Role of instantaneous force magnitude and duration on particle entrainment

Manousos Valyrakis, Panayiotis Diplas, Clint L. Dancey, Krista Greer, and Ahmet O. Celik

Received 31 December 2008; revised 30 October 2009; accepted 16 November 2009; published 14 April 2010.

[1] A new criterion for the onset of entrainment of coarse sediment grains is presented here. It is hypothesized that not only the magnitude, but also the duration of energetic near bed turbulent events is relevant in predicting grain removal from the bed surface. It is therefore proposed that the product of force and its duration, or impulse, is a more appropriate and universal criterion for identifying conditions for particle dislodgement. This conjecture is investigated utilizing two theoretical models, representative of two modes of entrainment: saltation and rolling. In these models, instantaneous, highly fluctuating turbulent forces are simulated as short-lived pulses of characteristic magnitude and duration, which transfer adequate fluid momentum to the particle, to trigger its entrainment. The analytical solution of the respective equations of motion is employed in deriving representations of threshold conditions in terms of the impulse characteristics. It is shown that hydrodynamic forces of sufficiently high magnitude are capable of entraining a particle only when they last long enough so that their impulse exceeds a critical value. To illustrate further the validity of the critical impulse concept, as well as extend and generalize its application to different entrainment levels of an individual grain, a novel experimental setup is utilized. This setup facilitates observations of angular displacement of a steel mobile particle in air due to electromagnetic pulses of different magnitude and duration. The experimentally obtained conditions for partial or complete entrainment support the concept of a critical impulse.


1. Introduction

[2] The dynamic interplay between fluctuating turbulent fluid forces and particle dislodgement for flows over an erodible boundary constitutes a central problem in earth surface dynamics and engineering. Soil erosion in riverine, estuarine, and aeolian environments provides a good example. One of the first attempts to highlight the importance of turbulence on particle movement, albeit in an elementary way, is attributed to Varenius [1664]. Varenius concluded from observations that the random movement of sediment is related to the fluctuating motion of the water in the stream. Since then, a large number of researchers have advocated this point of view based on detailed field, laboratory, and other studies [Leighly, 1934; Einstein and El-Samni, 1949; Sutherland, 1967; Paintal, 1971b; Grass, 1970, 1983; Apperley and Raudkivi, 1989; Nelson et al., 1993; Papanicolaou et al., 2001; Nelson et al., 2001; Sumer et al., 2003; Paiement-Paradis et al., 2003; McEwan et al., 2004; Schmeekle et al., 2007]. Evidently, the role of the instantaneous stress tensor is very important, especially at conditions near the threshold of movement. This point has been well demonstrated for both smooth [e.g., Sutherland, 1967; Grass, 1971; Sumer et al., 2003] and rough boundaries [e.g., Apperley and Raudkivi, 1989; Schmeekle and Nelson, 2003], as well as in the presence of bed forms [e.g., Raudkivi, 1966; Nelson et al., 1993; Sumer et al., 2003; Paiement-Paradis et al., 2003].

[3] In spite of this widespread recognition, attempts to link the characteristics of turbulent flow to particle entrainment have not typically surpassed qualitative descriptions. Even in the very few and most sophisticated cases where researchers have resorted to the solution of the governing flow equations, they have predominantly employed the time-averaged Reynolds equations. This is a reflection of the difficulties in accounting for the intricate ways that turbulence influences, or even dominates, particle threshold conditions. Shields was among the first researchers to formulate a quantitative criterion for inception of grain motion. His empirically obtained diagram [Shields, 1936] remains the standard method for identifying threshold of motion conditions and considerable effort has been spent to explain...
deviations from it, and to devise alternative plots for a variety of flow/sediment cases [e.g., Bettes, 1984; Buffington and Montgomery, 1997; Shvidchenko and Pender, 2000; Paphitis, 2001; Paphitis et al., 2002]. As of this time, the development of a precise and universal criterion for determining the initiation of sediment movement remains elusive.

[4] To make the problem more manageable, in this study we consider the phenomenon of initiation of motion for particles that are sufficiently large so that \( R_e > 100 \), which corresponds to the Reynolds independent region of Shields’ diagram, where \( R_e = \frac{u^* D_{50}}{v} \) is the boundary Reynolds number, \( u^* \) the shear velocity, \( D_{50} \) the bed material median size, and \( v \) the kinematic viscosity of water. Even under these conditions the scatter of the threshold of motion results reported in the literature [e.g., Lavelle and Moffield, 1897; Buffington and Montgomery, 1997] exceeds an order of magnitude. The conventional Shields parameter, \( \tau^* = \frac{\tau}{\rho g \phi D_{50}} \), where \( \tau \) is the boundary shear stress, \( g \) the acceleration of gravity, and \( \rho_f \) and \( \rho_s \) the density of fluid and sediment, respectively, is applicable to averaged en masse entrainment by steady uniform flow, as it has been reported by several researchers [e.g., Paintal, 1971a; Coleman and Nikora, 2008]. It is therefore evident that \( \tau^* \), which represents a temporal, and usually spatial, average of the rapidly fluctuating instantaneous stresses encountered near the boundary, is an incomplete measure of the turbulent flow processes responsible for particle dislodgement. The wide range of grain parameters, such as size and shape, pocket geometry, particle exposure, and bed geometries further complicate the overall picture.

[5] Many researchers, in an effort to overcome the limitations of the approach proposed by Shields, have advocated the important role that peak values in the instantaneous velocity or pressure field in the vicinity of the boundary, and resulting hydrodynamic forces, play on particle dislodgement, particularly for flow conditions near threshold of movement [e.g., Sutherland, 1967; Hofland et al., 2005; Schmeeckle et al., 2007; Vollmer and Kleinhans, 2007; Gimenez-Curto and Corniero, 2009]. More details about the role of such peak forces have been provided by Diplas et al. [2008], who demonstrated for the first time that the concept of impulse associated with an energetic turbulent event is the more suitable parameter for determining particle entrainment, by saltation (using theoretical arguments) and rolling (by analyzing experimental data obtained via an electromagnet). The present study seeks to extend and generalize the validity of the impulse concept proposed by Diplas et al. [2008] by accounting for a variable slope in the saltation formulation of the problem and by analytically considering the case of particle entrainment via rolling motion. The present results are elaborated further to describe different levels of movement, other than the critical for full entrainment, and corroborated through a series of appropriately designed laboratory experiments.

2. Interpretation of Incipient Motion Condition From a New Perspective

[6] Einstein and El-Samni [1949], Sutherland [1967] and Cheng and Clyde [1972] were among the first researchers who emphasized the role of fluctuating forces, exceeding the temporal mean hydrodynamic force, on inception of particle entrainment. By the traditional interpretation of incipient motion, a sediment grain can be displaced from a particle configuration, if sufficient hydrodynamic force, above a critical level, is applied to it. However, these “extreme” high-magnitude turbulent events are usually limited in their duration, which may render them ineffective for completely dislodging a grain from its initial position in the bed matrix. In contrast, turbulent events of not as high magnitude (still exceeding a critical minimum value) but of sufficient duration may transfer enough momentum over this longer duration for the particle to not only initiate motion but be fully displaced from its local arrangement in the bed matrix. This notion implies that the duration of turbulent structures, as well as the magnitude of the force they exert, is of significance when the full displacement of the grain is considered.

[7] In an effort to identify the relationship between local instantaneous forces and particle movement, a laboratory flume experiment was performed at near threshold conditions. A tilting flume 0.6 m wide and 20.5 m long, located in the Baker Environmental Hydraulics laboratory was used for this purpose. A two-component laser Doppler velocimeter (LDV) with average sampling rate of about 500 Hz, was used to determine the instantaneous streamwise velocity history one diameter upstream and along the centerline of a 12.7 mm diameter spherical Teflon® particle (Figure 1a). The particle is fully exposed, resting on top of four layers of 8 mm glass beads. The particle is free to move downstream due to the near bed flow. However, further entrainment is retained by a carefully placed pin, which sends the particle back into its initial local bed configuration, so that a continuous record of displacement events can be obtained (Figure 1b). For a flow having a depth averaged velocity of 0.2 m/s, friction velocity of 0.025 m/s and depth of 7.2 cm, the entrainment rate of the exposed particle was on average close to two full entrainments per minute. A He-Ne laser and photomultiplier were used simultaneously with the LDV to identify and encode the displacement of the mobile sphere, in real time (Figure 1a). The He-Ne laser beam targets the photomultiplier and is partially blocked by the initially resting particle (Figure 1a). At the displaced position the particle completely blocks the beam affecting the reading of the photomultiplier. In this manner, grain displacement is automatically and accurately detected. Excerpts from the complete velocity (LDV) and dislodgement (He-Ne) record are shown in Figure 1b. In Figure 1b the instants of full grain dislodgement is indicated with dashed vertical lines. The results from these tests consistently indicated that only a few of the high-magnitude local velocity events resulted in full particle dislodgement (examples in Figure 1b are events B, C, E, and F), even though their magnitude was well in excess of the minimum velocity required for a net unbalanced force on the particle, assuming the standard drag formula parameterization. For example, over the duration (approximately 30 min) of the experiment, only 65 out of 1465 above threshold events resulted in entrainment [Diplas et al., 2008]. Closer observation of the record suggests that although all the velocity spikes have relatively short duration, of the order of several or tens of milliseconds (ms), the ones that did not cause dislodgement appear to be particularly short lived (e.g., events A and D in Figure 1b). Apparently, these extreme
magnitude, but short-lived, events may initiate grain movement but do not last sufficiently long to fully dislodge it. This vibration or pivoting of a particle has been observed in the laboratory from our own flume experiments and in the field as well [e.g., Garcia et al., 2007].

Given the highly fluctuating nature of turbulent flow, and associated hydrodynamic forces, there is a need to refine the currently used definition for initiation of motion. In addition to a local, instantaneous force magnitude that exceeds a critical value ($F_{crit}$ – the horizontal dashed line in Figure 1c), particle dislodgement requires that the force be applied for an adequate period of time, to transfer sufficient momentum to the particle to carry it out of its initial grain configuration. Thus as shown in Figure 1c, though both events (D, E) are strong enough to commence particle movement, only event E lasts sufficiently long to accomplish dislodgement. A parameter that captures both force magnitude and duration simultaneously is impulse ($I_i$), defined as the integral of the hydrodynamic net force over the duration of the force ($T_i$):

$$I_i = \int_{t_i}^{t_i+T_i} F(t) dt$$

3. Impulse Criterion Formulation

The threshold force characteristics leading to full displacement of a spherical grain by either saltation or rolling is the focus of this paper. Both of these modes have a probability of occurrence, depending on local grain arrangement and flow conditions. For the limiting case of an entirely hidden particle, the forces that may trigger its initially upward movement (or incipient saltation) are lift forces, while for the case of an exposed grain both drag and lift play a role in mobilizing the grain, usually by rolling, for conditions very close to threshold (Figures 2a and 2b, respectively). In the analytical models developed here, dislodgement via saltation is considered accomplished when the grain is lifted a characteristic height, which for saltation of coarse grains in water is of the order of one particle diameter [e.g., Nino and Garcia, 1994]. Full dislodgement by rolling is achieved when the grain reaches the topmost location in its local configuration. In both cases the grain reaches a position of higher exposure to the near bed flow which eventually results in its further entrainment downstream.

3.1. Incipient Saltation

The interaction of lift forces with sediment particles has been pursued analytically both from a deterministic [e.g., Jeffreys, 1929; Benedict and Christensen, 1972; Ling, 1995; Nino et al., 2003] and probabilistic viewpoint [e.g., Cheng and Chiew, 1998; Wu and Lin, 2002; Wu and Yang, 2004] by means of statistical treatment of flow and grain parameters. Saltation of coarse bed material has also been investigated experimentally providing useful statistics such as lift intensities [Chepil, 1958; Cheng and Clyde, 1972; Mollinger and Nieuwstadt, 1996] and even grain trajectories [Abbott and Francis, 1977; Nino and Garcia, 1994]. Einstein and El-Samni [1949] experimentally measured the lift forces generated due to pressure differences between the top and bottom of hemispheres. They were the first to emphasize the significance of rapidly fluctuating lift forces due to the instantaneous variation of the near rough bed pressure field, varying over a range of the same order of magnitude as the mean lift. More recently, Mollinger and Nieuwstadt [1996] measured directly lift forces on a sphere lying on a smooth bed, finding intensities (ratio of root mean square to mean values) of 2.8. Thus for the accurate prediction of incipient saltation, one should account for...
The two limiting cases of local bed particle—

\[ F_{L} = C_{L} \beta_{L} \rho_{f} \rho_{m} \frac{V_{g}^{2}}{2} \] 

where \( C_{L} \) is the lift coefficient, \( \beta_{L} \) is the lift factor, \( \rho_{f} \) is the fluid density, \( \rho_{m} \) is the particle density, \( V_{g} \) is the particle velocity, and \( \frac{V_{g}^{2}}{2} \) is the particle energy.

such peaks in the applied fluid forces. Here, the above rationale is extended to include not only the peaks in magnitude, but also the duration of the applied fluctuating hydrodynamic lift. This is accomplished by considering the temporal history or duration of such peak events and treating them as square pulses capable of imparting sufficient momentum to the grain to cause its full displacement normal to the boundary.

3.1.1. Equation of Motion

For the analytical model of grain saltation, spherical particles are considered. The only forces applied upon the grain in its initial position are the hydrodynamic lift and buoyancy forces and its weight (Figure 2a). This is representative of the case when the grain under consideration is surrounded by similar size particles forming a closely packed configuration, rendering hydrodynamic drag ineffective, due to underexposure. In this context, the trajectory of a saltating grain consists of a path that is initially nearly normal to the boundary [Bagnold, 1973]. The more the grain becomes exposed into the flow, the more the drag forces become effective in its downstream entrainment, which results in a projectile-like motion (e.g., path OA, in Figure 2a). The initial portion of the trajectory (OA in Figure 2a) due to the application of an impulsive lift force is modeled here. Consistent with the definition of full displacement by saltation, the particle must reach or exceed a threshold height, of the order of one grain diameter \( z_{\text{max}} = 2R_{m} \), where \( R_{m} \) is the radius of the mobile sphere. This may be seen as the minimum required displacement for the grain to pass over neighboring grains of about the same size [Abbott and Francis, 1977]. Lift forces triggering entrainment by saltation are assumed to be of high magnitude \( (F_{L}) \) compared to the resisting forces and of relatively short duration \( (T_{\text{salt}}) \) compared to the response time of the particle or \( t_{OA} \), which is the time required for the particle to reach the maximum displacement height, \( z_{\text{max}} \). As in Diplas et al. [2008] such forcing events can be modeled as square pulses, \( F_{L}(1 - H(t - T_{\text{salt}})) \) (shown as an inset in Figure 2a), where \( H(t - T_{\text{salt}}) \) is the unit step (Heaviside) function, defined by

\[ H(t - T_{\text{salt}}) = \begin{cases} 0, & t < T_{\text{salt}} \\ 1, & t \geq T_{\text{salt}} \end{cases} \]

\[ V(\rho_{s} + \rho_{f}C_{m}) \frac{d^2z(t)}{dt^2} = F_{L}(1 - H(t - T_{\text{salt}})) - W \cos \alpha + B_{f} \] 

where \( C_{m} \) is the added mass coefficient, which equals 0.5 for spheres in water [Auton et al., 1988], \( W = \rho_{s}V_{g}g \), is the particle’s weight, \( V \) its volume (\( V = \frac{4}{3}\pi R_{m}^{3} \)), \( B_{f} = \rho_{f}V_{g}\cos \alpha \), is the buoyancy force (acting normal to the bed surface for uniform or gradually varied flows following Christensen [1995]) and \( z(t) \) is the normal to the bed trajectory component of the particle’s center of mass at time \( t \).

3.1.2. Results

The equation of motion along the \( z \) direction (using the Cartesian coordinate system shown in Figure 2a), accounting for added mass and variable bed slope, \( \alpha \), is

\[ z(t) = \frac{\cos \alpha(\rho_{s} - \rho_{f})g t^{2}}{2(\rho_{s} + \rho_{f}C_{m})} \left[ F_{L} \left(1 - \left(1 - \frac{T_{\text{salt}}}{t}\right)^{2}\right) - 1\right], \quad \text{for} \quad t \geq T_{\text{salt}} \] 

with \( F_{R} = \cos \alpha(\rho_{s} - \rho_{f})V_{g} \), the sum of resisting forces in \( z \) direction. Equation (4) describes the deceleration phase of the grain after the cessation of impulsive lift force application. The maximum displacement height, \( z_{\text{max}} \), is obtained from equation (4), considering at that position the grain has no upward velocity (\( \frac{dz}{dt} = 0 \)):

\[ z_{\text{max}} = \frac{\cos \alpha(\rho_{s} - \rho_{f})g T_{\text{salt}}^{2}}{2(\rho_{s} + \rho_{f}C_{m})} \left[ \frac{F_{L}}{F_{R}} \left(1 - \frac{F_{L}}{F_{R}}\right)\right] \]
Equation (5) can be used to determine the required duration of the applied lift force, $F_L$, given specified dislodgement height, $z_{\text{max}}$:

$$T_{\text{salt}} = \frac{F_R}{F_L} \sqrt{\frac{(\rho_s + \rho_f C_m)}{(\rho_s - \rho_f) g}} \frac{2}{(1 - \frac{F_R}{F_L})} \cos \alpha \frac{z_{\text{max}}}{d_{\text{max}}^2}$$

Assuming that full entrainment by saltation occurs for $z_{\text{max}} = 2R_m$, the required duration $T_{\text{salt}}$ becomes:

$$T_{\text{salt}} = \frac{F_R}{F_L} \sqrt{\frac{4R_m (\rho_s + \rho_f C_m)}{\cos \alpha (\rho_s - \rho_f) g}} \left(1 - \frac{F_R}{F_L}\right)$$

[14] Utilizing the component of resisting force in $z$ direction ($F_R$) and the time required for the free fall of the grain from elevation $h_{\text{max}} = 2R_m \cos \alpha$, $t_{\text{ff}} = \sqrt{\frac{2h_{\text{max}} (\rho_s + \rho_f C_m)}{g \rho_s - \rho_f}}$, equation (5) can be normalized:

$$\frac{\dot{z}}{\dot{z}_{\text{max}}} = \frac{\dot{T}_{\text{salt}}}{T_{\text{salt}}^2} \left(1 - \frac{F_R}{F_L}\right)$$

where $\dot{z} = \frac{z_{\text{max}}}{t_{\text{ff}}}$ is the normalized displacement and $\dot{F}_L = \frac{F_R}{F_L}$ and $\dot{T}_{\text{salt}} = \frac{T_{\text{salt}}}{t_{\text{ff}}}$ denote normalized lift force and duration, respectively. Equation (8) may be used to illustrate the significance of both duration of the applied lift force as well as its magnitude, as shown in Figure 3, for complete entrainment ($\dot{z} = 1$). This normalized threshold curve (Figure 3), represents combinations of normalized lift force and duration which identify events that are barely sufficient to lead to full displacement. Events above the threshold curve lead to complete entrainment of the grain while events below the curve are insufficient to dislodge the grain, according to the definition employed here. The lift force required to dislodge the grain increases in magnitude as its duration decreases, in an almost inverse fashion. A 50% decrease in normalized duration (from 0.50 to 0.25, which for coarse gravel is of the order of tens of milliseconds), requires an 80% increase of lift force magnitude (from 2.5 to 4.5). Near bed turbulent flows exhibit a range of different time scales of coherent structures. Due to the variability in the duration of such turbulent forcing events, prediction of entrainment based solely on criteria relevant to the magnitude of flow alone, cannot be sufficient.

[15] However, the normalized impulse, $I_{\text{salt}}$, which for saltation can be expressed as the product of normalized lift force with normalized duration, is observed to remain almost constant, for infinitesimally small normalized durations (from equation (8): $\lim_{I_{\text{salt}} \to 0} I_{\text{salt}} = 1$, for $z_{\text{max}} = 2R_m$). For the previously mentioned examples, normalized impulse has a value of 1.25 and 1.125, respectively. While for a range of possible durations, the critical lift forces vary over an order of magnitude (Figure 3), critical impulse exhibits only a small relative change. The significance of this observation becomes more apparent when the variability of the instantaneous fluctuating lift forces is considered. As an example Schmeeckle and Nelson [2003] measured “upward events” of magnitude close to six times the average lift force, for the case of a sheltered grain in a gravel bed. Since high normalized lift forces are typical for entrainment by saltation, the constancy of impulse to a value close to one renders it a robust criterion for prediction of this type of movement.

[16] For lower values of normalized lift force (or higher values of normalized duration), the displacement $z(T_{\text{salt}})$ will be greater, considering equations (4) and (7). Higher values of $z(T_{\text{salt}})$ imply greater exposure to the mean flow when the lift has ceased to be applied. Subsequently, the drag force may impart additional momentum for the grain’s entrainment. This observation conceptually illustrates that even in such cases that lie in between pure saltation and pure rolling, the constancy of normalized saltation impulse to a value close to one is not necessarily a conservative approximation.

3.2. Incipient Rolling

[17] Similar to saltation, incipient movement of a sediment grain by rolling has also been studied in the past, using either a theoretical deterministic [White, 1940; Coleman, 1967; Komar and Li, 1988; James, 1990; Ling, 1995] or stochastic approach [Papanicolaou et al., 2002; Wu and Chou, 2003; Wu and Yang, 2004; Hofland and Battjes, 2006]. Other researchers have experimentally investigated the effect of hydrodynamic forces acting on spherical particles along with the effects of the local grain arrangement or flow parameters, such as relative grain protrusion [Fenton and Abbott, 1977; Kirchner et al., 1990; Chin and Chiew, 1993] and relative flow depth [Shvidchenko and Pender, 2000] and slope [Lam Lau and Engel, 1999; Bey and Debnath, 2000; Gregoretti, 2001, 2008; Recking, 2009], or solved the equations of motion of individual grains to obtain their trajectories, considering the instantaneous forces acting on them [McEwan and Heald, 2001; Schmeeckle and Nelson, 2003; Valyarakis et al., 2008]. Recently the effect of turbulent fluctuations of the local velocity and pressure field on sediment mobilization has received a lot of attention [Dancey et al., 2002; Zanke, 2003; Hofland et al., 2005; Schmeeckle et al., 2007; Smart and Habersack, 2007; Vollmer and Kleinhans, 2007]. In the following analysis the role of duration of applied hydrodynamic forces in addition to their magnitude is investigated.
where $F_N$, represents the reaction force, acting normal to the contact surfaces, with direction toward the center of the base particles. The mobilization condition can be expressed as

$$\frac{d^2 \theta}{dt^2} > 0 \quad (9b)$$

[19] Pigozzi et al. [2007] have theoretically shown that the particle follows the minimum resistance path (OA'), rolling between the two downstream supporting particles for the arrangement shown in Figure 2b. Similar to the incipient saltation analysis, the short-lived hydrodynamic forces acting on the mobile sphere can be represented by a step (Heaviside) function of certain duration ($T_{roll}$) and magnitude (Figure 2b inset). Then the equations of motion in polar coordinates, $\theta$ and $\xi$, [e.g., Schmeeckle and Nelson, 2003] considering equation (9a), obtain the following form:

$$L_{arm} \left( \frac{7}{5} \rho_s + \rho_f C_m \right) V \frac{d^2 \theta}{dt^2} = [F_D \sin(\theta - \alpha) + F_L \cos(\theta - \alpha)]$$

$$\cdot \left[ 1 - H(t-T_{roll}) \right] + B_f \cos(\theta - \alpha) - W \cos \theta \quad (10a)$$

$$L_{arm}(\rho_s + \rho_f C_m) V \left( \frac{d\theta}{dt} \right)^2 + F_N = [F_L \sin(\theta - \alpha) - F_D \cos(\theta - \alpha)]$$

$$\cdot \left[ 1 - H(t-T_{roll}) \right] + B_f \sin(\theta - \alpha) - W \sin \theta$$

$$\quad (10b)$$

where $L_{arm}$ is the lever arm defined as the distance between the center of gravity (O) and point of rotation (D) of the rolling grain ($L_{arm} = OD = \sqrt{R^2 + 2RmR_b}$, Figure 4).

3.2.2. Results

[20] The pure rolling of the grain while retaining contact with the base particles can be described from equation (10a). This occurs only for certain combinations of drag and lift which considering the equilibrium of forces in Figure 4 can be expressed as

$$\Sigma F_\theta = F_D \sin(\theta_0 - \alpha) + (F_L + B_f) \cos(\theta_0 - \alpha) - W \cos \theta_0 > 0 \quad (11a)$$

$$\Sigma F_\xi = -F_D \cos(\theta_0 - \alpha) + (F_L + B_f) \sin(\theta_0 - \alpha) - W \sin \theta_0 < 0 \quad (11b)$$

where $\Sigma F_\xi$ and $\Sigma F_\theta$ represent the sum of forces in the radial, $\xi$, and tangential, $\theta$, directions at the rest position, respectively.

[21] Pure rolling can be described in two phases; an accelerating phase due to the action of short-lived, high-magnitude, hydrodynamic forces and a decelerating one considering only the resisting forces.

[22] If the duration of the acceleration phase ($t < T_{roll}$), is relatively short, small angular displacements will occur.
which are functions and
\[ \cos \theta \sin \alpha, \quad \text{equation } 16 \]
and 
\[ \sqrt{\frac{2W\rho_0}{\Sigma F_\theta}} \]
with 
\[ \rho_0 = (1 - \sin \theta_0) \left( \frac{\cos \theta_0}{\sin \alpha - \cos \alpha \frac{\Sigma F_\theta}{\Sigma F_\xi}} \right), \]
a coefficient incorporating the effects of initial geometrical arrangement and the relative density of fluid and solid grain. Equation (16) is the counterpart to equation (7) in the salitation analysis, now for rolling. The components of total force in both tangential and radial directions as well as their duration can be normalized utilizing the weight of the grain and the time required for free fall from a height proportional to \((1 - \sin \theta_0)\rho_0\), which is the elevation difference between the initial and topmost positions, respectively. Thus, using the normalized variables 
\[ T_{\text{roll}} = \sqrt{2W\rho_0 W_{\text{mod}} / L_{\text{arm}}}, \quad \Sigma F_\xi = W_{\text{mod}} / L_{\text{arm}}, \quad \Sigma F_\theta = \Sigma F_\xi, \quad \text{equation } 16 \]
becomes
\[ T_{\text{roll}} = \sqrt{\frac{1}{L_{\text{arm}} m_{\text{mod}}} - \Sigma F_\xi \arc \sin \left( \sqrt{\frac{-\Sigma F_\xi}{\Sigma F_\theta}} \right)} \]

Figure 5. Threshold surface predicting the critical pulse characteristics (normalized hydrodynamic drag, \( \Sigma F_\theta \), and lift, \( \Sigma F_\xi \), forces and normalized duration, \( T_{\text{roll}} \)) for onset of entrainment.

\((\Delta \theta \ll \theta_0)\). With this approximation the equation (10a), can be linearized to allow for an analytical solution (see Appendix A):

\[ L_{\text{arm}} m_{\text{mod}} \frac{\text{d}^2 (\Delta \theta)}{\text{d} t^2} = \Sigma F_\theta - (\Delta \theta) \Sigma F_\xi \geq 0 \]

where 
\[ m_{\text{mod}} = (\frac{2}{3} \rho_s + \rho_f C_m) V, \]
which holds for \( t < T_{\text{roll}} \). Assuming \( \Delta \theta(t = 0) = 0 \) and \( \Delta \theta'(t = 0) = 0 \), as the initial conditions the angular displacement, \( \Delta \theta \), (for \( t < T \)) is obtained from the solution of equation (12):

\[ \Delta \theta = \frac{\Sigma F_\theta}{-\Sigma F_\xi} \left\{ \cosh \left( \sqrt{\frac{-\Sigma F_\xi}{L_{\text{arm}} m_{\text{mod}}}} \right) - 1 \right\} \]

[23] At time \( t = T_{\text{roll}} \), when the impulsive forces cease, the particle will have a new position \( \theta(t = T_{\text{roll}}) = \theta_0 + \Delta \theta \), with angular velocity, \( \Delta \theta'(t = T_{\text{roll}}) \), calculated by taking the derivative of equation (13):

\[ \Delta \theta'(t = T_{\text{roll}}) = \frac{\Sigma F_\theta}{\sqrt{L_{\text{arm}} m_{\text{mod}}} \sqrt{-\Sigma F_\xi}} \sin \left( \sqrt{\frac{-\Sigma F_\xi}{L_{\text{arm}} m_{\text{mod}}}} \right) \]

[24] In this analysis, for the sake of simplicity, only the impulsive drag and lift forces responsible for mobilizing the grain are considered. Thus for \( t > T_{\text{roll}} \), the particle decelerates, due to the action of the gravitational force component. The equation of motion in the \( \theta \) direction becomes

\[ L_{\text{arm}} m_{\text{mod}} \frac{\text{d}^2 \theta}{\text{d} t^2} = B_f \cos \theta - W \cos \theta \]

[25] According to the definition of entrainment by rolling, the applied impulse has to impart enough momentum so that the grain at time \( t = T_{\text{roll}} \), will reach the topmost position, \( \theta(t = T_{\text{roll}}) = \pi/2 \), with zero tangential velocity, \( \Delta \theta'(t = T_{\text{roll}}) = 0 \). With these conditions and equations (13) for \( t = T_{\text{roll}} \) and (14), the duration of impulse necessary for grain dislodgement, as a function of the force components, particle and local geometry characteristics is obtained from equation (15), by multiplication of all parts with the angular velocity, \( \Delta \theta' \) and subsequent integration (see Appendix):

\[ T_{\text{roll}} = \sqrt{\frac{L_{\text{arm}} m_{\text{mod}}}{\Sigma F_\xi} \arc \sin \left( \sqrt{\frac{-\Sigma F_\xi}{\Sigma F_\theta}} \right)} \]

[26] The result is shown in Figure 5, as a surface plot of normalized duration \( T_{\text{roll}} \), which are functions of lift and drag normalized by the weight of the mobile grain. Points at or above the threshold surface denote combinations of duration, drag and lift force, which will result in complete entrainment by rolling. A weak dependence of \( T_{\text{roll}} \) for a wide range of negative values of \( \Sigma F_\xi \) is observed (Figure 5). This implies that the role of the normalized radial component of the total force is limited to setting a constraint for the ratio of effective hydrodynamic forces, so that movement occurs in rolling mode. However, a rather significant dependence on the normalized tangential component of the resultant forces is seen. \( \Sigma F_\theta \) is almost inversely related to \( T_{\text{roll}} \) (Figure 5), meaning that for increasing magnitude of this component, the required duration for triggering particle motion is reduced. For instance, for the limiting case of \( \Sigma F_\xi = 0 \), equation (17) predicts (by application of l’Hospital’s rule) that \( T_{\text{roll}} \) and \( \Sigma F_\theta \) are inversely related:

\[ \lim_{\Sigma F_\xi \to 0} T_{\text{roll}} = \frac{1}{\Sigma F_\theta} \]

[27] Even though the magnitude of \( \Sigma F_\theta \) may exhibit a variation of more than an order of magnitude, the normalized impulse, \( I_{\text{roll}} \), defined as the product of \( T_{\text{roll}} \) and \( \Sigma F_\theta \) retains a constant value of unity, according to equation (18). Similar to the case of entrainment by salitation, the observed constancy or small variation of normalized impulse for a wide range of forcing conditions makes impulse a valuable criterion for coarse grain entrainment prediction.

[28] The threshold conditions for complete grain entrainment are highly dependent on the local particle configuration, such as lever arm and pivoting angle as shown from equation (16). A practical dimensional example is provided in the following. For instance, assume an 8 mm
quartz sphere ($\rho_s = 2650$ kg/m$^3$), submersed in water ($\rho_f = 1000$ kg/m$^3$), resting on an arrangement of three closely packed base particles of the same radius (8 mm), with zero bed slope ($\alpha = 0$). For different levels of lift force the critical combinations of drag force and duration are determined from equation (16), while satisfying the constraints for rolling mode. The relation between drag force magnitude and duration for zero lift force is plotted in Figure 6, for a range of durations. Similar to the case of entrainment by saltation, only combinations of the driving force (here drag) and duration which fall above the threshold curve correspond to complete entrainment.

The accuracy of the analytical solution of the linearized equations of motion for rolling depends on the linearization approximation that the angular displacement, $\Delta \theta$, is relatively small during the application of the hydrodynamic forces. This assumption is correct for relatively short durations or high magnitudes of force, which is assessed by numerically solving the equations of motion (e.g., equation (10a)). The critical impulse calculated from the theoretical linearized model, is slightly overpredicted compared to the numerical obtained solution. The ratio of excess impulse (due to positive grain velocity at the topmost position) over the imparted impulse can be used as an indicator to quantitatively evaluate the magnitude of error, as shown in Figure 7. It is observed that for relatively high magnitudes of drag force (e.g., 3 times the particle submerged weight) the error is about 5% (with a maximum error of 12% for drag force of the order of the particle submerged weight). Considering that such extreme forces are expected when displacing a grain by saltation, the accuracy of the linearized solution is high.

The required duration is almost inversely related to the applied drag force (not qualitatively different from the case of saltation). Thus as in the case of displacement by saltation, the magnitude of the force alone is insufficient to predict the complete removal of a spherical grain from its local position in the bed matrix. That is, given the short-lived nature of the applied hydrodynamic forces, information about the duration is relevant and necessary to fully capture the physics of the entrainment process. This conclusion is verified experimentally in the section 4 and the role of impulse is demonstrated to be the appropriate parameter to characterize the critical condition for entrainment.

4. Electromagnet Experiments

To test the theoretical analysis, and demonstrate the relevance of impulse to grain dislodgment, a set of controlled bench top experiments, modeling the incipient motion of a fully exposed spherical grain by rolling, were performed with the use of an electromagnet.

4.1. Description of Setup and Experimental Process

The setup consisted of a mobile, steel sphere, located on top of a horizontal layer of fixed Teflon® spherical particles and an electromagnet through which forces of defined duration and magnitude could be applied in a controlled manner [Diplas et al., 2008]. The triangular array of base particles was fixed on a horizontal plate of adjustable height and lateral position, permitting the precise positioning of the center of the mobile grain along the centerline of the electromagnet, with a known distance from its face ($h$). The centers of the particles in the test fixture form a tetrahedron, which is the same local arrangement employed in the theoretical rolling analysis (Figure 4 inset). The experiments were performed in still air. The setup also included a data acquisition board (DAQ), a signal processor and a circuit used for voltage amplification. Appropriate software was used on a personal computer to generate user defined series of electromagnetic pulses of varying duration and magnitude. The circuit amplifies the signal sent by the DAQ system to the magnet, reads the voltage drop across the electromagnet as well as the current and then deamplifies these signals to an appropriate range to be read back in by the DAQ. A total of seven different local arrangements were tested (Table 1), either by changing the distance of the electromagnet from the mobile grain ($h$), or by varying the ratio of the radii of mobile to base particles ($R_m/R_b$). Each of the runs is generally named after the combination of diameters of mobile and base particles. A high-speed camera was used to accurately monitor the displacement of the steel particle. Careful observation of the
videos verified the assumption that entrainment of the spherical particles occurred only by rolling (in the absence of sliding).

When current passes through the electromagnet coil, an electromagnetic field is generated, resulting in a force which, depending on its magnitude and duration, may mobilize the steel particle. Here, the electromagnetic force is considered to simulate, in a simplified way, the drag force acting on a grain due to a peak in the local streamwise velocity component. However, rather than rapidly fluctuating hydraulic forces of unknown magnitude and durations, typically encountered in near boundary turbulent flow, these parameters are accurately controlled through suitable software and circuitry. Electromagnetic forces are applied in the form of square pulses characterized by certain values of force magnitude, $F_e$, and duration, $T_e$. For a given particle configuration, the magnitude of force is related to the voltage across the electromagnet, $V_e(t)$, via the following relation:

$$F_e(t) = \frac{N_e^2 \mu_e A_e}{2h_e r_e} V_e(t)^2$$

(19)

where $N_e$ is the number of turns of the wire in the coil of the electromagnet, $A_e$ is the characteristic area of the face of the electromagnet, $\mu_e$ is the magnetic permeability of the medium (here air) and $r_e$ the resistance of the electromagnet. The above equation is derived by considering the energy contained in the electromagnetic field, assuming that the magnetic flux induced by the electromagnet does not change when the particle is displaced from its initial location. Since for a given grain arrangement (i) the values of the coefficients of the electromagnet as well as the parameters specific to the grain arrangement, $A_e$ and $h_e$, are fixed, the combination of terms may be represented by a constant, $c_e$, and the applied force is directly proportional to the square of applied voltage ($F_e = c_e V_e^2$). Thus it is possible to apply an electromagnetic pulse of specific duration and magnitude with this design, by controlling the voltage across the electromagnet.

For a particular grain arrangement a search for the critical $F_e$, $T_e$ combinations was performed, from which threshold curves could be obtained. The applied voltages ranged from 2.5 to 15 Volts, with a minimum voltage step size of 0.25 Volts, while the duration of their application varied from 3 to 75 ms with a time step of 2–4 ms. First, a series of electromagnetic pulses of sufficiently long duration (1.5 s), representing nearly steady state forcing conditions, were applied with increasing amplitude, until the particle is displaced. In this manner, the minimum voltage, $V_{\text{min}}$, required for particle removal was determined for each configuration. For a given particle configuration, two different strategies can be followed to find the minimum pulses that fully displace the particle; either the voltage ($V_e > V_{\text{min}}$), is steadily increased for constant duration (Figure 8a) or for constant voltage ($V_e > V_{\text{min}}$) the duration is gradually increased (Figure 8b). For both cases the search is completed when the particle is entirely displaced from its initial location.

Figure 9, is typical of the results of the followed procedure. In Figure 9, ($T_e$, $V_e^2$) data pairs are presented for a 4 mm steel ball placed on a triangular arrangement of 4 mm base particles (run 4–4). For fixed pulse duration voltage was gradually increased until the sphere was barely dislodged from its initial arrangement. Then the duration was adjusted to a new fixed value and the above procedure was repeated. After a wide range of $T_e$, $F_e$ combinations have been explored, the threshold curve indicative of the necessary and sufficient impulsive force characteristics for complete grain dislodgment, should lie between the points signifying “movement” and those just “prior to movement” (Figure 9). Following the same procedure, the seven sets of runs corresponding to different particle arrangements with varying mobile and base particle diameters (from 4 to 8 mm), were completed. A total of 1709 data points were obtained (Table 1), which are used to derive the threshold curves for each particle configuration. Each experiment was performed up to three times to ensure repeatability and accuracy.

### Table 1. Characteristics of Different Particle Configurations and the Best Fit Curves Derived From the Data Points for Which Movement Was Observed

<table>
<thead>
<tr>
<th>Set of Runs</th>
<th>$2R_m$ (mm)</th>
<th>$2R_b$ (mm)</th>
<th>$h_e$ (mm)</th>
<th>Number of Runs</th>
<th>Best Fit Equation</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4–4</td>
<td>4</td>
<td>4</td>
<td>3.15</td>
<td>224</td>
<td>$F_e^2 = 449.37T_e^{1.03}$</td>
<td>0.96</td>
</tr>
<tr>
<td>6–6</td>
<td>6</td>
<td>6</td>
<td>4.73</td>
<td>184</td>
<td>$F_e^2 = 745.37T_e^{1.00}$</td>
<td>0.98</td>
</tr>
<tr>
<td>8–8</td>
<td>8</td>
<td>8</td>
<td>6.31</td>
<td>286</td>
<td>$F_e^2 = 958.17T_e^{1.04}$</td>
<td>0.98</td>
</tr>
<tr>
<td>6–8</td>
<td>6</td>
<td>6</td>
<td>6.31</td>
<td>273</td>
<td>$F_e^2 = 1714.57T_e^{1.07}$</td>
<td>0.98</td>
</tr>
<tr>
<td>6–6b</td>
<td>6</td>
<td>6</td>
<td>6.31</td>
<td>260</td>
<td>$F_e^2 = 1048.07T_e^{1.09}$</td>
<td>0.98</td>
</tr>
<tr>
<td>4–6</td>
<td>4</td>
<td>4</td>
<td>6.31</td>
<td>239</td>
<td>$F_e^2 = 1109.77T_e^{0.99}$</td>
<td>0.98</td>
</tr>
</tbody>
</table>

Figure 8. Representation of the programmable voltages for the generation of electromagnetic pulses by (a) varying the voltage levels for fixed duration and (b) changing the duration for fixed magnitude.
4.2. Results

From the presentation of obtained data points as well as the threshold curves it is evident that a trend consistent with the theoretical results is observed (e.g., Figure 9, for run 4–4). Representative best fits for the group of data points for which movement was observed are obtained by matching a power equation of the form $V_e^2 = k T_e^q$, with $k, q$ coefficients characteristic of each arrangement. For all sets of runs the correlation coefficient of the best fit curves is quite high ($R^2 > 0.95$, Table 1). For instance, the best fit equation for run 4–4 is given by equation $V_e^2 = 449.3 T_e^{-0.03}$ (continuous curve in Figure 9), with a correlation coefficient $R^2 = 0.96$. Comparison of the critical curves for entrainment obtained from a best fit to the observed data for each run, with the threshold curves predicted from rolling analysis, equation (16), reveals the accuracy of the analytical approach. As an example for the case of run 4–4, an almost perfect match between the two curves is seen. In agreement with the results of the theoretical analysis, while the range of critical electromagnetic force ($F_e$ parameterized by $V_e^2$), changes over an order of magnitude (e.g., from 13 to 195 $V^2$ – Figure 9), the impulse computed as the product $F_e T_e$, remains almost invariable over the range of possible durations (e.g., for 4–4: $I_e = 449.3 T_e^{-0.03}$, for $T_e$ from 6 to 46 ms, the change in $I_e$ is less than 6%).

Figure 9. Plot of threshold curves and $T_e, V^2$ combinations for run 4–4. Two critical curves are shown: the first obtained from the best fit of the data points for which entrainment was observed (continuous line) and the second predicted from equation (16) (dashed line, scaled appropriately to account for $c_i$).

Figure 10. Data points of normalized magnetic force (parameterized by $\hat{V}^2$) and duration, $\hat{T}_e$, for which movement was observed for all the runs. The best fit (dashed) curve and the threshold curve predicted from equation (18) (continuous line) are also shown.
of $T_e$ and $F_c$ with the maximum duration and smallest force magnitude required for particle dislodgment ($T_{c,max}$, $F_{c,min}$). Thus the normalized values of magnetic force and duration are $\hat{F}_c = \frac{F_{c}}{F_{c,min}} = \frac{c V_{c}^2}{V_{c,min}^2} = \hat{V}_c^2$ and $\hat{T}_c = \frac{T_{c}}{T_{c,max}}$, respectively, where $V_{c,min}$ and $T_{c,max}$ vary for each combination of mobile and base grains. Diplas et al. [2008] provide a best fit for these data ($\hat{V}_c^2 = 0.96 T_c^{0.99}$, see dashed curve in Figure 10), with a coefficient of determination of $R^2 = 0.96$. The normalized threshold curve predicted from the rolling analysis (continuous line in Figure 10), from equation (18), provides a good fit to the normalized experimental data and almost overlaps with the best fit curve. In this context and in agreement with the analytical treatment of the phenomenon, a constant normalized impulse, defined as the product of normalized force magnitude and normalized duration of its application ($I_c = F_c T_c = 0.96 T_c^{0.01} \sim$ constant), successfully describes the full entrainment of a particle.

5. Discussion

In the following, a comparison of the critical conditions for entrainment of a particle by saltation and rolling is explored, for a specific grain arrangement. Then, the critical conditions, for both modes, are expressed in terms of the normalized impulse, to generalize the results irrespective of the geometrical characteristics of particle configuration. Finally, the impulse concept is extended to describe different levels of motions.

5.1. Comparison of Saltation and Rolling Thresholds: Force-Duration Representation

The modes of grain movement are not only dependent on the local geometrical configuration, but also on the ratio of the relative magnitude of instantaneous drag and lift forces acting on it. This implies that even for the limiting case of a fully exposed particle, saltation could occur under particular flow conditions. Rolling occurs when the exposed particle retains contact with the downstream base particles, throughout its movement (e.g., $O'A'$, in Figure 2). This is true for ratios of drag to lift that satisfy equations (9a) and (9b), or for simplicity considering static equilibrium of forces, equations (11a) and (11b); $\sum F_D > 0$ and $\sum F_L < 0$. Based on the same rationale, for greater lift forces movement will occur by saltation ($\sum F_D > 0$ and $\sum F_L > 0$), while the case of no motion is described by $\sum F_D < 0$. Equation (11a) may be recast into a more meaningful form $F_{D}^{e}(F_L + B_f - W \cos(\theta_0 - \alpha)) > \tan(\theta_0 - \alpha)$, in which the term in the left hand side can be seen as the ratio of drag to a net lift force. Thus, the value of this ratio with respect to $\tan(\theta_0 - \alpha)$, or equivalently the sign of $\sum F_L$, determines the initial mode of entrainment.

This can be illustrated by considering the example of an 8 mm particle, discussed in the section 3.2.2 of the rolling analysis. The different regimes of no motion and movement by rolling or saltation can be defined in the plane of lift and drag forces, normalized by the mobile particle’s weight (Figure 11). The line $\sum F_L = 0$ (continuous line, Figure 11), separates the two modes of dislodgement, while the line $\sum F_D = 0$ (dashed line, Figure 11), partitions the movement from the no-movement regime.

Even though the effect of the instantaneous magnitude of drag and lift on grain entrainment has been extensively investigated [Einstein and El-Samni, 1949; Sutherland, 1967; Nelson et al., 1995; Cheng and Chiew, 1998; Sechet and Le Guennec, 1999; Schmeeckle and Nelson, 2003; Sumer et al., 2003; Zanke, 2003; Hofland et al., 2005; Smart and Habersack, 2007], the role of duration of force is analytically and experimentally demonstrated to be of equal significance. Considering the analytical solutions for the two modes of entrainment (equations (7) and (16)), along with the constraint equations that delimit their boundaries, Figure 11 can be expanded to include duration as a third dimension (Figure 12), for the dimensional example of an exposed particle. In Figure 12, the surface plots representative of the critical $(F_D, F_L, T)$ combinations for saltation and rolling modes are illustrated (Figures 12a and 12b, respectively).

A comparison of the thresholds of the two different modes for the case of a fully exposed grain reveals the need of higher magnitude of forces or duration for the case of saltation as opposed to the rolling mode (Figures 12a and 12b). For instance for the same drag and lift forces (e.g., along line $\sum F_D = 0$), saltation requires longer durations compared to rolling. Similarly for a constant level of force duration, the critical values of both lift and drag are higher if entrainment occurs by saltation (Figures 12a and 12b). Thus the saltation mode of entrainment requires greater impulses compared to the rolling mode, which is intuitively correct. This observation may be generalized to various degrees of exposure (here controlled by angle of repose), for a given particle size. Qualitatively, these results agree with the increase in magnitude of critical Shields’ stress for saltation compared to incipient rolling [Ling, 1995; Wu and Chou, 2003].

The relation of critical force to duration is easier to examine by using vertical cut planes e.g., of constant lift force. For example the plane $F_L = 1.5 F_R = 0.0045N$, is shown to intersect with both saltation and rolling threshold surfaces (thick continuous lines in Figures 12a and 12b, respectively). The resulting threshold curves, characteristic of each mode, are illustrated in Figure 13a. The saltation model does not account for drag force (equation (7)), which
is illustrated by the constancy of duration for different values of this parameter (Figure 13a). Beyond a certain value of drag force (here for $F_D > 1.5F_R$), entrainment occurs in rolling mode. It is worth noticing the relative difference in the magnitude of impulse duration ($T$), between the saltation and rolling regimes. The duration required for entrainment by saltation mode, is almost seven times greater compared to the rolling mode (for the same magnitude of lift and drag forces), which renders displacement of the completely exposed particle by saltation under relatively low impulse values, significantly less probable. An increase of the lift force to a value of $F_L = 1.7F_R$ (Figure 13b), reduces both required durations for saltation and rolling. As a result their ratio is retained almost the same (to a value of seven). The minimum drag force, separating saltation from rolling regimes, also increases (compare dashed vertical lines in Figures 13a and 13b), which is also supported from equation $\Sigma F_D = 0$. For negative values of $\Sigma F_D$, entrainment is possible only by rolling (Figure 12b) and saltation regime vanishes. For instance, for the cut plane $F_L = −1.5F_R$, the characteristic almost inverse relation of drag forces and duration is seen (Figure 13c). Relative to the previous cases of higher impulsive lift forces, relatively higher levels of drag forces are required for entrainment by rolling, which is in agreement with the stabilizing role of negative lift.

5.2. Comparison of Saltation and Rolling Thresholds: Normalized Impulse Representation

[44] The analytically derived normalized solutions of the equations of motion by saltation and rolling (equations (8) and ((17))) can be expressed in terms of the normalized critical impulse, as the product of magnitude of the appropriate driving force with the duration of its application. The normalized critical impulse for saltation, $I_{sal}$, is the product of normalized lift force calculated from equation (8) and normalized duration is

$$I_{sal} = \bar{T}_{sal} \bar{F}_L = \bar{T}_{sal} \bar{F}_L \left(1 + \frac{4z}{\bar{T}_{salt}} \right) \quad (20)$$

[45] According to equation (20), the critical level of impulse for saltation ($z = 1$), depends on normalized dura-

**Figure 12.** Threshold surfaces indicating the critical drag, lift, and duration combinations, for the modes of entrainment by (a) saltation and (b) rolling, for the case of an exposed particle ($R_m = 4$ mm, $R_b = 4$ mm, $\alpha = 0$). The intersection of the saltation and rolling critical surfaces with the vertical cut plane, $F_L = 1.5F_R = 0.0045$N, is emphasized with a thick line to illustrate the separation between the two modes.

**Figure 13.** Threshold curves for movement by saltation or rolling for the case of an exposed particle ($R_m = 4$ mm, $R_b = 4$ mm, $\alpha = 0$) and different values of lift force: (a) $F_L = 1.5F_R$, (b) $F_L = 1.7F_R$, (c) $F_L = −1.5F_R$. 

[44] According to equation (20), the critical level of impulse for saltation ($z = 1$), depends on normalized dura-

**Figure 12.** Threshold surfaces indicating the critical drag, lift, and duration combinations, for the modes of entrainment by (a) saltation and (b) rolling, for the case of an exposed particle ($R_m = 4$ mm, $R_b = 4$ mm, $\alpha = 0$). The intersection of the saltation and rolling critical surfaces with the vertical cut plane, $F_L = 1.5F_R = 0.0045$N, is emphasized with a thick line to illustrate the separation between the two modes.

**Figure 13.** Threshold curves for movement by saltation or rolling for the case of an exposed particle ($R_m = 4$ mm, $R_b = 4$ mm, $\alpha = 0$) and different values of lift force: (a) $F_L = 1.5F_R$, (b) $F_L = 1.7F_R$, (c) $F_L = −1.5F_R$. 

12 of 18
Figure 14. Critical normalized impulse for full dislodgement versus normalized duration as predicted by saltation theory (dashed line), rolling theory (continuous line), and as a linear best fit of the observed data from electromagnet experiments (dashed-dotted line).

The constancy of impulse can be further illustrated from the experimental results contained in this study. Expressing the normalized data (Figure 10) of drag force magnitude (square of voltage) and duration, in terms of their product or impulse, provides the data points shown in Figure 14. A nearly constant value of unity for the critical impulse is computed for different levels of $\Sigma F_\xi$ and $\bar{T}_{roll}$ that are of practical importance. For example for the case of $\Sigma F_\xi = -1$ the divergence from $I_{roll} = 1$ over the range of $\bar{T}_{roll}$ from 0 to 1, is less than 12%. For the limiting case of $\Sigma F_\xi = 0$, equations (18) and (23)) predict a constant value of $I_{roll} = 1$, independent of duration (solid line, in Figure 14). The constancy of impulse can be further illustrated from the theoretical model, to best match the forcing conditions in the electromagnet experiments ($F_L = 0$ and $\alpha = 0$).

5.3. Generalization of Various Degrees of Movement

Consistent with lab and field observations the degree of movement of sediment grains depends on the intensity or strength of the flow. For flow conditions below threshold, the particle may vibrate or slightly hop within its pocket. For near threshold flow conditions these movements become stronger and at certain instances the critical level is reached or exceeded triggering a complete entrainment. Further increase of the flow intensity will result in more frequent and
strong particle entrainments. Saltation or rolling impulse models can be utilized to describe this wide range of grain motions. For instance in accordance to equation (8), different saltation heights ($z$) are expected, depending on the value of the normalized force, $\hat{F}_L$, and duration, $\hat{T}_{salt}$, (or impulse $\hat{I}_{salt}$, considering equation (20)). Even though a definition of those phases of incipient motion is subjective, a classification may be attempted, in order of increasing $z$, as follows (Figure 15):

\begin{align}
0 < z < 0.3, & \text{ vibration} \\
0.3 \leq z < 1, & \text{ hoping} \\
1 < z \leq 2, & \text{ weak entrainment} \\
2 < z, & \text{ strong entrainment}
\end{align}

Vibration is a term conventionally used to characterize weak movement that is difficult to be discerned with bare eyes, while hoping can be used to describe short hops which however do not result in full displacement. Similarly, higher than critical saltation impulses $\hat{I}_{salt}$ result to relatively greater values of $\hat{z}(>1)$. This observation is useful to extend the concept to entrainment of nonuniform bed material. If for instance the particles surrounding the mobile grain have $n_s$ times its diameter ($D_b = 2n_sR_m$), complete entrainment by saltation requires that the saltation height will exceed $D_b$ or $\hat{z} > n_s$.

Using the same rationale different impulse (or $\hat{F}_L$, $\hat{T}_{salt}$) levels, can be used to characterize different degrees of angular motion for the case of rolling. Equation (16) can be modified to describe partial grain motions:

$$T_{roll} = \frac{l_{surf} \cdot m_{mod}}{-\frac{\hat{F}_L}{\hat{F}_e} \cdot \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \cdot \sin \left( \frac{\sqrt{2W_p\lambda} \cdot \frac{\sqrt{-\frac{\lambda F_e^2}{\hat{F}_e}}}{\frac{\lambda F_e^2}{\hat{F}_e}} \cdot \sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)}{2} \right) \cdot \sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)}{\sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)} \cdot \sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)$$

with $\lambda$, the ratio of partial angular dislodgement to the angular displacement for complete entrainment:

$$\lambda = \frac{\sin(\theta_{fin}) - \sin(\theta_0)}{\sin(\frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}}) - \sin(\theta_0)}$$

where $\theta_{fin} < \pi/2$, signifies the final angular position of the particle. As in the case of saltation, curves predicting various levels of angular motion, $\lambda$, are illustrated in Figure 16, for run 8–8. The effect (angular displacement) of the offered impulses is recorded with a high-speed camera, and the data points are classified with respect to $\lambda$, as shown in Figure 16. In general a satisfactory agreement is seen from both observed and predicted results.

In this manner, knowledge of the impulse content through the probability density function (PDF) of flow impulses or equivalently the joint PDF of the magnitude and duration of impulses, for a particular flow, will allow the estimation of the probability that a certain fraction of bare eyes, while hoping can be used to describe short hops which however do not result in full displacement. Similarly, higher than critical saltation impulses $\hat{I}_{salt}$ result to relatively greater values of $\hat{z}(>1)$. This observation is useful to extend the concept to entrainment of nonuniform bed material. If for instance the particles surrounding the mobile grain have $n_s$ times its diameter ($D_b = 2n_sR_m$), complete entrainment by saltation requires that the saltation height will exceed $D_b$ or $\hat{z} > n_s$.

Using the same rationale different impulse (or $\hat{F}_L$, $\hat{T}_{salt}$) levels, can be used to characterize different degrees of angular motion for the case of rolling. Equation (16) can be modified to describe partial grain motions:

$$T_{roll} = \frac{l_{surf} \cdot m_{mod}}{-\frac{\hat{F}_L}{\hat{F}_e} \cdot \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \cdot \sin \left( \frac{\sqrt{2W_p\lambda} \cdot \frac{\sqrt{-\frac{\lambda F_e^2}{\hat{F}_e}}}{\frac{\lambda F_e^2}{\hat{F}_e}} \cdot \sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)}{2} \right) \cdot \sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)}{\sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)} \cdot \sin \left( \frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}} \right)$$

with $\lambda$, the ratio of partial angular dislodgement to the angular displacement for complete entrainment:

$$\lambda = \frac{\sin(\theta_{fin}) - \sin(\theta_0)}{\sin(\frac{\sqrt{2W_p\lambda}}{\sqrt{\frac{\lambda F_e^2}{\hat{F}_e}}}) - \sin(\theta_0)}$$

where $\theta_{fin} < \pi/2$, signifies the final angular position of the particle. As in the case of saltation, curves predicting various levels of angular motion, $\lambda$, are illustrated in Figure 16, for run 8–8. The effect (angular displacement) of the offered impulses is recorded with a high-speed camera, and the data points are classified with respect to $\lambda$, as shown in Figure 16. In general a satisfactory agreement is seen from both observed and predicted results.

In this manner, knowledge of the impulse content through the probability density function (PDF) of flow impulses or equivalently the joint PDF of the magnitude and duration of impulses, for a particular flow, will allow the estimation of the probability that a certain fraction of

![Figure 15](attachment:image1.png)

**Figure 15.** Generalized threshold curves for saltation, providing different levels of normalized linear grain displacement ($\hat{z}$) as a function of the offered impulse characteristics (normalized magnitude, $\hat{F}_L$, and duration, $\hat{T}_{salt}$).

![Figure 16](attachment:image2.png)

**Figure 16.** Generalized threshold curves for rolling, providing different levels of normalized angular grain displacement, $\lambda$, as a function of the offered impulse characteristics (normalized magnitude, parameterized by $V^2$, and duration, $T_e$).
bed material will respond according to a defined mobility level (2).

5.4. Generalization to Other Grain Arrangements

[53] In section 3, the analytical formulation of the problem focused on the two limiting cases, completely exposed and fully hidden spherical grain, as shown in Figure 2. In both cases the mobile particle was resting on grains of the same size and shape. These two particle configurations were utilized because they are simpler to investigate the mechanics of particle entrainment, dominated by lift in the former case and drag in the latter. However, the electromagnet experiments further validated the impulse concept for particles of the same shape, spherical, but different size for the case representing drag-induced initiation of motion. The degree of particle exposure in these experiments ranged from 65% (for the 4 mm on 6 mm case) to 89% (for the 8 mm on 6 mm case).

[54] For a particle arrangement more representative of field conditions, the grain shape, size, exposure and packing density will vary significantly for both the mobile and its neighboring particles [e.g., Kirchner et al., 1990]. In the more general case, particle dislodgement will be due to the combined effect of drag and lift forces. Because impulse is the relevant criterion for describing particle dislodgement for each of the two limiting cases, it is expected to remain valid for the more general case of variable local grain topography [Diplas et al., 2008]. Nevertheless, additional work will be necessary to demonstrate this result conclusively.

6. Conclusions

[55] A set of flume experiments have been performed wherein the magnitude and duration of hydrodynamic forcing events along with the incipient entrainment of coarse grains were recorded. These experiments demonstrated that peak values in the instantaneous drag force record are necessary but not sufficient to trigger particle entrainment. It was observed that the duration of the peak values was a factor in particle entrainment. It is therefore conjectured that impulse rather than just the magnitude of hydrodynamic forcing, is relevant to the description of the incipient motion phenomenon.

[56] This hypothesis was investigated by considering separately the analytical formulation of particle dislodgement for the two basic modes of entrainment, namely saltation and rolling. The problem was made more tractable by simplifying the grain parameters and microtopography, assuming spherical, imbedded or fully exposed particles. For both modes of entrainment the significance of force duration has been demonstrated and the relevance of impulse has been established.

[57] Novel experiments that employ an electromagnet to control the magnitude and duration of the mobilizing force are in agreement with the theoretical results for different degrees of particle exposure. The relevance of the impulse criterion is demonstrated further by applying this concept for the case of different levels of grain mobility.

[58] The impulse criterion is physically sound since it appropriately accounts for the rapid fluctuations of the turbulent forces acting on grains. The developed equations can be used to characterize the mode of incipient motion of grains as well as signify the instances or frequency of their entrainment subject to applied hydrodynamic forces.

Appendix A

A1. Linearization of Equation (10a)

[59] For short impulse durations the angular displacement, \( \Delta \theta \), at any time instance \( t < T_{roll} \) is small. Thus considering \( \theta = \theta_0 + \Delta \theta \) and assuming \( \Delta \theta \ll \theta_0 \):

\[
\cos \theta = \cos(\theta_0 + \Delta \theta) = \cos \theta_0 \cos \Delta \theta - \sin \theta_0 \sin \Delta \theta
\]

\[
= \cos \theta_0 - \Delta \theta \sin \theta_0 \quad (A1a)
\]

\[
\sin \theta = \sin(\theta_0 + \Delta \theta) = \sin \theta_0 \cos \Delta \theta + \cos \theta_0 \sin \Delta \theta
\]

\[
= \sin \theta_0 + \Delta \theta \cos \theta_0 \quad (A1b)
\]

Equation (10a), for \( t < T_{roll} \), becomes

\[
L_{arm} \left( \frac{7}{5} \rho_s + \rho_f C_m \right) \frac{d^2 \theta}{dt^2} = F_D \sin(\theta - \alpha) + (F_L + B_f) \cos(\theta - \alpha) - W \cos \theta \quad (A2)
\]

Considering equations (A1a) and (A1b), the linearized version of equation (A2) is

\[
L_{arm} \left( \frac{7}{5} \rho_s + \rho_f C_m \right) \frac{d^2 \theta}{dt^2} = [F_D \sin(\theta_0 - \alpha) + (F_L + B_f) \cos(\theta_0 - \alpha) - W \cos \theta_0 - \Delta \theta (-F_D \cos(\theta_0 - \alpha) + (F_L + B_f)) \sin(\theta_0 - \alpha) - W \cos \theta_0] \quad (A3)
\]

Collecting the terms in equation (A3) and using equations (11a) and (11b), the linearized form of equation of motion for rolling (equation (12)) is obtained.

A2. Derivation of Equation (16)

[60] Multiplication of all parts of equation (15) with the angular velocity, \( \Delta \theta' \) results in

\[
L_{arm} m_{mod} \Delta \theta' \frac{d^2 \Delta \theta}{dt^2} = [W \cos(\theta_0 + \Delta \theta) - W \cos(\theta_0 + \Delta \theta_0)] \quad (A4)
\]

Integrating equation (A4) from \( t = T_{roll} \) to \( t = T_{roll} + \tau_{roll} \), yields

\[
L_{arm} m_{mod} \frac{1}{2} \left( \Delta \theta_{roll}^2 - \Delta \theta_{roll0}^2 \right) = [W \cos(\theta_0 + \Delta \theta_{roll}) \sin \alpha + B_f \sin(\theta_0 + \Delta \theta_{roll}) - W \cos \alpha \sin(\theta_0 + \Delta \theta_{roll})] - [W \cos(\theta_{roll} + \Delta \theta_{roll0}) \sin \alpha + B_f \sin(\theta_{roll0}) - W \cos \alpha \sin(\theta_{roll0})] \quad (A5)
\]
Considering \( \theta(t = t_{\theta,r}) = \pi/2 \), \( \Delta \theta(t = t_{\theta,r}) = 0 \) and \( \Delta \theta \ll \theta_0 \), equation (A5) is simplified as follows:

\[
L_{\text{arm} \text{mod}} \frac{1}{2} \Delta \theta_{\text{mod}}^2 = \left[ W \cos \theta_0 \sin \alpha + B_f \sin \theta_0 - W \cos \alpha \sin \theta_0 \right] - \left[ B_f - W \cos \alpha \right]
\]

(A6)

Substituting in equation (A6) the angular velocity at time instance \( t = T_{\text{roll}} \) (from equation (14)), and solving for impulse duration, \( T_{\text{roll}} \), the equation for entrainment of a particle by rolling is obtained (equation (16)).

Notation

- \( A_e \) area of the face of the electromagnet [L²]
- \( B_f \) buoyancy force [M L t⁻²]
- \( c_i \) coefficient of electromagnetic force (dimensionless)
- \( C_m \) added mass coefficient (dimensionless)
- \( D_b \) diameter of base particles [L]
- \( D_{50} \) median diameter of base particles [L]
- \( F \) net hydrodynamic force acting on solid particle [M L t⁻¹]
- \( F_D \) hydrodynamic drag force [M L t⁻²]
- \( F_{D0} \) normalized hydrodynamic drag force (dimensionless)
- \( F_e \) electromagnetic force [M L t⁻²]
- \( F_{L} \) hydrodynamic lift force [M L t⁻²]
- \( F_{L0} \) normalized hydrodynamic lift force (dimensionless)
- \( F_{\text{NE}} \) reaction force [M L t⁻²]
- \( F_R \) sum of resisting forces in z direction [M L t⁻²]
- \( h_e \) distance of the mobile particle from the face of the electromagnet [L]
- \( h_{\text{max}} \) maximum elevation of particle [L]
- \( H \) heaviside function
- \( I_i \) impulse due to hydrodynamic force (generic definition) [M L t⁺]
- \( I_{\text{roll}} \) normalized impulse for rolling (dimensionless)
- \( I_{\text{salt}} \) normalized impulse for saltation (dimensionless)
- \( I_{e} \) normalized electromagnetic impulse (dimensionless)
- \( q \) regression coefficient (dimensionless)
- \( L_{\text{arm}} \) lever arm [L]
- \( m_{\text{mod}} \) modified mass of particle (including added mass effects) [M]
- \( n_s \) coefficient of relative magnitude of base to mobile particle size (dimensionless)
- \( N_e \) number of turns of the wire in the coil of the electromagnetic (dimensionless)
- \( k \) regression coefficient (dimensionless)
- \( r_e \) resistance of the electromagnet [Ω]
- \( R^2 \) coefficient of determination (dimensionless)
- \( R_b \) radius of base particle [L]
- \( R_m \) radius of the mobile particle [L]
- \( R_s \) particle Reynolds number (dimensionless)
- \( t \) time [t]
- \( t_{\text{ff}} \) free fall time [t]
- \( t_{\text{salt}} \) duration of upward movement of saltating particle [t]

\( i_{\text{salt}} \) normalized duration of upward movement of saltating particle (dimensionless)

\( T_e \) duration of electromagnetic force [t]

\( T_i \) duration of hydrodynamic force [t]

\( T_{\text{roll}} \) duration of drag force (dimensionless)

\( T_{\text{salt}} \) duration of lift force [t]

\( T_{\text{roll}} \) normalized duration of lift force (dimensionless)

\( u^* \) shear velocity [L t⁻¹]

\( V \) solid particle’s volume [L³]

\( V_e \) voltage across the electromagnet [V]

\( W \) minimum required voltage for particle removal [V]

\( \dot{V} \) normalized voltage across the electromagnet (dimensionless)

\( W \) solid particle’s weight [M L t⁻²]

\( z \) Cartesian coordinate (normal to the bed) [L]

\( z_{\text{max}} \) maximum saltation height [L]

\( \dot{z} \) normalized saltation height (dimensionless)

\( \alpha \) bed slope [°]

\( \Delta \theta \) angular displacement [°]

\( \theta_i \) polar coordinates [°]

\( \theta_{\text{fin}} \) final angular position of the particle (for incomplete motion) [°]

\( \theta_0 \) pivoting angle [°]

\( \lambda \) ratio of partial to complete angular displacement (dimensionless)

\( \nu \) kinematic viscosity of water [M² t⁻¹]

\( \mu_e \) magnetic permeability of the medium (here air) [M L t⁻² A⁻²]

\( \rho_f \) density of fluid [M L⁻³]

\( \rho_s \) density of solid particles [M L⁻³]

\( \rho_0 \) coefficient including effects of local geometry and relative density of solid-fluid phases (dimensionless)

\( \Sigma F_{\xi} \) sum of forces in the radial direction [M L t⁻²]

\( \Sigma F_{\phi} \) sum of forces in the tangential direction [M L t⁻²]

\( \Sigma F_{\psi} \) sum of forces in the radial direction (dimensionless)

\( \Sigma F_{\phi} \) normalized sum of forces in the tangential direction (dimensionless)

\( \tau^* \) Shields stress (dimensionless)

Acknowledgments. The support of the National Science Foundation (EAR-0439663 and EAR-0738598) and Army Research Office is gratefully acknowledged. The presentation as well as the substance of this paper have been improved through the comments and insights of three reviewers and the Associate Editor.

References


A. O. Celik, C. L. Dancey, P. Diplas, and M. Valyrakis, Baker Environmental Hydraulics Laboratory, Department of Civil and Environmental Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA.

K. Greer, Rummel, Klepper, and Kahl, LLP Consulting Engineers, 601 North Calvert St., Baltimore, MD 21217, USA.