Generic Mathematical Model of a Propeller Driven Fixed-Wing Aircraft

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Introduction

- Flight simulation is extensively involved in the design of modern aircrafts and is less costly than wind tunnel and flight testing.
- It is also well established among the community of airline pilots for their training.
- The aerospace department wishes to have a generic model that can be adapted to different aircraft configurations.
- A first model was built last year that simulated the longitudinal dynamics of a North American/Ryan Navion (see picture [1]).

Objectives

- Build a 6-degree of freedom (DOF) model from the previous work (i.e. take into account the lateral dynamics).
- Trim the aeroplane for different flight conditions and assess the model accuracy and sensitivity.
- Simulate the response to control and wind disturbance.

Frame of Work

- The aircraft dynamics are governed by the equations of motion [3] relative to the body axis in Figure 1:

\[ m \ddot{V} + m \dot{W} = F \]  
\[ m \ddot{W} + m \dot{V} = N \]  
\[ I_\theta \ddot{\theta} + I_{\phi} \dot{\phi} = Q \]  
\[ I_{\theta} \ddot{\theta} + I_{\phi} \dot{\phi} = Q \]  

\[ \eta_{\theta} \ddot{\theta} + \eta_{\phi} \dot{\phi} = T \]

- The model of the aeroplane is made of different sub-systems (e.g. wings, tail); see Figure 2. Each component forces and moments are estimated [4]:

\[ X = X_{air} + X_{prop} + X_{wind/airs} \cdots \]
\[ N = N_{air} + N_{prop} + N_{wind/airs} \cdots \]

Testing

- The mathematical model must be tested to assess its accuracy and sensitivity.
- Trimming the aeroplane and studying its response to disturbance are two tools to test the model.

Trim (wings-level flight) at service ceiling 18,000 ft

- At lower speeds than \( V_{cruise} \approx 140 \) kts, a bigger \( \delta_\alpha \) is required to level the Navion (i.e. \( \theta \) remains close to 0 ± 1.5° throughout the flight envelope).
- An aft c.g. gives more manoeuvrability: a smaller \( \delta_\alpha \) is required. This is the opposite for a forward c.g.
- c.g. displacements sensitive.

Lateral Dynamics Enhancement

- The fin is responsible for the lateral motion in the xz plane. It also contributes to the directional and lateral trim.
- The forces are assumed to act at its aerodynamic centre (a.c.) \( ac_f \), (see Figure 3). A non-zero sideslip angle \( \beta \), at the fin creates lift (coefficient \( C_{l_{ac_f}} \)) and drag forces (coefficient \( C_{d_{ac_f}} \)) in Figure 4:

\[ F_{l_{ac_f}} = \frac{1}{2} \rho c \alpha c_{ac_f} \sin(\beta) \]  
\[ D_{ac_f} = \frac{1}{2} \rho c \alpha c_{ac_f} \cos(\beta) \]

- The total moment created by the fin is:

\[ M_{ac_f} = M_{\alpha} + \frac{1}{2} \rho c \alpha c_{ac_f} \frac{L}{L_{ac_f}} \]

\[ M_{\alpha} = \delta \tau_{ac_f} \times F_{ac_f} \]

\[ N_{ac_f} = \delta \tau_{ac_f} c_{ac_f} M_{ac_f} \]

Response to Disturbance

- No satisfactory response to a control/wind disturbance: unstable behaviour for \( \phi \) and \( \psi \) attitudes.
- The non-linear and the linear model (linearised about the trim states) give similar response to a same disturbance.

Conclusion and Future Work

- A 6-DOF model is fully built and can be trimmed for different flight conditions specified by the user (i.e. altitude, flight speed, climb angle, sideslip, flaps deflection).
- The aircraft behaviour is satisfactory in trim, although sensitive to c.g. displacement, which can be an issue when a different aeroplane is used.
- Unstable lateral response to disturbance is the main problem.
- Possible future work (except solving this issue): account for a stall angle, for trim tabs and for downwash/sidewash effects.

References