Characterisation and Modelling of a Melt-Extruded LDPE Closed Cell Foam

Qusai Hatem Jebur\textsuperscript{1,a}, Philip Harrison\textsuperscript{1,b}, Zaoyang Guo\textsuperscript{2,c}, Gerlind Schubert\textsuperscript{1,d} & Vincent Navez\textsuperscript{3,e}

\textsuperscript{1}School of Engineering, University of Glasgow, Glasgow, G12 8QQ, Scotland
\textsuperscript{2}School of Civil Engineering and Geosciences, Newcastle University, Newcastle upon Tyne NE1 7RU, UK
\textsuperscript{3}NMC, Gert-Noël-Straße, B-4731, Eynatten, Belgium

\textsuperscript{a}q.jebur.1@research.gla.ac.uk, \textsuperscript{b}philip.harrison@glasgow.ac.uk, \textsuperscript{c}zaoyang.guo@newcastle.ac.uk, \textsuperscript{d}g.schubert.1@research.gla.ac.uk, \textsuperscript{e}vincent.navez@nmc.be

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Abstract

This paper describes uniaxial compression tests on a melt-extruded closed-cell Low-Density Polyethylene (LDPE) foam. The stress-strain response shows the mechanical behaviour of the foam is predominantly transversely isotropic viscoelastic and compressible. Images analysis is used to estimate the Poisson’s ratio under large strains. When the deformation is less than 5 percent, the kinematics and mechanical response of the polymer foam can be well-described by a linear compressible transversely isotropic elastic model. For large strain, a method of manipulating experimental data obtained from testing in the principal and transverse directions (stress vs strain and Poisson’s ratio) in order to estimate the uniaxial compression response of the foam at any arbitrary orientation is proposed. An isotropic compressible hyperfoam model is then used to implement this behaviour in a finite element code.

Introduction

Polymeric foams are widely used in various areas such as packaging, impact energy absorption and cushioning, for example in seating and training shoes [1, 2]. Closed-cell Low Density Polyethylene (LDPE) foams have found widespread application in impact energy absorption and packaging applications [1]. While in general foam materials are not isotropic, for most design purposes they are treated as such. The validity of this assumption will depend on the degree of anisotropy of the foam and in some cases may be questionable. Most polymeric foams tend to display at least some degree of anisotropy that can usually be related back to their manufacture [2]. An important manufacture route for LDPE closed cell foams is the melt extrusion process. Here, the extrusion and foaming process can have a significant effect on the microstructure and consequently on the mechanical response of the foam. This is because the flow field through the extruder and the subsequent gas expansion during the foaming process can produce elongated cells along the direction of extrusion and potentially molecular orientation within the skeletal polymeric foam structure itself. The degree of anisotropy can be manipulated by controlling the process conditions.

In this investigation a LDPE closed-cell foam (density = 40kgm\textsuperscript{-3}), manufactured using a melt extrusion process, has been characterised and found to exhibit three important properties including: (i) a strongly transversely isotropic viscoelastic response (ii) compressibility and (iii) a variable non-zero Poisson’s ratio which changes with strain. Constitutive models that can correctly describe the response of such materials would be of great help in design optimisation of applications based on this important class of material. To the best of the authors’ knowledge, such a model has yet to be published though steps in this direction have been taken: For the case of small strain, the linear elastic transversely isotropic compressible model [3] can provide accurate results as demonstrated by Wei et al. [4]. For large strain conditions which must necessarily include a non-linear material response, alternative approaches have been devised: Isotropic elastic compressible foams can be
modelled using Ogden’s hyperelastic model for isotropic compressible rubber [5-7] and transversely isotropic incompressible hyperelastic models have been proposed for biological materials; the high water content of which means the incompressibility condition is appropriate [8]. To create a model for transversely isotropic compressible materials under large strain a linear elastic/plastic approach has been demonstrated by Tagarielli et al. [9] (for balsa wood); though beyond small strains, the deformation was modelled as irreversible and while appropriate for balsa wood is not suitable for describing the recovery of predominantly viscoelastic materials such as LDPE foam. In this paper we demonstrate a simple hyperelastic approach using a limited set of experimental data to model a transversely isotropic compressible foam compressed uniaxially at an arbitrary orientation to the material’s preferred direction.

Experimental Results

Uniaxial compression and shear tests. A Zwick/Roell Z250 compression machine with a maximal load of 25 KN and a load cell with 0.01% full-scale accuracy were used. Cubic specimens (50 x 50 x 50 mm) were cut for compression tests using a band saw from the original rectangular extruded sections (approximately 2000 x 100 x 100 mm). The mechanical response of the principal and the two transverse directions for the cube are shown in Fig. 1-(a). Each test was repeated five times. The foam was significantly stiffer in the principal direction than in the transverse directions. Following the relevant test standards [10] experiments were carried out at a strain rate of $1.6 \times 10^{-3}$ s$^{-1}$. Strains of 90% were applied after which the material was completely unloaded at the same rate of strain.

During loading, the average Young’s modulus (small strain) of the principal direction was $2.039 \pm 0.390$ MPa (error is given by the standard deviation). This is much larger than the average modulus of the transverse direction; $0.741 \pm 0.157$ MPa, see Fig. 1-(b). Thus, the material shows significant transverse isotropy [11]. Also after compression to 25% strain, Fig. 1-(b) shows hysteresis in the load/unload curves suggesting viscoelasticity and at least some degree of semi-permanent deformation, though given enough time the specimen almost completely recovers its original shape. Repeat tests after several days indicate permanent changes in the loading curve, suggesting damage to the foam’s internal structure. As the strain rate increases, rate effects (hysteresis) become more apparent and as the maximum applied strain is increased the degree of permanent change to the loading curve (damage) also increases. Thus, the general behaviour of this material is complex and results from a combination of viscoelastic and plastic deformation in the skeletal framework of the polymer foam coupled with compression and diffusion of the gas within and between the closed cell microstructure and most likely, some degree of permanent damage. As a first step we aim to model the foam using a constitutive model currently implemented in Abaqus$^\text{TM}$ [12] based on Ogden’s isotropic compressible hyperelastic model [6] and we also suggest a method to enhance the use of this model with respect to the material in question. Future work will involve incorporation of internal variables in the constitutive model to account for viscous effects and possibly also for plasticity and damage [13].

![Figure 1](image-url)
The Poisson’s ratio of the foam was also characterised, Fig. 2-(a-b), and results are shown in Fig. 2-(c). Images were taken at regular intervals using a Canon EOS 1000D digital camera and measured using simple image analysis software Image-J [14]. Vertical and horizontal dimensions of the material were measured from each image. Specimen dimensions were (50 x 50 x 50 mm) and compression tests were conducted both along and orthogonal to the principal direction in order to calculate the two different Poisson’s ratios (see Eq’s. 1-3 and Fig. 2). The strain in each experiment was determined using Eq. 1 & Eq. 2

\[
\varepsilon_1 = \frac{kr - KR}{KR}, \quad \varepsilon_2 = \frac{mn - MN}{MN}
\]

(1) & (2)

where subscripts 1 and 2 refer to the principal and transverse directions. In Figs. 2-(a) & (b) the foam is orientated with the principal direction aligned with the direction of compression. The two Poisson’s ratios are determined using Eq. 3. Fig. 2-(c) clearly shows that rather than being constant, the Poisson’s ratio in both directions is a function of strain. Sixth order polynomials have been fitted to the data (also shown in Fig. 2-(c)).

\[
\nu_{12} = -\frac{\varepsilon_2}{\varepsilon_1}, \quad \nu_{21} = -\frac{\varepsilon_1}{\varepsilon_2}
\]

(3)

Off-axis large strain kinematics. It is interesting to observe the effect of the foam’s transverse isotropy on its deformation kinematics. As expected, Fig. 3-(a) shows that when compressed uniaxially with the direction of compression misaligned with the foam’s principal directions, the foam’s kinematics are also anisotropic. As there are currently no large-strain transversely isotropic compressible models implemented in Abaqus™, accurately predicting both stress and kinematics for this material is not yet possible (without implementing an entirely new model using the user subroutine facility). However, Fig. 3-(b) shows how a linear-elastic compressible transversely isotropic model [15] can successfully approximate the observed kinematics (though not stress) under large strain uniaxial deformation. To do this the \(G_{12}\) shear modulus of the foam was determined for use in this linear model using test specimens measuring (250 x 50 x 25 mm) each glued to a steel shear plate and tested at a strain-rate of \(2 \times 10^{-4}\) s\(^{-1}\), as per the British standard [16]. Shear tests were repeated four times. In future, digital image correlation may provide a more explicit comparison of the deformation kinematics.
Fig. 3. Off-axis uniaxial compression of foam to 26% engineering strain. (a): experimental observation with principal direction at 45° to the direction of compression (b): Displacement predictions of linear compressible transversely isotropic model.

**Off-axis stress-strain results** Stress-strain results from the tests shown in Fig. 2 & 3 are shown in Fig. 4-(a). Each test was repeated four times. As expected, the stress-strain curves of the specimen, orientated at 45°, lies approximately mid-way between the 0° and 90° curves. However, the equivalent curves for the specimens orientated at θ°, the rotation angle of the specimen at 22.5° and 67.5°, are clearly shifted in a non-linear manner.

![Graph showing stress-strain results](image)

Fig. 4. (a): Uniaxial compression for cube size 50x50x50mm with different degree of rotation of the foam’s principal axis. (b): Principal and transverse uniaxial compression curves with ‘apparent’ modulus shown at 0.2 strain.

**Predicting the off-axis stress strain curves**

The aim here is to predict the approximate uniaxial compression stress versus strain curves for arbitrary off-axis specimen orientations using just the principal and transverse uniaxial compression data. To do this we use the same rotational transformation used to determine the stiffness of a linear compressible transversely isotropic material when loaded off-axis (see Eq. 4 [3]. However, for large strains we have used an ‘apparent’ stiffness for $E_1$ and $E_2$, the modulus in the principal and transverse directions, as indicated for a strain of 0.2 in Fig. 4-(b). These apparent moduli are therefore functions of the strain, i.e. $E_i(\varepsilon)$ where $i = 1$ or 2.

\[
\left[ \frac{1}{E_\phi(\varepsilon)} \right] = \cos^2(\theta) \left( \cos^2(\theta) - \sin^2(\theta)v_{12}(\varepsilon) \right) + \frac{\sin^2(\theta)}{E_1(\varepsilon)} \left( \sin^2(\theta) - \cos^2(\theta)v_{12}(\varepsilon) \right) + \frac{\cos^2(\theta)\sin^2(\theta)}{G_{12}(\varepsilon)} \quad (4)
\]

\[
G_{12}(\varepsilon) = \frac{E_1(\varepsilon)}{2(1 + v_{12}(\varepsilon))} \quad (5)
\]
here $G_{12}(\varepsilon)$ is the small strain shear modulus measured as a function of the finite uniaxial compressive strain, $\varepsilon$. To actually measure this function is not a trivial procedure and so in its place we make a first approximation using Eq (5) which is an adaptation of the shear modulus for an isotropic compressible elastic material under small strains and where $\nu_{12}(\varepsilon)$ is a polynomial function fitted to the data of Fig. 2. To give an idea of the accuracy of this prediction for LPDE foam, the measured value of $G_{12}$ under zero compressive strain is $0.83 \pm 0.2$ MPa, whereas Eq 5 predicts a value of $0.58 \pm 0.1$ MPa; about 30 percent difference. Using Eq. 4 the stress versus strain curves for arbitrary off-axis measurements can be generated. The resulting predictions are shown in Fig. 5-(a) (dashed lines) and compared with the experimental measurements (continuous lines). The method produces good agreement between the two sets of curves. Finally, the isotropic hyperfoam model implemented in Abaqus™ was fitted to these predictions (using data up to a strain of 0.6). The resulting hyperfoam predictions for uniaxial compression are compared against the original experimental data in Fig. 5-(b). Simulations employed just one element using the static standard nonlinear solver and the default element provided by Abaqus™ for fitting a hyperfoam model; a continuum 8-noded reduced integration element (C3D8R) [17]. The hyperfoam model requires a single and constant value for the Poisson’s ratio. However, the data in Fig. 2-(c) demonstrate two different Poisson’s ratio variables (not constants). In order to provide a single and constant value for the model at each orientation, the two small strain values of the Poisson’s ratio, $(\nu_{12} = 0.22, \nu_{21} = 0.12$ see Fig. 2-(c)) were taken as the extreme values and intermediate angles were calculated using a simple rotation transformation.

![Graph](image1.png)

Fig. 5. (a): Experimental uniaxial compression curves (continuous lines) with different degrees of rotation of rotated foam’s principal axis and predicted uniaxial compression curves (dashed lines) using Eq. (4) for $\theta = 22.5^\circ$, $45^\circ$ and $67.5^\circ$. (b): Experimental uniaxial compression curves (continuous lines) with different degrees of rotation of rotated foam’s principal axis and curves predicted by compressible isotropic Hyperfoam model fitted to the predicted curves.

Conclusions

A method of predicting one-way uniaxial compression behaviour of strongly transversely isotropic compressible foam, using an isotropic hyperelastic model currently implemented in Abaqus™ has been demonstrated. Using the method a reasonable approximation of the material’s response can be predicted in FE simulations for arbitrary angles based on data from uniaxial compression tests in the principal and transverse directions. The method is restricted to uniaxial compression and therefore not suitable for more complex deformation but may be of use under certain limited scenarios.

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References


