Monolithic Multiple Colliding Pulse Mode-Locked Quantum-Well Lasers: Experiment and Theory

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Abstract—We report here on a monolithic multisectioned quantum-well laser that operates as multiple colliding pulse mode-locking source. With this type of operation several pulses are present within a laser simultaneously and it produces high-repetition rate pulse trains (up to 375 GHz) and pulse widths of around 1–3 ps, as deduced from linear correlation technique measurements. By changing the configuration of the sections 1, 2, 3, or 4 pulse operation can be selected; frequency- and time-domain theories have been developed to explain this type of behavior.

I. INTRODUCTION

SEMICONDUCTOR lasers are compact and efficient sources of picosecond and subpicosecond pulses. Due to their small size, low pumping power requirement, low cost and robustness they are the most suitable laser device for integration in optical circuits for ultrashort photonic applications, such as high-speed communication systems, ultrafast data processing, optical computing and opto-microwave-electronic interfacing. Among the different methods of short pulse generation in semiconductor lasers mode-locking is able to produce shorter pulses, with better spectral characteristics at higher repetition rates [1]–[3]. The most common approaches to mode-lock semiconductor lasers are passive, active and hybrid mode-locking, in monolithic or external extended cavity configuration [1], [2]. Monolithic mode-locked semiconductor lasers devices offer the possibility of very low cost sources of ultrashort pulses which are compact, reliable, robust, efficient and can be mass-produced. The monolithic passive mode-locking scheme is attractive for achieving high-repetition rates, above 100 GHz since it does not require the injection of a high-frequency RF signal. This scheme can be very simply implemented by integrating a saturable absorber in the laser cavity [1]–[3]. The fast dynamics of the nonlinear behavior of the saturation and recovery of both absorber and gain sections of the laser is responsible for its ultrafast characteristics [4]–[6].

Portnoi et al. first observed mode-locking at 200 GHz, by using proton bombardment to form the saturable absorber section in the laser [7]. Chen et al. used a 250 μm long monolithic passive colliding pulse mode-locked (CPM) laser to obtain 0.65 ps pulses at 350 GHz [8]. The CPM configuration consists of placing a saturable absorber section precisely in the middle of the laser cavity, which makes the laser generate two counter propagating pulses that collide in the absorber section [3], [8]. The saturable absorber section is implemented by simply applying a reverse bias to the central part of the split contact of the laser [8], [9].

More recently a growing interest has been apparent in the generation of ultrashort pulses at ultra-high-repetition rates [10]–[13], reaching terahertz rates [14], [15]. Such high speed is achieved by inducing the laser to operate at harmonics of the fundamental inverse round-trip time of the cavity. Generation of harmonics of the repetition rate can be achieved in actively mode-locked lasers by suitable choice of the repetition frequency of the driving signal [2], [3]. In passive mode-locked lasers, several harmonics have been observed in a multistable behavior [16], [17], where the operating harmonic is dependent on the gain section current [10], [14]–[17].

In this paper, we present the progress on generation of short pulses at high-repetition rates by multiple colliding pulse mode-locked (MCPM) quantum-well lasers. These devices are monolithic multi-sectioned passive mode-locked semiconductor lasers that can generate short pulses at harmonics of the cavity round trip frequency, depending on the electrical bias applied to each of their sections. Fig. 1(a) shows the top...
metalllic contact (NiCr/Au)
SiO₂ injection
blocking layer
over, and below, an active region (4 QW)
metallic contact (Au/Ge/Au/Ni/Au)

Fig. 2. Transversal cross section of the laser, showing waveguide and material structure.

view diagram of the MCPM laser and Fig. 1(b) shows its longitudinal cross section with the electrical connections for forward and reverse biasing. The laser has a contiguously electrically connected gain section and 3 separated sections placed at every quarter of the cavity length. Each of the three sections (labeled a, b, and c on the diagram) is separately electrically addressable and when reverse biased it behaves as a saturable absorber and when forward biased as a gain section. Therefore, by selectively biasing, one can choose the number and position of saturable absorbers in the laser cavity, which causes the laser to have 1, 2, 3, or 4 pulses circulating in the cavity, giving first to fourth harmonic of the repetition rate, respectively. In contrast to the lasers reported in [10], [14]–[17] the multiple pulse operation (or harmonics generation) here does not depend on the current applied to the gain section. To date our highest repetition has been achieved with a 400-μm long MCPM laser, where we obtained 1-ps pulses at 375 GHz [18].

Section II of this paper describes the fabrication of the MCPM lasers. Details of the quantum-well material and waveguide structure are discussed. Special attention is given to the design and mechanisms of operation of the monolithically integrated saturable absorber sections. Section III presents the experimental results on the different modes of operation of the MCPM laser, whereas Section IV is concerned with theoretical models and results. Finally, in Section V, we give the conclusion of the paper.

II. DEVICE FABRICATION

The MCPM laser is fabricated on a GaAs–AlGaAs 4 quantum-well material grown by atmospheric pressure metal organic vapour phase epitaxy (APMOVPE). The semiconductor material structure consists of 0.2 μm heavily p-doped (2 × 10^{19} cm^{-3}) GaAs cap layer followed by 1 μm p-type carbon doped (2.2 × 10^{17} cm^{-3}) Al₀.₄₃Ga₀.₅₇As upper cladding layer. The quantum-well structure consists of four 10 nm GaAs wells spaced by 10 nm Al₀.₂₀Ga₀.₈₀As barriers and they are surrounded by two 0.1 μm Al₀.₂₀Ga₀.₈₀As layers in a separate confinement structure. The lower cladding layer is formed by Al₀.₄₃Ga₀.₅₇As silicon n-doped (1.4 × 10^{17} cm^{-3}) layer 1.7 μm thick.

The laser waveguide structure used is a 3-μm wide, 0.6-μm high stripe loaded waveguide formed by SiCl₄ reactive ion etching and it is defined by photolithography and NiCr masking prior to etching. A 200-nm thick SiO₂ layer is deposited on the material to serve as a current injection blocking layer and a contact window is open on the waveguide by HF chemical etching. Fig. 2 shows the transversal cross section of the laser waveguide, as well as the layers of the material structure. Gain and absorber sections are defined by photolithography, top metallization (NiCr–Au) and lift-off, leaving 10-μm gaps between sections (see Fig. 1). Substrate thinning, metallic contact deposition (Au–Ge–Au–Ni–Au) and laser cleaving complete the fabrication process. The lasers are mounted and wire bonded for CW operation. No contact annealing is used since it deteriorates the adhesion of the bonding pads to the SiO₂ layer underneath it, making bonding very difficult. The fabrication technique described above is simpler and of lower cost than the buried heterostructure used in previous publications [3], [10], [14] since it does not involve any regrowth process.

In our devices, saturable absorption is obtained by applying a negative bias between the absorber sections a, b, and c in Fig. 1 and the ground contact. This reverse biasing technique has been successfully employed by a number of authors [2], [3], [9]. An electric field is applied transversally to the quantum wells, which induces a shift of the semiconductor band edge, known as quantum confined Stark effect [19], [20]. It is shown in Fig. 3 where we obtain the spectra of light transmission through the laser waveguide for different reverse bias. To obtain those we coupled the output of a tuneable Ti:sapphire laser into a quantum-well laser waveguide, which was fully reverse biased for better signal contrast. The output intensity was measured using a photodiode and lock-in amplifier. Spectrum for TE and TM polarization of the Ti:sapphire
laser were obtained. Fig. 3 shows that one can introduce a controlled amount of absorption in the semiconductor laser cavity using the reverse bias technique. The optical intensity of the mode-locked pulses inside the laser cavity can be estimated to be of the order of 1.6 MW/cm². In this estimation we assume a laser emitting 1.2-ps pulses at 240-GHz repetition rate, with 8-mW average power. We also assume an effective waveguide cross-sectional area of $3 \times 1 \mu m$ or $3.0 \times 10^{-8} \text{cm}^2$ and a facet reflectivity of 0.3. This intensity is high enough to saturate the absorption of the absorber section by mostly band filling effect [19], [20]. The applied reverse bias acts once again, now sweeping out the generated carriers from the absorber sections, helping the recovery to the unsaturated state [2], [3], [9]. The saturation and recovery processes described above may not be the only ones present in ultrafast mode-locked lasers. Faster nonlinear effects, necessary for achieving high-repetitions rates and short pulses, will be discussed in the subsequent sections.

The electrical isolation between gain and absorber sections is an important issue on the design and fabrication of mode-locked lasers. A poor isolation allows current leak from the gain to the absorber section, decreasing the efficiency of the reverse bias on the absorber. It also drains current from the gain section in the vicinity of the absorber section, making the latter effectively wider. A short and efficient absorber section is important for its saturation and recovery characteristics. We improve the electrical isolation by removing the highly doped GaAs cap layer from the gap regions between sections using a selective dry etching technique [21], [22] that stops on the AlGaAs layer. By measuring the resistance between gain and absorber sections when no bias is applied we obtain a dc electrical isolation of 1.5 kΩ. We observe that this technique gives about 50% improvement on the original isolation (without etching). Better isolation can be achieved by deeper etching into the upper cladding layer or by using wider gaps between sections. The isolation also depends on the doping level and thickness of the cap and upper cladding layers and the figures presented here are for our particular material structure.

III. EXPERIMENTAL RESULTS

The characterization of the MCPM lasers is performed by measuring the time averaged optical spectra and the linear cross-correlation trace. The linear cross-correlation method for time-domain measurement uses a Michelson interferometer to scan a pulse through itself and its neighbors in a train of pulses, obtaining pulse width, repetition rate and coherence length of the laser [23]. This technique detects the mixing signal between light from each arm of the interferometer at a low speed photodetector and it does not employ second harmonic generation. The mixing technique employing a lock-in amplifier and two choppers (one in each arm of the interferometer) removes the dc level generated by the direct detection of light from each individual arm of the interferometer. Therefore, only the interference signal is detected [23]. Although this technique has some limitations concerning pulse width measurement [11], [23], it is attractive for characterization of mode-locked semiconductor lasers since these devices generally generate low optical powers, which makes second harmonic generation (SHG) more difficult in practice. The limitation of the technique consist of its insensitivity to the effect of frequency chirp on pulses [11], [23]. Therefore measurements of time-bandwidth product could be misleading. However, it is reasonable for these lasers because there has been no evidence of significant chirping from CPM lasers and measurements have generally indicated transform limited pulses [3], [8]. A detailed study of pulse structure is out of the scope of this paper since it involves more sophisticated measurement schemes than correlation methods. Nevertheless we are implementing the SHG autocorrelation technique in the near future as an attempt to have more information about chirp in pulses. Fig. 4 shows the correlation and optical spectra of a MCPM laser for its different modes of operation, including single mode, multimode, 1, 2, 3, and 4 pulse mode-locking.
and to be complete we also show the spontaneous emission of the laser. The total cavity length of the laser is 600 μm and each one of its 3 independent sections a, b, and c in Fig. 1 is 30-μm long. The laser operates on CW regime and when every section is forward biased the threshold current is 30 mA. Fig. 1(b) shows the electrical connections to bias the laser in the case of 4 pulse mode-locking. A current source is used to bias the gain sections, whereas a voltage source applies the reverse bias to the absorber sections. A 50 Ω resistor is used to limit the current flowing from the absorber sections preventing damage.

Fig. 4(a) and (b) show the cross-correlation and optical spectrum of the spontaneous emission of the laser when it is operating below threshold, at 25 mA. The laser emits in a very broad spectral range, with no observable longitudinal modes. The only feature observed in the correlation trace is a short double-sided exponential peak at zero delay. The fast exponential decay with delay from the zero point of the interferometer indicates that the emission is highly incoherent, as expected from spontaneous emission. The typical fringes produced in interferograms are not resolved here.

When every section of the laser is forward biased and it operates at currents not far beyond threshold the laser emits in a single longitudinal mode. It is shown in Fig. 4(c) and (d) for 60 mA pumping current. The side mode suppression is of about 25 dB (not measurable from the Fig. 4(d)). The correlation trace is flat in the observed range of 45 ps. No decay can be noticed, indicating the laser has a long coherence length [24].

The laser can also emit several longitudinal modes. Multi-mode (free-running, nonlocked) operation occurs for currents very close to threshold and also for high currents, above the single-mode operation region. Fig. 4(e) and (f) show the correlation and optical spectrum for multimode operation when 35 mA is applied to the laser. Fig. 4(e) shows peaks at the fundamental repetition rate of the cavity (60 GHz), which correspond to 0.15 nm spacing between modes in Fig. 4(f). These peaks arise from beating of the several longitudinal modes observed in the spectra. This mode beating effect means that, in time domain, the laser produces a sequence of spikes in the form of a pulse train [24], [25]. Therefore, there is here an apparent ambiguity between multimode and mode-locked operation. The ambiguity is removed by analyzing the coherence properties of the laser. The difference between mode-locking and multimode behavior is that with a mode-locked laser the modes add together coherently and that there is a strong phase relationship between pulses in the pulse stream [24]. In an ideal mode-locked case, where the phase of the modes are exactly the same, the coherence length of the laser would be infinite. A laser is considered to be mode-locked if its coherence time is much longer than the round-trip-time of the cavity, or photon lifetime—for the structures studied those two times are of the same order of magnitude. One can observe in Fig. 4(e) a pronounced exponential decay on the amplitude of the peaks, indicating that the laser has a short coherence length, of about 20 ps (6 nm), which is very close to the round trip time (16.4 ps). This means that there is no constant phase relationship between modes and the sequence of spikes produced by them is formed and shaped in a rather chaotic and noisy fashion, not having the high regularity in shape and amplitude typical of mode-locked pulses. This kind of ambiguity also exists in the nonlinear correlation methods [24], [26]. In the autocorrelation by second harmonic generation technique peaks also appear for multimode operation and the ambiguity is solved by observing the contrast ratio (peak to background), which is 3:1 for ideal mode-locking and 3:2 for multimode operation [24], [26].

Mode-locking can be achieved by reverse biasing one of the three sections a, b, or c in Fig. 1. If section a is reverse biased, providing saturable absorption, while sections b and c are forward biased, providing gain, the MCPM laser has one saturable absorber in the cavity and in this case it mode-locks at the repetition rate corresponding to the cavity round trip time. For the 600 μm long laser it is 60 GHz, which is the first harmonic of the repetition rate. Since only one pulse is present in the cavity at a time we call this mode of operation 1 pulse mode-locking. Fig. 4(g) and (h) show this type of operation when we apply −0.95 V to section a and 60 mA to the rest of the laser. The average width of the correlation trace pulses is 3.2 ps, obtained by measuring the width of each pulse shown in Fig. 4(g), apart from the one at zero delay, which is the auto-correlated pulse and contains less reliable information about pulse width [23]. Assuming in this case a hyperbolic secant pulse shape the deconvolution factor of 1.56 is used to obtain the actual pulse width [26], which is 2.1 ps FWHM. The same procedure will be applied to pulse width measurements thereafter. The longitudinal mode space measured in Fig. 4(h) is 0.15 nm, which correspond to the 60 GHz repetition rate obtained from Fig. 4(g). In contrast to the case of multimode operation discussed before, the coherence time of the laser observed in Fig. 4(g) is much longer than in Fig. 4(e). The measurement of the coherence length is limited by the continuous scanning range of the interferometer, which is 15 mm (50 ps) in our setup. But we have made a noncontinuous scan measurement, up to pulses 10 round-trips apart (150 ps), which showed no noticeable change in pattern. This long coherence time compared to the round-trip time (16.4 ps) and the photon lifetime (about 5 ps) is an indication of mode-locking operation. Due to symmetry similar results are obtained by reverse biasing section c and forward biasing sections a and b.

Two pulse CPM operation is obtained when the central section of the laser (section b in Fig. 1) is reverse biased whereas the others (sections a and c) are forward biased by connecting them to the gain section. This configuration corresponds to standard CPM operation of lasers, which has one saturable absorber in the middle of the cavity [3], [8], [27]. Fig. 4(i) and (j) show the cross-correlation and optical spectra for 2 pulse CPM operation when 0.37 V reverse bias is applied to section b and 54 mA is applied to the rest of the laser. It can be seen in Fig. 4(i) that three longitudinal modes are apparent and the mode spacing is 0.3 nm which is twice the original cavity mode space (0.15 nm) and corresponds to a pulse repetition rate of 120 GHz. From previous work [3], [27] it is clear that this is the standard behavior of a semiconductor CPM laser and it indicates that two pulses are circulating in the cavity and colliding in the saturable absorber. With only three
modes locked it is difficult to fit a typical envelope shape to the
optical spectra and the pulse shape can be close to a quasi-
sinusoidal shape. But for the sake of simplicity, we assume
here and thereafter a hyperbolic secant pulse shape as well,
which is the most commonly found in the literature for this
type of device [2], [3]. From Fig. 4(i) we obtain a deconvolved
pulse width of 3 ps, at 120 GHz repetition rate.

The Fig. 4(k) and (l) are obtained when the sections a
and c are reverse biased with 0.74 V. The section b and
the gain section are forward biased with 54 mA. In this
configuration the laser has two saturable absorbers in the
cavity. It can be seen in Fig. 4(l) that in this case the separation
of the longitudinal modes is 0.45 nm, which is three times
the original cavity mode space, corresponding to 180 GHz.
Fig. 4(k) gives 2 ps pulse width and 180 GHz, which is the
third harmonic of the repetition rate. As an extension of the
two-pulse CPM operation, this is explained by assuming that
the laser is operating in a three-pulse regime, where three
pulses are present in the cavity. Two pulses collide at every
1/3 of the cavity length, while the other is 2/3 of the cavity
length distant from them, that is at the facet. Although
the collision points do not correspond to the exact position
of the saturable absorbers, the pulses are wide enough (2 ps
pulsewidth is 150 μm long inside the laser) to overlap in the
saturable absorber section. Therefore, the pulses still collide
in the absorber sections.

Four pulse MCPM operation is achieved when the three
sections (a, b, and c) are reverse biased. As a result the laser
has three saturable absorbers in the cavity. Fig. 4(m)
and (n) show the cross-correlation and optical spectrum for
0.33 V reverse bias applied to the three sections, whereas the
gain section is forward biased with 65 mA. The longitudinal
mode separation is 0.6 nm which corresponds to a pulse
repetition rate of 240 GHz, which is the fourth harmonic. The
pulse width measured is 1.6 ps. This configuration has four
pulses circulating in the cavity and they collide exactly in the
saturable absorber sections. Two of them collide with other
two pulses in sections a and c, and after moving a quarter
of the cavity length two pulses collide in the section b, while
the other two pulses are at each facet of the laser [11]. From
a shorter laser, 400-μm long with 15-μm saturable absorber
sections, operating in four-pulse mode we obtained 1-ps pulse
width at 375 GHz repetition rate, as it is shown in Fig. 4(o)
and (p).

For the devices studied here, we could not find any change
of harmonics (multistability) when the current is varied (up
to 90 mA) for each laser configuration. It indicates that,
in contrast to previous work [10], [14]–[17], the multipulse
operation discussed here is due to the number and position
of the saturable absorbers in the cavity and not the current level.
It will be further discussed in Section IV.

We also obtained ranges of current and absorber bias in
which the laser operates in four-pulse MCPM mode. It is
shown in the light versus current curve of Fig. 5. The output
optical power is measured as a function of the gain section
current for different absorber biasing conditions. One can
clearly see the increase of the threshold current of the laser
as the absorber bias is decreased from 0–4 V. It is due to
the insertion of loss in the cavity, which comes from the
increasing absorption in the absorber sections as a function
of the applied reverse bias (see Fig. 3). The dark lines on the
$L \times I$ curves show where stable 4 pulse mode-locking (at 240
GHz) is found. Up to 7 mW average power, corresponding to
25-mW peak power can be achieved. The pulse energy is 0.03
pJ. These values are very similar to the ones reported in the
literature for this type of device [2], [3]. We also observed the
range of mode-locking in a gain current versus absorber bias
graph and it is shown in Fig. 6. In this graph, the absorber
bias is set at zero gain section current and we measure the
change of the bias as the gain current is increased, which gives
the dashed lines in Fig. 6. The lasing threshold and the areas
of CW, mode-locking or self-pulsation operation are shown.
Again the dark lines on the curves indicate where stable four-
pulse MCPM is found. We can notice two main features that
differ this range of mode-locking from the ones found in the
literature [3]. This range is rather small in current and reverse
bias, and it shows two distinct areas where four-pulse MCPM
is observed. From Fig. 6 it is clear that there are two points
where mode-locking is found for each absorber bias curve with
initial values between –2.8 and –3.3 V. We believe that the
explanation for these features lies on the need of very fast
absorption and gain recovery in order to sustain mode-locking
at 240 GHz (4.1 ps duty cycle). This fast dynamics would be
achieved with the help of the light and heavy hole excitonic
non-linearities, which would give rise to the two distinct
regions of mode-locking. The previously reported ranges of
mode-locking are for longer lasers, generating lower repetition
rates, at about 80 GHz [3]. They naturally have more relaxed
requirements on fast dynamics of gain and absorption, which
provokes the widening and merging of the two regions into a
continuous one.

We have obtained strong indications that the same laser
device can also operate Q-switched. Using a fast photodetector
and a RF spectrum analyser we observed that the laser can
self-pulsate at about a few gigahertz repetitions rate range,
depending on the forward current applied to the gain section
and the reverse bias applied to the absorber sections. The laser
In the $L \times I$ curves shown in Fig. 5, Q-switching occurs at
lower currents than mode-locking, for a given reverse bias.
Fig. 6 shows the area where Q-switching (self-pulsation) is
found. A comprehensive study and characterization of the Q-
switching operation of these devices is out of the scope of this
paper and it will be subject of future work.

IV. Theory

The aim of this section is to study the origin of multiple-
colliding-pulse mode locking and to assess the roles played
by the physics of gain and saturable absorber (SA) media and
the geometry of the laser in establishing this regime. First, we
shall concentrate on determining conditions for the regime to
take place. The usual way of doing this is to describe the lasing
regime in terms of mode amplitudes and phases, that is, to use
a frequency-domain approach [5], [28]–[30]. Here, we shall
extend it to colliding-pulse configurations, including MCPM.
A. The Frequency-Domain Model

We start with a system of coupled-mode equations for complex mode amplitudes. Systems of that kind were applied to semiconductor lasers by Lau [5], [28]–[30] for the case of standard AM mode locking and also by Shore and Yee [31] for the case of the so-called frequency-modulation mode locking, caused by gain nonlinearities in a single-section laser. Here, we use a system [32] which is an extension of both these models. The derivation of the model will be published elsewhere [33] and is skipped here since the structure of the resulting equations is close to those presented in [5], [28]–[30] and fairly physically transparent

\[
\frac{\partial}{\partial t} E_k = \frac{1}{2} \left[ A g \gamma + A a \gamma - i a c \right] E_k + \sum G_m E_{k+m},
\]

and phase, \( \Gamma \) is the confinement factor, \( \gamma = c/\eta g \) the group velocity of light (\( \eta \) and \( \eta g \) being the phase and group refractive indices, respectively). \( k \) and \( k \) stand for gain in the gain section of the laser and the absorption coefficient in the saturable absorber (SA) section at the spectral position of the \( k \)th mode. They should be understood as the “slow” components of corresponding quantities which vary in time only on non-ultrafast timescale, i.e., slower than the round-trip time of the cavity, for instance due to relaxation oscillations or self-pulsing, but not to mode locking. \( f g \) and \( f a \) are fractions of the cavity length occupied by gain and SA sections, \( a c \) is the cavity loss, \( \Delta \omega k \) is the frequency detuning of the mode from its spectral position in the passive cavity. The last term in (1) takes into account the interaction between modes separated in frequency by \( m \) fundamental intermodal intervals \( A = \pi v g / L \). The strength of the interaction \( G \) is given by the following expression

\[
G_m = \left( \sum E_k E_{k+m} \right) \left\{ \varepsilon_m \left[ v g (1 + i \alpha g) \cdot A g \cdot \tau \Sigma a \gamma + \varepsilon_m \right] \right\}.
\]

In (2), the first factor is the component of the light intensity oscillating at the frequency \( m \Delta \Omega \) due to beating between modes; the second factor is the response of the active medium of the SA section (the first term in the figure brackets) and of the gain section (the second term) to these light intensity oscillations. \( A g = \partial g / \partial N \) and \( A a = -\partial a / \partial N \) are gain and SA cross sections (carrier density derivatives), \( \alpha g \) and \( \alpha a \) are the linewidth enhancement factors in corresponding sections. The necessary assumption in (2) is that, although mode number (i.e., frequency) dependence of the net mode gain \( f g \gamma A g - f a \gamma A a - a c \) is important for mode competition and therefore retained in (1), the values \( g \) and \( a \) separately may be considered weakly dependent on mode number (frequency). Similarly, the dispersion of \( \alpha g \) and \( \alpha a \) is neglected, hence no mode subscript is considered assigned in (2) to \( g, a, A g, A a, \alpha g \), and \( \alpha a \).

\[
\tau \Sigma a,g = \left( \frac{1}{\tau \alpha g} + v g A a,g \sum \frac{E_m^2}{E_m} \right)^{-1}
\]

stand for the effective recombination times in SA and gain sections, including both spontaneous/nonradiative and stimulated recombination. The first term in each square brackets of (2) is then the “slow” term describing the light-induced oscillations of saturable absorption and gain due to oscillations of the total carrier density in the corresponding section. The second, “fast,” term \( \varepsilon_m \), is introduced to take into account pulsations of saturable absorption and gain due to pulsations of the deviation of the carrier energy distribution from quasiequilibrium [34], of the carrier temperature [35], [36] and, specifically for QW absorbers, of the degree of exciton ionization [37]. All these mechanisms (nonlinearities) are essentially fast, with characteristic relaxation times on subpicosecond scale, hence the expression “fast term.” We
assume for simplicity that in both gain and SA sections one of these nonlinearities dominates over the others; otherwise one should sum over all mechanisms involved. Similarly to the slow term, the fast term may then be written as

$$\varepsilon_m^{(a,g)} = \frac{\varepsilon_m(a,g)}{1 - im\Delta\Omega r_{nl}^{(a,g)}}$$

where \(\varepsilon_m^{(a,g)}\) are the phenomenological absorption/gain suppression coefficients and \(r_{nl}^{(a,g)}\) are the nonlinearity relaxation times.

All the values discussed so far are independent on the geometry of the laser cavity. The only way this geometry enters the model is via the quantities \(\xi_{nl}(m)\). These quantities are overlap factors characterizing the spatial overlap between the two interacting modes and the net gain (gain and saturable absorption) pulsations that lead to this interaction. The expression for these factors is [32]

$$\xi_{nl}(m) = \int d\varphi \cdot (u_k u_{k+m}^*) \cos \left(\frac{m\pi}{L} \left( z + \frac{L}{2} \right) \right). \quad (3)$$

In (3), the integration is over the fraction of the laser length occupied by the corresponding section, the zero of \(z\) is assumed in the center of the cavity, \(u_k\) are the longitudinal mode profiles (wave functions) of the cavity. The cosine factor in (3) may be understood as the spatial profile of the net gain pulsations. This factor is the important difference between the expressions (3), derived from the first principles, and the less accurate expressions empirically introduced by previous authors [30]—not all the features discussed below are predicted by that earlier model. For more details on mode profiles and overlap factors, see Appendix.

**B. Mode-Locking Condition**

The usual way of defining the condition for mode locking analytically is to consider a single-mode solution of (1) and study the stability of this solution with respect to the excitation of weak side modes. If the single-mode solution is unstable then the mode locking is expected to occur. A formally different, but equivalent procedure was also performed by Haus and Silberberg [37]. In [5] and [28]–[30], the side modes are taken as the modes nearest to the initially existing main mode, i.e., one interval \(\Delta\Omega\) apart from it. For our purposes, we must allow the side modes to be separated from the main mode by \(m = 1, 2, 3, 4 \ldots\) intervals \(\Delta\Omega\), and shall then check the mode locking condition for various \(m\). If for some range of parameters this condition is satisfied for a certain \(m > 1\) but not satisfied for \(m = 1\), this will mean that the MCPM regime (presumably with \(m\) pulses coexisting in the cavity) is indeed realized within this parameter range. Following the usual procedure of the (instability) analysis, we consider the system (1) with only three modes, that is, the main mode \((k = 0)\) and the side modes \((k = \pm m)\), taken into account. The single-mode solution \(E_0 > 0, E_{\pm m} = 0\) is the trivial solution of the system. The range of parameters within which this solution is unstable should be derived from an obvious condition

$$\max \left( \frac{d\ln E_m}{dt}, \frac{d\ln E_{-m}}{dt} \right)_{E_{\pm m} = 0} > 0. \quad (4)$$

A straightforward application of the condition (4) to the system of coupled-mode equations in the general form of (1) leads to an apparently unexpected conclusion that the condition (4) may be satisfied without a saturable absorber. Indeed, it is known (see e.g., [31], [34], [36]) that even without the SA, gain nonlinearities can still ensure multimode lasing. But multimode emission, generally speaking, is not the same as mode locking, at least as amplitude-modulation mode locking, i.e. generation of short pulses, as discussed in Section III. This presents a problem for the interpretation of (4) which, however, is clearly not specific to MCPM (the same problem obviously arises when trying to accurately define conditions for ordinary mode locking in monolithic-cavity laser diodes) and will therefore be addressed elsewhere [33]. Here, we shall restrict ourselves to a simplified approach [28]–[30], ignoring the contribution of the refractive index related terms in (2), i.e. setting \(\alpha_g = \alpha_a = 0\); \(\mathfrak{m}(\varepsilon) = 0\), and also ignoring gain/absorber nonlinearities hidden in \(g\) and \(a\) (which may lead to multimode emission due to spectral hole burning). Then, indeed, multimode emission is synonymous with mode locking and, after some trivial calculations, (1) and (4) yield an approximate condition for this regime in the form

$$\varepsilon_m^{(a)} \left[ \frac{v_g A_{\alpha} r_{\alpha} \varepsilon(a)}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} + \varepsilon(a) \right]$$

$$\varepsilon_m^{(a)} \left[ \frac{1}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} \right]$$

$$\frac{\Delta E_{m}}{E_0} = \Delta E_{m}^{(a)} \left[ \frac{v_g A_{\alpha} r_{\alpha} \varepsilon(a)}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} \right]$$

$$\varepsilon_m^{(a)} \left[ \frac{1}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} \right]$$

$$\frac{\Delta E_{m}}{E_0} = \Delta E_{m}^{(a)} \left[ \frac{v_g A_{\alpha} r_{\alpha} \varepsilon(a)}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} \right]$$

$$\varepsilon_m^{(a)} \left[ \frac{1}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} \right]$$

where the coefficients have been assumed real (as is usually the case); and \(\Delta E_{m}^{(a)} = f_{g}(g_0 - g_m) - f_{a}(a_0 - a_m)\).

For monolithic-cavity lasers with the cavity lengths of the order of hundreds of micrometers and realistic stimulated lifetimes, one may usually assume \(m\Delta\Omega r_{\alpha} \varepsilon(a) \gg 1\). Also, given subpicosecond nonlinearity relaxation times, \(m\Delta\Omega r_{\alpha} \varepsilon(a) \ll 1\). So (5a) is simplified to

$$\varepsilon_m^{(a)} \left[ \frac{v_g A_{\alpha} r_{\alpha} \varepsilon(a)}{(m\Delta\Omega)^2 r_{\alpha} \varepsilon(a)} + \varepsilon(a) \right] - \varepsilon_m^{(g)} g \left[ \frac{v_g A_{\alpha} r_{\alpha} \varepsilon(a)}{(m\Delta\Omega)^2 r_{\alpha} \varepsilon(a)} + \varepsilon(a) \right]$$

$$\frac{\Delta E_{m}}{E_0} = \Delta E_{m}^{(a)} \left[ \frac{v_g A_{\alpha} r_{\alpha} \varepsilon(a)}{(m\Delta\Omega)^2 r_{\alpha} \varepsilon(a)} + \varepsilon(a) \right]$$

$$\varepsilon_m^{(a)} \left[ \frac{1}{1 + (m\Delta\Omega r_{\alpha} \varepsilon(a))^2} \right]$$

Relating the expressions (5) directly to experimentally observed current and voltage ranges for mode locking would require more microscopic studies into the physics of the active media, most importantly of the SA. For instance, as was pointed in Section III, the two separate sub-regions of mode locking in the experimental diagrams (Fig. 6) strongly suggest that two exciton peaks may be involved in the mode locking, which in terms of this model would mean strong changes in both \(A_{\alpha}\) and \(\varepsilon(a)\) with pumping conditions. Presently, the value \(\varepsilon(a)\) is the least well-known parameter in the model and may be used as a fitting parameter. For instance, let us consider the two absorber-related terms in the first square brackets of (5b) (the terms responsible for more locking) and estimate which of them is likely to be the most important. With \(A_{\alpha} = 10^{-15} \text{cm}^2\) (a usual value in the literature), \(\tau_{\Sigma} = 20\)
ps (of the order of the round-trip time), \( \Delta \Omega = 2\pi \times 60 \text{ GHz} \) (corresponding to the cavity length of the laser studied), and \( m = 1 \) the value of the absorption suppression coefficient required for the fast term to dominate over the slow term is estimated as \( \varepsilon(a) = 10^{-17} \text{ cm}^3 \). This is a reasonably modest value, often quoted for gain nonlinearity coefficients, and absorber nonlinearities are likely to be stronger [38]. For more pronounced MCPM \( (m > 1) \) the requirement for \( \varepsilon(a) \) to dominate becomes increasingly more liberal. One may therefore suggest that for mode locking of laser diodes with short monolithic cavities, and particularly for MCPM, fast nonlinearities may be the dominating contribution in establishing the condition for mode locking regime. This somewhat contradicts the conclusion made in [37] that, although the fast component of the saturable absorption largely defines the parameters of the mode locking pulses, the condition for the regime to take place is likely to be defined by the slow SA. However, one notices that a) the authors of [37] distinctly kept in mind lasers with extended cavities, with values of \( \Delta \Omega \) 1–2 orders of magnitude smaller, which makes the conclusion of [37] much more likely to hold, and b) in [37], only one specific absorber nonlinearity was considered, namely excitonic ionization, for which the value \( \varepsilon(a) \) is not a free parameter, but is known to be small. An estimate similar to that made above suggests that the gain term in (5b) may also be dominated by gain nonlinearities, rather than pulsations of total density. Then the most important \( m \)-dependent variables in (5b), determining whether mode locking will be normal or (multiple) colliding pulse, are the overlap factors \( \xi_m \) (the margin \( \Delta \xi_m \) of the net gain is approximately proportional to \( m^2 \), but is likely to be not very significant for well-developed lasing). Since the factors \( \xi_m \) scale the strength of interaction between modes, they should affect not only the limits of the regime but also the characteristics of the well-developed MCPM. Therefore, we find it instructive to study the overlap factors in some detail.

C. Overlap Factors: The Effect of Laser Geometry

Fig. 7 shows the overlap factors \( \xi_m \) calculated for different configurations studied in the experiment. The absorber sections were taken to be 40 \( \mu \text{m} \) long, i.e., including the area under both the contacts and the separating grooves. Changing the SA length while keeping the positions of the section centers constant scales the absolute values of \( \xi_m \) proportionally to the absorber length, but their relative values change very slightly, as long as the fraction \( f_a \) of the cavity occupied by the SA remains small.

The clearest case for interpretation is the case of a single SA positioned in the middle of the cavity (CPM configuration, Fig. 7(a)); here, one clearly sees that overlap factors over the SA (but not gain) section for odd values of \( m \) are close to zero (for an ideal cavity with 100% reflection, they would be exactly zero). Therefore, when the transition from single-mode lasing to mode-locking takes place, modes separated by an odd number of \( \Delta \Omega \) from the main mode should not be excited and, as the regime develops, may be expected to remain very strongly inhibited. Therefore mode locking spectra in this configuration will always be dominated by a group of modes separated by \( 2\Delta \Omega \), which in the time-domain means that two pulses coexist in the cavity, i.e., CPM takes place.

For the case of three absorbers spaced by a quarter of the cavity length (Fig. 7(b)), one sees that the overlap factors over the absorber section \( \xi_m \) at \( m = 4 \) are approximately three times higher than the values for \( m = 1, 2, 3 \). Conversely, the overlap over the gain section sees a dip at \( m = 4 \). One can expect therefore that modes separated by \( 4\Delta \Omega \) are, indeed, likely to be the first to be excited and then remain favored in mode locking spectrum. This implies that this should,
indeed, be the four-pulse configuration, which is observed experimentally. However, since $\xi_{m}^{(a)}$ at $m = 1, 2, 3$ are nonzero, some modes with smaller amplitudes may be excited in between the main modes, therefore the four pulses within one period may be expected to have different amplitudes. This is also corroborated by experimental evidence (note an envelope modulation in the correlation function) and will be confirmed hereafter by time-domain calculations.

For the case where we experimentally observed "normal" mode locking with one side absorber a quarter of the cavity length apart from the facet (Fig. 7(c)), the picture is qualitatively the same as in Fig. 7(b), but the difference between $\xi_{4}^{(a)}$ and other values of $\xi_{m}^{(a)}$ is much smaller. What one may expect in this case is therefore a series of modes separated by the fundamental interval $\Delta \Omega$, but with each fourth mode being somewhat larger in amplitude. In time domain, this means one strong pulse per repetition period with small leading and/or trailing pulses a quarter of the round-trip period apart from the main one. Note that the values of $\xi_{m}^{(a)}$ in this case, which, unlike all other cases studied, is strongly asymmetric, should be treated as absolute values. The actual values for $m = 1, 3$ are complex.

Finally, the most difficult to explain is the case when two side absorbers are biased (Fig. 7(d)). Experimentally, this is the three-pulse case, which means that the largest overlap coefficient should be $\xi_{3}^{(a)}$. However, if one assumes that the SA is exactly confined to the area beneath the corresponding contact or even the area including the separating grooves, then the dominating value is still $\xi_{4}^{(a)}$. To make the coefficient $\xi_{3}^{(a)}$ the largest, one has to assume that the absorbers actually stretch strongly beyond the contacts, approaching the points $|z| = L/6$ (i.e., one third of the cavity length apart from the facets). This is corroborated by time-domain simulations; furthermore, one notices that in the experiments, this is the case when the MCPM is least clearly rendered.

The main advantage of describing MCPM with the use of the overlap factors $\xi_{m}^{(g,a)}$ (3) is that these factors are defined almost entirely by the geometrical positions and lengths of the gain and SA sections and are virtually independent of the parameters of the active medium such as $A_{g}, A_{a}, \varepsilon^{(g)}, \varepsilon^{(a)}$, which are often not very well known. We believe this makes the predictions made in this section general and reliable. However, the results obtained so far may indirectly prove only the fact that MCPM should indeed take place, but cannot predict the parameters of the well-developed regime. So we proceed with some numerical simulations of the mode locking dynamics. The easiest way to perform such simulations would be to fully numerically solve the multimode system (1), which for this purpose should be completed with usual rate equations for the ("slow") carrier densities in gain and SA sections. One notes, however, that the model (1)–(3) is essentially a small-signal model and its applicability, at least for high intensities, needs further investigation. Also, in its present form the model does not take into account the colliding pulse effect (interaction of counterpropagating pulses via self-induced gratings in the SA [3]), which may be expected to be of some significance for a regime such as MCPM, where the pulses are indeed colliding in the SA. Therefore, in order to avoid these limitations and to double check the results obtained from the frequency-domain model, we use a distributed time-domain model for full-scale numerical calculations.

D. The Time-Domain Model

The model we use is a variation of the model used in [13] and is similar in its main features to those of [39] and [40]. The light propagation in the cavity is described by a propagation equation of the form

$$\pm \frac{\partial E_{RL}}{\partial z} + v_{g} \frac{\partial E_{RL}}{\partial t} = \Gamma \left( \hat{g} E_{RL} + i \alpha g E_{RL} + i K_{LR,RL} E_{L,R} \right) + F_{\text{rand}}$$ (6)

where $E_{RL}$ are complex amplitudes of the field of the light wave propagating to the right and to the left respectively. The operator $\hat{g}$ includes a digital filter simulating gain dispersion [39]

$$\hat{g} E = g \cdot \Delta \Omega_{g} \cdot \int_{0}^{\infty} E(t - \tau) \exp(-\Delta \Omega_{g} \cdot \tau) d\tau$$

where $\Delta \Omega_{g}$ is the gain linewidth and $g$ is the usual gain value used in the previous subsection (for the values of $z$ corresponding to the SA subsections, $g$ is obviously changed to $a$). The list of parameters, together with the values used, is given in the Table I. The last term in (6) represents the random noise source [39] and is essential for running the model. A self-induced grating (colliding-pulse effect) is included by the distributed reflection terms $KE$, which are taken into account only in SA sections. The dynamic coupling coefficients $K_{LR,RL}$ are calculated from the equations

$$\frac{dK_{LR}(z)}{dt} = -\left( \frac{1}{\tau_{N}} + v_{g} A_{a} S(z) + \frac{16\pi^{2}Dv_{g}^{2}}{\lambda^{2}} \right)$$

$$K_{RL} = -K_{LR}^{*}$$ (7a)

where $D$ is the ambipolar diffusion coefficient, and $\lambda$ the lasing wavelength (in vacuum); the term proportional to $D$ takes into account smoothing out of the grating by carrier diffusion. The linewidth enhancement factor for the SA region has been taken as zero.

At the facets, the usual reflection boundary conditions are imposed. The dynamics of the carrier density is described by local rate equations with photon density defined as the squared absolute value of the local field

$$\frac{\partial N(z,t)}{\partial t} = \frac{J(z,t)}{ed} - \frac{N}{\tau_{N}(z)} - v_{g} g(|E_{RL}(z,t)|^{2})$$

$$+ |E_{L}(z,t)|^{2}$$ (8)

where the recombination time $\tau_{N}(z)$ is defined differently for gain and SA sections.
TABLE I
LASER DIODE PARAMETERS USED IN COMPUTATIONS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>electron lifetime (in gain section)</td>
<td>$\tau_n$</td>
<td>$1/\alpha_n$</td>
<td>s</td>
</tr>
<tr>
<td>cavity losses</td>
<td>$\kappa_c$</td>
<td>$2(\kappa_n+\kappa_s)$</td>
<td>cm</td>
</tr>
<tr>
<td>pumping current in gain section</td>
<td>$J_g$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>pumping current in saturable absorber (SA) section</td>
<td>$J_a$</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>active layer thickness</td>
<td>$d$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>bimolecular recombination coefficient</td>
<td>$B$</td>
<td>$1.5 \times 10^{19}$</td>
<td>cm$^{-3}$s$^{-1}$</td>
</tr>
<tr>
<td>spontaneous emission factor</td>
<td>$\delta$</td>
<td>$10^4$</td>
<td></td>
</tr>
<tr>
<td>nonradiative recombination time in SA section</td>
<td>$\delta_n$</td>
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<td>ns</td>
</tr>
<tr>
<td>group velocity of light</td>
<td>$v_g$</td>
<td>$0.75 \times 10^8$</td>
<td>cm/s</td>
</tr>
<tr>
<td>confinement factor</td>
<td>$\Gamma$</td>
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<td></td>
</tr>
<tr>
<td>gain cross section</td>
<td>$A_g$</td>
<td>$4 \times 10^{16}$</td>
<td>cm$^2$</td>
</tr>
<tr>
<td>SA cross section</td>
<td>$A_a$</td>
<td>$8 \times 10^{16}$</td>
<td>cm$^2$</td>
</tr>
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<td>effective transparency carrier density, gain section</td>
<td>$N_{Og}$</td>
<td>$1.2 \times 10^{18}$</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>effective transparency carrier density, SA section</td>
<td>$N_{Oa}$</td>
<td>$1.2 \times 10^{18}$</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>gain saturation (nonlinearity) coefficient</td>
<td>$g_0$</td>
<td>$5 \times 10^{18}$</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>gain nonlinearity relaxation time</td>
<td>$\tau_{g NL}$</td>
<td>0.5</td>
<td>ps</td>
</tr>
<tr>
<td>SA saturation (nonlinearity) coefficient</td>
<td>$g_{0a}$</td>
<td>$5 \times 10^{17}$</td>
<td>cm$^{-3}$</td>
</tr>
<tr>
<td>SA nonlinearity relaxation time</td>
<td>$\tau_{g NL}$</td>
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<td>ps</td>
</tr>
<tr>
<td>intensity reflection coefficients (both sides)</td>
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<td></td>
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<tr>
<td>linewidth enhancement factor (gain section)</td>
<td>$g_{NL}$</td>
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<td></td>
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<tr>
<td>laser cavity length</td>
<td>$L$</td>
<td>600</td>
<td>$\mu$m</td>
</tr>
</tbody>
</table>

Finally, to introduce gain nonlinearities with finite relaxation time, we use an additional equation of the type

$$\frac{dg}{dt} = \frac{g_0 - g(1 + e^{(g)})S}{\tau_{g NL}(g)}; \quad g_0(N) = A_g(N - N_{Og}), \quad (9)$$

a similar equation being used for the SA section.

E. Simulation Results

Fig. 8 shows some examples of the simulated temporal profiles of the MCPM regime for different geometries corresponding to those studied theoretically in the previous subsection and experimentally in Section III.

For the case of "classical" colliding pulse mode locking (Fig. 8(a)) one observes a well-developed train of pulses at the repetition rate of $2/T(T = 2\pi/\Delta\Omega = 2L/v_g)$ being the fundamental repetition period) with some period doubling present. This agrees reasonably well with both the frequency-domain results (some of the overlap coefficients for odd number of mode intervals being nonzero) and the experimental observations (note the envelope modulation in the measured correlation curve).

Fig. 8(b) illustrates the case with all three absorbers involved. Four approximately evenly spaced ($T/4$ apart from one another) pulses are clearly observed within each fundamental repetition period, as expected from the experimental results. The amplitudes of the pulses within one repetition period are different, and very strongly depend on the parameter values used. For instance, for shorter SA sections (20 $\mu$m instead of 40 $\mu$m as in the figure), the amplitude of one of the pulses becomes virtually zero. More accurate measurements could determine which of the two cases is closer to the experimental observations.

Fig. 8(c) shows the case of a single SA a quarter of the cavity length away from the facet. The pulse train in this case has a period of $T$, with a smaller trailing pulse following a quarter of the period after each main pulse (seen as a shoulder due to the gain dispersion and finite nonlinearity relaxation time).

Fig. 8(d) and (e) show the expected three-pulse case (two absorbers biased). Here, we are also able to simulate a stream of pulses separated in time by $T/3$, but to do so, as in the frequency-dependent model, we have to shift the absorbers
from the positions fixed by the contact towards the more even spacing \((L/3)\), as seen in Fig. 8(e). For the absorbers centered as grown (Fig. 8(d)) we still observe a leading pulse approximately \(\text{not exactly} \) a third of the period ahead of the main one, which also gives two intermediate peaks within each period of the correlation function, but can hardly explain the experiment quantitatively, since the leading pulse amplitude is very weak.

Pulse durations in most cases varied within the range of 1–3 ps (broader structures in Fig. 8(c) and (d) are apparently due to overlap of consecutive pulses). Such durations are in reasonable agreement with the estimates made from the experimental observation; detailed comparison seems premature at this stage of studies due to both the limitations of the experimental technique and the large number of fitting parameters in the simulations.

Finally, we briefly discuss the behavior of the results when some of the parameters are changed. Variation of nonlinearity recovery times within the subpicosecond limits changed the relative amplitudes of multiple pulses but brought no qualitative changes. With very small nonlinearity coefficients in the SA section, when (5b) was not fulfilled, the simulated laser exhibited steady single-mode lasing, as expected. For some range of pumping levels, the model predicted an apparently chaotic slow envelope of the pulse train, not observed in the experiment. We believe the discrepancies may be due to the fact that the characteristic parameters of the gain and SA sections may vary with current in realistic circumstances (for instance, due to imperfect electric isolation, the SA voltage decreases by absolute value as the current increases, thus probably decreasing the absorption suppression coefficient), which is not taken into account by the model. Taking into account the dynamic gratings (colliding pulse effect) shortens the pulses by approximately 10–20\%, but otherwise is not very significant.

V. DISCUSSIONS AND CONCLUSION

The operation of a monolithic multisectioned semiconductor laser has been described. The laser can be operated as either single mode, multimode, self-pulsing (or \(Q\)-switched) and mode-locked in either 1, 2, 3, or 4 pulse operation. This device is a very versatile source of high repetition rate (up to 375 GHz) ultrashort pulses (around 1–3 ps) of light. The various modes of operation of the device have been experimentally investigated and we have found experimental indications of the presence of excitonic nonlinearities in its operation.

The theoretical results from the frequency domain model strongly suggest that for short monolithic cavities, fast nonlinearities play a major role in mode-locking operation and that the device geometry is highly favourable to MCPM operation. Numerical simulations with a time-domain model were performed, using saturation coefficients and fast recovery times estimated from the frequency domain model. Good agreement between the models and the experimental results for MCPM operation was obtained, although there are still some features which require further investigation for a complete understanding of the laser. In particular the nature of the very rapid recovery of the saturable absorber, which we postulate here (the time-domain simulations use a recovery time of around 0.5 ps), has been observed in pump-probe experiments on reverse biased multiple quantum-well material [6], but has not been fully understood. Also, it is not absolutely clear why the three-pulse operation occurs given that the saturable absorbers are not placed 1/3 along the laser cavity.

The device is a monolithic semiconductor chip which can be mass produced; it is low-cost, robust, efficient, and reliable. These are crucial features if ultrashort pulses of light are to find wide-spread application outwith the laboratory.

APPENDIX

LONGITUDINAL MODE PROFILES AND OVERLAP FACTORS OF THE CAVITY

The coupled-wave (1) are obtained by distributing the lasing light in the cavity into longitudinal modes:

\[
E(z, t) = \sum_k E_k(t) u_k(z) + \text{c.c.} \tag{10}
\]

The technique for such distribution for realistic lasers with open cavities was proposed by Siegman [41] and developed by Hamel and Woerdman [42]; a simple generalization for the case of lasers with intracavity absorbers was performed in [32] and [33] to derive (1) and (3). The wave functions of the cavity are defined as

\[
\begin{align*}
\psi(z, t) &= u_k(t) u_k(z) + u_k^*(t) u_k^*(z) \\
\beta(z) &= \frac{k \pi}{L} + i \gamma(k)(z)
\end{align*}
\]

where \(u_k \) and \(u_k^* \) are left-hand and right travelling waves, \(q_k \) is the complex mode vector

\[
\begin{align*}
u_k &= u_k + u_k^* \\
u_k' &= \frac{L}{N} \exp \left( -i \int_{-L/2}^{L/2} q_k(z') dz' \right) \\
u_k'' &= \frac{1}{N} \exp \left( i \int_{-L/2}^{L/2} q_k(z') dz' \right)
\end{align*}
\]

where \(u_k' \) and \(u_k'' \) are left-and right travelling waves, \(q_k \) is the complex mode vector

\[
\begin{align*}
u_k &= \frac{k \pi}{L} + i \gamma(k)(z) \\
\beta(z) &= \frac{k \pi}{L} + i \gamma(k)(z)
\end{align*}
\]

where, in steady state, \(\gamma_k = g_k \) in gain sections and \(\gamma_k = -g_k \) in absorber sections, so that

\[
\tau_{LR} \exp \left( -i \int_{-L/2}^{L/2} q_k(z') dz' \right) = 1.
\]

The normalization factor \(N \) is chosen so that

\[
\int_{-L/2}^{L/2} dz u_k(z) u_k^*(z) = 1.
\]

Substituting (11)–(13) into (3), one obtains, after some trivial though cumbersome calculations, the explicit expressions for the overlap factors \(\xi_m^{(g, a)} \) [33]

\[
\xi_m^{(g, a)} = \sum_{\ell \in \beta, \alpha} \xi_{\ell m}^{(g, a)} \tag{14}
\]
where the summation is over all the appropriate (gain or absorber) segments and

\[
\xi_{lm} = \frac{1}{2N^2L} \left\{ \frac{\exp (\gamma_1 L_l) - 1}{\gamma_1} \left[ \exp \left( \sum_{p=1}^{l-1} \gamma_p L_p \right) \right] \right. \\
+ \frac{1}{\tau L} \left[ \exp \left( \gamma_1 + \frac{2\pi n}{L} \right) L_l - 1 \right] \\
\left. \left[ \exp \left( \gamma_1 + \frac{2\pi n'}{L} \right) L_l \right] \right. \\
+ \frac{1}{\tau L} \left[ \exp \left( - \left( \gamma_1 + \frac{2\pi n_{L+1}}{L} \right) \right) \right] \right\}. \tag{15}
\]

In (15), \( L_l \) and \( z_l \) are the length of the \( l \)-th segment and the co-ordinate of its left border, and the mode index dependence of \( \gamma \) has been dropped, as discussed in Section IV-A. For the normalizing factor, to satisfy (13), one uses

\[
\mathcal{N}^2 = \frac{1}{L} \sum_{all} \left\{ \frac{\exp (\gamma_1 L_l) - 1}{\gamma_1} \left[ \exp \left( \sum_{p=1}^{l-1} \gamma_p L_p \right) \right] \right. \\
+ \frac{1}{\tau L} \left[ \exp \left( - \left( \gamma_1 + \frac{2\pi n_{L+1}}{L} \right) \right) \right] \right\}. \tag{16}
\]

Expressions (14)–(16) are used to calculate the results plotted in Fig. 7.

Notes Added in Proof

Recent experiments using conventional SHG autocorrelation technique are consistent with the results presented here. In particular, with 2 pulse CPM operation we observe transform limited (chirp free) pulses.

REFERENCES


Eugene A. Avrutin for a photograph and biography, see this issue, p. 460.

C. N. Ironside was born in Aberdeen, Scotland, in 1950. His Ph.D. work, at Heriot-Watt University, was on a type of tunable infrared semiconductor laser the Spin-flip Raman Laser which used InSb as the gain medium.

Work in this area led on to a study of the resonant nonlinear optical properties of InSb subsequently employed in optically bistable devices. He moved to Oxford University to work on time resolved spectroscopy of solids which included optical energy transfer processes in solids and exciton dynamics.

He developed some new time-resolved spectroscopy techniques in this area and developed an interest in ultrashort pulses. Since 1984, he has been at Glasgow University working mainly on the generation and application of ultra-short pulses in integrated optics. His research interests now include, high-speed all-optical switching in semiconductor waveguides, optoelectronic properties of RTD’s and modelocked semiconductor lasers.

J. S. Roberts, photograph and biography not available at the time of publication.

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