NUMERICAL EVALUATION OF A RATE DEPENDENT MODEL FOR VISCOUS TEXTILE COMPOSITES

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SUMMARY: A predictive approach to modelling the forming of viscous textile composites has been implemented in two commercial finite element codes, Abaqus StandardTM and Abaqus ExplicitTM. A multi-scale energy model is used to predict the shear force – shear angle – shear rate behaviour of viscous textile composites, at specified temperatures, using parameters supplied readily by material manufacturers, such as fibre volume fraction, weave architecture and matrix rheology. The output of the energy model is then fed into finite element simulations to provide in-plane shear properties of the material constitutive model implemented in the finite element code. The resulting predictions are rate dependent. Picture Frame and Bias Extension test simulations are used to evaluate the predictions.

KEYWORDS: Finite Element, Press Forming, Viscous Textile Composite, Virtual Process Optimisation

INTRODUCTION

Finite Element (FE) technology has proved itself to be a valuable tool for design and manufacture process-optimisation in both the plastics and metals industries. Certain manufacture processes within the composites industry are ideal candidates for such virtual optimisation. However, fundamental to this goal is the prior development of suitable material constitutive models and the subsequent successful implementation of these models in FE codes. The objective of this work is to enable virtual process optimisation for the forming of viscous textile composites, particularly in relation to the press-forming operation.

In this paper we describe how, using a Multi-Scale Energy Model (MSEM), the shear behaviour of textile composites can be predicted from easily obtained experimental data and also, how these predictions can be incorporated directly into FE simulations. The motives for such constituent-based predictive modelling are two-fold. First, once the material behaviours of the constituent components are known then the rheology of any composite comprised of matrix and continuous inextensible fibres can be predicted, allowing the pre-manufacturing optimisation of a composite to suit a potential application. Second, characterising the rheological behaviour of, for example, a thermoplastic matrix polymer at different shear rates and temperatures is relatively easy using modern rheometers, compared with the equivalent but more difficult task of characterising the rheology of textile composites using Picture
Frame (PF) and Bias Extension (BE) tests [1]. Success in both these issues would lead to significant reductions in time and cost during the manufacture process.

The paper is organised as follows. The FE model is described before presenting an overview of the MSEM. A Non-Orthogonal Constitutive Model (NOCM) implemented in the FE user subroutine is outlined and a detailed description of the method of incorporating predictions of the MSEM into the NOCM is given. Both PF and BE test simulations are conducted and the force predictions from each of these two types of simulation are analysed. Particular attention is paid to the rate-dependency of the model.

**FINITE ELEMENT MODEL**

A NOCM implemented previously in commercial FE codes, Abaqus Standard™ and Explicit™, [2, 3] was used previously to incorporate predictions from a constituent-based MSEM [4-6] through the use of either membrane or shell elements. In this paper an alternative approach is adopted [7-8], replacing the tensile contribution of the NOCM with truss elements. Thus, the final FE model is a hybrid of truss elements, representing the high tensile stiffness fibres, and membrane elements, representing the shear properties of the viscous textile composite (see Figure 1). The material properties of the truss elements are given using an isotropic elastic model while those of the membrane elements are given using a user-defined NOCM with the tensile moduli of the two fibre directions set to negligible values. By performing this investigation, the relative advantages of the two alternative modelling approaches are explored and a summary of the advantages and disadvantages of each approach is presented at the end of the paper.

![Figure 1. FE unit cell representation of textile structure of (a) previous modelling approach and (b) current modelling approach](image)

The resulting rate dependent model is tested using both the implicit and explicit FE methods through implementation in both Abaqus Standard™ and Abaqus Explicit™. The implicit method provides an exact solution to the momentum equations though has the drawback of being computationally expensive. The explicit method is faster and is well suited to the analysis of highly non-linear problems e.g. large deformation, contact and dynamic phenomena, though it is generally less accurate than the implicit method. Thus, simulation predictions for relatively simple deformations, such as those produced during PF and BE tests, each of which are accessible to simulation by either numerical method have been made. Comparison of results of the two methods serves both as an evaluation of the modelling approach and also as mutual verification of the consistency of the results.
MULTI-SCALE ENERGY MODEL

The role of the energy model is to predict the shear force – shear angle – shear rate behaviour of viscous textile composites using parameters supplied readily by material manufacturers, such as fibre volume fraction, weave architecture and matrix rheology. The model has been described in detail previously [4] and so only a brief description is given here.

The meso-scale kinematics observed in many types of viscous textile composites have motivated the use of a novel two-phase material model structure to analyse the energy dissipation within viscous textile composites. These kinematics have important consequences for the deformation occurring during shear, both within tow (fibre bundle) and inter-tow regions, and also between tow crossovers. Namely, the rate of deformation tensor must be derived separately for both the tow and inter-tow regions and a further dissipative energy term must be derived to account for viscous energy loss at the tow crossovers.

The stress-power in the tow and inter-tow regions can be calculated using the constitutive equation for a uniaxial ideal fibre reinforced fluid [9]. Within this model appear terms describing the so-called ‘longitudinal’ and ‘transverse’ viscosities of the tows. These terms describe the dynamic interaction occurring between fibres and matrix on a micro-scale. Using micro-mechanical modelling principals [10-11] reasonable predictions for both these viscosity terms can be made from the matrix viscosity and fibre volume fraction within the tows. Thus, given the rate of deformation and stress tensor for the tow and inter-tow regions, the stress power generated by these regions can be determined.

The velocity field between crossovers is calculated by analysing the in-plane kinematics of tow deformation during shear. Using the velocity field and matrix film thickness the shear strain rate in the matrix film separating tows at crossovers can be estimated. From these calculations an estimate of the rate of energy dissipation can be determined due to shear between tow crossovers. By combining the energy contributions from both tow and inter-tow shear together with crossover shear, the total rate of energy dissipation during shear of the textile composite can be estimated and from this the resistance to shear deformation can be determined.

NON-ORTHOGRAPHONAL CONSTITUTIVE MODEL

The role of the NOCM is to relate stress and strain in a form suitable for convenient implementation in a FE code. Again, the model has been described in detail previously [2-3] and so only a brief description is given here.

To model the stress and strain relationship dependent on the fibre directional properties, a non-orthogonal equation has been developed in an explicit mathematical form using a homogenization method. The resulting 2-D equation is based on a structural net concept and its derivation follows two distinct steps. Firstly, the contribution to total stress due to tensile strain in the non-orthogonal fibres is derived in a materially embedded orthogonal reference frame. In this particular investigation, this contribution is replaced by truss elements. Thus, for the sake of brevity, the discussion of this part of the NOCM is omitted.

In a second step, shear stress versus shear angle data is related to shear stiffness using a non-orthogonal analysis. The incremental stress due to shear is calculated by linearising the stress equation which is transformed into the orthogonal fibre bisector frame and related to the fibre shear angle increment, $\Delta \theta$, as
\[
\Delta \sigma = \begin{bmatrix}
\Delta \sigma_{xx} \\
\Delta \sigma_{yy} \\
\Delta \sigma_{xy}
\end{bmatrix} = \begin{bmatrix}
0 & 0 & 2G_1g_1^1g_2^1 + G_2\left(g_1^1g_2^2 - g_1^2g_2^1\right) \\
0 & 0 & 2G_1g_2^1g_2^2 + G_2\left(g_1^1g_2^2 - g_1^2g_2^1\right) \\
0 & 0 & G_1\left(g_1^1g_2^2 + g_1^2g_2^1\right)
\end{bmatrix} \Delta \theta
\]  

where \(G_1\) and \(G_2\) are model parameters that can be related explicitly to the shear force versus shear angle curve and \(g_1^1, g_2^2\) are components of \(\mathbf{g}_1\) expressed in the orthogonal fibre bisector frame. Similarly, \(g_2^1, g_2^2\) are components of \(\mathbf{g}_2\). An attractive feature of this non-orthogonal model is that, given the shear angle the two shear parameters \(G_1\) and \(G_2\) can be related analytically to the shear force versus shear angle response of textile composites as

\[
G_1 = \frac{1}{lh} \left\{ \frac{dF_s}{d\theta} \sqrt{g^{1\!1}} + F_s \sqrt{g^{1\!1}(g^{1\!1} - 1)} \right\}
\]  

\[
G_2 = \left(\frac{F_s}{lh}\right) \sqrt{g^{1\!1}}
\]  

where \(F_s\) is the shear force versus shear angle which can be approximated using a polynomial function of \(\theta\) the material shear angle, \(dF_s/d\theta\) is the gradient of the shear force versus shear angle curve which is also a function of \(\theta\), \(l\) and \(h\) are, respectively, the side length and thickness of the material and \(g^{ij}\) are components of the conjugate tensor. Since the NOCM expresses the relationship solely between 2-D stress and shear strain, as opposed to rate of shear strain, then implemented in this form, the model is a purely rate independent model.

**INTERFACE BETWEEN MULTI-SCALE ENERGY MODEL PREDICTIONS AND NON-ORTHOGONAL CONSTITUTIVE MODEL**

The role of the interface between the MSEM and the NOCM is both to facilitate prediction of forming behaviour of viscous textile composites using a predictive constituent based approach and also to introduce rate dependency into the FE predictions. An example of the predicted shear force surface for a thermoplastic composite is shown in Figure 2. Details of the material for which the predictions were made are given in the figure caption. In order to incorporate these predictions in the FE code and consequently introduce rate dependency in the simulation, polynomial fits are made to the shear force surface at specific angular velocities. In Figure 2, five different fits to the MSEM predictions are indicated by the thick black lines superimposed on the shear force surface. The polynomial coefficients of these fits and corresponding angular velocities are stored in a matrix. This is the output of the MSEM and is stored in a text file in a specified location on the computer for reference by the material user subroutine of the FE code.

The white cross on the shear force surface of Figure 2 indicates the shear angle (\(= 35^\circ\)) and angular velocity (\(= 0.5 \text{ rads}^{-1}\) or \(29^\circ \text{ s}^{-1}\)) of a given element at a given time. In determining the shear parameters of this element, \(G_1\) and \(G_2\), the code assigns to the element the coefficients of the polynomial curve fitted at an angular velocity higher but contiguous with the angular velocity of the element. In this case the element would be assigned the polynomial coefficients of the 0.6 rads\(^{-1}\) (or \(34^\circ \text{ s}^{-1}\)) curve.
While the function of the user sub-routines of Abaqus Standard™ and Explicit™ is the same, to calculate the stress increment from a given strain increment, the two codes operate in slightly different ways. The implicit method requires that the tangent stiffness matrix, \( d\sigma_y / d\theta \) be passed to the FE code, from which the stress increment is calculated within the code as

\[
\Delta\sigma_y = \frac{d\sigma_y}{d\theta} \Delta\theta
\]  

i.e. the implicit method requires linearization of \( \sigma_y \) for calculation of \( \Delta\sigma_y \). Contrary to the implicit method, in the explicit method \( \Delta\sigma_y \) is calculated within the user subroutine and then passed to the FE code. Thus, in this case \( \Delta\sigma_y \) can be calculated by a choice of two methods, either using Eq (4) or simply by

\[
\Delta\sigma_y = \sigma_y(\theta + \Delta\theta) - \sigma_y(\theta)
\]  

In order to maintain the same implementation technique in both FE codes (implicit and explicit) and thus facilitate mutual validation between predictions of the two codes, Eq (4) is implemented in both Abaqus Standard™ and Explicit™. However, the process of linearising the stress equation means that Eq (4) is sensitive only to the gradient of the SF-SA input curves and insensitive to step changes in these curves. The meaning of this last statement is illustrated in the following example. Consider a PF test [1] undergoing a constant displacement rate, \( \dot{d} \). The angular shear rate of the material in the test, \( \dot{\theta} \), is related to \( \dot{d} \) by

\[
\dot{\theta} = \frac{\dot{d}}{\sin(\pi/4 - \theta/2)}
\]
Thus, during simulation of a PF test with constant \( \dot{d}, \dot{\theta} \) increases with increasing \( \theta \) and the interface described above selects gradually steeper input curves. As an idealised example, Figure 3 shows four constant \( \dot{\theta} \) SF-SA curves, represented by four thin linear lines (the higher the \( \dot{\theta} \), the steeper the gradient). As the PF simulation progresses \( \dot{\theta} \) increases according to Eq (6) and the interface selects gradually steeper input curves. In Figure 3 the gradient of the successive input curves increases by a factor of two. The desired ideal prediction of the code is shown by the thick black line in Figure 3(a). However, as mentioned previously the linearised form of the NOCM is sensitive only to the gradient of the input SF-SA curve and insensitive to the step changes in the shear force. In practice this means that the actual prediction of the code using these four linear SF-SA input curves is shown in Figure 3(b) (thick black line).

**Figure 3.** The four linear lines represent four input curves referenced by the rate-effect interface. (a) Desired ideal PF simulation prediction (thick black line) using the four linear SF-SA input curves (b) Actual PF simulation prediction using the four linear SF-SA input curves

Thus, in this state, the final interface prediction is much less rate-sensitive than desired. In order to introduce full material rate dependency in the model a modification of the original interface has been implemented. The method is based on a feedback algorithm and works as follows:

- The angular rate in an element, \( \dot{\theta} \) is calculated and the appropriate SF-SA input curve is chosen accordingly (as with the original rate-effect interface)
- Within the user subroutine the current 2-D stress prediction at the integration points within a given membrane or shell element, \( \sigma'_{ij} \), is rotated into the local reference frame of the textile (the \( x \)-axis of the local system is co-linear with one of the fibre directions of the textile) to calculate \( \sigma'_{ij} \)
- \( \sigma'_{ij} \) is used to calculate the normalised shear force of the element, \( F'_{s} = \sigma'_{ij} \times h \) in Nm\(^{-1}\)
- \( F'_{s} \) is compared to \( F_{s} \), the normalised shear force calculated from the SF-SA input curve. If \( F'_{s} \) is smaller than \( F_{s} \), then the gradient of the SF-SA input curve, \( dF/d\theta \), is multiplied by a factor \( N \), before calculating the tangent stiffness matrix, Eq (1).

In this way the size of \( \Delta \sigma_{ij} \) is increased until \( F'_{s} \) equals \( F_{s} \). The previous example is used to demonstrate the functionality of the modified interface, see Figure 4. In the following section both PF and BE test simulations are used to evaluate the rate dependent force predictions of the interface.
PICTURE FRAME AND BIAS EXTENSION TEST SIMULATIONS

Picture frame test simulations have been conducted, see Figure 5. 16 quadrilateral linear membrane and 40 linear truss elements were used to model the blank (the predictions are almost insensitive to mesh density and further refinement is unnecessary). The boundary conditions imposed during the simulations were such that nodes along sides of the material were constrained to move along lines connecting the corner nodes. The bottom left corner was pinned while the upper right corner moved at a constant displacement rate in the diagonal direction.

For the PF simulation, Eq. (6) shows that for a constant $\dot{d}$, $\dot{\theta}$ increases with increasing $\theta$. Thus, during the course of the simulation the elements undergo progressively faster angular shear and are consequently assigned polynomial SF-SA curves of increasing value as the shear angle increases. A velocity scale factor has been introduced in the user subroutine. This velocity scale factor affects the FE results only via its influence regarding assignment of polynomial SF-SA curves to the elements during the simulation. Otherwise the velocity scale factor has no bearing on the FE calculations. In this simulation 35 polynomial curves were fitted to the MSEM predictions at equal increments over an angular shear rate range of $=10^\circ \text{s}^{-1}$ to $26^\circ \text{s}^{-1}$. Elements undergoing angular shear rates higher than $26^\circ \text{s}^{-1}$ were automatically assigned the highest polynomial SF-SA curve. In all subsequent simulations the tensile modulus of the truss elements was set at 30,000 MPa and the cross sectional diameter at $2 \times 10^{-5} \text{mm}^2$, while $E_1$ and $E_2$ in the NOCM were assigned a negligible value of 6.5 Pa with a membrane section thickness of 0.5 mm.
Three different methods can be used to analyse the force predictions of the simulation:
A. The first method is by examining the normalised shear force of the membrane elements (calculated directly from the integration points as outlined in the previous section)
B. A second method is by examining the reaction force on the corner node
C. An alternative method to the above is via calculation of the gradient of the internal strain energy versus distance curve

The first method neglects the energy contribution to the total force of the model from the truss elements. For implicit simulations Methods B and C produce very similar results. However, in the case of the explicit method, the reaction force produces extreme oscillations due to the force concentration on the single corner node. In this case the energy gradient, Method C is a better option of analysing the data, since the total strain energy of the entire model is less prone to rapid oscillations. (The force is calculated by fitting a high order polynomial to the energy versus distance curve and differentiating). Note however that some error is introduced due to the curve fitting procedure.

In Figure 6 force predictions of both the implicit (left) and explicit (right) FE methods are shown. The 35 polynomial input curves are also presented in each diagram. In order to demonstrate rate dependence the implicit simulation was conducted at a slightly higher displacement rate than the explicit simulation. The shear force calculated from the membrane elements using Method A show the interface works very well (continuous lines). The alternative methods of plotting the data show a significant contribution to the apparent shear force from the truss elements. Increasing the stiffness of the truss elements failed to decrease the size of this contribution.

![Figure 6. Force results from the PF simulations. The 35 polynomial input curves are shown in both figures. Force predictions of implicit are shown on the left and explicit are on the right. The thick black curves were produced using Method A. The broken curves were produced using Method B (left) and Method C (right)](image)

The model has also been validated using BE test simulations using the implicit method, see Figure 7 and Figure 8. The mesh is constructed from 52 linear quadrilateral elements, 24 triangular membrane elements plus 138 truss elements. The bottom nodes were pinned and the top nodes were constrained to move directly upwards. Nodes along the sides of the mesh were constrained to lie along a straight line between the corner and mid-side nodes. Care was taken to align the mesh with the fibre directions [12]. The data produced using Method B have been normalised using the procedure described in [1]. Results from the BE test simulation show the interface works very well but again an extra contribution to the apparent shear force is produced by the truss elements. The size of this contribution remained constant irrespective of increases in the stiffness of the truss elements.
Figure 7. (a) Normalised shear force of the elements, \( F' = \sigma'_y \times h \text{ Nm}^{-1} \) (b) maximum principal stress in the truss elements Nm\(^{2}\). As with the PF simulations the boundary conditions also produce tensile stress concentrations in the truss elements near the corner nodes.

Figure 8. Bias Extension test simulation. The 35 polynomial input curves are shown together with the two force predictions. The thick black curve was produced using Method A. The grey curve was produced using Method B.

**CONCLUSIONS**

A rate dependent predictive model for viscous textile composites has successfully been implemented in finite element software. Close agreement between model predictions using two different numerical methods serves to validate the rate dependent force predictions of the model. An extra contribution to the apparent shear stress due to the truss elements is observed, possibly the result of concentrated nodal loads. Future work will involve examining the influence of rate dependence in complex forming situations.

The method of using hybrid unit cell elements, coupling elastic truss elements with a non-orthogonal constitutive membrane element (with negligible tensile stiffness) has proved to have certain advantages. Namely:

- the solution to the stress field is extremely stable, even at very high shear angles
- the method is simple to implement
- the method automatically avoids problems due to in-plane changes in the fibre direction [12]

However, this approach has a number of disadvantages including:

- longer simulation times due to a large increase in the number of elements
- coupling stresses and strains in the two different fibre directions represented by the truss elements is not an easily soluble problem
meshing can be problematic. Commercial meshing software can be used though in this case an in-house meshing algorithm was implemented and found to be much more convenient

the method is restricted to quadrilateral element formulations

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