An All-Optical Switch Employing the Cascaded Second-Order Nonlinear Effect

C. N. Ironside, Member, IEEE, J. S. Aitchison, and J. M. Arnold

Abstract—A new, ultrafast, low loss, all-optical switch based on the cascaded second-order nonlinearity is proposed. The device is based on an integrated Mach-Zehnder interferometer where quasi-phase matching produces a nonlinear phase shift with a different sign in each arm of the interferometer. A full treatment of the nonlinear phase shift indicates that the proposed device, if fabricated in AlGaAs, could switch with powers around 1.3 W in a length of approximately 1 cm.

I. INTRODUCTION

INTEGRATED guided wave, all-optical switches are generally designed to operate via an intensity-dependent refractive index resulting from the third order nonlinear susceptibility, $\chi^{(3)}$ see for example [1] and [2]. Recently, De Salvo et al. [3] have shown that an effective intensity-dependent refractive index can be obtained from cascading the second order nonlinearity $\chi^{(2)}$. They demonstrated that the effective intensity-dependent refractive index $n_{\text{eff}}$ produced by the cascaded second order nonlinearity could be positive or negative depending on the phase matching conditions, and they also measured $n_{\text{eff}}$ as $\pm 2 \times 10^{-14}$ cm/W$^{-1}$ for KTP (KTiOPO$_4$), which has a $d_{33}$ of $3.1 \times 10^{-12}$ m V$^{-1}$. The approximate expression they derived for $n_{\text{eff}}$ was as follows:

$$n_{\text{eff}} = \frac{4\pi}{c\lambda} \frac{d_{33}^2}{m_{1}m_{2}} \frac{1}{\Delta k L}$$

where $L$ is the length of the device, $d_{\text{eff}}$ is the effective second-order nonlinear coefficient, $\Delta k$ is the phase mismatch, $n_{\text{eff}}$ is the refractive index at the second harmonic, and $n_{\text{i}}$ is the refractive index at the fundamental frequency; $\lambda$ is the wavelength. De Salvo et al. also showed $n_{\text{eff}}$ is not constant with intensity (i.e., not strictly Kerr law $n = n_0 + n_1 I$), but tends to saturate at phase changes of around $\pi/4$.

We propose here an integrated all-optical switch that uses the cascaded second order effect and is based on the integrated Mach-Zehnder interferometer, illustrated in Fig. 1. A similar device configuration has already been demonstrated utilizing the third-order nonlinearity in AlGaAs [2] where an asymmetric splitting of the input intensity was employed to obtain a differential phase shift in the interferometer. The asymmetric splitting reduces the overall throughput of the device. However, with the cascaded second order nonlinearity not only is the intensity dependency of the nonlinear phase shift increased but it is also possible to control the sign of the nonlinear phase shift by appropriate choice of phase matching. The phase matching scheme proposed is the so-called quasi-phase matching (QPM) scheme which has been used to phase match for second harmonic generation in LiNbO$_3$ [4], [8], LiTO$_3$ [5], and KTP [6] and has also been used in a semiconductor, GaAs [7]. With QPM the phase mismatch is given by

$$\Delta k = k_{2\omega} - 2k_{\omega} - K$$

where $k_{2\omega}$ is the phase propagation coefficient of the second harmonic light, $k_{\omega}$ is that of the fundamental light and $K = 2\pi/\Lambda$ where $\Lambda$ is the periodicity of the grating. By arranging to have different grating periodicities in each arm of the interferometer it is, therefore, possible to have a different sign of the effective $n_0$ in each arm of the interferometer. In other words, with the cascaded second-order effect it is possible to operate the device in a "push-pull" mode, where symmetric splitting of the light between the arms of the interferometer can be employed with a subsequent increase in the overall transmission of the device compared to the asymmetric Mach-Zehnder of [2]. We call the proposed device a cascaded, second order, push-pull switch. The phase change required in each arm for an on-to-off transition, or conversely, if the device is biased differently, an off-to-on transition, is $\pi/2$. For this amount of nonlinear phase shift, (1) may not be valid and therefore the approximate theory of [3] has to be extended to large nonlinear phase shifts.

In this paper the theory of the second-order cascade is...
extended to large nonlinear phase shifts and design examples based on LiNbO$_3$ and AlGaAs are given.

II. THEORY OF OPERATION

It was shown in [3] that the fundamental field $E_1$ and the second-harmonic field $E_2$ satisfy the coupled differential equations

$$\frac{dE_1}{dz} = i\frac{k\chi^{(2)}}{2n_o} E_2^* e^{i\Delta k z}$$ \hspace{1cm} (3a)

$$\frac{dE_2}{dz} = i\frac{k\chi^{(2)}}{2n_o} E_1 e^{-i\Delta k z}$$ \hspace{1cm} (3b)

where $z$ is the propagation distance, $k$ is the free space wavenumber, $\chi^{(2)}(2\omega; \omega, \omega)$, $n_o$ and $n_{2o}$ are the refractive indices at $\omega$ and $2\omega$, respectively, and $\Delta k$ is the phase mismatch defined previously in (2). (Here our conventions are such that the equations of (3) are actually the complex conjugates of those in [3], which is slightly more convenient.) The second harmonic field $E_2$ can be eliminated from (3) to give

$$\frac{d^2E_1}{dz^2} - i\Delta k \frac{dE_1}{dz} - \Gamma^2(1 - 2|E_1^2/E_0^2|)E_1$$ \hspace{1cm} (4)

with

$$\Gamma = \frac{kdE_0}{\sqrt{n_o n_{2o}}}$$ \hspace{1cm} (5)

where $d = \chi^{(2)}/2$.

Equation (4) was analyzed in [3] by approximate linearization for small conversion efficiency, and by numerical integration for other cases. In fact, (2) can be integrated exactly in terms of Jacobian elliptic functions [11], [12], from which a much larger range of approximations can be deduced. In [11], only the amplitude of the second harmonic is given explicitly, whereas here we give expressions for the full variation of the fundamental with distance including its phase, which was not considered in [11]. In [12], although (2) appears, the physical problem addressed is completely different to that considered here. An approximate treatment is also given in [13].

The exact results are summarized below

Let $\gamma = ((\Delta k/2)^2 - \Gamma^2)^{1/2}$, $\epsilon = (2\Gamma/\Delta k)^2$, and let

$$E_1 = E_0 e^{i\phi/(\Delta k)^2}.$$

Then, $\Phi$ forms a suitable normalized dependent variable. Further, let $\Phi = Re^\theta$ where $R$ and $\theta$ are real-valued functions of $z$. Then integration of (4) gives

$$\theta = \Gamma B \int_0^z R^{-2} \, dz$$ \hspace{1cm} (7a)

$$\Gamma z = \int_0^{\infty} X \{ (X^2 - R_1^2)(X^2 - R_2^2)(R_3^2 - X^2) \}^{-1/2} \, dx.$$ \hspace{1cm} (7b)

In these equations, $R_1$, $R_2$, and $R_3$ are roots of

$$R^6 + \frac{\gamma^2}{\Gamma^2} R^4 - \frac{\epsilon}{\Gamma} R^2 + B^2 = 0$$ \hspace{1cm} (8)

ordered so that $R_1^2 < 0 \leq R_2^2 \leq R_3^3$, and $A$ and $B$ are integration constants to be determined from the appropriate boundary conditions. These boundary conditions are derived from the physical initial conditions $E_2|_{z=0} = 0$ and $E_1|_{z=0} = E_0$, which implies from (6) that $R|_{z=0} = 1$. The integration variable $X$ in (7) oscillates in the range $R_1^2 \leq X^2 \leq R_3^2$ in the case of interest here, from which it can be deduced that $R_1 = 1$ to obtain $R|_{z=0} = 1$. Applying the boundary conditions one finds, after some tedious but straightforward algebra,

$$A = 0$$ \hspace{1cm} (9)

$$B = -\frac{1}{\sqrt{\epsilon}}$$ \hspace{1cm} (10)

$$R_1^2 + R_2^2 + R_3^2 = -\frac{1 - \epsilon}{\epsilon}$$ \hspace{1cm} (11)

and explicit expressions for the roots are as follows:

$$R_1^2 = \frac{1}{2\epsilon} \{ -1 - (1 + 4\epsilon)^{1/2} \}$$ \hspace{1cm} (12a)

$$R_2^2 = \frac{1}{2\epsilon} \{ -1 + (1 + 4\epsilon)^{1/2} \}$$ \hspace{1cm} (12b)

$$R_3^2 = 1$$ \hspace{1cm} (12c)

with the parameter $\epsilon = (2\Gamma/\Delta k)^2$. Equation (7b) can be evaluated in terms of elliptic functions as

$$R^2 = R_3^2 - (R_1^2 - R_2^2)sn^2(Z', m)$$ \hspace{1cm} (13)

where $Z' = (R_1^2 - R_2^2)/(R_3^2 - R_1^2)$; physically $R^2$ is the proportion of power in the fundamental. These parameters translate into

$$Z' = \frac{\Delta k L}{2\sqrt{\epsilon}} \{ 1 + 2\epsilon + (1 + 4\epsilon)^{1/2} \}^{1/2}$$ \hspace{1cm} (14a)

$$m = \frac{1 + 2\epsilon - (1 + 4\epsilon)^{1/2}}{1 + 2\epsilon + (1 + 4\epsilon)^{1/2}}$$ \hspace{1cm} (14b)

The elliptic function in (13) is periodic with a period $Z'_p = 2K(m)$, where $K(m)$ is the complete elliptic integral of the first kind, which translates into

$$z_p = \frac{4\sqrt{\epsilon}}{\Delta k} \{ 1 + 2\epsilon + (1 + 4\epsilon)^{1/2} \}^{-1/2} K(m)$$ \hspace{1cm} (15)

for the physical periodic distance $z_p$. The physical interpretation of this behavior is that a certain fraction of the incident power is periodically exchanged with the second-harmonic. At distances that are multiples of $z_p$ from the initial input point $z = 0$ the power in the fundamental wave recovers its initial magnitude, leaving no power in the second-harmonic field. We call this period the depletion period and it should be noted that, unlike the usual
analysis of second harmonic generation, in this full treatment it is clear that the depletion period is intensity dependent. For vanishingly small $\epsilon$ the period (15) reduces to $z_p = 2\pi / \Delta k$, in agreement with the numerical results of [3].

Having obtained the exact form of the magnitude of the fundamental wave $R$, it then follows using (7a) that

$$\theta = -\frac{1}{\alpha} \Pi(D, \phi|m)$$

with

$$\phi = \arcsin (\sqrt{n(Z', m)})$$

$$\alpha = \{1 + 2e + (1 + 4e)^{1/2}\}^{1/2} / \sqrt{2}$$

where $\Pi$ is the elliptic integral of the third kind and the depletion parameter $D = (R_1^2 - R_2^2) / R_1^2$ represents the proportion of the fundamental power which is exchanged with the second harmonic field. Care has to be taken with (17) in selecting the correct principal value of the arcsin function, particularly if the device is several depletion lengths long. The nonlinear phase shift is given by

$$\theta_{NL} = \theta + \frac{\Delta k \xi}{2}$$

where the second term in (19) cancels the linear phase shift at $\epsilon = 0$. In fact, $\theta_{NL}$ is the total phase-shift experienced by the fundamental field according to (6). The proportion of light remaining in the fundamental at a given power and length is given explicitly by (13).

In the case studied analytically in [3] the parameter $\epsilon$ is very small ($\epsilon << 1$); our formulae are valid for any $\epsilon$ without approximation. The most drastic approximation for the phase change produced by the mismatched $\chi^{(2)}$ grating can be obtained by approximating $R$ in (8) by the constant value $R = R_1 = 1$, thereby neglecting any depletion of the fundamental wave due to its conversion to second-harmonic. After substituting in (6) the expressions (10) for the coefficient $B$ and (12c) for $R$, the result is $\theta = -\Delta k \xi / 2$, which shows no nonlinear effects, and exactly cancels an identical phase factor introduced when scaling the fundamental field in (6). The next level of approximation for small $\epsilon$ is obtained by approximating the elliptic integral in (16), using $D = \epsilon$ and $m = \epsilon^2$. It can then be verified that (17) reduces to

$$\theta = -\frac{\Delta k \xi}{2} \left(1 + \frac{\epsilon}{2}\right)$$

when $z$ is a multiple of the depletion period $z_p$, in agreement with [3]. The effective nonlinear refractive index $n_{eff}$ is calculated from (20), noting that $\epsilon$ is proportional to the initial intensity of the fundamental wave $I_0 = n_0 E_0^2 / 2Z_0$, with the result already quoted in (1).

There are two distinct regimes of operation of the grating device: when $\epsilon \ll 1$ the fundamental is only weakly depleted. In these two regimes the depletion parameter $D$ is $O(\epsilon)$ and $O(1)$, respectively. In the weakly depleted regime the formulae of [3] are adequate, but only a small nonlinear phase change per unit length can be generated; in the strongly depleted regime much larger phase changes can be generated, but the formulae derived here must be used to design the device.

If we let the phase change in arm 1 of the device be $\theta_{NL1}$ and that in arm 2 of the device be $\theta_{NL2}$ then the transmission $T$ of the device at the fundamental wavelength is determined by the usual $\cos^2$ behavior of a Mach-Zehnder interferometer and is given by

$$T = R^2 \cos^2 \left(\frac{\theta_{NL1} - \theta_{NL2}}{2} + \xi\right)$$

where $\xi$ is a phase offset resulting from geometrical optical path differences in the arms of the interferometer.

The theory presented above is a full treatment of the nonlinear phase shift as a result of the cascading of the second nonlinear order effect that gives rise to the exchange of energy between the fundamental and the second harmonic fields. Exact solutions in terms of elliptic functions have been derived for both the nonlinear phase shift and the proportion of energy remaining in the fundamental. The above theory is not fully comprehensive in as much as effects such as mode-overlap and linear losses are not included. However, in the context of the above theory design studies can be undertaken for various material systems to obtain estimates of the switching powers required in the proposed device.

III. DESIGN EXAMPLES

As design examples we will consider both LiNbO$_3$ and AlGaAs based devices. These materials have been chosen because their nonlinear properties are well understood and there is an existing technology for the fabrication of gratings and waveguides in these materials. In the case of LiNbO$_3$, gratings can be fabricated which are $\chi^{(2)}$ phase reversal gratings; with AlGaAs only refractive index gratings have as yet been produced and Fejer et al. [10] have shown that if the refractive index modulation is comparable to the dispersion, which can be achieved with sufficient Al fraction, then efficient phase-matching can be obtained. First we consider LiNbO$_3$, for which the relevant materials parameters are $d_{33} = 32.2$ pm V$^{-1}$ (the $d_{33}$ coefficient can be accessed with QPM), $n_{33} = 2.258$ and $n_a = 2.137$ for the fundamental wavelength of $\lambda = 1.55$ $\mu$m (this wavelength is chosen because it corresponds to a low loss, optical communications wavelength). The arms are assumed to consist of single mode waveguides of length $L = 1$ cm, and effective cross section area, $A = 1.6 \times 10^{-2} \text{m}^2$. We design the device so that the phase mismatch condition $\Delta kL = +2\pi$ in one arm and $\Delta kL = -2\pi$ in the other arm. The phase offset $\xi$ sets the bias point of the device. With the phase offset $\xi = \pi / 2$ the device behaves as an off-to-on switch; in the GaAs example below the phase offset is $\xi = 0$, in which case the device behaves as an on-to-off switch. The transmission of the device as a function of coupled input power is
shown in Fig. 2(c); also shown in Fig. 2(a) and Fig. 2(b) are the fundamental as a fraction of the input from (13) and the nonlinear phase change in one arm (the \( \Delta kL = \pm 2\pi \) arm) of the interferometer both as a function of input power to the device.

For AlGaAs, the relevant material parameters are estimated as \( d_{31} = 150 \text{ pm V}^{-1} \), \( n_{\text{GaAs}} = 3.6 \), and \( n_{\text{AlGaAs}} = 3.5 \), \( \lambda = 1.55 \text{ µm}, L = 1 \text{ cm}, A = 1.6 \times 10^{-11} \text{ m}^2 \), \( \gamma = 0 \) and \( \Delta kL = \pm 2\pi \). The parameters here are chosen as typical; the \( d_{31} \) parameter is estimated from results reported for GaAs for fundamental wavelengths of 10.6 µm [9] and, given the resonant enhancement of the second-order nonlinearity as the band-edge is approached, this figure is likely to be an underestimate. The Al fraction should be selected so that the second harmonic light is not absorbed; an Al fraction of around 0.2 will ensure that the second harmonic light at around 750 nm is not absorbed. The transmission for the AlGaAs device as a function of coupled into power is illustrated in Fig. 3(c); again the fundamental fraction and the phase change in one arm of the interferometer are shown in Fig. 3(a) and (b). In this device the phase offset is also set to zero to illustrate the device operating as an on-to-off switch.

The powers required to achieve the \( \pi/2 \) phase shift necessary for the first off-to-on transition in Fig. 2(c) and on-to-off transition in Fig. 3(c) are, respectively, 6.5 W and 1.3 W; this is equivalent to a switching intensity of 40.6 MW cm\(^{-2}\) for LiNbO\(_3\) and 8.1 MW cm\(^{-2}\) for AlGaAs. To make the comparison with the third-order nonlinearity we can translate these figures to an effective intensity-dependent refractive index or \( n_{\text{eff}} \) for LiNbO\(_3\) \( n_{\text{eff}} = 2.95 \times 10^{-11} \text{ cm}^2 \text{ W}^{-1} \), and for AlGaAs \( n_{\text{eff}} = 14.8 \times 10^{-11} \text{ cm}^2 \text{ W}^{-1} \). For AlGaAs the comparison can be made with [2] where an optimal value of the third-order nonlinearity in AlGaAs was employed in an asymmetric Mach-Zehnder switch configuration and \( n_{\text{eff}} \) was measured to be \( 5.4 \pm 0.1 \times 10^{-14} \text{ cm}^2 \text{ W}^{-1} \). Therefore for the proposed device employing the cascade second-order nonlinearity a reduction of approximately three orders of magnitude in the switching intensity can be expected. Furthermore, the nonresonant second-order nonlinearity is intrinsically ultrafast and low loss.

IV. CONCLUSIONS

In conclusion, we have proposed an all-optical switch based on an integrated Mach-Zehnder interferometer that employs the cascaded second-order optical nonlinearity and quasi-phase matching to produce a positive and neg-
ative $n_{zeff}$ in opposite arms of the interferometer. A full mathematical treatment of the second-order cascaded process has yielded exact solutions for the nonlinear phase shift and for the fraction of power remaining in the fundamental. The switching characteristic of the device has been calculated and has been shown to be capable of switching with modest powers available from pulsed semiconductor lasers.

The continuous wave theory derived here assumes that the fundamental and second harmonic waves are strictly monochromatic which will not be the case if ultrashort pulses are employed. These pulses have a range of Fourier frequency components. The most significant consequence of this will probably be in the phase mismatch criterion which will be difficult to achieve for all pulse frequency components. It may be possible to mitigate this potential problem by employing a chirped phase mismatch grating. However, at this stage of its development the prediction of the full CW theory should be confirmed before pulsed investigations are undertaken.

The device could find applications which include all-optical switching in high bit-rate optical communication systems, where the particularly useful features are the ultrafast response and the modest switching power required. Low switching power is an important consideration particularly in high bit-rate applications where the average power approaches the peak power. For fundamental physics studies in quantum optics the device could be employed in quantum nondemolition measurements.

The device concept is applicable to other material systems which have the appropriate material properties; the material figure of merit is the same as for second-harmonic generation, namely $d_{zeff}^2/n^3$. The other requirements are a waveguide and a grating technology.

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