SHEAR TENSION COUPLING IN BIAXIAL BIAS EXTENSION TESTS

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SUMMARY

Biaxial Bias Extension tests have been performed on a plain-weave carbon fibre engineering fabric. An energy based normalisation technique has been used to investigate the effect of in-plane tension on the shear resistance by using specimens of various sizes and by applying transverse forces of varying weight.

Keywords: Shear characterisation, biaxial bias extension test

INTRODUCTION

The coupling between shear resistance and in-plane tension is of interest when modeling the forming behaviour of engineering fabrics. During the forming process, the magnitude of in-plane tension in a fabric sheet is determined by the boundary conditions applied to the material via the use of a blank-holder. The optimum forming condition can be predicted using CAE tools and may involve a variable pressure distribution around the edges of the material [1] resulting in a variable tensile field through the forming sheet. Since the dominant mode of deformation of an engineering fabric during forming is through trellis-shear, and the fabric’s shear compliance is influenced by the size of the in-plane tension, it is important to capture the coupling between shear and tension when modeling the forming process. Attempts to develop constitutive models that incorporate such a coupling have recently been published [2]. However, establishing a simple and reliable test method to characterise the shear-tension coupling has yet to be fully realized. This paper is aimed both at developing such a test method and also to establish a convenient procedure to analyse the results.

Several prior studies have examined the issues surrounding the shear characterisation of textile composites and engineering fabrics during forming [3, 4]. Two tests in particular have received considerable interest; the Picture Frame (PF) and the Uniaxial Bias Extension (UBE) test. Each of these tests has its relative merits and disadvantages, for example the PF test induces approximately homogeneous deformation throughout the test specimen but is known to be susceptible to errors resulting from its boundary conditions; sample misalignment may produce large forces due to tensile strain along the fibre directions. The UBE test is less susceptible to sample misalignment but can be used only to characterise textile-based materials to relatively low shear angles, typically around 40 degrees, before test specimens begin to deform through mechanisms other
than trellis shear, such as intra-ply slip. Such mechanisms are a result of the specific conditions imposed by the UBE test and are not particularly representative of actual press or diaphragm forming conditions. In neither of these tests is it possible to control simply the in-plane tension during characterisation, though attempts to do this in PF tests have been reported [5]. The Biaxial Bias Extension (BBE) test can potentially avoid many of the problems associated with the PF and UBE tests, i.e. results are less influenced by the test’s boundary conditions and test samples are less prone to intraply slip. The BBE test can also be used to investigate the effects of in-plane tension on shear behaviour by varying the size of the transverse load applied to the specimen. However, as with the UBE test the deformation field induced throughout the test sample is not homogeneous. This fact leads to added complexity when normalising test data. An energy-based approach to normalisation was taken in a prior investigation concerning UBE test data [3]. The normalisation scheme takes into account the sample’s non-homogeneous strain field. The present work uses a similar energy-based analysis in deriving expressions that can be used to normalize BBE test data [6]. Test specimens of various sizes together with transverse forces of varying magnitude have been employed in this investigation in an attempt to use the energy normalisation method to isolate the influence of in-plane tension on the shear behaviour of an engineering fabric, in this case a plain-weave carbon fabric. Results are analysed and discussed and methods to improve the test procedure are suggested.

**EXPERIMENTAL METHOD**

**Test Set-Up**

At least two designs have been used in the past for BBE tests (see Fig 1). One aims to keep the centre of the test specimen stationary using custom made apparatus [7] while a simpler design involves modification of a standard universal testing machine and applies a constant transverse force on a fabric specimen undergoing a tensile test [6, 8]. This investigation uses the latter design, Fig 1(right).

![Figure 1(a) Custom designed rig where centre of sample is stationary [7]. (b) Universal test machine with pulleys installed to provide a transverse force [6, 8].](image-url)
Correction for Reaction Force

In both cases the force response measured by the test machine is comprised of two components:

1. the force due to deformation of the sheet, $F_M$
2. the reaction force due to work done against the transverse load transmitted through the fabric to the loadcell, $F_R$

In order to correctly analyse the material behaviour the reaction force has to be removed from the total measured force. If ideal kinematics are assumed it can be shown that for the first type of set-up (Fig 1: left)

$$F_R = \frac{F_I}{\tan\left(\frac{\pi - \theta}{4}\right)}$$

where $\theta$ is the fabric shear angle or for the second set-up (Fig 1: right)

$$F_R = \frac{F_I \cos \alpha}{\tan\left(\frac{\pi - \theta}{4}\right)} + F_I \sin \alpha$$

where $\alpha$ is the angle the transverse cord (in Fig 1 a fishing line) makes with the horizontal axis and increases during the course of the test. $\alpha$ is zero at the start of the test. Thus, before normalisation can take place the material response, $F_M$, has to be extracted from the total measured force, $F_T$.

$$F_M = F_T - F_R$$

ENERGY NORMALISATION THEORY

In a previous investigation [6] it was shown that the energy normalisation method described here provides normalised results that sit between the upper and lower normalisation limits suggested previously by Potluri et al. [8]. The aim of the theory is to determine the force versus shear angle response of the material such that the force can be compared directly with results from say a PF test or a UBE test, i.e. to generate test-independent results for the material response. The analysis is complicated by the non-homogeneous shear field occurring throughout the specimen. Thus, the theory aims to decouple the different contributions to the total force from Regions A and B in the specimen (see Fig 3) and begins with a result from analysis of the PF test [4].
For a given shear angle, $\theta$, 

\[ \frac{c_A T_A}{k_2} = \frac{F_{pf1}}{L_{pf1}} = \frac{F_{pf2}}{L_{pf2}} \]  

(4)

where the forces, $F_{pf1}$, $F_{pf2}$ and lengths $L_{pf1}$, $L_{pf2}$ correspond to those produced using two picture frames of different sizes, as shown in Fig 2. $T_A$ is the instantaneous thickness of the sheet at a given shear angle, $c_A$ is the power storage/dissipation per unit current volume at a given shear angle and angular shear rate and $k_2$ is a function of the shear angle and angular shear rate and is based on the PF geometry.

In Fig 3, two different biaxial specimen geometries are considered. The relative volumes of material in Regions A and B for the two geometries can be determined, i.e. 

\[ V_A = V_B / n \]  

(5)

where for Fig 3(a) $n = 1$ and for Fig 3(b) 

\[ n = \frac{4L_c}{L_f - L_c} \]  

(6)

where $L_c$ is the clamping length and $L_f$ is the total side length of the square specimen, $V_A$ and $V_B$ are the initial volumes of material in Regions A and B respectively. In Fig 3(b) the Region $C_2$ can be omitted or included when cutting the specimen as theoretically it does not deform and hence does not contribute to the measured force. Initial tests [8] suggest Region $C_2$ may improve the integrity of the sample such that the specimen conforms more closely to ideal kinematics for a greater duration of the test (to higher shear angles).

**Assumptions used in normalisation theory**

Certain assumptions are made in the analysis. The main assumption of ideal kinematics tends to be correct at lower shear angles of up to around 30° of shear. The assumptions include:

1) the shear angle in Region A, $\theta_A$ is always twice that in region B, $\theta_B$
2) the angular shear rate in Region A, $\dot{\theta}_A$ is always twice that in region B, $\dot{\theta}_B$
3) Regions $C_1$ and $C_2$ remain undeformed throughout the test
4) Region B generates the same proportion of the total stress-power of the material at a given $\theta$ and $\vartheta$ irrespective of the size of the sample.
5) At a given $\theta$ the thickness in Region B will become equal to the thickness in Region A such as it was when the shear angle in Region A was $\theta/2$.

![Figure 3. Various geometries used for biaxial tests. The relative volumes of material in Regions A and B in (a) is 1 while in (b) is given by Eq (5) & (6).](image)

By considering the contribution to the measured force from Region B as compared to a PF test (which, theoretically contains only Region A) it can be shown that

$$\frac{c_A T_A}{k_x} = \frac{F_M}{L [1 + nX]}$$

where $X$ represents the relative additional contribution to the measured material force due to Region B. Assumption 5 is used to determine the form of $X$ to find

$$\frac{c_B T_B \cos(\theta/2)}{c_A T_A \cos \theta} = X(\theta)$$

hence in general $X$ is purely a function of $\theta$ and where $c_B$ and $T_B$ are similar functions as those defined earlier but with the subscript $B$ indicating these functions apply to Region B. Determining the relationship between $c_A$ and $c_B$ uses the idea that the shear resistance in Region A when the shear angle in Region A is $\theta/2$ is the same as the shear resistance in Region B when the shear angle in Region B is $\theta/2$. It can be shown that

$$c_B(\theta) = \frac{\dot{\theta}_B(\theta)}{\dot{\theta}_A(\theta/2)} c_A(\theta/2) = \frac{\sin \left( \frac{\pi - \theta}{4} \right)}{2 \sin \left( \frac{\pi - \theta}{4} \right/ 2} c_A(\theta/2)$$

where the factor introduced in Eq (9) accounts for the different angular shear rate experienced by Region A when the shear angle in this region is $\theta/2$ and Region B when the shear angle in Region B is $\theta/2$ (i.e. when the shear angle in Region A is $\theta$). Eq (9) neglects the changing shear resistance of the material as a function of in-plane tension, i.e. as the test progresses the shear force tends to increase and the in-plane tension in the sheet increases. Using Eq (9) and Assumption 5 it follows that
\[ c_B(\theta)T_B(\theta) = \frac{\dot{\theta}_B(\theta)}{\dot{\theta}_A(\theta/2)} c_A(\theta/2)T_A(\theta/2) \]  

(10)

which can be re-written as

\[ \xi(\theta) = \frac{\dot{\theta}_B(\theta)}{\dot{\theta}_A(\theta/2)} \psi(\theta/2) \]  

(11)

where \( \psi \) and \( \xi \) represent the power dissipation / storage per unit area of Regions A and B respectively. Combining Eq (7), (9) and (11) find

\[ \psi(\theta) = \frac{F_M k_2}{L_1} \left[ \frac{n}{2} \right] \frac{\cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\pi - \theta}{4}\right)}{\cos(\theta) \sin\left(\frac{\pi - \theta}{2}\right)} \psi\left(\frac{\theta}{2}\right) \]  

(12)

If \( \psi(\theta) \) can be found using Eq (12) then a direct comparison between the BBE and PF test can be made using Eq (4). To do this, examining Eq (12), \( F_M \) and \( d_1 \) are measured during BBE tests and \( L_1 \) and \( n \) are known from the initial sample geometry. Assuming ideal kinematics (Assumptions 1-3) the shear angle can be found from \( d_1 \) and \( L_1 \) using

\[ \theta = \frac{\pi}{2} - 2\cos^{-1}\left[\frac{d_1}{2L_1} + \frac{1}{\sqrt{2}}\right] \]  

(13)

and

\[ k_2 = \frac{\dot{d}_1}{L_1 \cos \theta} \]  

(14)

Thus, all the terms on the right hand side of Eq (12) are obtained apart from \( \psi(\theta/2) \). In order to evaluate Eq (12) an iterative scheme can be implemented [3].

**MATERIAL**

A plain woven carbon fabric has been chosen for the tests [6]. Details regarding the fabrics structure, areal density and thickness include: Tex Count = Warp 800.9 & Weft 798.6, ends per cm = 3.00, picks per cm = 3.29, areal density = 528gm\(^2\), width of a pick = 3.33 mm, width of an end = 3.04 mm, thickness at 2g force /cm\(^2\) = 0.889 mm, and thickness at 50g force/cm\(^2\) = 0.725 mm.

**RESULTS**

Several preliminary tests have been conducted on two different sized specimens (16x16 & 24x24cm) using transverse loads of varying weight (100, 300 & 400g).
Figure 4. Axial force versus experimentally measured shear angle for 16x16cm specimen (a) with 100g transverse weight (b) with transverse weight component of force removed

Fig 4(a) shows typical raw axial force versus experimentally measured shear angle data, note that the transverse load was placed on the sample after the start of the test hence the sudden increase in force at low shear angles. The shear angle was measured using an automated image analysis system using a video camera positioned behind the specimen. In some cases the material was unintentionally sheared slightly in the negative shear direction at the start of the test due to handling of the sample when positioning in the clamps. Fig 4(b) shows the modified data, i.e. with the reaction force due to the transverse load removed.

Fig 5 shows data normalised using the energy normalisation method (normalisation applied after the reaction force from the transverse load has been removed). Comparing the data from the same sized specimens suggests a correlation between in-plane tension (resulting from the transverse load) and a higher shear resistance.

The reason that the 24x24cm specimen shows one of the highest shear resistances despite not being subjected to the largest transverse force may be due to the fact that the larger specimen results in higher axial forces, creating larger in-plane tensions within the sample, i.e. the magnitude of the in-plane tension is determined not only by the size of the transverse load but also by the force due to shearing the specimen itself. However, the size of the variation between results in Fig 5 makes it difficult to draw firm conclusions. In future, with more experimental data at hand it will be interesting to
compare the size of the normalised axial force against various possible measures of in-plane tension, e.g. the transverse force, the axial force or perhaps the axial force per unit length at a given shear angle.

Fig 6 shows the measured shear angle versus the theoretical shear angle for each of the tests conducted. The theoretical angle is determined from the cross-head displacement by assuming ideal shear kinematics, i.e. trellis shear in Region A (see Fig 3) and no deformation in Region C₁. Fig 6 suggests that the specimens undergo mixed-kinematics from relatively early on in the test, i.e. the data fall below the ideal kinematics line indicated in Fig 6. It is impossible to determine whether this is due to extension of Region C₁ or extension of the yarns throughout the sample due to, for example, a decrease in yarn crimp. However, past experience shows it is likely that Region C₁ has extended, at least to some degree, and so the kinematics in Region A are probably closer to ideal than indicated by Fig 6. By measuring the extension of Region C₁ it is possible to correct the data of Fig 6, a procedure that could be explored in future experiments. Another possibility is to ensure Region C₁ remains un-deformed throughout the test. This could possibly be achieved either by selectively impregnating this region with resin prior to testing, creating a solid composite in Region C, or possibly through using custom designed clamps that trap Region C₁ during the test.

The kinematics in the 24x24cm specimen suggest the shear angle in Region A is greater than that predicted by ideal kinematics. It may be that larger specimen possesses greater material integrity and therefore follow ideal kinematics more closely than the smaller 16x16cm specimens, though it seems unlikely that the angle should be higher than the ideal case and so further tests will be conducted to examine this unexpected result.

Figure 6. Measured verses theoretical shear angle for various tests.
CONCLUSION

A preliminary investigation has been conducted to determine the utility of BBE tests in exploring the sensitivity of an engineering fabric’s shear resistance to in-plane tension. Initial results suggest the test can indeed be used for this purpose though care has to be employed in order to generate repeatable data. In particular, the shear kinematics should be monitored and recorded in order to improve the subsequent data analysis. Possible adaptations of the test have been suggested in order to freeze the deformation of Region $C_1$ during the test in order to produce a more idealised shear behaviour in the specimen.

References


