Nonlocal elasticity based magnetic field affected vibration response of double single-walled carbon nanotube systems

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The behaviour of carbon nanotubes in a magnetic field has attracted considerable attention in the scientific community. This paper reports the effects of a longitudinal magnetic field on the vibration of a magnetically sensitive double single-walled carbon nanotube system (DSWNTS). The two nanotubes of the DSWNTS are coupled by an elastic medium. The dynamical equations of the DSWNTS are derived using nonlocal elasticity theory. The two nanotubes are defined as an equivalent nonlocal double-Euler-Bernoulli beam system. Governing equations for nonlocal bending-vibration of the DSWNTS under a longitudinal magnetic field are derived considering the Lorentz magnetic force obtained from Maxwell's relation. An analytical method is proposed to obtain nonlocal natural frequencies of the DSWNTS. The influence of (i) nanoscale effects and (ii) strength of longitudinal magnetic field on the synchronous and asynchronous vibration phase of the DSWNTS is examined. Nonlocal effects with and without the effect of magnetic field are illustrated. Results reveal the difference (quantitatively) by which the longitudinal magnetic field affects the nonlocal frequency in the synchronous and asynchronous vibration modes of a DSWNTS. *© 2012 American Institute of Physics*. [http://dx.doi.org/10.1063/1.4720084]

I. INTRODUCTION

As controlling every parameter in experiments at the nanoscale is difficult, and molecular dynamics (MD) simulations[1–4] are computationally expensive, classical continuum theory is becoming popular for vibration characterisation of nanostructures. Over the past decade, extensive literature has reported evidence of classical continuum models being able to predict the performance of "large" nanostructures. However, classical continuum mechanics is a scale-free theory and cannot account for effects arising from small size. Experimental[5–8] and atomistic simulations[9,10] have evidenced a significant "size-effect" in mechanical and physical properties when the dimensions of the structures become "small." Size effects are related to atoms and molecules that constitute the materials. The application of classical continuum models thus may be questionable in the analysis of "smaller" nanostructures. Therefore, there have been research efforts[11–14] to bring in scale-effects within the formulation by amending the traditional classical continuum mechanics. One widely used size-dependent theory is the nonlocal elasticity theory pioneered by Eringen.[15]

Nonlocal elasticity accounts for the small-scale effects arising at the nanoscale level. In nonlocal elasticity theory, the small-scale effects are captured by assuming that the stress at a point is a function of the strains at all points in the domain.[15] This is unlike classical elasticity theory. Nonlocal theory considers long-range inter-atomic interaction and yields results dependent on the size of a body. Some drawbacks of the classical continuum theory can be efficiently avoided and size-dependent phenomena can be reasonably explained by the nonlocal elasticity theory. The use of nonlocal elasticity in the mechanical analysis of nanostructures is extensively reported in the literature.[13,16–29]

Magnetic field effects in nanotubes and nanoplates (graphene) are important for exciting potential applications in nanotechnology such as in nanoelectromechanical systems (NEMS), microelectromechanical systems (MEMS), nanosensors, spintronics, and nanocomposites. In recent years, research interest has grown on studying the magnetic properties of nanotubes and behaviour of nanotubes within a magnetic field. Bellucci *et al.*[30] investigated the influence of a perpendicular magnetic field on the transport properties of carbon nanotubes (CNTs). Chang and Lue[31] considered multiwall carbon nanotubes (MWCNTs) and studied their magnetic properties by electron paramagnetic resonance and magnetization measurements. The effects of an external magnetic field on the magnetic properties of nanotubes and nanowires were investigated by Chen *et al.*[32] Jang and Sakka[33] examined the influence of the shape and size of CNTs on the alignment of multi-wall CNTs under a strong magnetic field. The study of electronic properties of nanotubes under a magnetic field has found interest among researchers. Kibalchenko *et al.*[34] examined the magnetic response of single-walled CNT under a magnetic field. Lee *et al.*[35] considered double-walled CNTs and illustrated discrete electronic states in a magnetic field. They confirmed the dependence of the magnitude and direction of the magnetic field on the electronic properties of double-walled CNTs. Roche and Saito,[36] Sebastiani and Kudin,[37] and Zhang *et al.*[38] investigated the electronic properties of carbon nanotubes in the presence of a magnetic field. Mechanical studies, viz., vibration of single-walled and multiwalled nanotubes (MWNTs) under a magnetic field have also been

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reported in the literature. Li *et al.*[39] investigated effects of a magnetic field on the dynamic characteristics of MWNTs. Coupled dynamic equations of MWNTs subjected to a transverse magnetic field were developed. Wang *et al.*[40] investigated the effects of a longitudinal magnetic field on wave propagation in CNTs embedded in an elastic matrix. Dynamic equations were derived by considering the Lorentz magnetic forces. Wang *et al.*[41] presented an analytical method to investigate van der Waals interaction effects on vibration characteristics of multi-walled carbon nanotubes embedded in a matrix under a transverse magnetic field. Wei and Wang[42] studied the different electromagnetic wave modes coupled in a longitudinal or transverse magnetic field.

One important technological step up from the concept of the single nanotube is that of complex nanotube systems. One complex nanotube system would be the double single-walled carbon nanotube system (DSWNTS).

This double nanobeam (nanotube) system could be used in nano-optomechanical systems (NOMS).[43] Elastically connected double-nanotube system can also be used for the acoustic and vibration isolation similar as macro double-beam-system. Vibration characteristics of multiple nanotubes dispersed in nanocomposites are important from the structural performance of nanocomposites. Vibration of double-nanotube-system coupled by elastic medium (matrix) in nanocomposites is worth understanding in this respect.

It should be noted that a DSWNTS is different from a double-walled carbon nanotube.[44] Vibration and instability studies of double nanobeam system (DNBS) are reported in Murmu and Adhikari.[45,46] To the best of authors' knowledge, the effects of magnetic fields on the vibration characteristics of a DSWNTS is unavailable in literature.

In the present paper, we study the effect of a magnetic field on the vibration of a DSWNTS. Nonlocal elasticity is used to address the small scale effects. The single-walled nanotubes in the DSWNTS are bonded by an elastic medium. The two nanotubes are defined as an equivalent nonlocal double Euler-Bernoulli beam system. Nonlocal effects in the longitudinal direction are considered and the nonlocal effects in the circumferential direction of the CNT are ignored. Governing equations for nonlocal bending-vibration of the DSWNTS under a longitudinal magnetic field are derived considering the Lorentz magnetic force obtained from Maxwell's relation. An analytical method is proposed to obtain natural frequencies of the DSWNTS as a function of nonlocal parameter and magnetic field parameter. Both synchronous and asynchronous vibration phases of the DSWNTS are highlighted. The present study will hopefully be useful for theoretical study and design of complex nanotubes systems in a magnetic field.

II. MAXWELL'S RELATION

Denoting $\mathbf{J}$ as current density, $\mathbf{h}$ as distributing vector of the magnetic field, and $\mathbf{e}$ as strength vectors of the electric field, the Maxwell relation according to Ref. 47 is given as

$$\mathbf{J} = \nabla \times \mathbf{h}, \tag{1}$$

$$\nabla \times \mathbf{e} = -\eta \frac{\partial \mathbf{h}}{\partial t}, \tag{2}$$

$$\nabla \cdot \mathbf{h} = 0, \tag{3}$$

$$\mathbf{e} = -\boldsymbol{\eta}\left(\frac{\partial \mathbf{U}}{\partial t} \times \mathbf{H}\right), \tag{4}$$

$$\mathbf{h} = \nabla \times (\mathbf{U} \times \mathbf{H}), \tag{5}$$

$$\mathbf{h} = \nabla \times (\mathbf{U} \times \mathbf{H}) = -H_x\left(\frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)\mathbf{i} + H_x \frac{\partial v}{\partial x}\mathbf{j} + H_x \frac{\partial w}{\partial x}\mathbf{k}, \tag{6}$$

where η is the magnetic field permeability. ∇ is the Hamilton operator and is expressed as $\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$. ($\mathbf{i}$, $\mathbf{j}$, $\mathbf{k}$) are the unit vectors. For the present analysis, we consider the longitudinal magnetic field as a vector $\mathbf{H} = (H_x, 0, 0)$ acting on the CNT. Let the displacement vector be $\mathbf{U} = (u, v, w)$, then

$$\begin{aligned}\mathbf{J} = \nabla \times \mathbf{h} = {} & H_x\left(-\frac{\partial^2 v}{\partial x \partial z} + \frac{\partial^2 w}{\partial x \partial y}\right)\mathbf{i} \\ & - H_x\left(\frac{\partial^2 v}{\partial y \partial z} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial z^2}\right)\mathbf{j} \\ & + H_x\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right)\mathbf{k}.\end{aligned} \tag{7}$$

The Lorentz force induced by the longitudinal magnetic field is given as

$$\begin{aligned}\mathbf{f} &= f_x\mathbf{i} + f_y\mathbf{j} + f_z\mathbf{k} = \eta(\mathbf{J} \times \mathbf{H}) \\ &= \eta\left[0\mathbf{i} + H_x^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right)\mathbf{j} + H_x^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right)\mathbf{k}\right].\end{aligned} \tag{8}$$

Therefore, the Lorentz forces along the x, y, and z directions are

$$f_x = 0, \tag{9}$$

$$f_y = \eta H_x^2\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y \partial z}\right), \tag{10}$$

$$f_z = \eta H_x^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 v}{\partial y \partial z}\right). \tag{11}$$

For the present vibrational analysis in DSWNTS, we assume that $w = w(x, t)$ only, so that the Lorentz force in the z direction is written as[40,48]

$$f_z = \eta H_x^2\left(\frac{\partial^2 w}{\partial x^2}\right). \tag{12}$$

It should be noted that in the present study the effective Lorentz force is a function of magnetic permeability and H_x.

III. NONLOCAL ELASTICITY

Here, we present a brief review of nonlocal elasticity. In nonlocal elasticity theory,[15] the basic equations for an isotropic linear homogenous nonlocal elastic body neglecting the body force are given as

$$\begin{aligned}
&\sigma_{ij,i} + \rho(f_j - \ddot{u}_j) = 0,\\
&\sigma_{ij}(\mathbf{x}) = \int_V \phi(|\mathbf{x}-\mathbf{x}'|,\alpha)\sigma_{ij}^C(\mathbf{x}')dV(\mathbf{x}'),\\
&\sigma_{ij}^C(\mathbf{x}') = C_{ijkl}\varepsilon_{kl}(\mathbf{x}'),\\
&\varepsilon_{kl}(\mathbf{x}') = \frac{1}{2}\left(\frac{\partial u_i(\mathbf{x}')}{\partial x_j'} + \frac{\partial u_j(\mathbf{x}')}{\partial x_i'}\right).
\end{aligned} \tag{13}$$

The terms σ_{ij}, σ_{ij}^C, ε_{kl}, C_{ijkl} are the nonlocal stress, classical stress, classical strain, and fourth order elasticity tensors, respectively. The volume integral is over the region **V** occupied by the body. The kernel function $\phi(|\mathbf{x}–\mathbf{x}'|, \alpha)$ is known as the nonlocal modulus. The nonlocal modulus acts as an attenuation function incorporating into the constitutive equations the nonlocal effects at the reference point **x** produced by local strain at the source $\mathbf{x}'$. The term $|\mathbf{x}–\mathbf{x}'|$ represents the distance in the Euclidean form and α is a material constant that depends on the internal (e.g., lattice parameter, granular size, distance between the C-C bonds) and external characteristic lengths (e.g., crack length, wave length).

The material constant α is defined as $\alpha = e_0a/\ell$. Here, e_0 is a constant for calibrating the model with experimental results and other validated models. The parameter e_0 is estimated such that the relations of the nonlocal elasticity model can provide a satisfactory approximation to the atomic dispersion curves of the plane waves obtained from atomistic lattice dynamics. According to Eringen,[15] the value of e_0 is reported as 0.39. Details on the various values of nonlocal parameter e_0 as reported by various researchers is discussed in Ref. 49. The terms a and ℓ are the internal (e.g., lattice parameter, granular size, distance between C-C bonds) and external characteristics lengths (e.g., crack length, wave length) of the nanostructure, respectively.

As Eq. (13) is difficult to solve, a differential form is popularly used as

$$(1-\alpha^2\ell^2\nabla^2)\sigma_{ij}(\mathbf{x}) = \sigma_{ij}^C(\mathbf{x}) = C_{ijkl}\varepsilon_{kl}(\mathbf{x}). \tag{14}$$

Here ∇ is the Laplacian. For one-dimensional nanostructures such as CNTs, Eq. (14) is simplified as

$$\left[1-(e_0a)^2\frac{d^2}{dx^2}\right]\sigma(x) = \sigma_{ij}^C(x). \tag{15}$$

Using the nonlocal relation (Eq. (15)), the governing equation for a single-walled CNT under the effect of longitudinal magnetic field can be written as

$$\begin{aligned}
&EI\frac{\partial^4 w(x,t)}{\partial x^4} + kw(x,t) - (e_0a)^2k\frac{\partial^2 w(x,t)}{\partial x^2}\\
&- q(x,t) + (e_0a)^2\frac{\partial^2 q(x,t)}{\partial x^2} + m\frac{\partial^2 w(x,t)}{\partial t^2}\\
&- (e_0a)^2m\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2} = 0,
\end{aligned} \tag{16}$$

where $q(x,t) = \int_A f_z dz = \eta A H_x^2 \frac{\partial^2 w(x,t)}{\partial x^2}$ is the magnetic force per unit length.[48] For derivation of beam equation for CNTs based on nonlocal elasticity, one can see Ref. 50. In the present work, the surface effects are ignored. In Sec. IV, we derive the equations for DSWNTS under magnetic field.

IV. GOVERNING EQUATIONS OF DSWCNT IN MAGNETIC FIELD

Consider a double single-walled carbon nanotube system as shown in Fig. 1. The DSWNTS is modelled as a DNBS.[45,46] The two single-walled carbon nanotubes are denoted SWCNT-1 and SWCNT-2. The nanotubes are bonded by an elastic medium. Vertically distributed springs model the elastic medium. The stiffness k of the springs is equivalent to the Winkler constant in a Winkler foundation model. The two nanotubes can be different where the length, mass per unit length, and bending rigidity of the ith beam are L_i, m_i, and E_iI_i $(i = 1, 2)$, respectively. In the present study, we consider two similar nanotubes. The transverse displacements of the two nanotubes are denoted by $w_1(x, t)$ and $w_2(x, t)$, respectively.

Using Eq. (16), the individual governing nonlocal equation for vibrating SWCNT-1 and SWCNT-2 is expressed as

SWCNT-1

$$\begin{aligned}
&EI\frac{\partial^4 w_1(x,t)}{\partial x^4} + k[w_1(x,t) - w_2(x,t)] - (e_0a)^2k\\
&\times\left[\frac{\partial^2 w_1(x,t)}{\partial x^2} - \frac{\partial^2 w_2(x,t)}{\partial x^2}\right] - \eta A H_x^2\frac{\partial^2 w_1(x,t)}{\partial x^2}\\
&+ \eta A(e_0a)^2H_x^2\frac{\partial^4 w_1(x,t)}{\partial x^4} + m\frac{\partial^2 w_1(x,t)}{\partial t^2}\\
&- (e_0a)^2m\frac{\partial^4 w_1(x,t)}{\partial x^2\partial t^2} = 0.
\end{aligned} \tag{17}$$

SWCNT-2

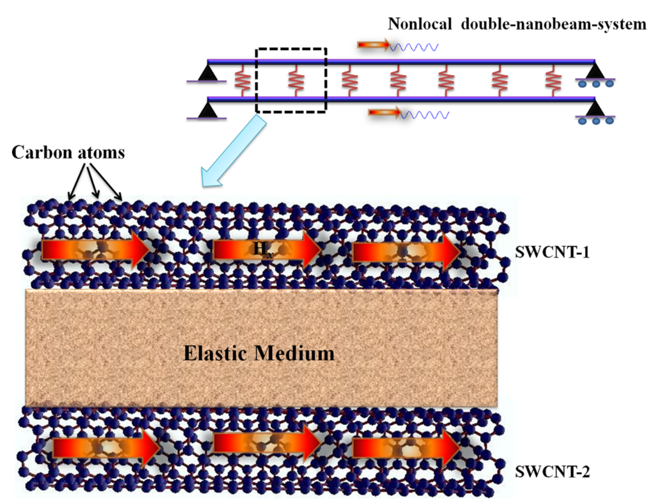

FIG. 1. DSWNTS modelled as nonlocal double nanobeam system in axial magnetic field.

$$EI\frac{\partial^4 w_2(x,t)}{\partial x^4} - k[w_1(x,t) - w_2(x,t)] + (e_0a)^2k \times \left[\frac{\partial^2 w_1(x,t)}{\partial x^2} - \frac{\partial^2 w_2(x,t)}{\partial x^2}\right] - \eta A H_x^2 \frac{\partial^2 w_2(x,t)}{\partial x^2} + \eta A (e_0a)^2 H_x^2 \frac{\partial^4 w_2(x,t)}{\partial x^4} + m\frac{\partial^2 w_2(x,t)}{\partial t^2} - (e_0a)^2 m \frac{\partial^4 w_2(x,t)}{\partial x^2 \partial t^2} = 0. \tag{18}$$

Considering a change of variables, as both nanotubes are similar

$$w(x,t) = w_1(x,t) - w_2(x,t), \tag{19}$$

so that

$$w_1(x,t) = w(x,t) + w_2(x,t). \tag{20}$$

Here, $w(x, t)$ is the relative displacement of the main nanobeam with respect to the auxiliary nanobeam.

Subtracting Eq. (18) from Eq. (17) gives

$$EI\left[\frac{\partial^4 w_1(x,t)}{\partial x^4} - \frac{\partial^4 w_2(x,t)}{\partial x^4}\right] + 2k[w_1(x,t) - w_2(x,t)] - 2(e_0a)^2k\left[\frac{\partial^2 w_1(x,t)}{\partial x^2} - \frac{\partial^2 w_2(x,t)}{\partial x^2}\right] - \eta A H_x^2\left[\frac{\partial^2 w_1(x,t)}{\partial x^2} - \frac{\partial^2 w_2(x,t)}{\partial x^2}\right] + \eta A(e_0a)^2 H_x^2\left[\frac{\partial^4 w_1(x,t)}{\partial x^4} - \frac{\partial^4 w_2(x,t)}{\partial x^4}\right] + m\left[\frac{\partial^2 w_1(x,t)}{\partial t^2} - \frac{\partial^2 w_2(x,t)}{\partial t^2}\right] - m(e_0a)^2\left[\frac{\partial^4 w_1(x,t)}{\partial x^2\partial t^2} - \frac{\partial^4 w_2(x,t)}{\partial x^2\partial t^2}\right] = 0. \tag{21}$$

By introducing Eqs. (19) and (20) and using Eq. (21), we get two equations

$$EI\frac{\partial^4 w(x,t)}{\partial x^4} + 2kw(x,t) - 2(e_0a)^2k\frac{\partial^2 w(x,t)}{\partial x^2} - \eta A H_x^2\frac{\partial^2 w(x,t)}{\partial x^2} + \eta A(e_0a)^2H_x^2\frac{\partial^4 w(x,t)}{\partial x^4} + m\frac{\partial^2 w(x,t)}{\partial t^2} - (e_0a)^2 m\frac{\partial^4 w(x,t)}{\partial x^2\partial t^2} = 0, \tag{22}$$

$$EI\frac{\partial^4 w_2(x,t)}{\partial x^4} - \eta A H_x^2\frac{\partial^2 w_2(x,t)}{\partial x^2} + \eta A(e_0a)^2H_x^2 \times \frac{\partial^4 w_2(x,t)}{\partial x^4} + m\frac{\partial^2 w_2(x,t)}{\partial t^2} - (e_0a)^2 m\frac{\partial^4 w_2(x,t)}{\partial x^2\partial t^2} = kw(x,t) - (e_0a)^2k\frac{\partial^2 w(x,t)}{\partial x^2}. \tag{23}$$

V. ASYNCHRONOUS AND SYNCHRONOUS VIBRATION

Now, we determine the solution of Eqs. (22) and (23). Let the solution of these equations be

$$w(x,t) = W(x)e^{i\omega t}, \tag{24}$$

where ω is the circular natural frequency and $W(x)$ is the mode shape. Term i is the conventional imaginary number $i = \sqrt{-1}$.

Substituting Eq. (24) into Eq. (22) yields

$$B_1\frac{d^4W(x)}{dx^4} + B_2\frac{d^2W(x)}{dx^2} - B_3W(x) = 0, \tag{25}$$

where

$$\begin{aligned} B_1 &= EI + (e_0a)^2\eta A H_x^2; \\ B_2 &= -m\omega^2(e_0a)^2 - \eta A H_x^2 - 2k(e_0a)^2; \\ B_3 &= -m\omega^2 - 2k. \end{aligned} \tag{26}$$

The general solution of Eq. (25) is given as

$$W(x) = C_1\sin\beta x + C_2\cos\beta x + C_3\sinh\gamma x + C_4\cosh\gamma x, \tag{27}$$

where

$$\beta^2 = \frac{1}{2B_1}\left(B_2 + \sqrt{B_2^2 + 4B_1B_3}\right), \tag{28}$$

$$\gamma^2 = \frac{1}{2B_1}\left(B_2 - \sqrt{B_2^2 + 4B_1B_3}\right). \tag{29}$$

The terms C_1, C_2, C_3, and C_4 are the constants of integration determined from the boundary conditions.

For simplicity, we consider the SWCNTs to be simply supported. The local and nonlocal boundary conditions are identical and expressed as[45]

$$W(0) = 0 \text{ and } W''(0) = 0, \quad W(L) = 0 \text{ and } W''(L) = 0, \tag{30}$$

Consider the case of the couple nanotubes when both vibrate in an out-of-phase sequence ($w_1 - w_2 \neq 0$). In out-of-phase vibration sequence, the nanotubes vibrates in opposite directions.

We introduce the following parameters for the sake of simplicity and generality

$$\Omega = \sqrt{\frac{m\omega^2L^4}{EI}}, \quad K = \frac{kL^4}{EI}, \quad \mu = \frac{e_0a}{L}, \quad MP = \frac{\eta A H_x^2 L^2}{EI}, \tag{31}$$

where Ω, K, μ, and MP are the frequency parameter, spring constant, nonlocal parameter, and magnetic parameter, respectively. Using Eq. (31), the expression for the natural frequency of the DSWNTS is evaluated as

$$\Omega = \sqrt{\frac{(n\pi)^4 + 2K + 2K(\mu)^2(n\pi)^2 + MP(n\pi)^2 + MP(\mu)^2(n\pi)^4}{1 + (\mu)^2(n\pi)^2}}, \quad n = 1, 2.... \tag{32}$$

Note that when the nonlocal parameter μ and magnetic field parameter M are set to zero, we get the expression of a classical double-beam-system

$$\Omega = \sqrt{(n\pi)^4 + 2K}, \quad n = 1, 2.... \tag{33}$$

Next, the in-phase sequence of vibration will be considered. In in-phase vibration sequence the nanotubes vibrate in same directions. For the present coupled nanotubes, the relative displacements between the two nanobeams are now absent ($w_1 - w_2 = 0$). Here, we solve Eq. (23) for the vibration of DSWNTS. The vibration of SWCNT-2 represents the vibration of the coupled vibrating system. We apply the same procedure for solving Eq. (23).

Using Eq. (31), we can obtain the natural frequencies. The natural frequencies for the DSWNTS in this case can be expressed as

$$\Omega = \sqrt{\frac{(n\pi)^4 + MP(n\pi)^2 + MP(\mu)^2(n\pi)^4}{1 + (\mu)^2(n\pi)^2}}, \quad n = 1, 2.... \tag{34}$$

Note that in the in-phase mode of vibration, the DSWNTS is independent of spring stiffness. When the magnetic field and nonlocal effects are neglected, we get the classical frequencies $\Omega = n^2\pi^2$.

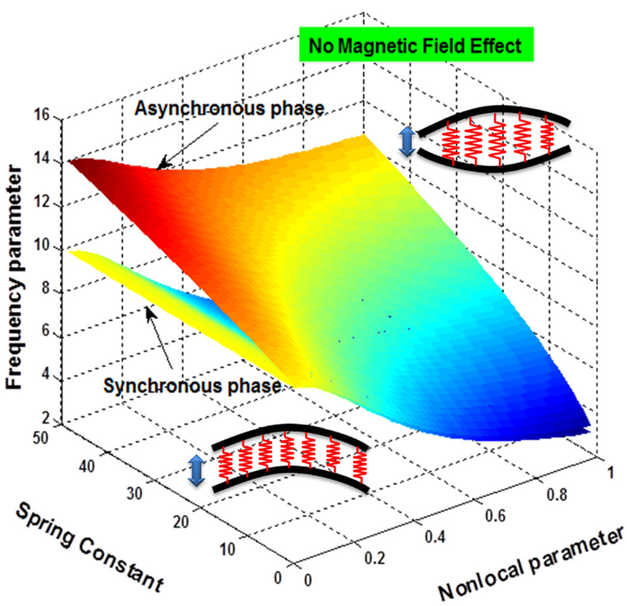

FIG. 2. Effect of coupling spring constant and nonlocal parameter on the natural frequencies of DSWNTS for first synchronous and asynchronous vibration in the absence of magnetic field.

VI. RESULTS AND DISCUSSION

The effect of increasing strength of magnetic field on the vibration characteristics of a DSWNTS is examined via nonlocal elastic model. A strong magnetic field is mathematically assumed in the analysis. The following properties are considered: radius $r = 3.5$ nm; Young's modulus, $E = 1$ TPa; $\rho = 2300\,\text{kg/m}^3$, wall thickness, $h = 0.34$ nm. The SWCNTs have equal length of 10 nm. The nonlocal theory illustrated here is a generalised theory and can be applied for the bending-vibration analysis of coupled multi-walled carbon nanotubes, double ZnO nanobeam systems, and double nanobeam systems for NOMS applications. Nonlocal effects in the axial direction are only considered. The frequency results, scale effects, and spring stiffness of the DSWNTS are presented in terms of the frequency parameter, nonlocal parameter, and spring stiffness parameter (spring constant).

In order to see the effect of nonlocal parameter (scale effects), coupling spring effects and magnetic field on the vibration response of the DSWNTS, three dimensional curves have been plotted. First, we study the vibration response of the DSWNTS in the absence of a magnetic field. Fig. 2 shows the effect of nonlocal parameter and the bonding spring stiffness on the frequency of the DSWNTS without a magnetic field effect. The nonlocal parameter is considered in the range of 0-1, while the spring constant is taken as $K = 0$–50. Both the synchronous and asynchronous vibration modes are considered here.

From the figure (Fig. 2), it is seen that increasing the nonlocal parameter decreases the frequency parameter. This is true for both the synchronous and asynchronous vibration modes (first modes). Natural frequencies for the asynchronous vibration phase are higher than the synchronous vibration phase. This is because only the asynchronous vibration of the DSWNTS is influenced by the coupling spring constant.

Fig. 3 shows the vibration response of the DSWNTS in the presence of a magnetic field. A very strong magnetic field is assumed, with a magnetic field parameter $MP = 25$ considered for the study. Comparing the vibration responses with and without magnetic field (Figs. 2 and 3, respectively), it is seen that the presence of a longitudinal magnetic field yields higher frequencies (both synchronous and asynchronous modes) for the DSWNTS. It can also be noted that the differences between the frequencies of the synchronous and asynchronous modes reduces in the presence of a magnetic field at lower nonlocal parameter.

Next, we study the effect of increasing strength of magnetic field on vibration characteristics of the DSWNTS. Figs. 4 and 5 show the effect of increasing magnetic field strength for $n = 1$ and $n = 2$, respectively. For the study we consider, the magnetic parameter varied from 0 to 50. The spring constant is considered to be $K = 50$. It can be seen that

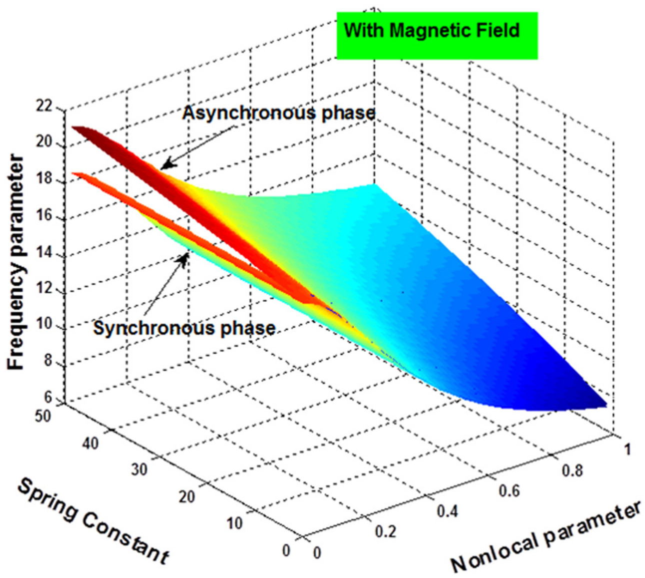

FIG. 3. Effect of coupling spring constant and nonlocal parameter on the natural frequencies of DSWNTS for first synchronous and asynchronous vibration in the presence of magnetic field.

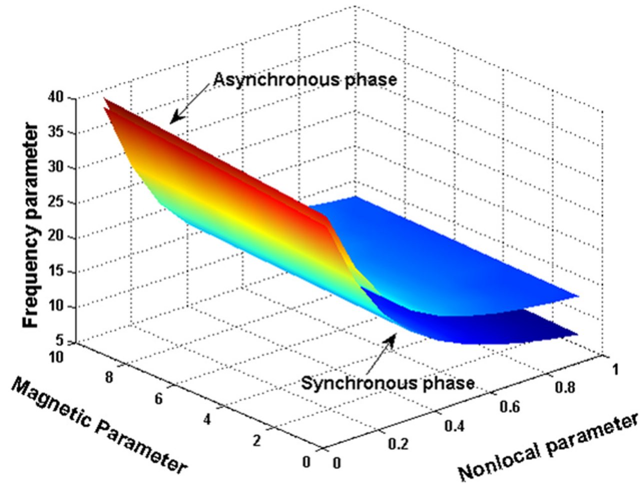

FIG. 5. Effect of magnetic parameter and nonlocal parameter on the natural frequencies of DSWNTS for second synchronous and asynchronous vibration.

increasing magnetic field strength yields increases in the frequencies of the DSWNTS. This trend is true for both synchronous and asynchronous modes. It should be also noted that the synchronous modes are more affected by the nonlocal parameter than asynchronous modes. Like the effect of spring constants, the magnetic field dampens the scale effects in asynchronous modes. For higher modes, $n = 2$, the vibration response is more affected by the nonlocal parameter. However, the differences between the frequencies between the synchronous and asynchronous modes become negligible under a magnetic field.

From the study, it is found that a longitudinal magnetic field increases the frequencies of the DSWNTS. This is explained as follows: the magnetic field can be considered as a shear layer parameter in a Pasternak foundation model.

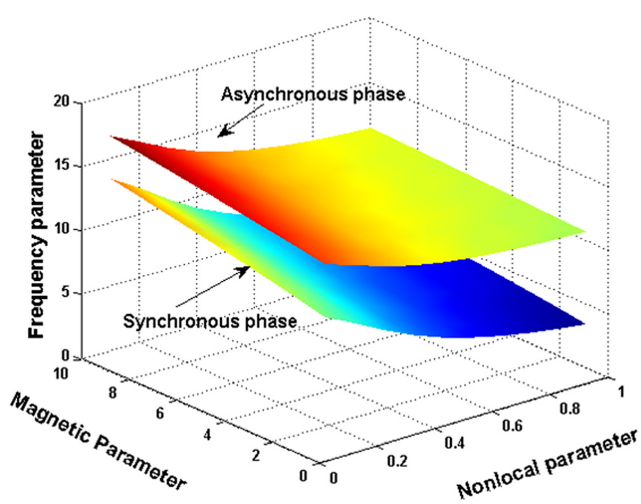

FIG. 4. Effect of magnetic parameter and nonlocal parameter on the natural frequencies of DSWNTS for first synchronous and asynchronous vibration modes.

The coupling elastic medium is thus analogous to a Pasternak foundation, where the Winkler modulus is the stiffness of the springs and the magnetic parameter is the Pasternak shear modulus (see Fig. 6). The Pasternak model can be seen as a membrane having a surface tension laid on a system of elastic springs. The Pasternak shear modulus is able to transfer load along the axial direction and hence increase the frequencies.

The desired fabrication of double carbon nanotube system subjected to magnetic field would be based on the data of natural frequencies and other factors. Eqs. (32) and (34) would provide an approximate expression to find the natural frequencies of the double carbon nanotube system subjected to magnetic field and with consideration of small-scale effects. The natural frequencies of the present system are a function of stiffness of elastic medium, nonlocal (small-scale) parameter, and magnetic parameter.

Experimental or molecular dynamics results for the present study of vibration of a DSWNTS are yet unavailable in literature, representing a scope for future study. For a macroscopic beam plate under the effect of an in-plane magnetic field, it is reported that the natural frequency increases as the strength of the in-plane (non-transverse) magnetic field increases.[51–54] In the present study, the longitudinal magnetic field increases the natural frequency. This may be attributed to the coupling effect of vibrating nanotubes and the longitudinal magnetic field.

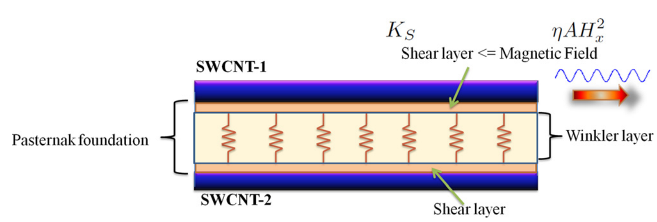

FIG. 6. DSWNTS under longitudinal magnetic field is shown analogous to double-nanobeam-systems coupled by equivalent Pasternak model.

VII. CONCLUSION

This paper presents an analytical model for studying the effects of a longitudinal magnetic field on the vibration of a magnetically sensitive DSWNTS. The dynamic equations of the DSWNTS are derived utilising nonlocal elasticity theory. The two nanotubes are defined as an equivalent nonlocal double Euler-Bernoulli beam system coupled by virtual springs. Governing equations for nonlocal bending-vibration of the DSWNTS under a longitudinal magnetic field are developed considering the Lorentz magnetic force obtained from Maxwell's relation. Analytical solutions are presented for the natural frequencies of the DSWNTS under a magnetic field. Results reveal that the nonlocal effects in the vibration response of the DSWNTS are dampened by the presence of a longitudinal magnetic field. The presence of a longitudinal magnetic field increases the natural frequencies of the DSWNTS. This is true for both synchronous and asynchronous modes of vibration. However, the differences between the frequencies of synchronous and asynchronous modes reduce in the presence of a magnetic field. The present study will hopefully be useful for theoretical study and design of nanotubes in a magnetic field.

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