node is considered in finite element formulation. The standard eigenvalue problem is solved by applying the QR iteration algorithm. The performance parameters of interest are natural frequencies, mode shapes and frequency response functions of the composite cantilever plate. The finite element code developed in FORTRAN language following the theoretical formulation described in Section 2 and the code is validated with published literature as shown in Table 1. Table 2 presents the comparative deterministic results for first five natural frequencies obtained using the FORTRAN code and commercial finite element analysis software (ANSYS). The results are found to be in good agreement. The mean value of plate material and geometric properties of the graphite-epoxy composite laminated cantilever plate used in the present analysis are given below:

(a) Deterministic mean values of material properties:

Material density, ρ = 1600 kg/m^3
Major in-plane Poisson's ratio, ν = 0.3
Young's modulus, E_1 = 138 GPa and E_2 = 8.9 GPa
Shear modulus, $G_{12} = G_{13}$ = 7.1 GPa and G_{23} = 2.84 GPa

(b) Dimension of composite plate:

Width, b = 1 m
Thickness, t = 0.004 m
Length, L = 1 m

Monte Carlo simulation (MCS) is employed in conjunction to the finite element model for quantifying uncertainty in the aforementioned three response parameters due to layer-wise individual and combined variations of the stochastic input parameters. Fig. 6 presents comparative results of the mean values and response bounds for first thirty natural frequencies using direct MCS and the proposed GHDMR based approach, wherein a good agreement is noticed between the two plots.

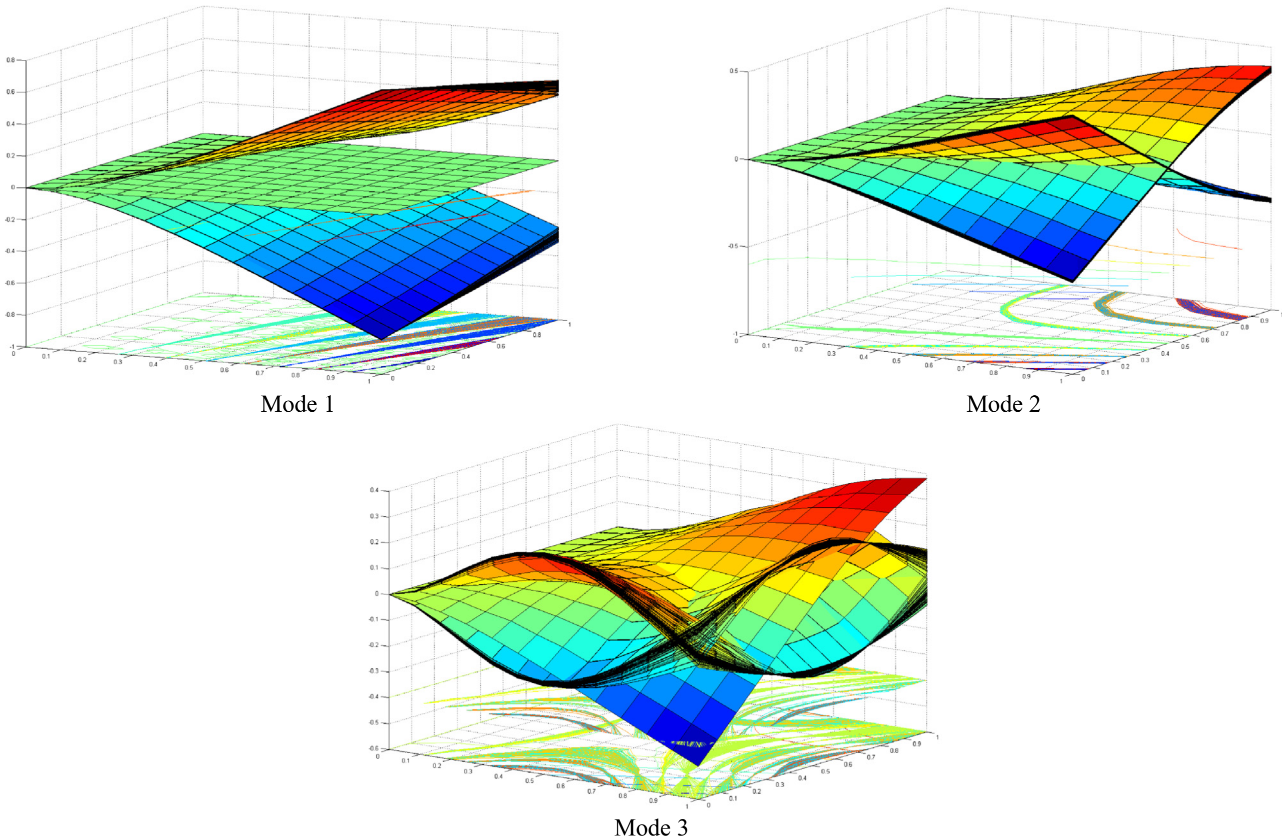

Fig. 9. First three stochastic mode shapes considering combined variation of ply orientation angle (45°/−45°/45°), elastic modulus and mass density for graphite-epoxy laminated composite cantilever plate.

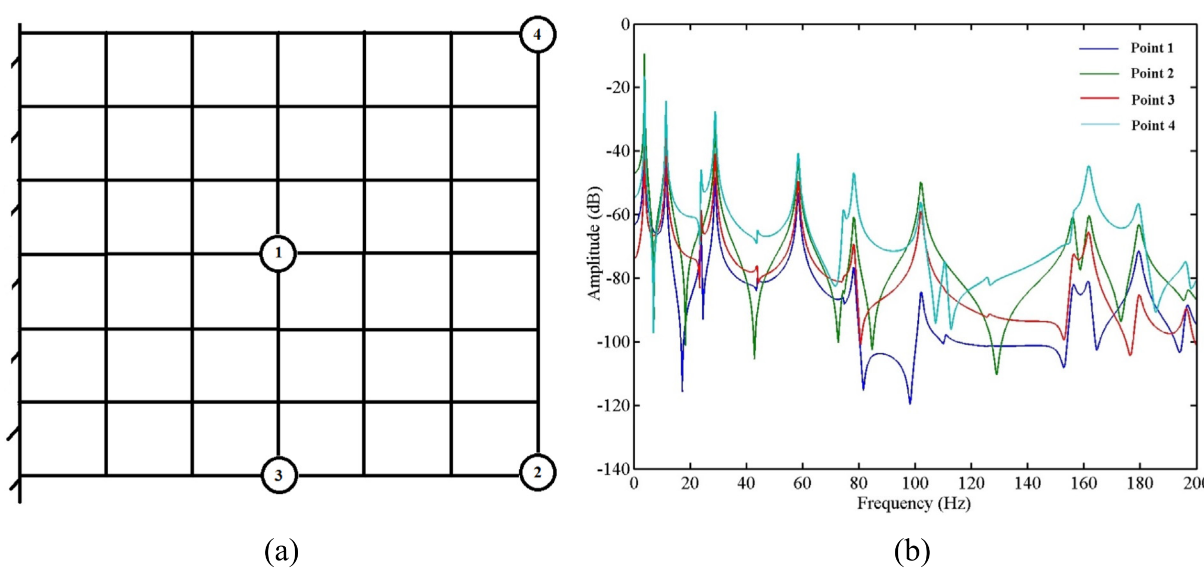

Fig. 10. (a) Driving point (point 2) and cross point (point 1, 3 and 4) for amplitude (in dB) of frequency response function. (b) Amplitude with respect to frequency corresponding to driving and cross points (deterministic responses).

To illustrate the validation of results of the proposed GHDMR based approach further with respect to direct MCS, natural frequencies corresponding to first and twentieth modes are selected arbitarily. The probability density function plots and the scatter plots corresponding to the two selected modes are presented in Fig. 7, wherein it is evident that the results of the two approaches are in quite good agreement corroborating accuracy of the GHDMR based approach. Fig. 8 shows the probability density function plots corresponding to first three natural frequencies obtained for combined variation of the stochastic input parameters using the proposed uncertainty quantification scheme using ANSYS. The results are compared with GHDMR based approach of uncertainty

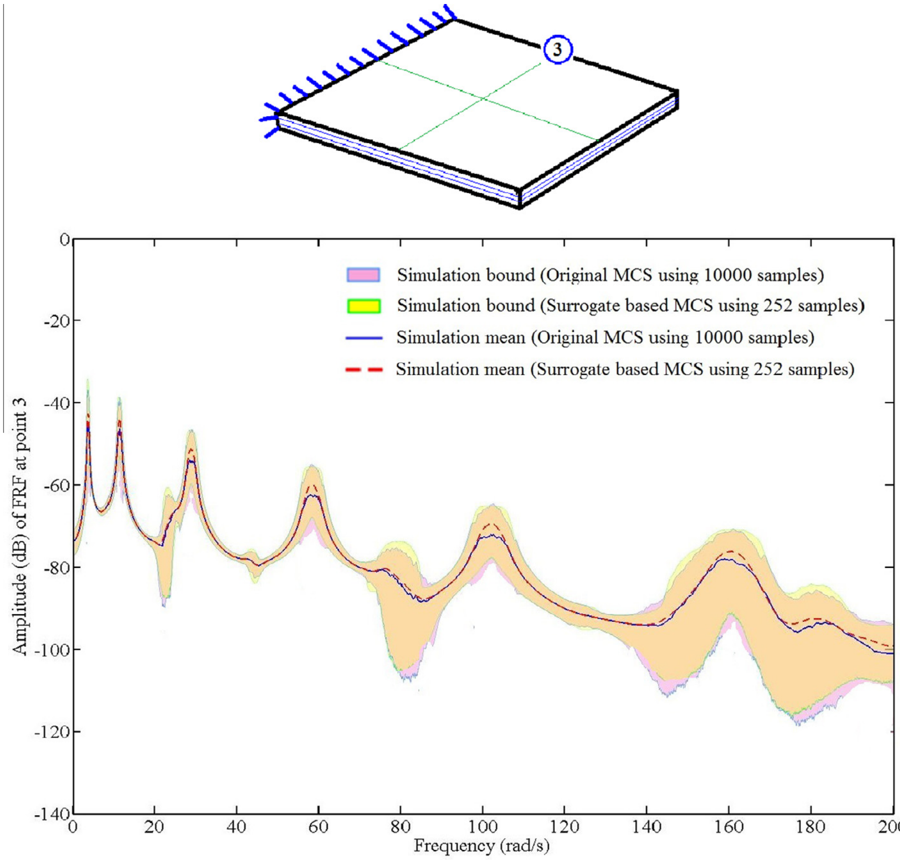

Fig. 11. Convergence study on amplitude (dB) with respect to frequency (rad/s) for simulation bound and simulation mean of MCS and surrogate (GHDMR) based approach considering combined variation of ply orientation angle, elastic modulus and mass density corresponding to point 3 of angle-ply (45°/−45°/45°) composite cantilever plate.

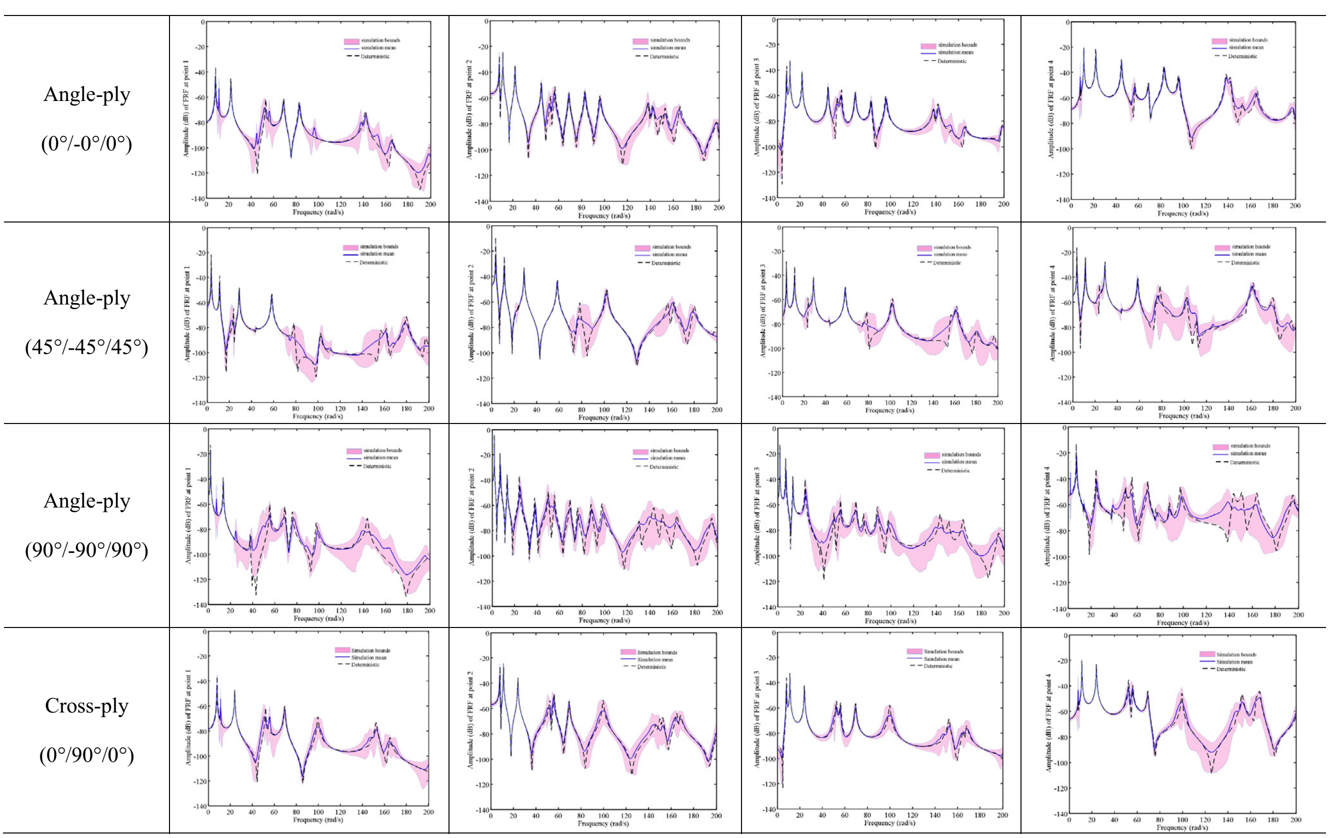

Fig. 12. Variation of only ply-orientation angle in $[\theta(\overline{\varpi})]$ each layer to plot the direct simulation bounds, direct simulation mean and deterministic values for amplitude (dB) with respect to frequency (rad/s) of points 1, 2, 3 and 4 considering graphite-epoxy composite cantilever plate.

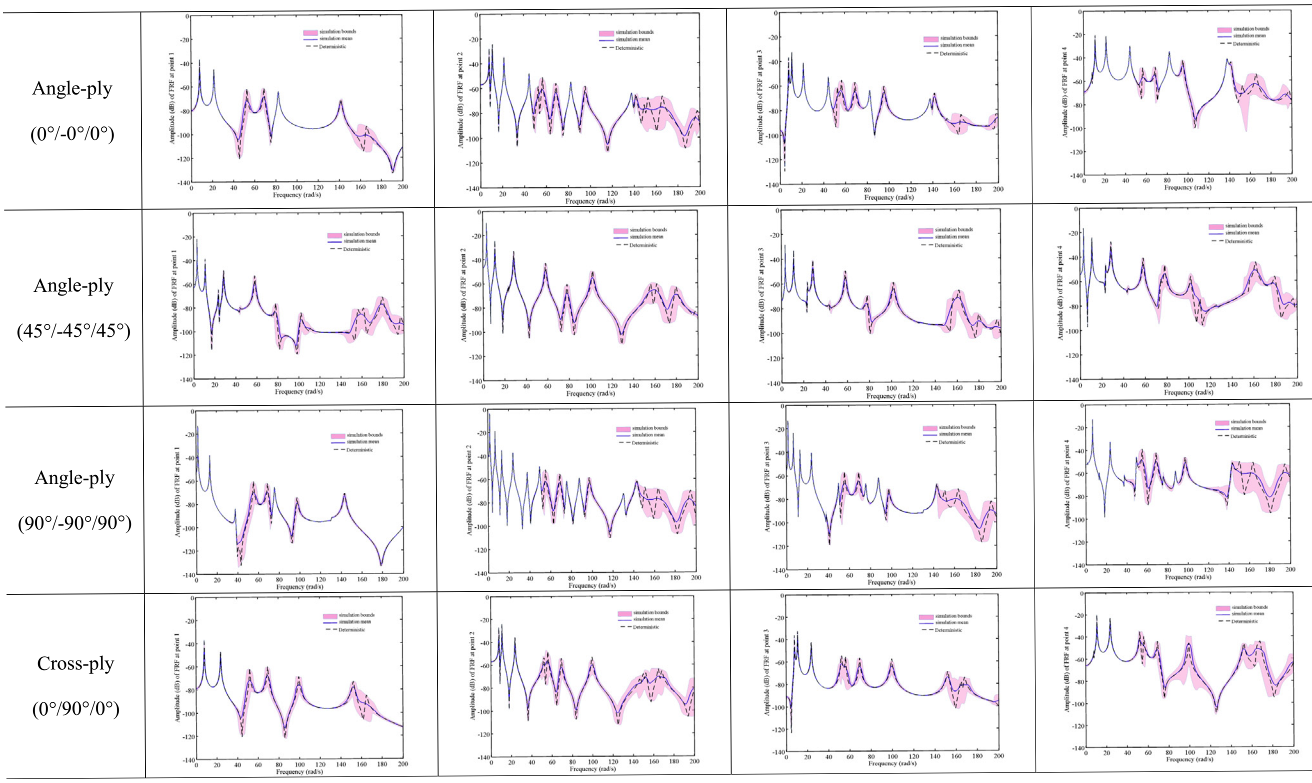

Fig. 13. Variation of only elastic modulus $[E_1(\overline{\omega})]$ in each layer to plot the direct simulation bounds, direct simulation mean and deterministic values for amplitude (dB) with respect to frequency (rad/s) of points 1, 2, 3 and 4 considering graphite-epoxy composite cantilever plate.

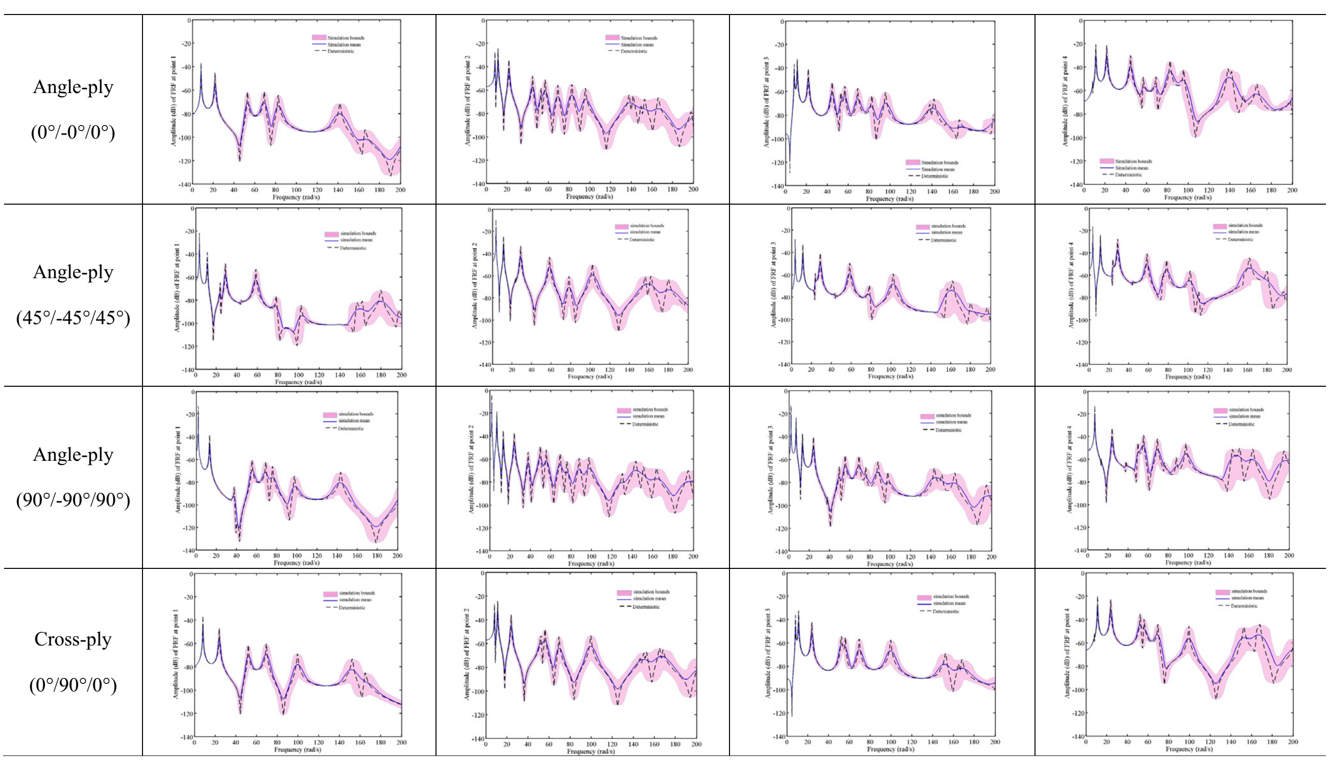

Fig. 14. Variation of only mass density $[\rho(\overline{\omega})]$ in each layer to plot the direct simulation bounds, direct simulation mean and deterministic values for amplitude (dB) with respect to frequency (rad/s) of points 1, 2, 3 and 4 considering graphite-epoxy composite cantilever plate.

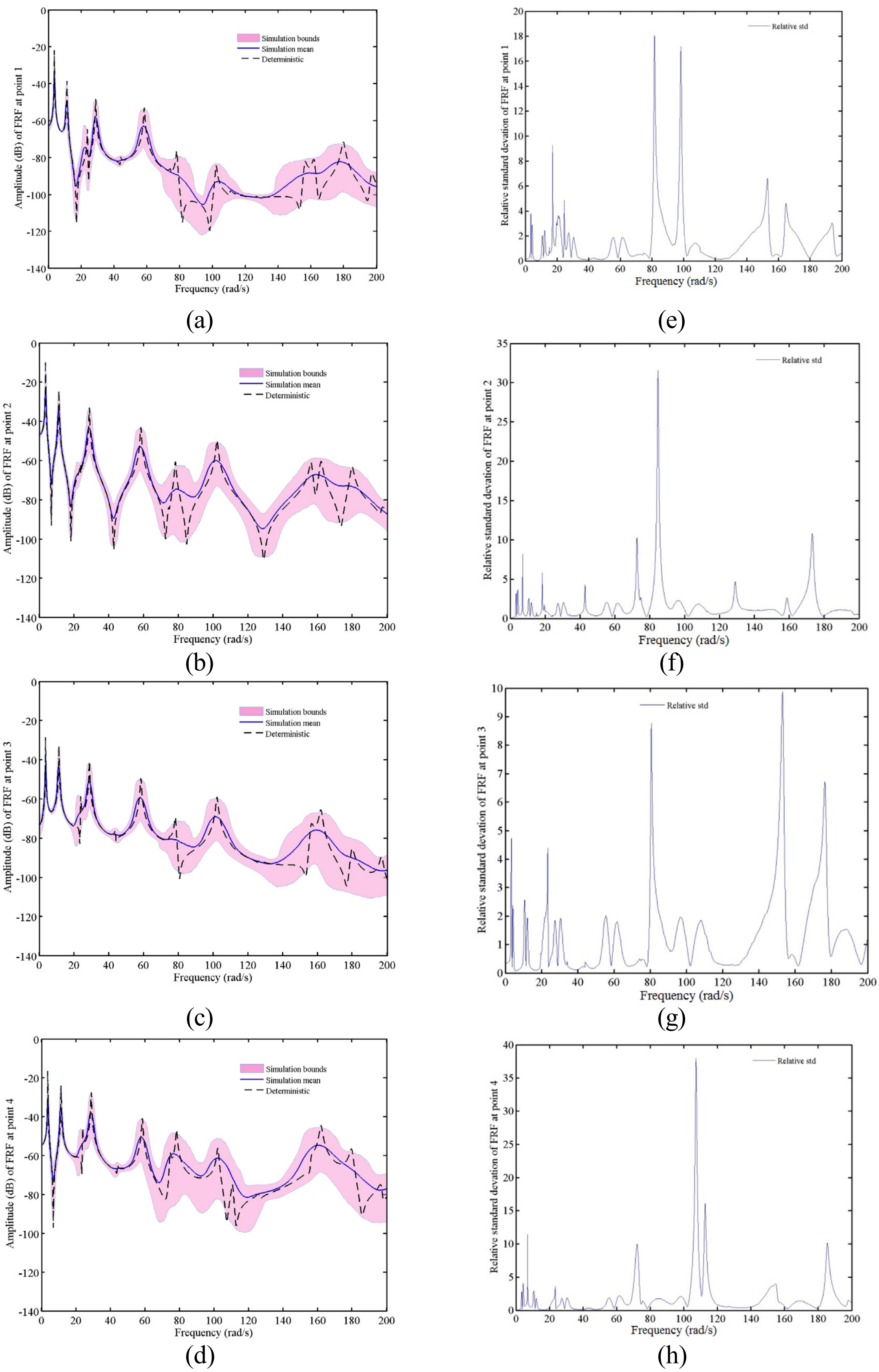

Fig. 15. (a–d) Amplitude with respect to frequency and (e–h) relative SD for points 1 to 4 for combined variation of ply orientation angle (45°/−45°/45°), elastic modulus and mass density.

quantification. A good agreement between these two approaches establishes the accuracy of the uncertainty quantification scheme using ANSYS.

In general, the frequency response function between two points on a composite structure indicate the a mathematical representation of the relationship between the input and the output of a system wherein an accelerometer and a force gauge hammer are used to measure the response and the excitation, respectively. In present study, the first three stochastic mode shapes are furnished in Fig. 9, wherein layer-wise combined variation of ply orientation angle (45°/−45°/45°), elastic modulus and mass density is considered for the graphite-epoxy laminated composite cantilever plate. From the figure it can be noticed that bending is the predominant factor in first mode and torsion is predominant in second mode, whereas a mixed behaviour of bending and torsion is present in the third mode. The frequency response functions (FRF) are obtained for both angle-ply and cross-ply laminated composite plates. Driving point (Point 2) and the cross points (Point 1, 3 and 4) considered in the present analysis are described in Fig. 10 (a). Fig. 10(b) shows the deterministic FRFs in a frequency range of 200 Hz. For the purpose of numerical calculations, 0.5% damping factor is assumed for all the modes. A typical validation plot for the FRF obtained for point 3 using both MCS and surrogate based (GHDMR) approach is furnished in Fig. 11, wherein a comparison with direct MCS is presented. From the figure it is evident that GHDMR is quite accurate with respect to direct MCS approach while high level of computational efficiency can be achieved using GHDMR in terms of number of finite element simulations. The sources of uncertainty in the FRFs are systematically investigated in this study following the proposed bottom up approach of uncertainty propagation with different configurations of ply orientation angles. The simulation bound, simulation mean and deterministic values of FRFs are portrayed in Figs. 12 and 13 for individual variation of ply orientation angle and elastic modulus (causing stochasticity in the stiffness matrix), respectively while Fig. 14 shows the same for individual variation of density (causing stochasticity in the mass matrix). In general, for a certain amount of variability in input parameter, higher volatility is observed in the higher frequency ranges. Fig. 15(a)–(d) shows the effect of combined random variation of ply orientation angle, elastic modulus and mass density (causing stochasticity in both the stiffness and the mass matrix) in the FRFs of an angle ply composite plate. The relative standard deviations are plotted for combined variation of stochastic input parameters as shown in Fig. 15(e)–(h), which gives a clear idea about the relative standard deviation (SD) of FRFs in different frequency ranges for the four considered points.

The representative results are presented in Fig. 16 showing the effect of noise on fundamental natural frequency of the angle ply (45°/−45°/45°) laminated composite plate considering combined variation of ply orientation angle, elastic modulus and mass density. The Gaussian white noise with a specific value of variance (Var) in the range of 0.2–2 is introduced in the set of fundamental natural frequency, which is used for GHDMR model formation. The results furnished in this article are obtained by using 1000 number of such noisy datasets, which involves formation of GHDMR model and thereby carrying out MCS for each dataset using corresponding surrogate models (refer to Fig. 5). As the variance increases, the bound of frequency responses (difference between upper and lower limit) are found to expand due to effect of noise as furnished in Fig. 16. The results presented for different values of variance are compared with the probability density function of noise-free case to provide a comprehensive idea about the performance of GHDMR in UQ under the influence of simulated noise for fundamental natural frequency of laminated composite plates. It is worthy to mention that all the results presented in this article are obtained using 10,000 simulations. The application of surrogate based approach using GHDMR allows to obtain these results by means of virtual simulations instead of actual finite element simulation. For layer-wise combined variation and individual variation of the stochastic input parameters, 252 and 64 samples, respectively are utilized to construct the GHDMR model. Thus for the purpose of uncertainty quantification, same number of actual finite element simulations are needed in the proposed approach, in contrast with 10,000 finite element simulations needed in direct MCS approach. Therefore, the proposed GHDMR based approach for uncertainty quantification in laminated composite structures is much more computationally efficient than conventional direct MCS approach in terms of finite element simulations.

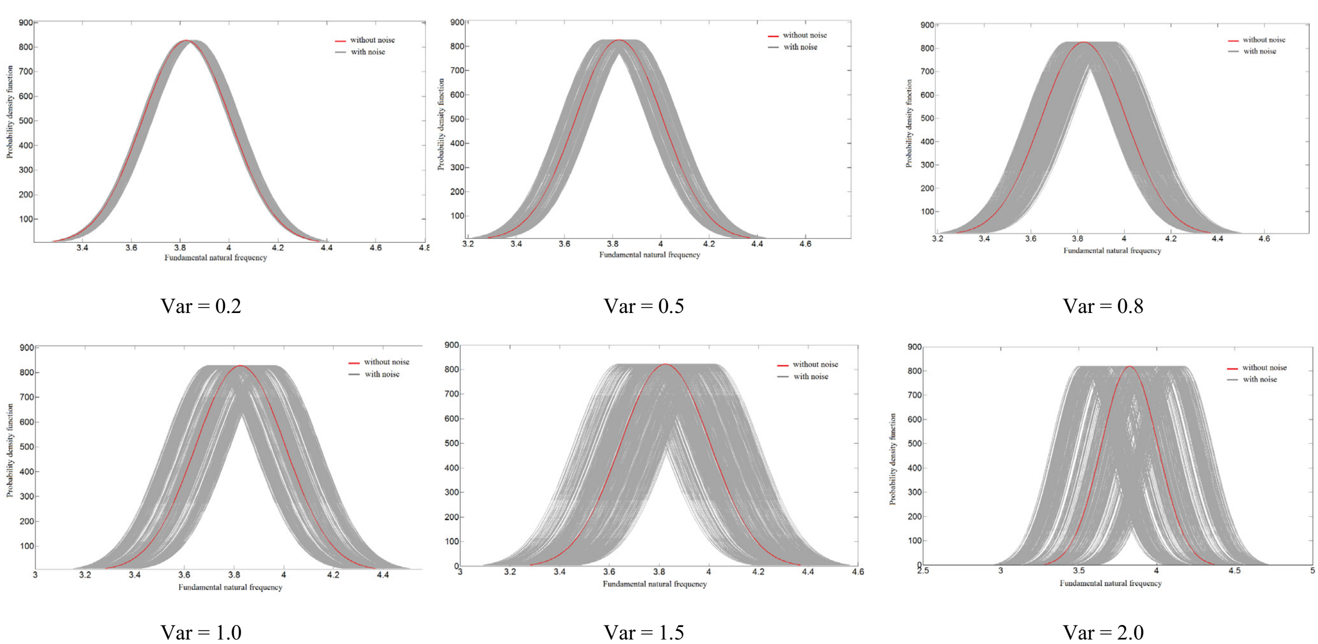

Fig. 16. Effect of noise on GHDMR based uncertainty quantification for fundamental natural frequency of angle ply (45°/−45°/45°) laminated composite plates considering combined variation of ply orientation angle, elastic modulus and mass density.

6. Conclusion

This article presents a bottom up uncertainty propagation scheme for laminated composite plates. An efficient surrogate (GHDMR) based approach is proposed to characterize uncertainty in different free vibration responses of the structure. Subsequently the performance of GHDMR based uncertainty propagation algorithm under the effect of simulated noise is investigated. The prime novelty of the present study lies in application of GHDMR approach for uncertainty quantification in natural frequency, mode shape and frequency response functions due to stochasticity in material properties and ply orientation angle. It is found that high level of computational efficiency can be achieved following the proposed approach compared to direct MCS without compromising the accuracy of results. The GHDMR based uncertainty propagation algorithm can be applied to more complex configurations of laminated composite structures and to quantify uncertainty for various other structural responses using the knowledge shared in this paper. Other sources of uncertainties in stochastic analysis of different responses can also be incorporated following the GHDMR approach. Another contribution of this article is the proposed industry oriented uncertainty quantification scheme using commercial finite element analysis software ANSYS, which is validated with the present problem of laminated composite plates using frequency responses. This approach has immense potential to be extended towards uncertainty quantification of large-scale complex structures in future investigations.

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