Analogue Electronics I (Aero)

§1 Basic Methods and Laws

Solution Methodology:

Electrical Circuit ⇒ Mathematical Model ⇒ written down and equations solved ⇒ Voltages and Currents in the circuit extracted (in steady state or time domain).

To get physical insight into the behaviour of electrical circuits we usually compare the flow of charge which constitutes an electric current to the flow of water in a pipe. The voltage applied is analogous to the water pressure while the resistance of the circuit is like the frictional losses which occur in the pipe.

Current: rate of flow of charge. 1 Coulomb per second (Cs-1) = 1 Ampere (A) ≈ 6.25×1018 electrons per second

Voltage: difference in potential energy between two points which drives the

movement of charge 1 Volt (V) = 1 Joule per Coulomb.
Resistance: the magnitude of the current flowing in a circuit in response to a potential difference is determined by its resistance.

Two types of analysis:

1. Direct current (d.c.) – no variation of voltage (V) and current (I) with time
2. Alternating Current (a.c.) – voltage (v) and current (i) are functions of time, and usually of the form sin ωt or cos ωt.

Techniques used in d.c. analysis are:

1. Combination of resistors in series and parallel
2. Potential and current dividers
3. Nodal Analysis
4. Thevenin and Norton's Theorems
5. Superposition principles
6. In a.c. analysis we can use the same techniques except that we use complex numbers to represent v, i and 'R'.

The underlying rules which form the basis of circuit analysis are:

A. **Ohm's Law** (Georg Ohm 1789-1854)
Voltage across a resistor (Volts, V) = Current through resistor (Amps, A) × Resistance (Ohms, Ω)

$V = IR$ or $I = V/R$ or $R = V/I$

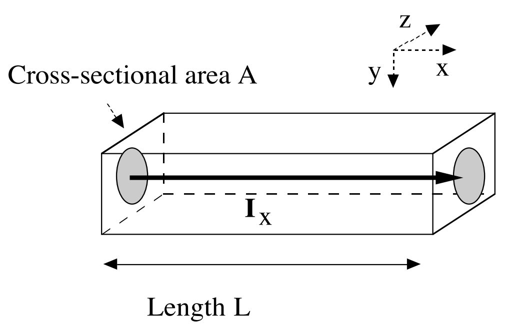
Why does this hold true ?

Here is a preview of how bad it gets (probably not examinable)

For d.c. $V = IR$ or $I = GV$

(R is resistance, $G = 1/R$ is conductance).

For internal device design need to work with (vector) electric field and with (vector) current density $J = I/A$ (or more correctly $J = dI/dA$) where $(d)A$ is the cross sectional area.



Use intensive quantities (valid at a point) such as resistivity ρ in Ωm.

In uniform conductor resistance

$R = \rho L/A$,

The electric field E is related to the Voltage V by $V = EL$. In general:

$$V_{ab} = \int_a^b EdL$$

so $V = EL = IR = I\rho L/A$ means

$E = \rho J$

"Internal" form of Ohms law.

In general E and J are vectors and the resistivity ρ is a matrix! Yeuch.

Worse: for a.c. work all of these may be complex! Yeuch^2

B. **Kirchhoff's Laws**
(Gustav Robert Kirchhoff 1824-1887)

a. Voltage Law

– round any closed loop in a circuit the rises in electrical potential (voltage) are balanced by the falls.

$\sum V_{rise} = \sum V_{fall}$

or $\sum V = 0$ where we must have a sign convention to distinguish rises from falls in potential

b. Current Law

– at any junction or node in a circuit the incoming currents balance the outgoing currents.

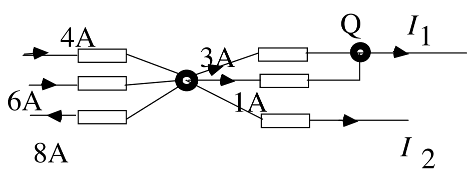
$\sum I_{in} = \sum I_{out}$

or $\sum I = 0$ where again we must have a sign convention to distinguish incoming and outgoing currents.

Kirchhoff's current law: Example

The sum of the currents into a node is zero.
$\Sigma I_{node} = \Sigma I_{in} + \Sigma I_{out} = 0$

Basis is conservation of current

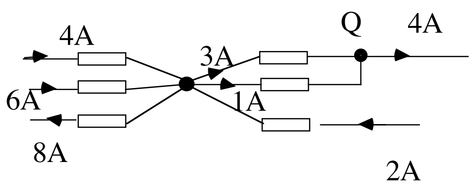


At Q, $I_1 = 3 + 1 = +4$ A

At main node , $4+6 = 8 + 4 + I_2$
$10 = 12 + I_2$
So $I_2 = -2$ A

The initial direction we chose for the current flow was wrong .

Complete solution from Kirchhoff's current law is:



Kirchhoff's voltage law: Example

Basis is conservation of energy

Since rises and falls in potential have opposite signs (yet) another way of looking at this is that the increase in potential produced by sources is balanced by the decrease in potential of components which consume energy.

or

The increase in potential produced by sources (more correctly sources transform mechanical energy (generators), or chemical energy(batteries) to electrical potential energy (eV)), is balanced by the decrease in potential of components which consume energy (more correctly, dissipate(e.g. resistor)) or (even better) transform to another form of energy (e.g. motor)).

We may write:

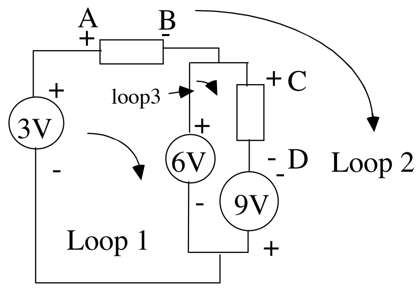
$\sum V_i = 0$ as

$\sum V_{rise} - \sum V_{fall} = 0$ or

$\sum V_{rise} = \sum V_{fall}$

Watch your signs !

Ok now for that example



Let the Unknown voltages be V_{AB} and V_{CD} and assign + and - signs arbitrarily.

Go round loops clockwise

Loop 1

$3 = V_{AB} + 6$ so $V_{AB} = -3V$

(i.e 3V with polarity reversed from initial choice)

Loop 2

$3 + 9 = V_{AB} + V_{CD}$

$3 + 9 = -3 + V_{CD}$ so

$V_{CD} = +15V$

Loop 3

$+6 + 9 = V_{CD}$ so $V_{CD} = +15$ V

Notation of reference directions :

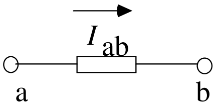
In analysing circuits, may not know, initially, the direction of currents and voltages. Need a notation to keep track of this. There are many in use. Usually the meaning is obvious from the context. Often, more than one will be used

at the same time (such as giving a current arrow from a to b and denoting the current by iab). The important thing is not which one you use, but to keep them internally consistent. Especially important is the sign!

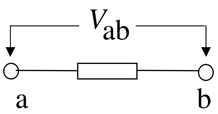
In the above example, the potential differences across the voltage sources were given, and only the sign of the potential differences across the resistors needed to be guessed. Usually, when dealing with both voltage and current, we start by assigning current variables and arbitrarily selecting a reference direction for each one. If its value turns out to be negative than the actual current is in the opposite direction to the reference direction. The sign of potential differences tend to be chosen based on the direction of the current guess.

Notation for currents and voltages :

The arrow denotes the direction of conventional current flow (from +ve to -ve).



Obviously $i_{ab} = - i_{ba}$



Positive if a is at higher potential than b.

Examples

V_a	V_b	V_{ab}	V_{ba}
+5V	+3V	+2V	-2V
-6V	-9V	+3V	-3V
+1V	-1V	+2V	-2V
+6V	+8V	-2V	+2V

Note : $v_{ab} = v_a - v_b$ also $v_{ab} = - v_{ba}$

Power dissipated in a resistor :

The power P dissipated by a current I in a resistor with voltage V across it is:
$P = IV$

We can also express this, by using Ohm's law, in terms of either the voltage and the Resistance,

$$P = IV = \frac{V}{R}V = \frac{V^2}{R}$$

or the current and the resistance.
$P = IV = IIR = I^2R$

To see why the power is given by VI we can deconstruct it. V is the energy per charge and I is the charge per second. multiplying them we have.

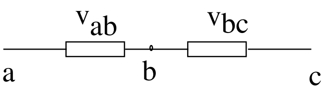
$$VI = \frac{\text{Energy}}{\text{Charge}} \times \frac{\text{Charge}}{\text{Time}} = \frac{\text{Energy}}{\text{Time}} = \text{Power}$$

The energy delivered per unit time is the power.

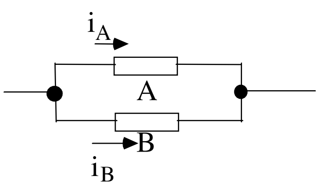
§2 Parallel & Series Circuits

Lets look at the classic first example of Ohm's law and Kirchoff's laws - Series and parallel resistors :

Circuit elements are said to be in series if they are connected end to end.

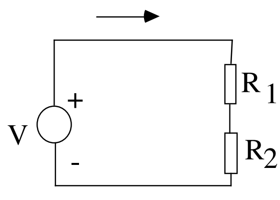


In such circuits the current in both elements is the same, but the voltage across them is different.
Elements are said to be in parallel if both ends of the elements are connected directly to corresponding ends of the other.



Here the voltage across both elements is the same but the current splits between them.

Resistors in Series:
same current flows in the two resistors, so different voltages.



Voltage across $R_1 = IR_1$
Voltage across $R_2 = IR_2$

So $V = IR_1 + IR_2 = I(R_1+R_2)$
if the combined resistance of the two resistors is R_T then

$$V = IR_T = I(R_1+R_2)$$

So

$$R_T = R_1+R_2$$

Also for n resistors in series , just add all the resistances.

$R_T = R_1 + R_2 + \ldots\ldots + R_n$

Notice also that (voltage divider) :

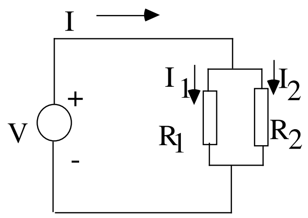
$$\frac{V_2}{V} = \frac{IR_2}{I(R_1+R_2)} = \frac{R_2}{R_1+R_2}$$

$$\frac{V_1}{V} = \frac{IR_1}{I(R_1+R_2)} = \frac{R_1}{R_1+R_2}$$

$$\frac{V_1}{V_2} = \frac{R_1}{R_2}$$

Resistors in Parallel:

Now have same voltage across both resistors, but a different current.
Let's analyse this:



Current in R_1 is $\frac{V}{R_1} = I_1$

Current in R_2 is $\frac{V}{R_2} = I_2$

Since the total current must flow through one of the two branches

Total current is $I = \frac{V}{R_1} + \frac{V}{R_2}$

but total current is also voltage divided by total resistance so

$$I = \frac{V}{R_T} = \frac{V}{R_1} + \frac{V}{R_2}$$

dividing throughout by the voltage we have:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

Which is in alternate form

$$R_T = \frac{R_1R_2}{R_1+R_2}$$

or for n resistors in parallel

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots \quad \ldots + \frac{1}{R_n}$$

Notice also that (current divider)

$$\frac{I_2}{I} = \frac{V}{R_2}\Big/\frac{V}{R_T} = \frac{1}{R_2}R_T = \frac{1}{R_2}\frac{R_1R_2}{R_1+R_2}$$

$$\frac{I_2}{I} = \frac{R_1}{R_1+R_2}$$

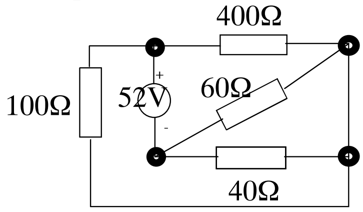
and similarly for I_1

Circuit Geometry :

Although we can often decompose a circuit into series and parallel elements, it is sometimes difficult to recognise them.
If all the current from one component (or network) goes into one and only one other component(or network), then they are in series. All the current from one thing goes into another thing-series.
If **both** ends of a particular component (or network) are connected to **both** ends of another component (or network) - with nothing in between - then they are in parallel. All the voltage across one thing is also across the other thing-parallel.
Eventually you will get used to this and see what is in series and what is in parallel. In the meantime one technique which can help is to redraw the circuits in standard form.

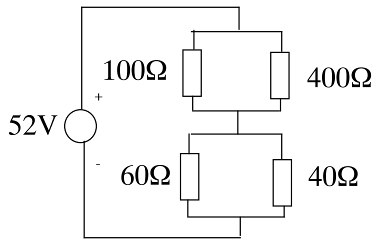
Start with the highest voltage at the top and the lowest at the bottom. At each node (junction point of components) draw a horizontal line representing a set of points at the same potential (voltage), connected to a branch. This method represent the cascade of voltage (electrochemical potential) as a cascade of height (gravitational potential), in a tree like structure. This picture appeals to the human animal, for some reason!

Example :



Chaotic, messy, harder to understand.
(but have a go ... which ones are in parallel?)

Is redrawn as:

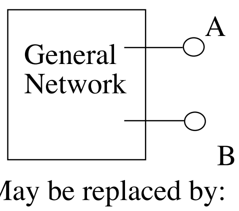


Restful, symmetric, easy to understand.
Obvious which ones are in parallel!

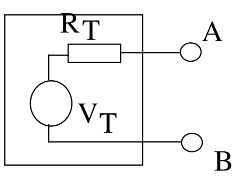
§3 Thévenin & Norton – Realistic Sources

Thévenin's Theorem

States that, as far as any external load is concerned, any two port network may be replaced by a voltage source V_T in series with a resistance R_T.
That is



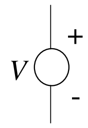
May be replaced by:



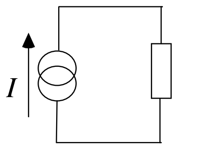
Where the magnitude of the voltage source V_T is the voltage which would appear across A and B on open circuit (the Thévenin voltage) and R_T (the Thévenin resistance) is the resistance which would be obtained if all the sources were made inoperative (that is if all voltage sources were replaced by short circuits and all current sources were replaced by open circuits).

This is of practical use, as we have implicitly assumed that energy is supplied to a circuit by either

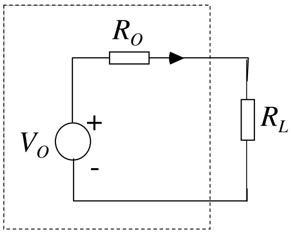
a) an ideal voltage source – creates a constant voltage or potential difference between its terminals irrespective of the current flowing through it.



b) an ideal current source – drives constant current irrespective of the voltage across its terminals.



In reality, practical generators cannot sustain voltages and currents regardless of the load because practical generators have internal resistance. For most purposes a real generator can be modelled as an ideal generator and a resistor.



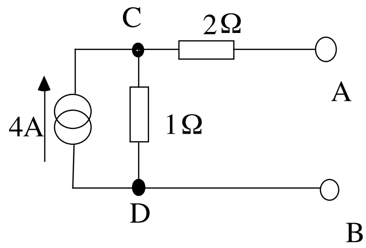
$V_O = IR_O + IR_L$ (Kirchhoff's Voltage Law)

$V_L = IR_L = V_O - IR_O$

The voltage which can be measured at the terminals is the ideal generator voltage minus the voltage dropped across the internal resistance due to the current taken from the generator.

(P.S. what's a loadline. P.P.S. generators in series and parallel)

Example:



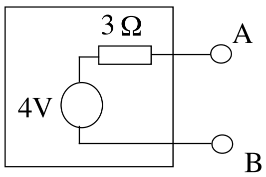
On open circuit no current can flow through the 2 Ohm resistor.

$V_{CD} = 4 \times 1 = IR = 4$ V

Because no current flows through the 2 Ohm resistor $V_A = V_C$ so on open circuit $V_{AB} = 4$ V $= V_T$

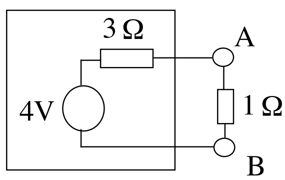
The resistance across AB when the current source is inoperative (i.e. open circuit for a current source) will be:
$1\Omega + 2\Omega + 3\Omega = R_T$

This means that the Thévenin equivalent circuit is:



Now confirm that the original and equivalent circuits give the same behaviour if connected to an external load.

Suppose the load between A and B is 1Ω.
In the Thévenin equivalent circuit

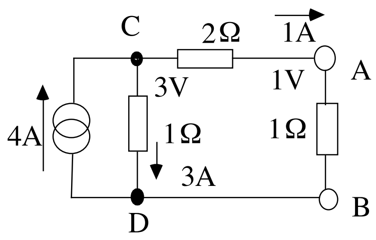


$$I = \frac{V}{R} = \frac{4}{3+1} = 1A$$

$V_{AB} = IR = 1 \times 1 = 1\ V$
or alternatively we can use the potential divider rule:

$$V_{AB} = \frac{1}{3+1} \times V = 1V$$

Original circuit:



In the parallel resistor network we have 1Ω in parallel with (2+1)Ω

$$\frac{1}{R_{tot}} = \frac{1}{1} + \frac{1}{3} = \frac{3+1}{3} = \frac{4}{3}\ \Omega^{-1}$$

So $R_{tot} = 3/4\ \Omega$

$V_{CD} = IR = 4 \times 3/4 = 3\ V$

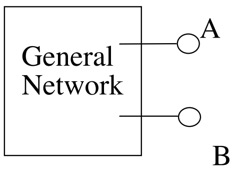
Then we could use the potential divider rule to get $V_{AB} = 1V$ and $V_{CA} = 2V$.

Otherwise we could use the fact that the current divides between the two branches in the ratio 3:1 and get the voltages that way.

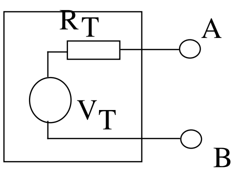
Norton's Theorem :

Any network with 2 accessible terminals, may, so far as external loads are concerned replaced by an ideal current source in parallel with a resistor. Although this is usually called Norton's theorem, it is not fundamentally different from Thévenin's Theorem.
So, as far as any external load is concerned, any two port network may be replaced by a Norton current source, …. I_N
in parallel with a Norton resistance R_N.

That is

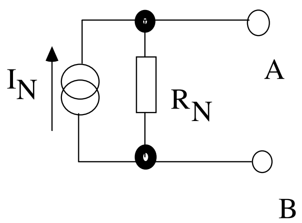


May be replaced by:



Thevenin equivalent circuit

Or alternatively by:



Norton equivalent circuit.

The open circuit voltage of the current source is $I_N R_N$ so

$V_T = I_N R_N$

The resistance, when the ideal current source is replaced by an open circuit is R_N.

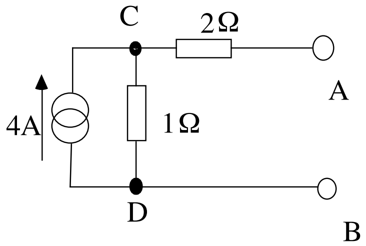
$R_N = R_T$

The procedure for replacing one of these circuits by the other is called the Thévenin-Norton transformation. It is often extremely helpful in solving circuit problems.

When we want to study what happens when one resistance in a large circuit is varied, it often simplifies the analysis if we replace the remainder of the circuit by it's Thévenin or Norton equivalent circuit.

This is why we are learning this!

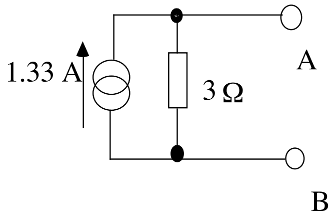
Previous example revisited:



Here, for A B open circuit we have 4V across the 1Ω resistor hence an open circuit output voltage of 4V, which is the Thévenin voltage. For the Thévenin or Norton resistance replace the current source by an open circuit (remember an ideal current source has infinite and an ideal voltage source has zero resistance).

This gives 1Ω in series with 2Ω = 3Ω which is the Norton (and Thevenin) resistance.

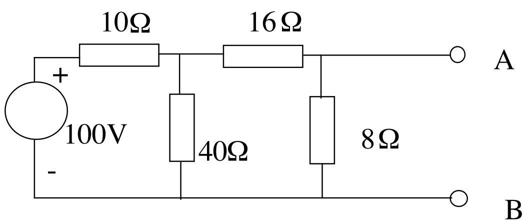
The Norton Current is that current which gives 4V over 3Ω which is 1.33A.



Norton equivalent circuit.

Another example:

Find the Thevenin and Norton equivalent of the following circuit.



If A and B are open circuit then the 16 and 8 ohm resistors are in series and total 24 Ω.

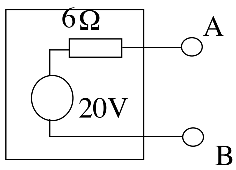
These are in parallel with the 40 Ω resistor the parallel combination resistance being

$$\frac{24 \times 40}{24 + 40} = 15\Omega$$

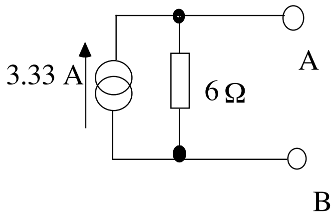
This 15Ω and the 10 Ω resistor form a potential divider with 3/5 of the voltage appearing across the 15Ω network(60V), of which 1/3 appears across the 8Ω resistor(20V). This is what appears at the output. Hence the Thévenin voltage is 20V.

To get the Thevenin resistance replace the voltage source with a short. Looking in to terminals A and B we can analyse the circuit as a resistance of 10 Ω in parallel with 40 Ω which is 8Ω. This is in series with 16Ω = 24Ω. This network is in parallel with 8Ω giving a resultant resistance of 6Ω. So the Thévenin resistance is 6Ω. Note that we must analyse the circuit twice in two different ways to get both the Thévenin voltage and the Thévenin resistance.

The Norton resistance is also 6Ω. The Norton Current is that current required to give 20V over 6Ω which is 3.33A.



Thevenin equivalent circuit.

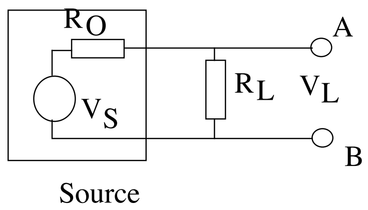


Norton equivalent circuit.

While, as far as any external circuit is concerned, the General network may be replaced by either it's Norton or Thevenin equivalent, the Norton and Thevenin equivalents are NOT the same internally.

If I gave you two identical boxes, one with a Norton quivalent circuit and one with a Thevenin equivalent circuit how could you tell them apart? Clue: no electrical measurement could tell the difference.

Maximum power theorem :



If we are are given a source with fixed voltage and internal resistance R_O, how do we maximise the power dissipated in the load?

$$V_L = \frac{R_L}{R_O + R_L} V_S$$

Power in load is

$$P = V_L I = V_L \frac{V_L}{R_L} = \frac{V_L^2}{R_L}$$

$$P = \left(\frac{R_L}{R_O + R_L} V_S \right)^2 \frac{1}{R_L} = \frac{R_L}{\left(R_O + R_L\right)^2} V_S^2$$

How do we calculate the maximum power?

We need to know how the power changes with R_L. This looks like a job for Calculus!

The rate of change of power with respect to R_L is

$$\frac{dP}{dR_L} = V_S^2 \frac{d}{dR_L} \left(\frac{R_L}{\left(R_O + R_L\right)^2} \right)$$

Now, for a very low resistance not much power will be dissipated in the load. For zero resistance there will be zero power. Also for a very high resistance there will be little power dissipated, for an infinite resistance this would be zero since the current would be zero.

So increasing the load resistance from zero the power will go up and up, reach a maximum and then go down and down. Mathematically then, if we can find the point where it just stops going up, and has not quite started going down, this will be the point of maximum power. That is the maximum power occurs when the rate of change of power is zero.

$$\frac{dP}{dR_L} = V_S^2 \frac{d}{dR_L} \left(\frac{R_L}{\left(R_O + R_L\right)^2} \right)$$

$$\frac{dP}{dR_L} = V_S^2 \frac{d}{dR_L} R_L \left(R_O + R_L\right)^{-2}$$

$$= V_S^2 \left(1 \times \left(R_O + R_L\right)^{-2} + R_L \times (-2) \times \left(R_O + R_L\right)^{-3} \times 1 \right)$$

$$= V_S^2 \left(\frac{1}{\left(R_O + R_L\right)^2} - \frac{2R_L}{\left(R_O + R_L\right)^3} \right)$$

$$= V_S^2 \left(\frac{R_O + R_L - 2R_L}{\left(R_O + R_L\right)^3} \right)$$

This equals zero provided that $R_O + R_L - 2R_L = 0$, *i.e.* $R_O = R_L$

So maximum power is delivered to the load if $R_O = R_L$

Under these conditions R_L is said to be matched to the output resistance (better: output impedance) of the source.Often will wish to design circuits and systems to achieve this.

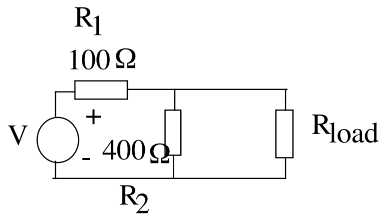
Superposition principle

The response in a linear circuit to independent sources is the sum of the response to each source alone, with the other sources zeroed (infinite Ω for current and 0Ω for voltage sources.)

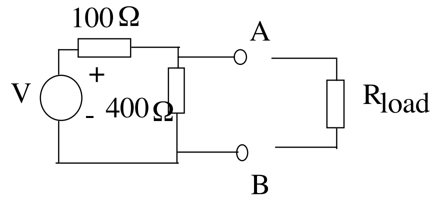
Maximum power example

Lets try an example of using a Thévenein equivalent circuit.

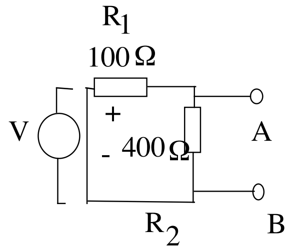
Find when the maximum power is dissipated in the load:



Split the circuit into the components under study and everything else. Find the Thévenin (or Norton, if convenient) equivalent of everything else.



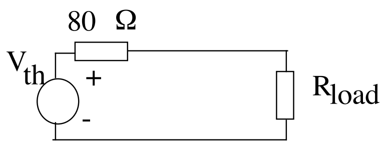
To determine the Thévenin resistance, short-circuit the source giving R1 and R2 in parallel.



Resistance between A and B is the Thévenin resistance

$$R_{th} = \frac{R_1 \times R_2}{R_1 + R_2} = 80\Omega$$

So our original circuit is equivalent to



So maximum power dissipation will occur at 80Ω.

Note that we do not need to know the voltage of the source to work this out.

The Thévenin voltage is the open circuit output voltage, which in this case will be

$$V_{th} = \frac{400}{100+400} V_s = \frac{4}{5} V_S$$

So will, for example, be 8V if V_S is 10V.

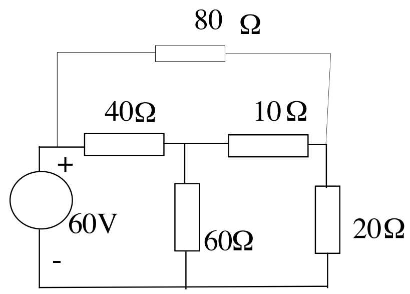
In this case the Maximum power will be

$$\frac{V_L^2}{R_L} = \left(\frac{80}{80+80} V_{th}\right)^2 \times \frac{1}{R_L} = 0.2\,W$$

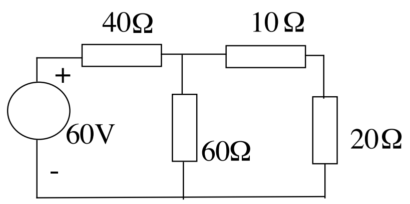
Circuits which cannot be decomposed into series and parallel elements :

Many circuits have elements which are neither in series or in parallel.

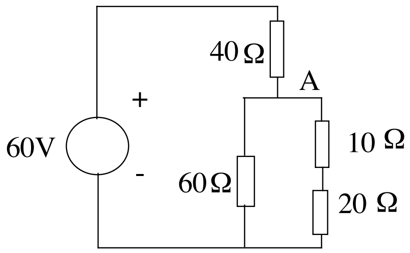
Lets look at an example:



First look at the circuit without the 80Ω resistor.



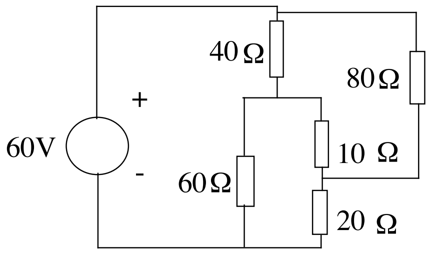
Redraw this in standard form as:



60||30 = (60x30)/(60+30) = 20 Ω.

$V_A = \left[\frac{20}{40+20}\right] \times 60 = 20V$

Now add 80Ω resistor:



We cannot decompose circuit into series and parallel components.

The most general method for solving this sort of circuit is to use Nodal Analysis. This method sets up (current) equations for the different nodes, using Kirchhoff's current law. We will return to this method later. In the meantime we will examine Thévenins theorem and apply it to a useful electrical circuit.