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LETTER TO THE EDITOR

Authors' reply to "Comments on 'Eigenderivative analysis of asymmetric non-conservative systems'"

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Jankovic [1] raises three issues relating to our paper on eigensystem derivatives. His two major charges are that the expressions for the first and second eigenvalue derivatives may be derived from his earlier paper [2]. The final charge is that expressing the eigenvector derivative as a linear combination of eigenvectors is not possible. We will now answer these charges in turn.

While it is true that it is possible to derive the expression for the first eigenvalue derivative given in our paper [3] from Jankovic's paper [2], Jankovic's derivation is for a general non-linear eigenvalue problem. The thrust of our paper was for a general linear second-order vibrating system, including viscous damping, and most of the development concerned the eigenvector derivatives. Indeed the calculation of eigenvalue derivatives has never been particularly difficult, and there is also some doubt about Jankovic's claim to be the first to derive the expression for the first eigenvalue derivative for a general system. Andrew *et al.* [4] not only gave the same expression as Jankovic, but preceded its introduction with the comment that it is a 'well-known formula'. The expression was given by Haftka and Adelman [5] and also by Andrew and Chu [6] in a comment on an earlier paper by Jankovic [7]. Our paper was never intended to be a review paper, and the literature on eigensystem derivatives covers such a vast range and depth that every paper cannot be included.

Most of the above comments also apply to the calculation of the second-order eigenvalue derivatives. We would also highlight that Jankovic [2] only considers the derivative with respect to a single parameter. For many applications the system matrices are functions of more than one parameter and thus the second-order derivative with respect to different parameters is required.

The last issue is whether one can write the eigenvector derivatives as a linear combination of the eigenvectors. Jankovic [1] seems to have misunderstood our development in that he thinks the derivatives for the non-linear eigenproblem were being considered. However, the second-order system had been rewritten as a first-order linear eigenproblem, and thus the

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requirement that the eigenvectors are independent and span the full $2n$-dimensional space is met. Proof of this is elementary and contained in many textbooks.

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